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PAPER

### Spin momentum-dependent orbital motion

#### Shaohui Yan<sup>1,3</sup>, Manman Li<sup>1</sup>, Yansheng Liang<sup>2</sup>, Yanan Cai<sup>1</sup> and Baoli Yao<sup>1,3</sup>

- State Key Laboratory of Transient Optics and Photonics, Xi'an Institute of Optics and Precision Mechanics, Chinese Academy of Sciences, Xi'an, People's Republic of China
- Shaanxi Key Laboratory of Quantum Information and Quantum Optoelectronic Devices, School of Science, Xi'an Jiaotong University, Xi'an, People's Republic of China
- <sup>3</sup> Authors to whom any correspondence should be addressed.

E-mail: shaohuiyan@opt.ac.cn and yaobl@opt.ac.cn

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#### Abstract

We present a theoretic analysis on (azimuthal) spin momentum-dependent orbital motion experienced by particles in a circularly-polarized annular focused field. Unlike vortex phase-relevant (azimuthal) orbital momentum flow whose direction is specified by the sign of topological charge, the direction of (azimuthal) spin momentum flow is determined by the product of the field's polarization ellipticity and radial derivative of field intensity. For an annular focused field with a definite polarization ellipticity, the intensity's radial derivative has opposite signs on two sides of the central ring (intensity maximum), causing the spin momentum flow to reverse its direction when crossing the central ring. When placed in such a spin momentum flow, a probe particle is expected to response to this flow configuration by changing the direction of orbital motion as it traversing from one side to the other. The reversal of the particle's orbital motion is a clear sign that spin momentum flow can affect particles' orbital motion alone even without orbital momentum flow. More interestingly, for dielectric particles the spin momentum-dependent orbital motion tends to be 'negative', i.e., in the opposite direction of the spin momentum flow. This arises mainly because of spin-orbit interaction during the scattering process. For the purpose of experimental observation, we suggest the introduction of an auxiliary radially-polarized illumination to adjust the particle's radial equilibrium position, for the radial gradient force of the circularly-polarized annular focused field tends to constrain the particle at the ring of intensity maximum.

#### 1. Introduction

It is well-known that light can carry both linear momentum and angular momentum (AM) [1–5]. When interacting with a small particle, the optical linear momentum or/and AM can be transferred to the particle, resulting in observable mechanical effects on the particle. For example, in optical tweezers, the linear momentum transfer is essential in providing the gradient force to bind particles at the center of light beam [6]; while in light-induced torque effects, the AM plays an important role in driving particles' orbital or/and spin motion [7–9]. In electromagnetism, the linear momentum density of light  $\mathcal{P}$  in free space is defined as the vector product of the electric field vector **E** and magnetic field vector **H**:  $\mathcal{P} = \mathbf{E} \times \mathbf{H}/c^2$  (*c* is the speed of light in free space). For monochromatic waves, the time-averaged momentum density  $\langle \mathcal{P} \rangle$  can, after some vector operations, be decomposed into a sum of its canonical part  $\mathbf{P}_0 \propto \text{Im}(\mathbf{E}^* \cdot (\nabla)\mathbf{E})$  and spin part  $\mathbf{P}_S \propto 1/2\nabla \times \text{Im}(\mathbf{E}^* \times \mathbf{E})$  [10–12]. (Here, to simplify the discussion, we adopt the electric-biased forms for optical orbital and spin momentum although there are more accepted dual-symmetry forms, i.e. electromagnetic-democracy forms). The cross products of the position vector  $\mathbf{x}$  with the orbital momentum density  $\mathbf{P}_O$  and spin momentum density  $\mathbf{P}_S$  give the corresponding orbital AM and spin AM, respectively.

In classical field theory, the orbital momentum density  $P_O$  appears naturally in the canonical stress tensor in the Lagrangian approach to electromagnetic fields [1, 11, 13]. The spin momentum density  $P_S$ 

presents itself as an additional tensor augmented to the canonical stress tensor to ensure conservation of angular momentum. For monochromatic fields, the orbital momentum density  $P_{O}$  is closely linked to the local wave vector  $\mathbf{k}_{loc}$  given by the phase gradient; while the spin momentum density  $\mathbf{P}_{S}$  is equal to the curl of the spin AM (the oriented ellipticity of the local polarization) of the field. As a result,  $P_S$  need not lie in the direction of  $\mathbf{k}_{loc}$ , leading to a deviation in direction of the optical linear momentum density  $\mathcal{P}$  from the orbital momentum density  $P_0$  [11–13]. The mechanical effect of the orbital momentum density  $P_0$  on a probe particle is easily observable. For example, a particle in a plane wave can experience a scattering force along the direction the vector  $\mathbf{k}_{loc}$  (so  $\mathbf{P}_{O}$ ) because of the presence of the dynamic phase factor  $\exp(i\mathbf{k}_{loc}\cdot\mathbf{x})$ . Recently, Bliokh et al [12] showed theoretically that in a structured evanescent field, there is a radiation force transverse to  $P_{O}$  on a probe gold Mie particle. This transverse force is  $P_{S}$ - or polarization-dependent, in a sharp contrast to the orbital momentum  $P_{O}$ -induced radiation force which is in general determined by phase gradient. This unconventional (spin-relevant) force was observed in experiment by Antognozzi et al [14] using a nano-cantilever placed in an evanescent wave above a total internal reflecting glass surface. The observed transverse force exhibits a clear polarization dependence. Besides evanescent fields, other structured fields also have transverse spin momentum (norm to orbital momentum). For example, Bekshaev et al [15] showed in theory that even in a simple field configuration like two-wave interference there exists a spin momentum-dependent force on probe Mie particles.

Here, we report a spin momentum-dependent orbital motion of particles in a circularly-polarized annular focused field. In such a field, the (azimuthal) spin momentum can form a closed azimuthal flow orbiting the optical axis. This spin momentum flow exhibits an obvious spin-dependence, its direction being reversed with the sign of the polarization ellipticity is swapped. Furthermore, the flow direction also depends on the radial derivative of the field intensity. For annular focused field discussed here, the radial derivative has different signs on two sides of the ring of intensity maximum, causing the spin momentum flow to reverse its direction when crossing this ring. When a gold Mie particle is placed in this field, an azimuthal radiation force emerges, as response to the spin momentum flow. As desired, the direction of this azimuthal force is in line with that of the spin momentum flow and is reversed when going from one side of the ring of intensity maximum to the other. Such a behavior of force suggests that we may verify in experiment the radial-position-dependence of spin momentum flow in an annular focused field by monitoring the orbital motion of a gold particle. For dielectric Mie particles, their orbital motions in the field are, in many cases, 'negative', that is, in the opposite direction of the spin momentum flow. This negative orbital motion is recently observed in a circularly-polarized (Gaussian-like) focused field in an indirect way [16]. The difficulty of observing directly such orbital motions (positive or negative) lies in the fact that the radial gradient force always tends to restrict the probe particle on the ring of intensity maximum where the spin momentum flow vanishes. To remove this barrier, we propose introduction of an auxiliary radially-polarized illumination (incoherent with the above circularly-polarized field) whose ring-like focused field have no any azimuthal momentum flow and can be used to adjust the particle's radial equilibrium position to facilitate spin momentum detection at different radial positions. With the introduction of this auxiliary field, a stable trapping along the longitudinal direction is also achievable, making it possible to watch with ease the particle's spin momentum-dependent orbital motion even its reversal on the two sides of the ring of intensity maximum.

The paper is organized as follows. Section 2 presents an analysis of the spin momentum in a ring-like focused field which is created by focusing an input field with circular polarization. By the cylindrical symmetry, the azimuthal component of the spin momentum in the focused field can form a closed circular flow circulating around the optical axis. One of the advantages of the ring-like focused field is that the converted orbital AM accompanying tight focusing can be suppressed to a relatively large extent. Moreover, the ring-like focused field has opposite azimuthal spin momentum flow, that is, on the two sides of the ring of intensity maximum the (azimuthal) spin momentum flow has opposite direction, an unusual property not pertaining to orbital momentum flow. For the further purpose, we investigate the focusing case. Section 3 gives numerically calculated forces on probe particles by the circularly-polarized annular focused field. Section 4 presents a discussion of radial and axial equilibrium positions of the probe particle under combined action of the circularly-polarized and the auxiliary radially-polarized illuminations. Section 5 gives an AM perspective on particles' orbital motion. Section 6 concludes the whole work.

#### 2. Spin momentum flow

Throughout this paper, the fields are assumed to vary in time harmonically. Hereafter, for simplicity, we drop the time-dependence factor. In general, optical trapping of particles is realized with a tightly focused field. In numerical calculations, the (complex) electric field  $\mathbf{E}(\mathbf{x})$  near the focus for an input field  $\mathbf{A}_0(\rho_0, \varphi_0)$ 

passing through a high numerical aperture (NA) objective lens can be expressed as the Richards–Wolf integral [17–19]

$$\mathbf{E}(\mathbf{x}) = C \iint_{\mathcal{D}} \mathbf{A}(\vartheta, \varphi) \exp(i\mathbf{k} \cdot \mathbf{x}) d\Omega$$
(1)

Here, *C* is a constant associated with the input power,  $(\vartheta, \varphi)$  are the spherical polar angles,  $d\Omega = \sin\vartheta d\vartheta d\varphi$  is the elemental solid angle and  $\mathbf{k} = k_1(\sin\vartheta\cos\varphi, \sin\vartheta\sin\varphi, \cos\vartheta)$  denote the wave vectors of the angular spectra components with wavenumber  $k_1 = 2\pi n_1/\lambda$  ( $n_1$  is the index of refraction of medium in image space and  $\lambda$  is the free space wavelength of light); the integral domain  $\mathcal{D} = \{(\vartheta, \varphi) | 0 \leq \vartheta \leq \vartheta_{\max}, 0 \leq \varphi \leq 2\pi\}$  with  $\vartheta_{\max}$  being the maximal converging angle determined by the NA, that is,  $\vartheta_{\max} = \arcsin(NA/n_1)$ ; the apodized field  $\mathbf{A}(\vartheta, \varphi)$  is related to the input field  $\mathbf{A}_0(\rho_0, \varphi_0)$ 

according the rule:

$$\mathbf{A}(\vartheta,\varphi) = \sqrt{\cos\vartheta}(\mathbf{e}_{\vartheta},\mathbf{e}_{\varphi}) \begin{pmatrix} \mathbf{e}_{\rho0} \cdot \mathbf{A}_0(f \sin\vartheta,\varphi-\pi) \\ \mathbf{e}_{\varphi0} \cdot \mathbf{A}_0(f \sin\vartheta,\varphi-\pi) \end{pmatrix}$$
(2)

where  $\mathbf{e}_{\rho 0}$  and  $\mathbf{e}_{\varphi 0}$  denote the respective unit vectors in the radial and azimuthal directions in the input plane while  $\mathbf{e}_{\vartheta}$  and  $\mathbf{e}_{\varphi}$  refer to the unit vectors in spherical polar angle directions with respect to the focus. In writing equation (2), we have used the geometry relations  $\rho_0 = f \sin \vartheta$  and  $\varphi = \pi + \varphi_0$ .

The time-averaged momentum density  $\langle \mathcal{P} \rangle$  of the focused field can be decomposed into the orbital momentum density  $\mathbf{P}_{O}$  and spin momentum density  $\mathbf{P}_{S}$  as [10–12]

$$\langle \mathcal{P} \rangle = (1/2) \operatorname{Re}(\mathbf{D}^* \times \mathbf{B})$$
  
=  $(\varepsilon_1/2\omega) \operatorname{Im}(\mathbf{E}^* \cdot (\nabla)\mathbf{E}) + (\varepsilon_1/4\omega)\nabla \times \operatorname{Im}(\mathbf{E}^* \times \mathbf{E})$ (3)  
=  $\mathbf{P}_{\mathrm{O}} + \mathbf{P}_{\mathrm{S}}$ 

with  $\epsilon_1$  denoting the permittivity of the image space medium (usually water) and the notation convention  $\mathbf{E}^* \cdot (\nabla)\mathbf{E} \equiv (\nabla \mathbf{E}) \cdot \mathbf{E}^*$  is adopted [10] (the repeated indices mean a summation). Here, we have adopted the Minkowski momentum definition:  $\langle \mathcal{P} \rangle = 1/2 \operatorname{Re}(\mathbf{D}^* \times \mathbf{B})$  and our decomposition of orbital and spin momentum is electric-biased. Other decompositions like dual-symmetry form contain both electric and magnetic contributions in each part [10–12]. The difference between the two forms disappears in the paraxial limit. Furthermore, in our numerical simulations, we find that for the focused field of a circularly-polarized illumination, the qualitative behaviors of the two forms are substantial similar.

It is shown that for a paraxial circularly-polarized field its azimuthal spin momentum  $P_{S,\phi}$  forms a (uniform) closed circular flow around the optical axis. For example, assume  $\mathbf{E}(\mathbf{x}) = \mathbf{e}_L f(\rho, z)$  or  $\mathbf{e}_R f(\rho, z)$  (here and below,  $\mathbf{e}_L = (\mathbf{e}_x + i\mathbf{e}_y)/\sqrt{2}$  and  $\mathbf{e}_R = (\mathbf{e}_x - i\mathbf{e}_y)/\sqrt{2}$  are two circular polarization basis vectors). It then follows that  $P_{S,\phi} \propto -\sigma \partial \rho |f|^2$ , where  $\sigma$  is the polarization ellipticity of field and is equal to 1 for  $\mathbf{e}_L$  and -1 for  $\mathbf{e}_R$ . Furthermore, for such a paraxial field,  $P_{O,\phi} = 0$ . As a result, a pure circularly-polarized paraxial field provides a good platform for investigation of spin momentum-dependent orbital motion. However, for tight focusing, the accompanying spin-to-orbit conversion leads to the following process  $[20-24]: |\mathbf{e}_L\rangle \rightarrow |\mathbf{e}_L\rangle + e^{i2\phi}|\mathbf{e}_R\rangle + e^{i\phi}|\mathbf{e}_z\rangle$  and likewise for a right-hand circular polarization illumination  $|\mathbf{e}_R\rangle$ . This shows that even the input field is in a pure circular polarization state without any vortex phase, the focused field contains other polarization components with non-zero vortex indices, a manifest of azimuthal orbital momentum.

In figure 1, we plot the azimuthal components of orbital and spin momentum density of the focused field for a uniform illumination ( $\lambda = 1.064 \ \mu m$ ) with left-hand circular polarization  $\mathbf{A}_0 \propto \mathbf{e}_L$  through an NA = 1.26 water immersed (thus  $n_1 = 1.33$ ) objective lens. It can be shown by direction calculation that the corresponding focused field takes the form  $\mathbf{E}(\mathbf{x}) = \mathbf{e}_L E_L(\rho, z) + \mathbf{e}_R e^{i2\phi} E_R(\rho, z) + e^{i\phi} \mathbf{e}_z E_z(\rho, z)$ , where ( $\rho, \phi, z$ ) are the cylindrical coordinates of the position vector  $\mathbf{x}$ . By substituting this expression into equation (3), one finds that either orbital or spin momentum forms a uniform closed circular flow orbiting the optical axis, that is,  $\partial_{\phi}(P_{O,\phi}) = 0$  and  $\partial_{\phi}(P_{S,\phi}) = 0$ . To simplify analysis, only the flows at a fixed azimuthal angle, say  $\phi = 0$ , are plotted. For purpose of discussion, the intensities of three electric field components are also plotted.

Figure 1 shows that the focused field contains a dominant left-circularly polarized component  $E_{\rm L}$ , a small axis component  $E_z$  and a negligible right-circularly polarized component  $E_{\rm R}$ . The  $E_z$  component carry a vortex phase factor  $e^{i\phi}$ , giving rise to the azimuthal orbital momentum with a relative maximum (to the spin momentum maximum) equal to ~0.27. From equation (3), we have  $P_{\rm O,\phi} \propto (I_z + I_{\rm R})/\rho \approx I_z/\rho$  with  $I_z = |E_z|^2$  and  $I_{\rm R} = |E_{\rm R}|^2$ . Thus,  $P_{\rm O,\phi}$  is positive over the whole radial range, a reflection of the fact that the sign of azimuthal orbital momentum is dependent only on the topological charge of vortex phase. The situation for the azimuthal spin momentum is different. It is shown from equation (3) that  $P_{\rm S,\phi} \propto -\sigma \partial \rho (I_{\rm L}-I_{\rm R}) \approx -\sigma \partial \rho I_{\rm L}$  with  $\sigma = 1$  for the left-circularly polarized illumination discussed here. For a







right-circularly polarized illumination, the result for  $P_{S,\phi}$  is obtained by setting  $\sigma = -1$  and the orbital momentum  $P_{O,\phi}$  also changes the sign correspondingly. Thus, the azimuthal spin momentum flow depends on the helicity as well as the radial derivative of the intensity. For a given value of  $\sigma$ , the sign of  $P_{S,\phi}$  will reverse if the radial derivative of the intensity changes its sign. This occurs usually on the two sides of a ring of (local) intensity extreme. In figure 1, the reversal of the sign of appears first on two sides of  $\rho \approx 0.52 \ \mu$ m, at which  $I_{\rm L}$  reaches a local minimum. However, this negative (azimuthal) spin momentum flow is in magnitude much less than that of the positive flow. To enhance the negative flow, we propose a ring-like focused field by means of so-called perfect vortex fields [25–28]. The key is to introduce to the input field a Bessel modulation factor  $J_0(\alpha k_1 \sin \vartheta)$ , where  $J_0(\cdot)$  denotes the first kind of cylindrical Bessel function of order zero. With this modulation, i.e.,  $\mathbf{A}_0 \rightarrow J_0(\alpha k_1 \sin \vartheta)\mathbf{A}_0$ , the focused field will exhibit a ring-like pattern with adjustable radius (controlled by the parameter  $\alpha$ ). The formation of the ring-like focused field can be understood by realizing that the field in the focal plane is in fact the inverse Fourier transform of the input field [see equation (1)] and the 2D Fourier transform of  $J_0(\cdot)$  is just a ring with a rather narrow width (ideally a delta-ring). This narrow-ring structure implies that the radial derivatives of intensity on two sides of the ring of intensity maximum have large magnitudes but opposite signs.

Figure 2 shows  $P_{S,\phi}$ ,  $P_{O,\phi}$ , and the three electric field component intensities as a function of radius  $\rho$  of the focused field of a modulated left-circularly polarized illumination  $\mathbf{A}_0 \propto \mathbf{e}_{\mathrm{L}} J_0(2.5k_1 \sin \vartheta)$  [here and below, we only take the numerical value of  $k_1$  (=  $2\pi n_1/1.064$ ) in the argument of  $J_0(\cdot)$ . The other parameters are as in figure 1. The focused field is dominated by the left-circularly polarized electric field component, whose intensity takes on an obvious ring-like pattern with central radius  $\rho_0 \approx 2.5 \,\mu\text{m}$  and width  $\Delta \approx 1\mu\text{m}$ . Under this field configuration, the azimuthal spin momentum has two appealing peaks





with opposite signs on the inner and outer sides of the ring of intensity maximum ( $\rho_0 \approx 2.5 \ \mu$ m). Moreover, the azimuthal orbital momentum has a local maximum value ~0.0412 near the ring of intensity maximum while the two peak values of the spin momentum are -1.024 and 1.0, implying that the orbital momentum may be neglected when considering the orbital motion of the probe particle. More interestingly, the occurrence of the two peaks with opposite signs of  $P_{S,\phi}$  facilitates greatly the observation of reversal of orbital motion of a probe particle on the two sides, which never be expected from orbital momentum.

#### 3. Mechanical effects of spin momentum

We now turn to mechanical effects of spin momentum on a probe particle. To detect spin momentum, it is necessary that the characteristic size of the probe particle a satisfies  $a > -1/k_1$  [12], that is, a finite size Mie particle. For incident light of 1.064  $\mu$ m wavelength and  $n_1 = 1.33$  as in figures 1 and 2, the particle's size needs to be larger than  $\sim 0.1273 \ \mu$ m. As an illustration, we consider a spherical polystyrene  $(n_2 = 1.59)$  particle of radius  $a = 0.5 \mu m$  located in the focused field as in figure 2. We use the Mie scattering theory and the Maxwell stress tensor to calculate the forces on the particle [29–32]. The Mie scattering theory can compute exactly the scattered fields by a uniform, isotropic spherical particle illuminated by a plane wave. With some appropriate modifications, it can handle the scattering problem involving tightly focused field given by the integral (1) [32]. Let the incident power  $P_0 = 200$  mW. The red curve in figure 3 shows the azimuthal force  $F_{\phi}$  on the polystyrene sphere. Obviously, this azimuthal force has two peaks (with opposite signs) on the two sides of the ring of intensity maximum ( $\rho_0 \approx 2.5 \ \mu$ m). Interestingly, the force is in the opposite direction of the azimuthal spin momentum flow (dashed curve): in the inner side near the ring the force is positive while the flow is negative, and the contrary situation arises in the outer side of the ring. This means that the polystyrene sphere will experience an orbital motion against the spin momentum flow, and will reverse the direction of orbital motion when crossing the ring of intensity maximum, which is not observed in orbital momentum-induced orbital motion.

The above counter-flow effect can be explained by considering the dipole–dipole coupling model [12, 33, 34], the next order correction to the dipole approximation. In the dipole approximation, it is shown that the scattering force on a dielectric particle  $\mathbf{F}_{\text{scat},\mathbf{p}} \propto \text{Im}(\alpha_e) \mathbf{P}_0$  [33]. Since we are interested in the region near the ring, the azimuthal orbital momentum there is neglectable as shown in figure 2, so is the azimuthal component ( $\mathbf{F}_{\text{scat},\mathbf{p}}$ ) $_{\phi}$ . The next order correction is the dipole–dipole term  $\mathbf{F}_{\text{scat},\mathbf{pm}}$  arising from the higher-order interaction between electric- and magnetic-induced dipoles [12, 34] whose azimuthal component ( $\mathbf{F}_{\text{scat},\mathbf{pm}}$ ) $_{\phi} \propto -\text{Re}(\alpha_e \alpha_m^*) P_{\text{S},\phi}$ . Here  $\alpha_e$  and  $\alpha_m$  are the electric and magnetic polarizabilities. Note that in writing ( $\mathbf{F}_{\text{scat},\mathbf{pm}}$ ) $_{\phi}$ , we have neglected the contribution from the orbital momentum  $P_{\text{O},\phi}$ , since it is negligibly small. Thus, we see that the azimuthal scattering force on the particle is, as far as dipole–dipole order, approximately proportional to  $P_{\text{S},\phi}$  with the proportionality  $-\text{Re}(\alpha_e \alpha_m^*)$ . For dielectric material, the proportionality  $-\text{Re}(\alpha_e \alpha_m^*)$  is usually negative, indicating the scattering force is opposite to the spin momentum flow as shown in figure 3 for the polystyrene sphere. However, for metal particles of high absorption, say gold particles ( $n2 \approx 0.4 + 7.36i$  at 1.064  $\mu$ m), the proportionality is positive [34]. The blue curve in figure 3 shows the (0.15 times) azimuthal force on a gold sphere of radius 0.5  $\mu$ m, the other parameters being the same as for the polystyrene sphere. As desired, the force is parallel to



the spin momentum flow. Again, the force reverses the direction on two sides of the ring of intensity maximum.

We now extend the analysis of azimuthal force to a broad range of particle size. Figure 4 describes the azimuthal force distribution in particle's radius and radial position for the polystyrene sphere and gold sphere. The black dashed line in each plot denotes the critical radial positions crossing which the azimuthal force reverses the direction. The starting point of each critical line is chosen such that the associated force is not too small. When the particle's radius is less than that at the starting point, the dipole force  $\mathbf{F}_{\text{scat},\mathbf{p}}$  dominates and the reversal of force cannot be observed. Note that the two critical lines are approximately two horizontal straight lines given approximately by  $\rho_0 \approx 2.5 \ \mu\text{m}$ , the value of radius of the ring. For each plot, the force is in the opposite direction of flow while for the gold sphere the situation is conversed. The results contained in figure 4 reveal that for a wide range of particle's size we can investigate the reverse of spin momentum flow of a circularly-polarized ring-like focused field by observing orbital motion reversal.

#### 4. Equilibrium-position consideration

The preceding sections reveal that the (azimuthal) spin momentum flow of a circularly-polarized ring-like focused field forms closed circular flow orbiting the optical axis and have two peaks with opposite signs on the two sides the ring of intensity maximum. When a probe particle is placed in such a flow, the azimuthal force on the particle, thus the particle' orbital motion, can manifest this two-peak feature of the flow by reversing the direction when crossing the ring along the radial direction. Moreover, the azimuthal force on the dielectric particle is always against the flow while for the gold particle the force is directed along the flow.

To detect the aforementioned radial position-dependent spin momentum flow, a simple and straightforward method is to observe the orbital motion of a probe particle placed into the flow. To this end, we need to first address the issue of equilibrium position of the particle, both radially and axially. We first consider the radial one. In optical trapping, the gradient force tends to hold a dielectric particle at the locus of intensity maximum. In the case of the polystyrene sphere discussed in figure 3, the radial equilibrium position is expected to be located on the ring of intensity maximum. The black dashed curve  $F_{\text{LP},\rho}$  in figure 5(a) describes the radial force on the same polystyrene sphere with the same illumination  $\mathbf{A}_0 \propto \mathbf{e}_L J_0(2.5k_1 \sin \vartheta)$  as in figure 3. It is seen that the radial equilibrium position, denoted by the circle  $C_0$ , is located roughly at  $\rho = 2.45 \ \mu$ m, very close to the ring of intensity maximum. At the position, the azimuthal force on the sphere is too small (see figure 3) to drive orbital motion in practice.

To observe the orbital motion of the probe particle and the corresponding reversal, it is desirable to develop a technique that can change the particle's radial equilibrium position. Here, we introduce an auxiliary radially-polarized illumination to realize this adjustment of radial equilibrium position. The radially-polarized illumination is assumed to be inherent with the circularly-polarized illumination. The focused field of radially-polarized illumination have many unique properties like strong axial component and smaller focal spot size [35–38]. Such focusing properties can be used to create an axially polarized optical needle [39], improve axial optical trapping efficiency [40–42] and break the diffraction limit in confocal microscopy [43]. One of the advantages of use of radially polarized illumination in our analysis is



**Figure 5.** Radial force of the focused field with a left-circularly illumination  $\mathbf{A}_0 \propto \mathbf{e}_I J_0(2.5k_1 \sin \vartheta)$  (black dashed curve) and two radially polarized illumination  $\mathbf{A}_{\text{Rad1}} \propto \mathbf{e}_{\rho 0} J_0(1.95k_1 \sin \vartheta)$  (red dashed curve) and  $\mathbf{A}_{\text{Rad2}} \propto \mathbf{e}_{\rho 0} J_0(3.25k_1 \sin \vartheta)$  (blue dashed curve). The coordinate  $\rho$  on the abscissa axis is the radial distance from the optical axis. The red solid curve presents the total radial force under the (incoherent) ( $\mathbf{A}_0$ ,  $\mathbf{A}_{\text{Rad1}}$ ) combination and the blue solid curve is the total radial force under the (incoherent) ( $\mathbf{A}_0$ ,  $\mathbf{A}_{\text{Rad1}}$ ) combination and the blue solid curve is the total radial force under the (incoherent) ( $\mathbf{A}_0$ ,  $\mathbf{A}_{\text{Rad1}}$ ) combination

that its focused field contains no azimuthal momentum flow, neither orbital nor spin, thus its introduction does not contaminate the spin momentum flow to be detected.

We now consider a radially-polarized illumination of the form

$$\mathbf{A}_{\mathrm{Rad}}(\rho_0,\varphi_0) = \begin{cases} \mathbf{e}_{\rho 0} \mathcal{A}_0 J_0(\beta k_1 \sin \vartheta), & 0.3 < \sin \vartheta < 1.26/n_1 \\ \mathbf{0}, & 0 < \sin \vartheta < 0.3 \end{cases}$$
(4)

where again  $\sin \vartheta = \rho_0 / f$  and  $A_0$  a constant associated with the incident power. The simple annulus pupil apodization function (4) is widely used in the calculations involving radially-polarized fields [19]. The amplitude modulation factor  $J_0(\beta k_1 \sin \vartheta)$  in the input field (4) is presented to produce a ring-like focused field pattern with adjustable radius controlled by the parameter  $\beta$ . When the ring radius of the radially polarized focused field is different from that of the circularly-polarized focused field presented in figure 2, the resulting force of the two radial forces may trap the probe particle at some intermediate position between the two rings. By this trick, we may change the radial equilibrium position as desirable. To verify this, we put the incident power of the radially-polarized illumination  $P_1$  equal to 600 mW and set  $\beta = 1.95$ . For simplicity, we denote this input field by  $\mathbf{A}_{\text{Rad1}} \propto \mathbf{e}_{\rho 0} J_0(1.95k_1 \sin \vartheta)$ . The input field  $\mathbf{A}_{\text{Rad1}}$  gives rise to a ring-like focused pattern with smaller radius than the left-hand circular polarization illumination  $A_0$ . The radial force  $F_{\text{Radl}}\rho$  under the individual illumination  $A_{\text{Radl}}$  is shown in figure 5 (red dashed curve), which attains its radial equilibrium position at  $\sim$ 1.92  $\mu$ m, roughly 0.53  $\mu$ m inwardly away from the original radial position  $C_0$  (~2.45  $\mu$ m) given solely by  $F_{LP,\rho}$  (black dashed curve) of the left-circularly-polarized illumination  $A_0$ . The total radial force  $F_{1,\rho}$  (red solid curve) under the incoherent ( $A_0, A_{Rad1}$ ) combination gives a final radial equilibrium position  $C_1$  (~1.92  $\mu$ m), where the azimuthal force on the particle is appreciable and positive. We next set  $\beta = 3.25$  and  $P_1 = 900$  mW with the corresponding input field denoted by  $A_{\text{Rad2}} \propto e_{\rho 0} J_0(3.25k_1 \sin \vartheta)$ . The radial force  $F_{\text{Rad2},\rho}$  (blue dashed curve) under this illumination has its radial equilibrium position exterior to  $C_0$ . The total radial force  $F_{2,\rho}$  (blue solid curve) under the combination ( $A_0$ ,  $A_{Rad2}$ ) results in a radial equilibrium position  $C_2$  (~2.76  $\mu$ m), where the azimuthal force on the particle is large and negative.

The above calculations show that the introduction of a radially-polarized illumination can confine stably the probe particle at specific radial position, where azimuthal spin momentum flow, thus the associated azimuthal force on the particle, is large and can be negative or positive. As a result, the detection of the reversal of spin momentum flow via particle's orbital motion is reliable, as long as radial trapping is concerned. We now turn to the axial equilibrium position issue with the parameters as given in figure 5. In figure 6, we plot the line scans of total axis forces at  $\rho = 1.92 \ \mu$ m for the ( $A_0, A_{Rad1}$ ) combination ( $F_{1,z}$  : red solid curve) and  $\rho = 2.76 \ \mu$ m for the ( $A_0, A_{Rad2}$ ) combination ( $F_{2,z}$  : blue solid curve), respectively. These two radial positions correspond to equilibrium positions for two combinations, respectively. For purpose of comparison, we also plot the axial forces at  $\rho = 1.92 \ \mu$ m for respective  $A_0$  ( $F_{LP1,z}$  : red dot-dashed curve) and  $A_{Rad1}(F_{Rad1,z}$  : red dashed curve), and those at  $\rho = 2.76 \ \mu$ m for respective  $A_0$  ( $F_{LP2,z}$  : red dot-dashed curve) and  $A_{Rad2}$  ( $F_{Rad2,z}$  : red dashed curve). We see that with  $A_0$  as the only illumination, there is hardly axial restoring force. While with the introduction of radially-polarized illumination, the effective axial



**Figure 6.** Axial forces at two radial positions  $\rho = 1.92 \ \mu m$  (red curves) and  $\rho = 2.76 \ \mu m$  (blue curves) of the focused field for different illuminations. The transverse coordinate *z* is the longitudinal distance from the focal plane. The left-circularly illumination  $\mathbf{A}_0 \propto \mathbf{e}_I J_0(2.5k_1 \sin \vartheta)$  provides almost no restoring force either at  $\rho = 1.92 \ \mu m$  (red dot-dashed curve) or at  $\rho = 2.76 \ \mu m$  (blue dot-dashed curve). The radially-polarized illuminations  $\mathbf{A}_{\text{Rad1}} \propto \mathbf{e}_{\rho 0} J_0(1.95k_1 \sin \vartheta)$  (red dashed curve) and  $\mathbf{A}_{\text{Rad2}} \propto \mathbf{e}_{\rho 0} J_0(3.25k_1 \sin \vartheta)$  (blue dashed curve) have effective restoring forces. The total axial force (red curve) under the ( $\mathbf{A}_0, \mathbf{A}_{\text{Rad1}}$ ) combination has an equilibrium position  $C_1 \ (\sim 0.39 \ \mu m)$  for  $\rho = 1.92 \ \mu m$  and total axial force (blue curve) with the ( $\mathbf{A}_0, \mathbf{A}_{\text{Rad2}}$ ) combination gives an axial equilibrium position  $C_2 \ (\sim 0.59 \ \mu m)$  for  $\rho = 2.76 \ \mu m$ .

restoring forces occurs, giving the axial equilibrium positions  $C_1$  (~0.39  $\mu$ m) for the ( $A_0$ ,  $A_{Rad1}$ ) combination and  $C_2$  (~0.59  $\mu$ m) for the ( $A_0$ ,  $A_{Rad2}$ ) combination. The maximal axial restoring forces for the two combinations are -8.47 pN and -3.25 pN, respectively. Thus, stable axial trapping is realized in either case: the ( $A_0$ ,  $A_{Rad1}$ ) combination used in detecting inner-side spin momentum flow or the ( $A_0$ ,  $A_{Rad2}$ ) used in detecting outer-side flow. The smallness of axial restoring force for the ( $A_0$ ,  $A_{Rad2}$ ) combination is a result of remarkable reduction in field intensity of the focused ring of the input field  $A_{Rad2}$  (although its input power is 900 mW). Enhancement of the axis restoring force is possible either by increasing the incident power or by decreasing the radii of all focused rings.

For gold particles, the axial equilibrium issue can hardly be addressed, since their high absorption causes a very strong scattering force. Even for radial trapping, the radial equilibrium can appear at several position, which complicate the situation very much.

#### 5. AM perspective on orbital motion

In section 3, we consider the particles' orbital motion by calculating the azimuthal force on them. In this section, we treat the particles' orbital motion by calculating angular momentum (AM) transfer. During the field-particle interaction, the total AM (field plus particle) is conserved. Mathematically, the AM conservation is written as [1]

$$\boldsymbol{\Gamma} = -\frac{n}{c} \oint_{S_{\infty}} (\mathbf{r} \times \langle \mathbf{S}_{\text{mix}} \rangle) d\sigma - \frac{n}{c} \oint_{S_{\infty}} (\mathbf{r} \times \langle \mathbf{S}_{\text{S}} \rangle) d\sigma$$

$$\equiv \mathbf{M}_{\text{mix}} - \mathbf{M}_{\text{S}}$$
(5)

where  $\Gamma$  is the torque on the particle,  $S_{\infty}$  is a closed spherical surface at infinity enclosing the particle, and  $\langle S_{mix} \rangle$  and  $\langle S_S \rangle$  are time-averaged Poynting vectors associated with extinction (arising from the interference between incident and scattered fields) and scattering processes, respectively [1, 30]. In light–matter interaction, their integrals give respective linear momenta per unit time removed from the incident light and scattered by the particle. Accordingly, the quantities  $\mathbf{M}_{mix}$  and  $\mathbf{M}_S$  in equation (5) define the AM per unit time removed from the incident field and scattered by the particle. Their difference gives the torque on the particle: a reflection of conservation of total AM (removed AM = scattered AM + particle's torque).

If the spherical surface  $S_{\infty}$  is centered at the particle's center of mass,  $\Gamma$  will be the spin torque on the particle; if the center of  $S_{\infty}$  coincides with the beam's center,  $\Gamma$  will give the total torque on the particle: the spin torque plus the orbital torque with respect to beam's center. Since we are dealing with particle's orbital motion in the focal plane, only *z*- component of equation (5) is of interest. Figure 7 shows the





*z*-components of the total torque  $\Gamma_z$  (solid), removed AM (blue dot-dashed)  $M_{\text{mix}, z}$ , scattered AM (red dot-dashed)  $M_{\text{S},z}$  for a gold sphere of radius  $a = 0.5 \,\mu\text{m}$  (all other settings are the same as in figure 3). In addition ,the spin torque  $\Gamma_z^{\text{spin}}$  (red dashed) on the particle is also plotted. This spin torque arises from absorption and is very small compared to the total torque. So, the gold sphere in fact mainly experiences an orbital torque. Moreover, we see that the removed AM is always positive but the scattered AM can either be positive or negative depending on the radial position, so that the total toque (mainly orbital) can also be positive or negative.

Likewise, we present the corresponding results for a PS sphere of radius  $a = 0.5 \mu m$  in figure 8. Unlike the gold sphere in figure 7, the PS sphere exhibits no absorption, thus the spin torque is zero, as desired. Moreover, the scattered AM for the PS sphere is almost positive, but its peak has a coordinate shift to that of the removed AM. As a result, their difference gives a torque on the PS sphere whose direction and magnitude still depend on radial position.

An interesting observation from the above results is that since our incident field has a dominant spin AM density, the removed AM from the incident field thus contains almost no orbital AM (thus the particle obtains no orbital torque from the incident field), suggesting no azimuthal linear momentum is removed in the extinction process (this can also be verified by directly calculating the azimuthal force associated with extinction process). Noting that the incident field is dominated by the spin momentum density, this result seems to confirm the 'virtual' nature of spin momentum.

#### 6. Discussions and conclusions

The orbital motion of the probe particles shown above is a result of higher-order moment interference in scattering, which gives rise to an azimuthal force  $(\mathbf{F}_{scat,pm})_{\phi} \propto -\text{Re}(\alpha_e \alpha_m^*) P_{S,\phi}$  as discussed in section 3. This force is closely related to the azimuthal spin momentum  $P_{S,\phi}$ , thus leading to an orbital motion. The direction of the motion is parallel or anti-parallel to  $P_{S,\phi}$  depending on the sign of proportionality factor  $-\text{Re}(\alpha_{e}\alpha_{m}^{*})$ . Alternatively, we can also examine the particles' orbital motion from angular-momentum point of view. For the purposes of understanding, we may view the interaction as a two-step process: first, the particle removes the AM from the incident light due to extinction effect and temporally obtains this AM; second, the particle scatters radiation, during which a certain amount of AM is taken away by the scattered field. The AM difference of the two processed gives the net torque on the particle to guarantee conservation of the total AM of the whole system (field plus particle). In our focused field configuration (a circularly-polarized annular focusing pattern), the orbital AM of the incident field is negligibly small so that the particle can only remove the spin AM, giving no orbital motion (with respect to the optical axis) on the particle. During the scattering, the redirection of spin AM gives rise to orbital AM in the scattered field, which may lead to an orbital motion on the particle. The whole process may be viewed as a typical example of spin-orbit interaction due to scattering. This spin-orbit interaction is related to incident light in two aspects: (a) polarization state of the incident light since different incident polarizations may give opposite orbital motions. (b) The gradient of the incident light (different radiation positions may also lead to reversal of the orbital motion). Moreover, electromagnetic properties of the particle (absorptive or non-absorptive) may also affect this interaction. In our calculations, we find that the scattering patterns are actually symmetry broken (not cylindrically symmetrical). The specific characteristic of asymmetry is affected by the above three factors (incident polarization, gradient of incident field and electromagnetic properties of the particle). These natures may help find potential applications in separation of light with different states of polarization and optical manipulation of small particles. The mechanism behind these effects are worthy of consideration in the near future.

In summary, we have revealed that the focused field of a uniform circularly-polarized illumination modulated by a Bessel function factor has a ring-like pattern, which exhibits interesting azimuthal momentum flow profile: the orbital momentum flow is negligibly small in the vicinity of the ring while the spin momentum flow has two peaks: one is interior to and the other exterior to the ring of intensity maximum. The two peaks have opposite signs. When a small probe particle (say a polystyrene sphere) is placed in this field, it can experience an azimuthal force that is closely related to the spin momentum flow, thus executing an orbital motion around the optical axis. Furthermore, the force, hence the orbital motion, reverses the direction when crossing the ring of intensity maximum, a manifest of spin momentum flow reversal. To observe such radial position-dependent orbital motion in practice, a radially-polarized illumination with proper amplitude modulation is introduced to shift the particle's radial equilibrium position. Given that this radial-position dependence is not expected from orbital momentum flow, the proposed spin momentum-dependent orbital motion may enrich our understanding of optical linear and angular momentum

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#### ORCID iDs

Shaohui Yan <sup>(b)</sup> https://orcid.org/0000-0003-3983-7206 Manman Li <sup>(b)</sup> https://orcid.org/0000-0002-6616-5284 Yansheng Liang <sup>(b)</sup> https://orcid.org/0000-0002-3561-1317

#### References

- [1] Jackson J D 1999 Classical Electrodynamics 3rd edn (New York: Wiley)
- [2] Loudon R and Baxter C 2012 Proc. R. Soc. London, Ser. A 468 1825
- [3] Bowman R W and Padgett M J 2013 Rep. Prog. Phys. 76 026401
- [4] Allen L, Beijersbergen M W, Spreeuw R and Woerdman J 1992 Phys. Rev. A 45 8185
- [5] Padgett M and Bowman R 2011 Nat. Photon. 5 343
- [6] Ashkin A, Dziedzic J M, Bjorkholm J and Chu S 1986 Opt. Lett. 11 288
- [7] He H, Friese M, Heckenberg N and Rubinsztein-Dunlop H 1995 Phys. Rev. Lett. 75 826

- [8] Simpson N, Dholakia K, Allen L and Padgett M 1997 Opt. Lett. 22 52
- [9] O'neil A, MacVicar I, Allen L and Padgett M 2002 Phys. Rev. Lett. 88 053601
- [10] Berry M V 2009 J. Opt. A: Pure Appl. Opt. 11 094001
- [11] Bliokh K Y, Bekshaev A Y and Nori F 2013 New J. Phys. 15 033026
- [12] Bliokh K Y, Bekshaev A Y and Nori F 2014 Nat. Commun. 5 3300
- [13] Soper D E 1976 Classical Field Theory (New York: Wiley)
- [14] Antognozzi M, Bermingham C, Harniman R, Simpson S, Senior J, Hayward R, Hoerber H, Dennis M, Bekshaev A and Bliokh K 2016 Nat. Phys. 12 731
- [15] Bekshaev A Y, Bliokh K Y and Nori F 2015 Phys. Rev. X 5 011039
- [16] Diniz K, Dutra R, Pires L, Viana N, Nussenzveig H and Neto P M 2019 Opt. Express 27 5905
- [17] Richards B and Wolf E 1959 Proc. R. Soc. London, Ser. A 253 358
- [18] Youngworth K S and Brown T G 2000 Opt. Express 7 77
- [19] Zhan Q 2009 Adv. Optic Photon 1 1
- [20] Bliokh K Y, Alonso M A, Ostrovskaya E A and Aiello A 2010 Phys. Rev. A 82 063825
- [21] Li C F 2009 Phys. Rev. A 80 063814
- [22] Nieminen T A, Stilgoe A B, Heckenberg N R and Rubinsztein-Dunlop H 2008 J. Opt. A: Pure Appl. Opt. 10 115005
- [23] Zhao Y, Edgar J S, Jeffries G D, McGloin D and Chiu D T 2007 Phys. Rev. Lett. 99 073901
- [24] Rodríguez-Herrera O G, Lara D, Bliokh K Y, Ostrovskaya E A and Dainty C 2010 Phys. Rev. Lett. 104 253601
- [25] Ostrovsky A S, Rickenstorff-Parrao C and Arrizón V 2013 Opt. Lett. 38 534
- [26] Vaity P and Rusch L 2015 Opt. Lett. 40 597
- [27] García-García J, Rickenstorff-Parrao C, Ramos-García R, Arrizón V and Ostrovsky A S 2014 Opt. Lett. 39 5305
- [28] Liang Y, Yan S, He M, Li M, Cai Y, Wang Z, Lei M and Yao B 2019 Opt. Lett. 44 1504
- [29] Bohren C F and Huffman D R 1983 Absorption and Scattering of Light by Small Particles (New York: Wiley)
- [30] Mishchenko M I, Travis L D and Lacis A A 2002 Scattering, Absorption, and Emission of Light by Small Particles (Cambridge: Cambridge University Press)
- [31] Barton J P and Alexander D R 1989 J. Appl. Phys. 66 2800
- [32] Yan S and Yao B 2007 Phys. Rev. A 76 053836
- [33] Chaumet P C and Nieto-Vesperinas M 2000 Opt. Lett. 25 1065
- [34] Chaumet P C and Rahmani A 2009 Opt. Express 17 2224
- [35] Dorn R, Quabis S and Leuchs G 2003 Phys. Rev. Lett. 91 233901
- [36] Yang L, Xie X, Wang S and Zhou J 2013 Minimized spot of annular radially polarized focusing beam Opt. Lett. 38 1331
- [37] Bauer T, Orlov S, Peschel U, Banzer P and Leuchs G 2014 Nat. Photon. 8 23
- [38] Cai Y, Liang Y, Lei M, Yan S, Wang Z, Yu X, Li M, Dan D, Qian J and Yao B 2017 Rev. Sci. Instrum. 88 063106
- [39] Wang H, Shi L, Lukyanchuk B, Sheppard C and Chong C T 2008 Nat. Photon. 2 501
- [40] Kawauchi H, Yonezawa K, Kozawa Y and Sato S 2007 Opt. Lett. 32 1839
- [41] Nieminen T A, Heckenberg N R and Rubinsztein-Dunlop H 2008 Opt. Lett. 33 122
- [42] Michihata M, Hayashi T and Takaya Y 2009 Appl. Opt. 48 6143
- [43] Xie X, Chen Y, Yang K and Zhou J 2014 Phys. Rev. Lett. 113 263901