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Generative framework for dimensionality reduction of large scale network of nonlinear dynamical systems driven by external input

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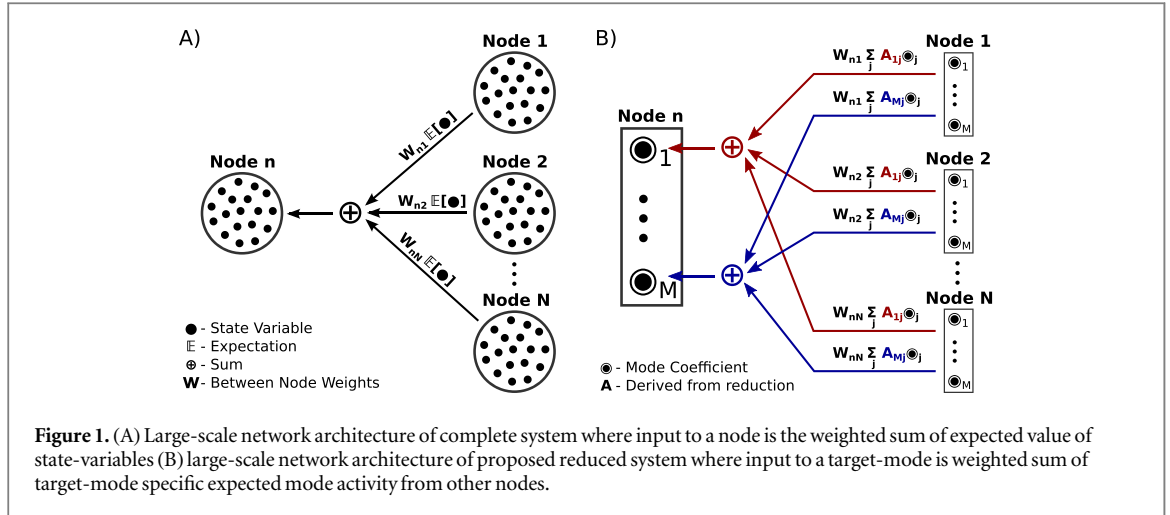
E-mail: 0shreydutta0@gmail.com, dipanjan.nbrc@gov.in and arpan@nbrc.ac.in**Keywords:** nonlinear dynamical systems, large-scale network dynamics, dimensionality reduction

Abstract

Several studies have proposed constraints under which a low-dimensional representation can be derived from large-scale real-world networks exhibiting complex nonlinear dynamics. Typically, these representations are formulated under certain assumptions, such as when solutions converge to attractor states using linear stability analysis or using projections of large-scale dynamical data into a set of lower dimensional modes that are selected heuristically. Here, we propose a generative framework for selection of lower dimensional modes onto which the entire network dynamics can be projected based on the symmetry of the input distribution for a large-scale network driven by external inputs, thus relaxing the heuristic selection of modes made in the earlier reduction approaches. The proposed mode reduction technique is tractable analytically and applied to different kinds of real-world large-scale network scenarios with nodes comprising of (a) Van der Pol oscillators (b) Hindmarsh–Rose neurons. These two demonstrations elucidate how order parameter is conserved at original and reduced descriptions thus validating our proposition.

Large-scale dynamical systems are useful tools to explain a wide variety of complex phenomena in nature e.g. financial markets [1], jamming transitions [2], human mobility dynamics [3], weather patterns [4] and brain dynamics [5]. While increase in scale or dimensions may increase the predictive power of the model system, nonetheless a reduction to simpler descriptions at lower dimensions is critical for having relevance to empirical observations and analytical tractability of underlying mechanisms governing empirical observations. One robust approach of reducing dimensions is defining modes on which the original system can be projected [4, 6]. The selection of a mode is often heuristically motivated, and the mode can also be an order parameter from the perspective of the paradigmatic framework of Synergetics [7]. In Neuroscience, reduction of dynamical systems with respect to modes constructed from distribution of external input has been performed earlier on small-scale network of linearly coupled excitable systems [6]. Since this reduction retain important network dynamics, large-scale networks were conceptualized by coupling these reduced systems with long-range coupling [8, 9], the later being heuristically argued from symmetry properties. In present work, we perform reduction on a large-scale network where connection among nodes involve global and local coupling mimicking a real-world system. Subsequently, long-range coupling term between modes in the reduced system is derived analytically as part of the reduction process. Global coherence (GC) is an order parameter that can be computed both at the level of original dynamical system as well as from the mode dynamics in the reduced system. Conservation of GC at both levels is used to validate the generality of our approach in two distinct networks. First, we simulate a large-scale network where each node is a Van der Pol oscillator having two-dimensional dynamics and coupled using local and global parameters. Second each node is a Hindmarsh–Rose (HMR) neuron, a three-dimensional dynamical system which can exhibit different time scales of oscillations resulting in bursting along with tonic spiking behavior.

Each i th excitable system in node n is represented by a vector of state variables (state vector) $\mathbf{x}_i^{(n)}$. Input to i th excitable system from interaction with other excitable systems within n th node is given by a vector function $\mathbf{g}(\{\mathbf{x}_j^{(n)}\}, K, i)$ where K is coupling constant and $\{\mathbf{x}_j^{(n)}\}$ is a set of J state-vectors ($j = 1, \dots, J$) in n th node. Input



to every excitable system in n th node from other nodes is given by another vector function $\mathbf{h}(\{\mathbf{x}_j^{(m)}\}, W, n)$ where elements of matrix W ($W \in \mathcal{R}^{I \times N}$) are the weights of connections between nodes and $\{\mathbf{x}_j^{(m)}\}$ is a set of state-vectors in N nodes ($m = 1, \dots, N$) (figure 1(A)). Vector function $\mathbf{f}(\mathbf{x}_i^{(n)})$ contributes to the local dynamics of i th excitable system in n th node. Then, the dynamics of the entire system is described by the following set of equations

$$\tau_n \dot{\mathbf{x}}_i^{(n)} = \mathbf{f}(\mathbf{x}_i^{(n)}) + \mathbf{g}(\{\mathbf{x}_j^{(m)}\}, K, i) + \mathbf{h}(\{\mathbf{x}_j^{(m)}\}, W, n) + \phi(I_i^{(n)}), \quad (1)$$

where $\dot{\mathbf{x}}$ is time derivative of state vectors, ϕ is a function of the external input (I) and τ_n is the time-constant of n th node which is a differentiating factor between nodes. However, within n th node, external current to i th excitable system ($I_i^{(n)}$) differentiates it from the rest. For a large scale network comprising of individual excitable nodes, (1) can be written as

$$\tau_n \dot{\mathbf{x}}^{(n)}(t, I) = \mathbf{f}(\mathbf{x}^{(n)}) + \mathbf{g}(\mathbf{x}^{(n)}, K) + \mathbf{h}(\{\mathbf{x}^{(m)}\}, W, n, i) + \phi(I), \quad (2)$$

where I is a continuous variable having normal distribution $\mathbb{N}(\mu, \sigma)$.

Now, we can represent $\mathbf{x}(t, I)$ as a superposition of M bi-orthogonal modes $\{v_i\}$

$$\mathbf{x}(t, I) = \sum_{i=1}^M \xi_i(t) v_i(I) + R(t, I), \quad (3)$$

where $R(t, I)$ is the residual and $M \ll J$ (figure 1(B)). The nature of reduction is such that dynamical system given in (1) is reduced to solving for the mode coefficients ξ_i as described in the following set of equations

$$\tau_n \dot{\xi}_i^{(n)} = F(\xi_i^{(n)}) + G(\{\xi_j^{(n)}\}, K, i) + H(\{\xi_j^{(m)}\}, W, n, i) + \Pi_i^{(n)}, \quad (4)$$

where

$$\begin{aligned} \xi_i &= \int_I \mathbf{x}(t, I) v_i^+ dI \\ F(\xi_i) &= \int_I \mathbf{f}(\mathbf{x}) v_i^+ dI \\ G(\{\xi_j\}, K, i) &= \int_I \mathbf{g}(\mathbf{x}, K) v_i^+ dI \\ H(\{\xi_j^{(m)}\}, W, n, i) &= \int_I \mathbf{h}(\{\mathbf{x}^{(m)}\}, W, n, i) v_i^+ dI \\ \Pi_i^{(n)} &= \int_I I v_i^+ dI \end{aligned}$$

and $\{v_i^+\}$ are the adjoint basis for the bi-orthogonal modes $\{v_i\}$.

Large-scale network of Van der Pol oscillators

A Van der Pol oscillator [10] has two state variables x and y which follows the following equations

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -a(x^2 - 1)y - x. \end{aligned} \quad (5)$$

A large-scale network where individual node is essentially a Van der Pol oscillator can be cast into equation (1) with the following relations

$$\begin{aligned} f(\mathbf{x}_i) &= \begin{bmatrix} y_i \\ -a(x_i^2 - 1)y_i - x_i \end{bmatrix} \\ g(\{\mathbf{x}_j\}, K, i) &= \begin{bmatrix} K(\mathbb{E}[\{\mathbf{x}_j\}] - \mathbf{x}_i) \\ 0 \end{bmatrix} \\ h(\{\mathbf{x}_j^{(m)}\}, W, n) &= \begin{bmatrix} \sum_{m=1}^N W_{nm} \mathbb{E}[\{\mathbf{x}_j^{(m)}\}] \\ 0 \end{bmatrix} \\ I &= \begin{bmatrix} I \\ 0 \end{bmatrix}, \end{aligned}$$

where a is a constant and $\mathbb{E}[\{\mathbf{x}_j\}]$ is the expected value of the state-variables $\{\mathbf{x}_j\}$. K is the coupling constant within each node between excitable units. For the reduced system described in (4) with $\xi_i = [\alpha_i, \beta_i]^T$, we derive

$$\begin{aligned} F(\alpha_i) &= \begin{bmatrix} \beta_i \\ -a_i \alpha_i^2 \beta_i + a \beta_i - \alpha_i \end{bmatrix} \\ G(\{\alpha_j\}, K, i) &= \begin{bmatrix} K \left(\sum_{j=1}^M A_{ij} \alpha_j - \alpha_i \right) \\ 0 \end{bmatrix} \\ H(\{\alpha_j^{(m)}\}, W, n, i) &= \begin{bmatrix} \sum_{m=1}^N W_{nm} \sum_{j=1}^M A_{ij} \alpha_j^{(m)} \\ 0 \end{bmatrix} \\ \Pi &= \begin{bmatrix} \Pi \\ 0 \end{bmatrix}, \end{aligned}$$

where the constants a_i and A_{ij} are computed by applying bi-orthogonal assumption of modes as stated in appendix A.

Large-scale network of HMR neurons

In Neuroscience, HMR neuron is a three-dimensional model of single neuron firing dynamics having three state variables [11]

$$\begin{aligned} \dot{x} &= y - ax^3 + bx^2 + I - z \\ \dot{y} &= c - dx^2 - y \\ \dot{z} &= r(s(x - x_0) - z), \end{aligned} \quad (6)$$

where a, b, c, d, r, s and x_0 are the constants. A network of excitatory and inhibitory HMR neurons have been used to describe a small-scale network of neurons [6]. Thus, a node in the brain (figure 1(a)) can be expressed as a six-dimensional state space with 3 excitatory and 3 inhibitory variables represented as $\mathbf{x} = [x_1, y_1, z_1, x_2, y_2, z_2]^T$. Extending this architecture, for the large-scale system in (1), we obtain

$$\begin{aligned} f(\mathbf{x}_i) &= \begin{bmatrix} y_{1i} - ax_{1i}^3 + bx_{1i}^2 - z_{1i} \\ c - dx_{1i}^2 - y_{1i} \\ r(s(x_{1i} - x_0) - z_{1i}) \\ y_{2i} - ax_{2i}^3 + bx_{2i}^2 - z_{2i} \\ c - dx_{2i}^2 - y_{2i} \\ r(s(x_{2i} - x_0) - z_{2i}) \end{bmatrix} \\ g(\{\mathbf{x}_j\}, K, i) &= \begin{bmatrix} (K_{11}(\mathbb{E}[\{\mathbf{x}_{1j}\}] - x_{1i}) - K_{12}(\mathbb{E}[\{\mathbf{x}_{2j}\}] - x_{1i})) \\ 0 \\ 0 \\ K_{21}(\mathbb{E}[\{\mathbf{x}_{1j}\}] - x_{2i}) \\ 0 \\ 0 \end{bmatrix} \end{aligned}$$

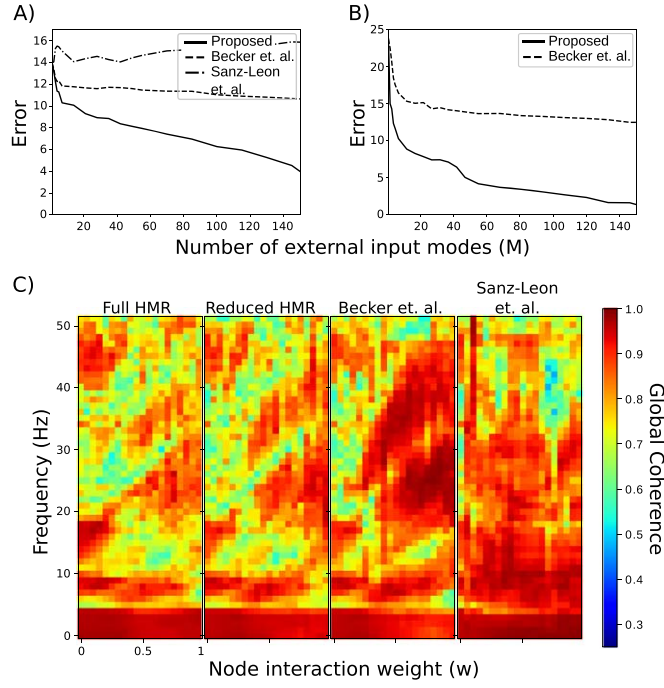


Figure 2. Improved error performance and preservation of global coherence spectra for the proposed reduction approach in comparison to previous heuristic approaches [8, 9] (A) error computed for Hindmarsh–Rose model for $a = 1, b = 3, c = 1, d = 5, r = 0.006, s = 4, x_0 = -1.6, K_{11} = 0.5, K_{21} = 0.5$, Inhibition – excitation ratio (IER) $\in (0, 2.5), \mu \in (0, 2.5), \sigma = 0.4, w \in (0, 1), M \in (1, 150), \tau_1 = 0.05$ ms, $\tau_2 = 1$ ms and $\tau_3 = 2.5$ ms (B) error computed for VanderPol model for $a = 0.1, K \in (0, 0.5), \mu \in (0, 0.5), w \in (0, 1), \tau_1 = \tau_2 = \tau_3 = 0.05$ ms, $\sigma = 0.4$ and $M \in (1, 150)$ (C) global coherence spectra is plotted for varying node interaction levels in the large-scale network for Hindmarsh–Rose model with $\mu = 1.75$ and IER = 2.0 for full and reduced cases (30 modes).

$$h(\{x_j^{(m)}\}, W, n) = \begin{bmatrix} \sum_{m=1}^N W_{nm} \mathbb{E}[\{x_j^{(m)}\}] \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$I = \begin{bmatrix} I1 \\ 0 \\ 0 \\ I2 \\ 0 \\ 0 \end{bmatrix},$$

where K_{11}, K_{12} and K_{21} are coupling constants between individual excitable units (excitatory as well as inhibitory) within a node. Each node is thus identical in terms of local connectivity, the only source of difference comes from the time constants which were set at different values (see figure 2 captions). For the reduced system in (4), $\xi_i = [\alpha_1, \beta_1, \gamma_1, \alpha_2, \beta_2, \gamma_2]^T$, we derive

$$F(\xi_i) = \begin{bmatrix} \beta_{1i} - a_{1i}\alpha_{1i}^3 + b_{1i}\alpha_{1i}^2 - \gamma_{1i} \\ c_{1i} - d_{1i}\alpha_{1i}^2 - \beta_{1i} \\ r\sigma\alpha_{1i} - r\gamma_{1i} - p_{1i} \\ \beta_{2i} - a_{2i}\alpha_{2i}^3 + b_{2i}\alpha_{2i}^2 - \gamma_{2i} \\ c_{2i} - d_{2i}\alpha_{2i}^2 - \beta_{2i} \\ r\sigma\alpha_{2i} - \gamma_{2i} - p_{2i} \end{bmatrix}$$

$$\begin{aligned}
G(\{\xi_j\}, K, i) &= \begin{bmatrix} K_{11} \left(\sum_{j=1}^M A_{ij} \alpha_{1j} - \alpha_{1i} \right) \\ K_{12} \left(\sum_{j=1}^M B_{ij} \alpha_{2j} - \alpha_{1i} \right) \\ 0 \\ 0 \\ K_{21} \left(\sum_{j=1}^M C_{ij} \alpha_{1j} - \alpha_{2i} \right) \\ 0 \\ 0 \end{bmatrix} \\
H(\{\xi_j^{(m)}\}, W, n, i) &= \begin{bmatrix} \sum_{m=1}^N W_{nm} \sum_{j=1}^M A_{ij} \alpha_{1j}^{(m)} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\
H &= \begin{bmatrix} H1 \\ 0 \\ 0 \\ H2 \\ 0 \\ 0 \end{bmatrix},
\end{aligned}$$

where the constants $a_{1i}, a_{2i}, b_{1i}, b_{2i}, c_{1i}, c_{2i}, d_{1i}, d_{2i}, p_{1i}, p_{2i}, A_{ij}, B_{ij}$ and C_{ij} are defined in appendix.

We simulated a network of three nodes with each functional unit in a node is governed by (1) Van der Pol (VDP) oscillator or (2) HMR neurons. Such three node-interactions are capable of addressing dynamical properties of perceptual behavior as well as neuronal oscillations [12]. The results are generalizable to arbitrary number of nodes since equation (4) is valid for any number of nodes with slight increase in mathematical complexity. We select the node connection matrix for the exemplar model of the given form

$$W = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix} \times w,$$

where w is a scalar in the range 0–1 representing negligible node interaction to strong node interaction. We also considered different values of K for VDP (and inhibition – excitation ratio, $IER = \frac{K_{12}}{K_{11}}$ for HMR) and for different values of mean current μ . For validating reduced system with M modes, we compare the GC evaluated using the complete system (C_p) with GC evaluated using the reduced system (R_p) using the following equation

$$\text{error}(M) = \sqrt{\sum_{p=\langle f, w, K, \mu \rangle}^T (C_p - R_p)^2}, \quad (7)$$

where f is frequency and T is total number of pairs $\langle f, w, K, \mu \rangle$. GC is computed between mean activities of each node. For the reduced system, the mean activity of each node is the mean of individual excitable systems' activities estimated using (3) without the residual.

As M increases, error in proposed reduction process decreases more rapidly as compared to previous approaches [8, 9] (figures 2(A) and (B)) for both large-scale networks using Van der-Pol and HMR models as nodes, thus, validating our reduction approach. For Van der-Pol model, reduction proposed in [9] generated numerical instability during simulation. GC calculated from proposed reduction for an exemplar parameter space matches closely with the original system unlike heuristic approaches (figure 2(C)). We further validate our framework by showing the reproduction of time-series of mean-field activity of each HMR and VDP node (figure 3) for several different parameters spaces. Finally, we validate our claim of preserving the long-range coupling by showing that the long-range coupling term in proposed reduction is preserved unlike heuristic approaches (figure 4).

In [6], local-interaction between state-variables was facilitated via the mean field activity of the node which by itself is a small-scale network. In the reduced model this interaction was preserved as the input to state variable of mode i was characterized via mode-specific output of the node ($\sum_{j=1}^M A_{ij} \alpha_j$) governed by matrix A which is obtained as a part of reduction. However, in the case of the large-scale network of these reduced nodes the input to state-variable of mode i of node m was either the activity of state-variable of mode i from other nodes [8] or it was the sum of activities of

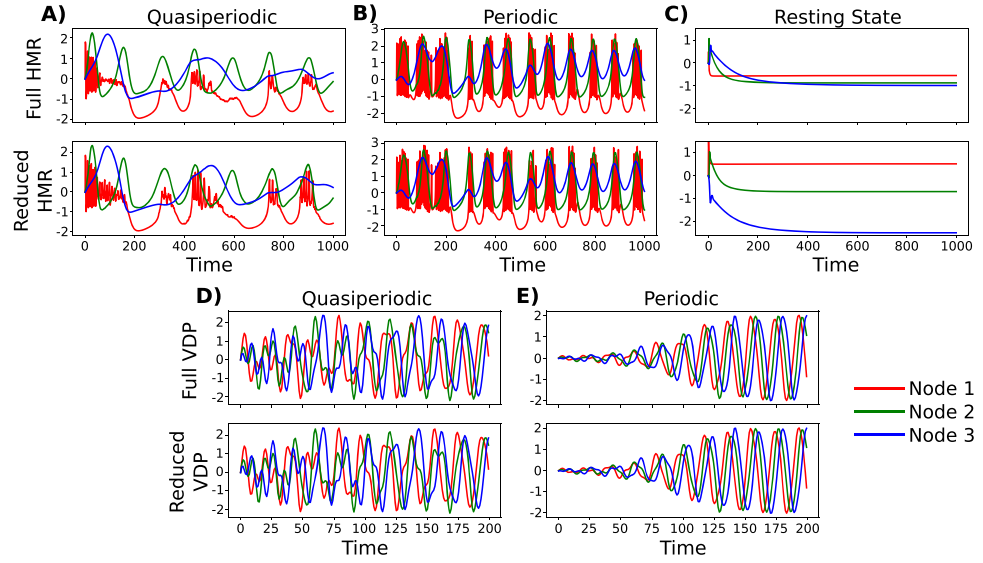


Figure 3. Preservation of mean-field activity of each node in the reduced model with 30 modes for Hindmarsh–Rose model (A) $w = 0.5$, $\mu = 2.5$, $IER = 0.8$ and $K11 = 0.5$ (B) $w = 0.5$, $\mu = 2.5$, $IER = 0.8$ and $K11 = 30$ and (C) $w = 0.5$, $\mu = 2.5$, $IER = 2.5$ and $K11 = 30$ and for Vander-Pol model (D) $w = 0.5$, $K = 0.1$ and $\mu = 1$ and (E) $w = 0.5$, $K = 0.1$ and $\mu = 0.1$.

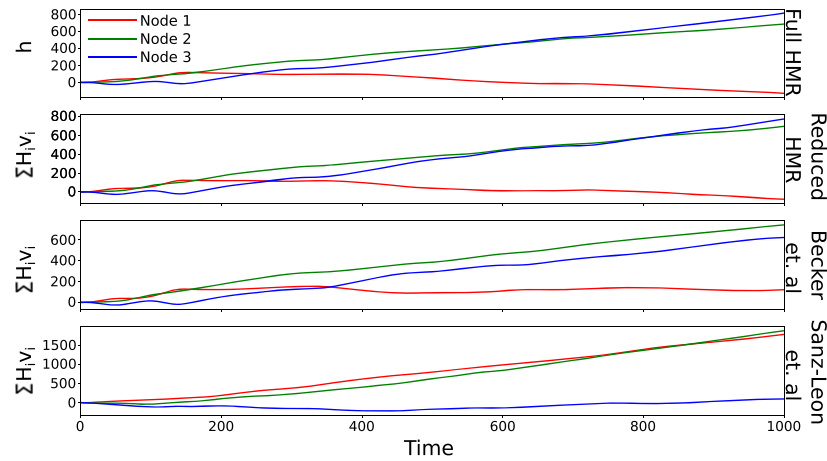


Figure 4. Preservation of input to each node due to long-range coupling term in the proposed reduced model with 30 modes for Hindmarsh–Rose model when $w = 1.0$, $\mu = 1.75$, $IER = 2.0$ and $K11 = 0.5$ as compared to heuristic approaches [8, 9].

state-variables of all modes from other nodes [9]. In this paper, the input to mode i of node m is derived by projecting the long-range interaction term of complete large-scale network to modes of the external input. Thus, the close resemblance of the results from our reduction approach to the complete network (figures 2 and 3) is due to the preservation of the long-range coupling (figure 4) which is derived from the complete network using (4).

In summary, we propose a generalized scheme for reduction of the dynamics of a large-scale network into lower dimensional mode description based on properties of the external input. Obviously, any such reduction lowers the computational complexity. However an important point to note is a model's benefit is not necessarily limited to mimicking the complex dynamics of real-world system. For example, a detailed model of the cortical layer will be highly informative [13], but not necessarily insightful for explaining the cortical interactions during a specific behavioral task. Since the functional properties of brain networks are related to its topology [14, 15], in future we would like to advance our approach by investigating the interaction of local-global connection topologies and external input distributions where implementing inhomogeneity in K may be very useful.

The proposed reduction approach is most relevant for studying the dynamical properties of interactions between external input and nonlinear network whose architecture is unambiguous. Our approach also validates that in the presence of an external input, complex large-scale networks show low-dimensional functional properties and thus can be reduced to lower dimensional modes shaped by the external input distribution without significant loss of information. Our reduction approach preserves the coherence spectra at different

frequencies when different nodes are weakly connected as well as when they are strongly connected (figure 2(C)). Such transition is seen in neural circuits during transition from deep sleep to light sleep [16], and hence the proposed reduction approach can explain how dynamical modes of brain networks are modulated by brain plasticity.

More generally, our approach is best-placed to any application where task inevitably involves reduction from high-dimensional state space to functionally well defined lower dimensional modes. Such problems are also not necessarily exclusively brain specific, but also pertinent for climate dynamics, traffic problems, or anywhere dynamical systems are driven by external time varying and distributed input. Hence, the approach presented in this article may be insightful for a wide range of scientific disciplines.

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Appendix A. Reduction coefficients for Van der Pol oscillator network

$$\begin{aligned} A_{ij} &= \int_{I'} g(I') v_j(I') dI' \int_I v_i^+ dI \\ a_i &= a \int_I v_i^3(I) v_i^+(I) dI \\ II_i &= \int_I I v_i^+(I) dI \end{aligned} \quad (8)$$

$g(I)$ is the pdf of external input.

Appendix B. Reduction coefficients for HMR neuronal network

$$\begin{aligned} A_{ij} &= \int_{I'} g1(I') v1_j(I') dI' \int_I v1_i^+(I) dI \\ B_{ij} &= \int_{I'} g2(I') v2_j(I') dI' \int_I v1_i^+(I) dI \\ C_{ij} &= \int_{I'} g1(I') v1_j(I') dI' \int_I v2_i^+(I) dI \\ a1_i &= a \int_I v1_i^3(I) v1_i^+(I) dI \\ a2_i &= a \int_I v2_i^3(I) v2_i^+(I) dI \\ b1_i &= b \int_I v1_i^2(I) v1_i^+(I) dI \\ b2_i &= b \int_I v2_i^2(I) v2_i^+(I) dI \\ c1_i &= c \int_I v1_i^+(I) dI \\ c2_i &= c \int_I v2_i^+(I) dI \\ d1_i &= d \int_I v1_i^2(I) v1_i^+(I) dI \\ d2_i &= d \int_I v2_i^2(I) v2_i^+(I) dI \\ p1_i &= rsx_0 \int_I v1_i^+(I) dI \\ p2_i &= rsx_0 \int_I v2_i^+(I) dI \\ II1_i &= \int_I I v1_i^+(I) dI \\ II2_i &= \int_I I v2_i^+(I) dI \end{aligned}$$

$g1(I)$ and $g2(I)$ are the pdfs of external input to excitatory and inhibitory sub-populations respectively.

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