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PAPER

Theory of BCS-like bogolon-mediated superconductivity in transition metal dichalcogenides

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Supplementary material for this article is available online

Abstract
We report on a novel mechanism of BCS-like superconductivity, mediated by a pair of Bogoliubov quasiparticles (bogolons). It takes place in hybrid systems consisting of a two-dimensional electron gas in a transition metal dichalcogenide monolayer in the vicinity of a Bose–Einstein condensate. Taking a system of two-dimensional indirect excitons as a testing ground of Bose-Einstein condensate we show, that the bogolon-pair-mediated electron pairing mechanism is stronger than phonon-mediated and single bogolon-mediated ones. We develop a microscopic theory of bogolon-pair-mediated superconductivity, based on the Schrieffer–Wolff transformation and the Gor'kov's equations, study the temperature dependence of the superconducting gap and estimate the critical temperature of superconducting transition for various electron concentrations in the electron gas and the condensate densities.

1. Introduction

The conventional microscopic Bardeen–Cooper–Schrieffer (BCS) superconductivity originates from the interaction between electrons and phonons (crystal lattice vibrations), which results in the attraction between electrons with opposite momenta and spins with the sequential formation of Cooper pairs [1, 2]. However, this phenomenon is usually observed at low temperatures (as compared with room temperature), of the order of several Kelvin since the phonon-mediated superconducting (SC) gap usually amounts to several meV. And superconductors with the critical temperature of SC transition T_c above 30 K are traditionally considered high-temperature superconductors [3].

In an attempt to increase the electron-phonon coupling and T_c, one immediately faces certain obstacles, one of which is the Peierls instability [4]. In the mean time, the search for high-temperature superconductivity is a rapidly developing area of research nowadays, especially in low-dimensional systems [5, 6]. In hybrid superconductor-semiconductor electronics and circuit quantum electrodynamics, two-dimensional (2D) superconductors might allow for scaling down the characteristic size of a device down to atomic-scale thickness for possible application in quantum computing [7–9]. Low-dimensional superconductors also provide such advantages as the robustness against in-plane magnetic fields due to the spin-valley locking [10] and an additional enlargement of T_c in the atomic-scale layer limit [11]. From the fundamental side, the SC phase in samples of lower dimensionality usually either co-exists or competes with other (coherent) many-body phases such as the quantum metallic or insulator states, the charge density wave, or magnetic phase, giving rise to richer physics than in three-dimensional systems [12]. The drawbacks and limitations of phonons as mediators of electron pairing for realizing high-T_c 2D superconductors motivate the search for other pairing mechanisms.

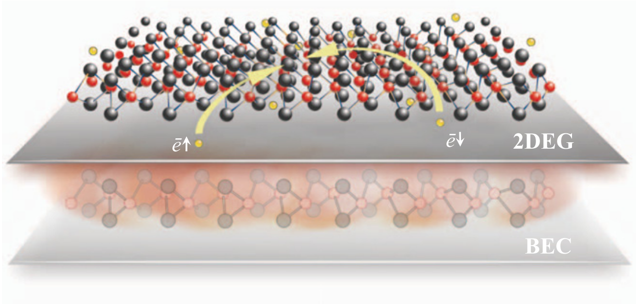

Figure 1. System schematic. 2DEG with parabolic dispersion in the vicinity of a 2D BEC. We consider the BEC of indirect excitons, which reside in a double quantum well: n-doped and p-doped layers of MoS_2 and WSe_2 separated by an hBN. Electrons and the condensate particles are coupled via the Coulomb forces, which allows electrons with opposite spins (yellow dots) form Cooper pairs.

There have been various attempts to replace regular phonons by some other quasiparticles aiming at increasing T_c and the SC gap. One of the routes is exciton-mediated superconductivity [13–15]. Photon-mediated superconductivity has also been recently predicted [16]. Another way is to use the excitations above a Bose–Einstein condensate (BEC), called the Bogoliubov quasiparticles (bogolons) in hybrid Bose–Fermi systems, where one expects the SC transition in the fermionic subsystem. The bosonic subsystem can be represented by an exciton or exciton–polariton condensate, which have been predicted [17–22] and studied experimentally [23–25] at relatively high temperatures sometimes reaching the room temperature. In systems of indirect excitons, spatially separated electron–hole pairs, achieving high-temperature condensation should be possible if using 2D materials based on transition metal dichalcogenides such as MoS_2 thank to large exciton binding energy [26]. Bogolons possess some of the properties of acoustic phonons and can, in principle, give electron pairing, as it has been theoretically shown in several works [27–29]. These proposals, however, operated with single-particle (single-bogolon) pairing, assuming that multi-particle processes belong to the higher orders of the perturbation theory and thus they are weak and can be safely disregarded. Is this widespread assumption true?

As the earlier work [30] points out, the bogolon-pair-mediated processes (2*b* processes in what follows) can give the main contribution when considering the scattering of electron gas in the normal state (above T_c). If we go down T_c, several questions arise naturally. Will there occur 2*b*-mediated pairing? What is its magnitude, as compared with single-bogolon (*1b*) processes? Is the parameter range [in particular, condensate density, concentration of electrons in two-dimensional electron gas (2DEG)] achievable experimentally? In this article, using the BCS formalism we develop a microscopic theory of 2b superconductivity and address all these questions.

2. Theoretical framework

Let us consider a hybrid system consisting of a 2DEG and a 2D BEC, taking indirect excitons as an example, where the formation of BEC has been reported [25, 31] (figure 1). The electrons and holes reside in n- and p-doped layers, respectively. These layers can be made of MoS_2 and WSe_2 materials separated by several layers of hexagonal boron nitride (hBN) [25]. The 2DEG and exciton layers are also spatially separated by hBN and the particles are coupled by the Coulomb interaction [32, 33] described by the Hamiltonian

$$\mathcal{H} = \int \mathrm{d}\mathbf{r} \int \mathrm{d}\mathbf{R}\Psi^\dagger_\mathbf{r}\Psi_\mathbf{r} g\left(\mathbf{r}-\mathbf{R}\right)\Phi^\dagger_\mathbf{R}\Phi_\mathbf{R}, \tag{1}$$

where $\Psi_\mathbf{r}$ and $\Phi_\mathbf{R}$ are the field operators of electrons and excitons, respectively, $g\left(\mathbf{r}-\mathbf{R}\right)$ is the strength of Coulomb interaction between the particles, $\mathbf{r}$ and $\mathbf{R}$ are the in-plane coordinates of the electron and the exciton center-of-mass motion.

Furthermore, we assume the excitons to be in the BEC phase. Then, we can use the model of a weakly interacting Bose gas and split $\Phi_\mathbf{R} = \sqrt{n_c} + \varphi_\mathbf{R}$, where n_c is the condensate density and $\varphi_\mathbf{R}$ is the field operator of the excitations above the BEC. Then, the Hamiltonian (1) breaks into three terms, two of which are

$$\mathcal{H}_1 = \sqrt{n_c}\int \mathrm{d}\mathbf{r}\Psi^\dagger_\mathbf{r}\Psi_\mathbf{r}\int \mathrm{d}\mathbf{R} g\left(\mathbf{r}-\mathbf{R}\right)\left[\varphi^\dagger_\mathbf{R}+\varphi_\mathbf{R}\right], \tag{2}$$

$$\mathcal{H}_2 = \int \mathrm{d}\mathbf{r}\Psi_{\mathbf{r}}^{\dagger}\Psi_{\mathbf{r}} \int \mathrm{d}\mathbf{R} g(\mathbf{r}-\mathbf{R})\varphi_{\mathbf{R}}^{\dagger}\varphi_{\mathbf{R}}. \tag{3}$$

The first term, $\mathcal{H}_1$, is responsible for electron-single bogolon interaction, and the second term, $\mathcal{H}_2$, is bogolon-pair-mediated. The third term reads $gn_c \int \mathrm{d}\mathbf{r}\Psi_{\mathbf{r}}^{\dagger}\Psi_{\mathbf{r}}$. It gives a shift $\delta\mu = gn_c$ of the Fermi energy $\mu = \hbar^2 p_F^2/2m$, where p_F is the Fermi wave vector and m is electron effective mass. Then p_F also becomes n_c-dependent, strictly speaking, but we disregard this correction in what follows.

We express the field operators as the Fourier series,

$$\varphi_{\mathbf{R}} = \frac{1}{L}\sum_{\mathbf{p}} \mathrm{e}^{\mathrm{i}\mathbf{p}\cdot\mathbf{R}}(u_{\mathbf{p}}b_{\mathbf{p}} + v_{\mathbf{p}}b_{-\mathbf{p}}^{\dagger}), \qquad \Psi_{\mathbf{r}} = \frac{1}{L}\sum_{\mathbf{k}} \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}}c_{\mathbf{k}},$$

where $b_{\mathbf{p}}(c_{\mathbf{k}})$ and $b_{\mathbf{p}}^{\dagger}(c_{\mathbf{k}}^{\dagger})$ are the bogolon (electron) annihilation and creation operators, respectively, and L is the length of the sample. The Bogoliubov coefficients read [34]

$$u_{\mathbf{p}}^2 = 1 + v_{\mathbf{p}}^2 = \frac{1}{2}\left(1 + \left[1 + \left(\frac{Ms^2}{\omega_{\mathbf{p}}}\right)^2\right]^{1/2}\right),$$

$$u_{\mathbf{p}}v_{\mathbf{p}} = -\frac{Ms^2}{2\omega_{\mathbf{p}}}, \tag{4}$$

where M is the exciton mass, $s = \sqrt{\kappa n_c/M}$ is the sound velocity, $\kappa = e_0^2 d/\epsilon_0\epsilon$ is the exciton–exciton interaction strength in the reciprocal space, e_0 is electron charge, ϵ is the dielectric constant, ϵ_0 is the dielectric permittivity, $\omega_p = \hbar sp(1 + p^2\xi_h^2)^{1/2}$ is the spectrum of bogolons, and $\xi_h = \hbar/2Ms$ is the healing length. Then equations (2) and (3) transform into

$$\mathcal{H}_1 = \frac{\sqrt{n_c}}{L}\sum_{\mathbf{k},\mathbf{p}} g_{\mathbf{p}} \left[(v_{\mathbf{p}} + u_{-\mathbf{p}})b_{-\mathbf{p}}^{\dagger} + (v_{-\mathbf{p}} + u_{\mathbf{p}})b_{\mathbf{p}}\right] c_{\mathbf{k}+\mathbf{p}}^{\dagger}c_{\mathbf{k}}, \tag{5}$$

$$\mathcal{H}_2 = \frac{1}{L^2}\sum_{\mathbf{k},\mathbf{p},\mathbf{q}} g_{\mathbf{p}} \left[u_{\mathbf{q}-\mathbf{p}}u_{\mathbf{q}}b_{\mathbf{q}-\mathbf{p}}^{\dagger}b_{\mathbf{q}} + u_{\mathbf{q}-\mathbf{p}}v_{\mathbf{q}}b_{\mathbf{q}-\mathbf{p}}^{\dagger}b_{-\mathbf{q}}^{\dagger} + v_{\mathbf{q}-\mathbf{p}}u_{\mathbf{q}}b_{-\mathbf{q}+\mathbf{p}}b_{\mathbf{q}} + v_{\mathbf{q}-\mathbf{p}}v_{\mathbf{q}}b_{-\mathbf{q}+\mathbf{p}}b_{-\mathbf{q}}^{\dagger}\right] c_{\mathbf{k}+\mathbf{p}}^{\dagger}c_{\mathbf{k}}, \tag{6}$$

where g_p is the Fourier image of the electron–exciton interaction. Disregarding the peculiarities of the exciton internal motion (relative motion of the electron and hole in the exciton), we write the electron-exciton interaction in direct space as

$$g(\mathbf{r}-\mathbf{R}) = \frac{e_0^2}{4\pi\epsilon_0\epsilon}\left(\frac{1}{r_{e-e}} - \frac{1}{r_{e-h}}\right), \tag{7}$$

where $r_{e-e} = \sqrt{l^2 + (\mathbf{r}-\mathbf{R})^2}$ and $r_{e-h} = \sqrt{(l+d)^2 + (\mathbf{r}-\mathbf{R})^2}$; d is an effective size of the boson, which is equal to the distance between the n- and p-doped layers in the case of indirect exciton condensate, and l is the separation between the 2DEG and the BEC [35]. The Fourier transform of (7) gives

$$g_{\mathbf{p}} = \frac{e_0^2\left(1 - e^{-pd}\right)e^{-pl}}{2\epsilon_0\epsilon p}. \tag{8}$$

Following the BCS approach [36], we find the effective electron s-wave [37] pairing Hamiltonian [see supplemental material (https://stacks.iop.org/NJP/23/023023/mmedia) [38]], considering $1b$ and $2b$ processes separately to simplify the derivations and draw the comparison between them,

$$\mathcal{H}_{\mathrm{eff}}^{(\lambda)} = \mathcal{H}_0 + \frac{1}{2L^2}\sum_{\mathbf{k},\mathbf{k}',\mathbf{p}} V_{\lambda}(p)c_{\mathbf{k}+\mathbf{p}}^{\dagger}c_{\mathbf{k}}c_{\mathbf{k}'-p}^{\dagger}c_{\mathbf{k}'}, \tag{9}$$

where $\mathcal{H}_0$ is a free particle dispersion term and

$$V_{1b}(p) = -\frac{n_c}{Ms^2}g_p^2, \tag{10}$$

$$V_{2b}(p) = -\frac{M^2 s}{4\hbar^3}\frac{g_p^2}{p}\left(1 + \frac{8}{\pi}\int_{p_{\min}}^{p/2}\frac{\mathrm{d}qN_q}{\sqrt{p^2 - 4q^2}}\right) \tag{11}$$

are effective potentials of electron–electron interaction. In equation (11), $N_q = \left[\exp(\frac{\omega_q}{k_B T}) - 1\right]^{-1}$ is the bogolon Bose distribution function. It gives the divergence of the integral at $q = 0$ typical for 2D systems

[31, 39, 40]. Therefore, we introduce a cutoff p_{min}, responsible for the convergence and associated with the finite size of the sample (or condensate trapping). The factor N_q emerges at finite temperatures and gives an increase of the exchange interaction between electrons. The number of thermally activated bogolons increases with temperature, which enhances the $2b$-mediated electron scattering.

Furthermore, we use the equation for the SC gap Δ_λ [36]

$$\Delta_\lambda(\mathbf{k}) = -\frac{1}{L^2}\sum_{\mathbf{p}} V_\lambda(p)\frac{\Delta_\lambda(\mathbf{k}-\mathbf{p})}{2\zeta^{(\lambda)}_{\mathbf{k}-\mathbf{p}}}\tanh\left(\frac{\zeta^{(\lambda)}_{\mathbf{k}-\mathbf{p}}}{2k_{\mathrm{B}}T}\right), \tag{12}$$

where $\zeta^{(\lambda)}_{\mathbf{k}} = \sqrt{\xi^2_{\mathbf{k}} + \Delta^2_\lambda(\mathbf{k})}$ with $\xi_{\mathbf{k}} = \hbar^2k^2/2m - \mu$ being the kinetic energy of particles measured with respect to the Fermi energy. Then, we change the integration variable and cancel out Δ_λ in both sides of equation (12) (since we consider the s-wave pairing when the SC gap is momentum independent). As a result, equation (12) transforms into

$$1 = -\int_0^\infty \frac{\mathrm{d}pp}{2\pi}\int_0^{2\pi}\frac{\mathrm{d}\theta}{2\pi}\frac{V_\lambda(|\mathbf{k}-\mathbf{p}|)}{2\zeta^{(\lambda)}_{\mathbf{p}}}\tanh\left(\frac{\zeta^{(\lambda)}_{\mathbf{p}}}{2k_{\mathrm{B}}T}\right), \tag{13}$$

where θ is the angle between the vectors $\mathbf{k}$ and $\mathbf{p}$. Furthermore, we switch from the integration over the momentum to the integration over the energy: $p \to 2m(\mu + \xi)$, and introduce an effective cut-off $\omega_b = \hbar s/\xi_{\mathrm{h}}$ in accordance with the BCS theory. This parameter appears by analogy with the Debye energy ω_{D} (in the case of acoustic phonon-mediated pairing), which is connected with the minimal sound wavelength of the order of the lattice constant and has obvious physical meaning. In the case of bogolons, this cut-off is less intuitive and, in principle, it remains a phenomenological parameter [29]. Its value $\hbar s/\xi_{\mathrm{h}}$ might be attributed to the absence of bogolon excitations with wavelengths shorter than the condensate healing length.

Let us, first, consider zero-temperature case, when the tanh function in equation (13) becomes unity and $N_q = 0$. Assuming that the main contribution into the effective electron-electron interaction comes from electrons near the Fermi surface and $p_{\mathrm{F}}d, p_{\mathrm{F}}l \ll 1$, we find analytical expressions,

$$\Delta_{1b}(T=0) = 2\omega_b\ \exp\left[-\frac{8Ms^2}{\nu_0 n_{\mathrm{c}}}\left(\frac{\epsilon_0\epsilon}{e_0^2 d}\right)^2\right], \tag{14}$$

$$\Delta_{2b}(T=0) = 2\omega_b\ \exp\left[-\frac{16\hbar^3 p_{\mathrm{F}}}{\tilde{\nu}_0 M^2 s}\left(\frac{\epsilon_0\epsilon}{e_0^2 d}\right)^2\right], \tag{15}$$

where $\nu_0 = m/\pi\hbar^2$ is a density of states of 2DEG, $\tilde{\nu}_0 = \nu_0\ \log(4p_{\mathrm{F}}L)/\pi$ is an effective density of states, and L is the system size. Note, that in equation (15) there emerges an additional logarithmic factor (as compared with the standard BCS theory). It happens due to the momentum dependence of the $2b$-mediated pairing potential V_{2b} and due to the integration over the angle θ in the self-consistent equation for the SC gap [equation (13)].

The SC critical temperature can be estimated from equation (13) exploiting the condition $\Delta_\lambda(T^\lambda_{\mathrm{c}}) = 0$. For $1b$ processes, it gives $T^{(1b)}_{\mathrm{c}} = (\gamma/\pi)\Delta_{1b}(T=0)$, where $\gamma = \exp C_0$ with $C_0 = 0.577$ the Euler's constant (see, e.g. [41]). The analytical estimation of $T^{(2b)}_{\mathrm{c}}$ this way is cumbersome due to the presence of N_q-containing term in equation (11).

3. Results and discussion

Full temperature dependence of Δ_λ can be studied numerically using equations (10)–(13). Here, we account for the temperature dependence of the condensate density using the formula, which describes 2D BEC in a power-law trap [40], $n_{\mathrm{c}}(T) = n_{\mathrm{c}}[1 - (T/T^{\mathrm{BEC}}_{\mathrm{c}})^2]$, where $T^{\mathrm{BEC}}_{\mathrm{c}}$ is a critical temperature of the BEC formation. We take $T^{\mathrm{BEC}}_{\mathrm{c}} = 100$ K in accordance with recent predictions [19, 25]. We also neglect the finite lifetime of bogolons, studied in works [42, 43] since in our case, the effective time of Cooper pair formation $\sim\Delta^{-1}_\lambda$ is smaller than the exciton scattering time on impurities τ, $\Delta_\lambda\tau/(\xi_{\mathrm{h}}k)^2 \gg 1$.

Figure 2 shows the comparison between the SC order parameters induced by $1b$- and $2b$-mediated pairings. At the same condensate density n_{c} and concentration of electrons in the 2DEG n_{e}, $2b$-induced gap $\Delta_{2b}(T)$ is bigger than $\Delta_{1b}(T)$. This drastic difference between them is caused by the ratio of two effective electron–electron pairing potentials, $V_{1b}/V_{2b} \sim (\xi_{\mathrm{h}}k_{\mathrm{F}})(n_{\mathrm{c}}\xi^2_{\mathrm{h}}) \ll 1$. Moreover, the finite-temperature correction to the $2b$-mediated pairing potential in equation (11) leads to dramatic enhancement of the SC gap with the increase of temperature. As a result, $2b$-induced order parameter reveals a pronounced

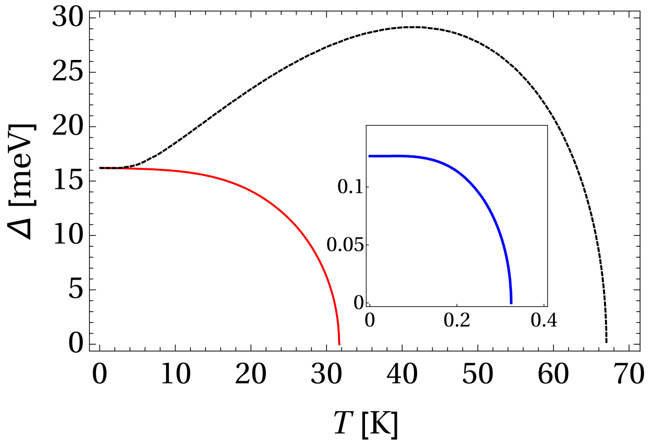

Figure 2. SC gap as a function of temperature. Red solid curve shows $2b$-mediated gap disregarding N_q-containing term in equation (11). Black dashed curve accounts for the full temperature dependence [including the influence of N_q-containing term in equation (11)]. Inset shows one-bogolon SC gap for comparison. We used the parameters, typical for MoS_2 and hBN: $\epsilon = 4.89$, $m = 0.46m_0$ (where m_0 is free electron mass), $M = m_0$, $d = 1$ nm, $l = 2.5$ nm, and $L = 10^{-6} \sim$ m. We also take $n_e = 1.2 \times 10^{12}$ cm^{-2} and $n_c = 5.0 \times 10^{10}$ cm^{-2}.

non-monotonous temperature dependence. We want to note, that non-monotonous dependence of the order parameter due to two-acoustic phonon-mediated pairing has been theoretically investigated in three-dimensional multi-band superconductors. There, however, the two-phonon processes were considered as a second-order perturbation [44] giving a contribution in the absence of single-phonon processes. In our case, $2b$ pairing belongs to the same order of the perturbation theory as $1b$ pairing [see equations (10) and (11)], as it will be discussed below.

We should also address the issue of Coulomb repulsion between electrons in 2DEG. A standard calculation [45] gives the following renormalization of the coupling constant: $\tilde{V}_\lambda(p_F) \to V_\lambda(p_F) - V'_C$, where $V_C{}' = V_C/[1 + \nu_0 V_C \log(\mu/\omega_b)]$ with V_C the momentum-averaged Coulomb potential [46]. Using the same parameters as in figure 2, we estimate $\nu_0 V_C{}' \approx 0.2$, while we consider $\nu_0 V_{2b}$ in the range 0.4–1 (along the text).

It should also be noted, that our approach is valid in the *weak electron–bogolon coupling regime* where the BCS theory is applicable [36, 47]. It corresponds to $\nu_0 V_{2b}(p_F) < 1$. Thus we only use $\nu_0 V_{2b}(p_F)$ in the range 0.4–1, where unity corresponds to a provisional boundary, where the weak coupling regime breaks and a more sophisticated strong-coupling treatment within the Eliashberg equations approach is required [46, 48–50]. However, we leave it beyond the scope of this article.

Figure 3 shows the dependence of the $2b$-mediated gap and the critical temperature on the condensate density. As it follows from equation (15) (and equation (14) for $1b$ processes), both Δ_λ and T_c grow with the increase of n_c (via the sound velocity s) or decrease of n_e (via the Fermi wave vector p_F in the exponential factor in g_{p_F}). A naive idea which comes to mind is to start increasing n_c up to the maximal experimentally achievable values and decreasing n_e while possible. However, the applicability of the BCS theory imposes an additional requirement: $n_e/n_c > d/a_B^{el}$, where $a_B^{el} = \pi\epsilon_0\epsilon\hbar^2/me_0^2$ is the Bohr radius of electrons in 2DEG. Meanwhile, considering only bogolons with a linear spectrum dictates another requirement: $k_F\xi < 1$, that gives the condition $n_e/n_c < d/a_B^{ex}$, where $a_B^{ex} = \pi\epsilon_0\epsilon\hbar^2/Me_0^2$ is the Bohr radius of exciton. It results in a condition imposed on the effective masses: the effective electron mass in 2DEG should be smaller than the mass of the indirect exciton. The optimal relation between n_e and n_c is $n_e/n_c \sim C_1\pi\epsilon_0\epsilon\hbar^2/m_0e_0^2$, where C_1 is a numerical constant and m_0 is a free electron mass.

Why is $2b$ superconductivity stronger than $1b$? The electron-single bogolon and electron–bogolon pair interactions are processes of the same order with respect to the electron–exciton interaction strength g_p due to the properties of weakly interacting Bose gas at low temperature. The full density of the Bose gas consists of three parts: (i) the condensate density n_c, (ii) density of excitations above the condensate $\varphi^\dagger_\mathbf{R}\varphi_\mathbf{R}$, and (iii) the 'mixed density' $\sqrt{n_c}(\varphi^\dagger_\mathbf{R} + \varphi_\mathbf{R})$. This last term here does not conserve the number of Bose-particles in a given quantum state and usually gives small contribution to different physical processes, such as electron scattering, since only the non-diagonal matrix elements of this operator are nonzero, see equation (2).

To understand the microscopic origin of this phenomenon, in figure 4 we show the Feynman diagrams, corresponding to $1b$ and $2b$ pairings, as it follows from the Schrieffer–Wolff transformation (see supplemental material [38]). The matrix elements of the electron–boson interaction g_p are multiplied by the Bogoliubov coefficients. In the $1b$ case, it is the sum $(u_\mathbf{p} + v_{-\mathbf{p}})$, while in the $2b$ case a product of the

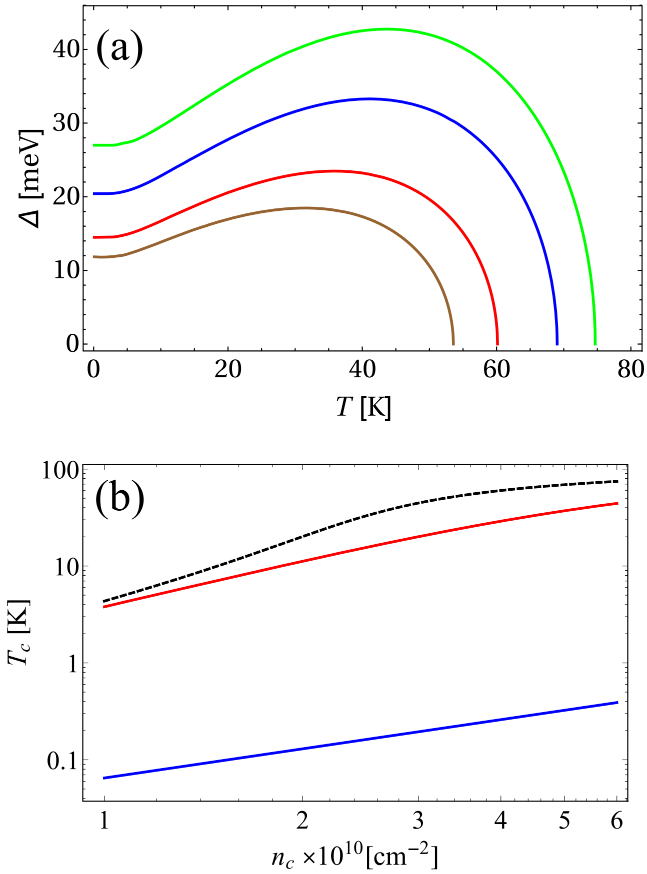

Figure 3. (a) SC gap due to bogolon-pair-mediated processes as a function of temperature for different condensate densities: $n_c = 3.5 \times 10^{10}$ cm^{-2} (brown), $n_c = 4.0 \times 10^{10}$ cm^{-2} (red), $n_c = 5.0 \times 10^{10}$ cm^{-2} (blue), and $n_c = 6.0 \times 10^{10}$ cm^{-2} (green). (b) Critical temperature as a function of condensate density for single-bogolon processes (blue), two-bogolon processes without the N_q-containing term in equation (11) (red), and two-bogolon processes with the N_q-containing term (black dashed). We used $n_e = 1.0 \times 10^{12}$ cm^{-2}. All other parameters are the same as in figure 2.

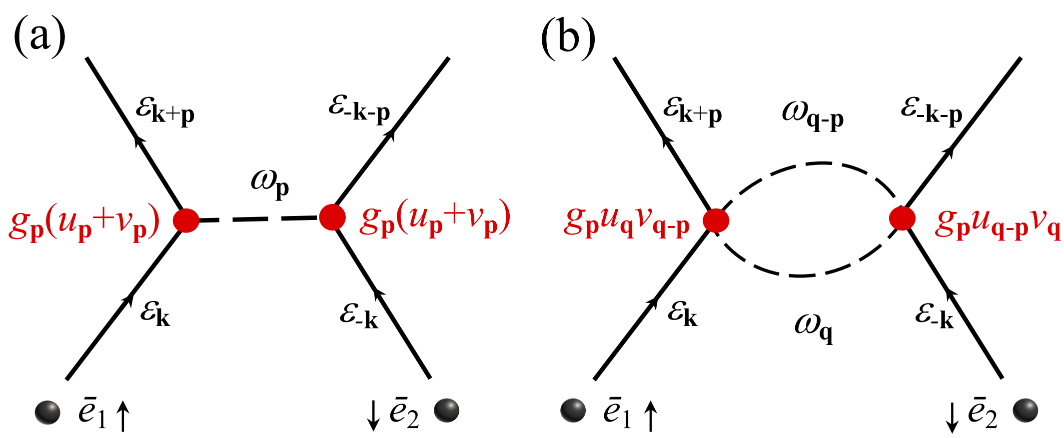

Figure 4. Effective Feynman diagrams of single-bogolon-mediated (a) and bogolon-pair-mediated (b) electron pairing.

kind $u_{\mathbf{q}}v_{\mathbf{q-p}}$. We see, that the key reason of suppression of the 1b processes is that there emerges a small factor $(u_{\mathbf{p}} + v_{-\mathbf{p}}) \sim (p\xi_h)^2 \ll 1$ [30]. Indeed, both $|u_{\mathbf{p}}|, |v_{\mathbf{p}}| \gg 1$, and they have opposite signs, thus negating each other in the sum. It can be looked at as a destructive interference of waves corresponding to $b_{\mathbf{p}}$ and $b^\dagger_{-\mathbf{p}}$. There is no such self-cancellation in the 2b matrix elements since $u_{\mathbf{p}}v_{\mathbf{p}} \sim (p\xi_h)^{-1} \gg 1$ (instead of $u_{\mathbf{p}} + v_{-\mathbf{p}}$). Here we can also recall the acoustic phonons, where such a cancellation effect does not take place, and hence the single-phonon scattering prevails over the two-phonon one, and thus the latter can be usually neglected. However, the physics in question is general and might be relevant to other proximity effects of the BEC phase. We want to mention also, that the processes involving three and more bogolons belong to the higher-order perturbation theory with respect to the electron–exciton interaction g_p and can be disregarded, as it has been discussed in [51].

We note, that performing the calculations and evaluating the gap and T_c, we assumed that the electron gas is degenerate at given n_e and temperature. We have to also note, that the approach discussed in this article is only valid as long as n_c is macroscopically large ($n_c \gtrsim 10^8$ cm^{-2}). Only under this condition, we can treat the bogolon dispersion as linear and use the mean field approach and the Bogoliubov transformations.

Certainly, SC T_c should be smaller than T_c^{BEC}. In GaAs-based excitonic structures, $T_c^{\mathrm{BEC}} \sim 1 - 7$ K [52] and it is predicted to reach $\sim$100 K or more in MoS_2 [19], which finds its experimental signatures [25]. If the temperature is above the critical one, there is no BEC but electrons are still coupled with excitons via Coulomb forces. However, we believe that in this case Bose gas-mediated superconductivity is strongly suppressed [53].

Usually, the conventional phonon-mediated superconductivity is explained the following quantitative way: one electron moving along the crystal polarizes the media due to the Coulomb interaction between this electron and the nuclei, and then another electron (moving with the opposite or close-to-opposite momentum to the first electron) feels this polarization of the media, and by that the electrons effectively couple with each other. In our case, the ions of the crystal lattice are replaced by indirect excitons. And here, the mechanism of electron–electron pairing is similar qualitatively but quantitatively different: instead of the deformation potential, one deals with the direct Coulomb interaction between electrons and excitons, which can be treated as dipoles. Thus, the effective matrix elements of this interaction are different. As the result, one electron disturbs the excitonic media in BEC, while another one (with opposite momentum) feels the polarization, and the SC pairing might occur.

4. Conclusions

We have studied electron pairing in a 2DEG in the vicinity of a two-dimensional BEC, taking a condensed dipolar exciton gas as an example. We have found that the bogolon-pair-mediated electron interaction turns out to be the dominant mechanism of pairing in hybrid systems, giving large SC gap and critical temperatures of SC transition up to 80 K. The effect is twofold. First, the bogolon-pair-induced gap is bigger than the single-bogolon one even at zero temperature due to the structure and magnitudes of the matrix elements of electron interaction. Second, we predict that, in contrast to single-bogolon-mediated processes, two-bogolon electron pairing potential acquires an additional temperature-dependent term, associated with the increase of the number of thermally activated bogolons with temperature. As a consequence, such term leads to non-monotonous temperature characteristics of the SC gap and a considerable increase of T_c. We expect this exotic feature to be observable experimentally. Moreover, instead of indirect excitons, one can employ microcavity exciton polaritons, where the BEC is reported to exist up to the room temperature [54], or other bosons.

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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