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PAPER

Skyrmion glass in a 2D Heisenberg ferromagnet with quenched disorder

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Abstract

A 2D Heisenberg ferromagnet with exchange J and random magnetic anisotropy of strength $D_R \ll J$ has been studied. Analytical theory for the dependence of the average size of a pinned skyrmion on the magnetic field H , and for stability of such skyrmions on a lattice, has been developed. It has been complemented by numerical studies of 2D lattices containing up to 40 million spins. At low fields the average size of the skyrmion, λ , is determined by the average size of Imry–Ma domains. On increasing the field the skyrmions first shrink, with $\lambda \propto D_R/H$, and then collapse at fields distributed around $H_c \propto D_R^{4/3}$. The concentration of the skyrmions goes down with the field as $\exp[-\frac{1}{2}(H/H_c)^{3/2}]$.

1. Introduction

Studies of systems with quenched randomness go back to the seminal paper of Imry and Ma (IM) [1] who observed that static disorder, no matter how weak, destroys the long-range order in a system with any continuous-symmetry order parameter in less than four dimensions. The resulting state has received the name of IM domains. The IM argument has been widely used in condensed matter in application to random magnets [2], arrays of magnetic bubbles [3], magnetic flux lattices in superconductors [4], superfluid He-3 [5], superconductor–insulator transition [6], etc. Later analytical studies based upon renormalization group and replica-symmetry breaking methods [7–9] suggested that the ordering can be more robust against static randomness, leading to a vortex-free Bragg glass state with an algebraic decay of correlations. Numerical work on systems with quenched randomness [10–12], from early on, uncovered strong non-equilibrium effects.

More recently it was found [13] that onset of the irreversible (or glassy) behavior in the presence of a static random field is determined by topology. It depends on the absence or presence of topological defects in the model, as well as on whether the defects are singular or not. The latter is determined by the relation between the dimensionality of space d and the number n of the components of the order parameter, e.g., a fixed-length spin field $\mathbf{S}(\mathbf{r})$. At $n > d + 1$, when topological defects are absent, the behavior is reversible and is characterized by the exponential decay of correlations in accordance with the IM argument. At $n \leq d$ the presence of topological defects leads to strong metastability and glassy behavior. At $n = d + 1$, when topological defects are non-singular, the system possesses weak metastability and exhibits a narrow hysteresis loop. This is the case of a two-dimensional ferromagnet in which topological defects are skyrmions.

Magnetic skyrmions have been subject of intensive recent theoretical and experimental research [14–16] due to their promise for developing topologically protected data storage. Mathematically they emerge as solutions [17–19] of the continuous-field Heisenberg exchange model in two dimensions with the energy $\frac{1}{2} \int d^2r (\partial \mathbf{s} / \partial r_\alpha)^2$, where \mathbf{s} is a three-component spin field of unit length and $r_\alpha = x, y$. An arbitrary spin-field configuration, $\mathbf{s}(\mathbf{r})$, is characterized by the topological charge,

$$Q = \int \frac{d^2r}{8\pi} \epsilon_{\alpha\beta} s_\alpha \epsilon_{abc} \frac{\partial s_b}{\partial r_\alpha} \frac{\partial s_c}{\partial r_\beta} = \int \frac{dx dy}{4\pi} \mathbf{s} \cdot \frac{\partial \mathbf{s}}{\partial x} \times \frac{\partial \mathbf{s}}{\partial y} \quad (1)$$

that takes values $Q = 0, \pm 1, \pm 2$, and so on. They describe topologically different non-singular mappings of the unit-radius sphere in the spin space onto the (x, y) geometrical space. $Q = \pm 1$ correspond to skyrmions and

antiskyrmions, while greater $|Q|$ describe more complex topological objects. In a pure exchange model on a lattice, skyrmions collapse due to violation of the scale invariance by the lattice [20]. Anisotropy, dipole–dipole interaction (DDI), and magnetic field can stabilize significantly large magnetic bubbles with skyrmion topology [21–23], while stability of small skyrmions requires other than Heisenberg exchange coupling or a non-centrosymmetric system with large Dzyaloshinskii–Moriya interaction [24–28]. It was shown that quantum fluctuations help stabilize skyrmions as well [29].

In this paper we show that weak quenched randomness provides stabilization of skyrmions in a generic 2D Heisenberg model on a lattice. To make our model closer to reality we consider random anisotropy (RA) rather than random field. In a film geometry the DDI effectively provides an easy-plane anisotropy. To elucidate the effect of the RA we shall assume that the RA energy and Zeeman interaction of spins with the external field applied perpendicular to the film are large compared to the DDI. In this case the effect of the DDI can be neglected. The physical mechanism of the stabilization of skyrmions in such a model is this. Fluctuations of the RA create randomly oscillating energy landscape of the average depth that scales as the square root of the number of spins, $N \propto \lambda^2$, on a spatial scale λ . Consequently a skyrmion of size λ finds a potential minimum of depth proportional to λ . The Zeeman energy of the skyrmion in the field opposite to its magnetization scales as λ^2 . This provides the minimum of the energy on λ . Discreteness of the crystal lattice adds negative exchange energy proportional to $1/\lambda^2$ [20]. This makes the skyrmions eventually collapse on increasing the field. We begin with the analytical approach and then confirm our picture by numerical studies of spin lattices containing up to 4×10^7 spins. Properties of the skyrmion glass that we observe in the numerical experiment agree well with our analytical findings.

2. Analytical theory

We consider a constant-length spin field $\mathbf{S}(\mathbf{r})$ with the energy

$$\mathcal{H} = \int d^2r \left[\frac{\alpha}{2} \left(\partial_\mu \mathbf{S} \right) \cdot \left(\partial_\mu \mathbf{S} \right) - \frac{\beta_R}{2} (\mathbf{n} \cdot \mathbf{S})^2 - \mathbf{H} \cdot \mathbf{S} \right], \quad (2)$$

where $\mathbf{n}(\mathbf{r})$ is a unit vector of the RA and \mathbf{H} is the magnetic field applied in the z -direction. Parameters S , α , and β_R are related to the parameters of the lattice model with the energy

$$\mathcal{H}_l = -\frac{J}{2} \sum_{ij} \mathbf{s}_i \cdot \mathbf{s}_j - \frac{D_R}{2} \sum_i (\mathbf{n}_i \cdot \mathbf{s}_i)^2 - \mathbf{H} \cdot \sum_i \mathbf{s}_i \quad (3)$$

via $S = s/a^2$, $\alpha = Ja^4$, and $\beta_R = 2D_R a^2$, where a is the lattice spacing. At $H = 0$ the system breaks into IM domains [1] which corresponds to a finite ferromagnetic correlation length [2, 19], $R_f/a \propto J/D_R$. The argument goes like this. If a significant rotation of the magnetization were to occur on a scale R , the density of the exchange energy would be of order J/R^2 , while fluctuations of the RA energy density, that provide the coherent anisotropy, would be of order $-(D_R/R^2)[(R/a)^2]^{1/2} = -D_R/(aR)$. Minimization of the sum of the two energies on R yields the above scaling of R_f .

This argument should be modified in the presence of topological defects. Such defects in the isotropic 2D exchange model are Belavin–Polyakov (BP) skyrmions. In a continuous spin-field approximation, their energy, due to the scale invariance, does not depend on the skyrmion size λ . Notice that any interaction that breaks the scale invariance, including the RA, destroys the BP solution [30]. However, when a weak interaction is turned on, the skyrmion may still be considered an approximate solution. It will begin to expand or collapse towards the value of λ that corresponds to the energy minimum. This must apply to the RA and Zeeman energies that are typically small compared to the exchange interaction.

The z -component of the spin-field of the skyrmion located at the origin in the background of spins looking up is given by [19] $S_z/S = (x^2 + y^2 - \lambda^2)/(x^2 + y^2 + \lambda^2)$. In the presence of the field $\mathbf{H} = H\hat{z}$ it generates Zeeman energy

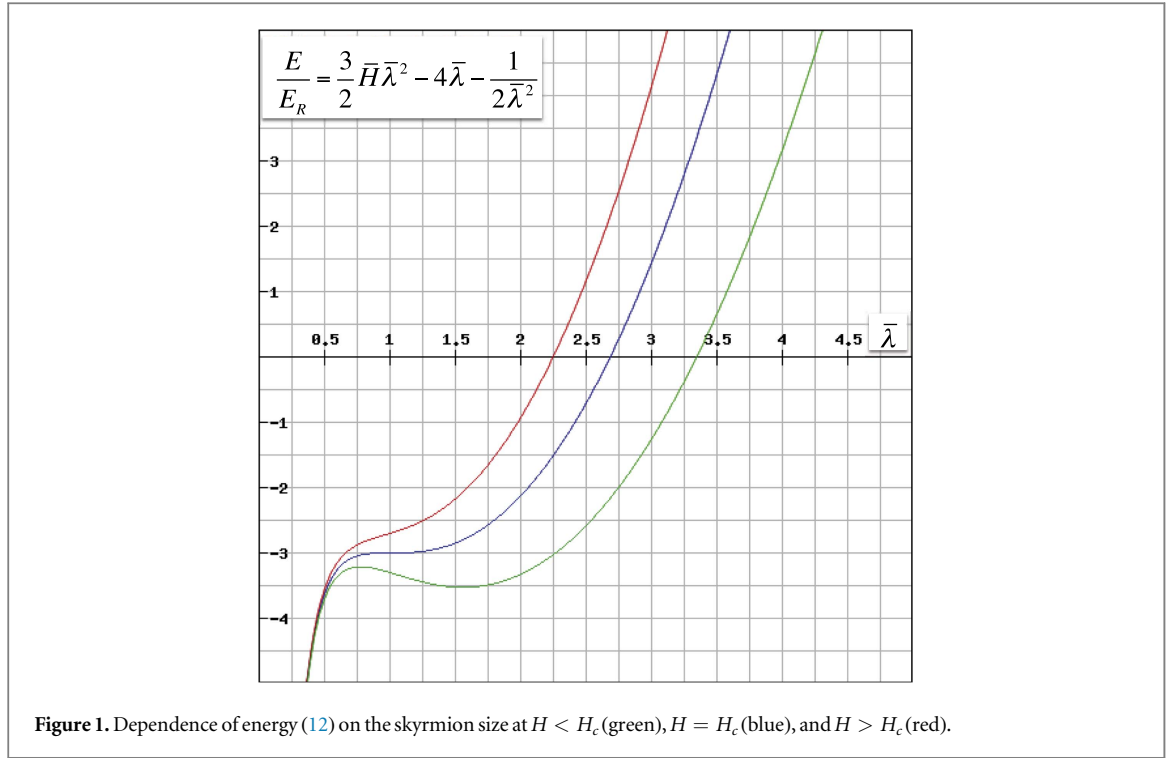
$$E_Z = -\int d^2r H(S_z - S) = 4\pi SH\lambda^2 \ln(L/\lambda), \quad (4)$$

where we assumed $\lambda \ll L$. Writing E_Z as $HS\pi R^2(\lambda)$ one can define the magnetic radius of the skyrmion, $R(\lambda) = 2\lambda \ln^{1/2}(L/\lambda)$. In the presence of the RA the system size L should be replaced by the length of order R_f or $Ja/(sH)$, whichever is smaller. In what follows we will approximate the magnetic size of the skyrmion and its Zeeman energy by

$$R(\lambda) = 2\lambda\sqrt{l}, \quad E_Z = 4\pi lHS\lambda^2 = 4\pi lsH(\lambda/a)^2, \quad (5)$$

where l is a logarithmic factor that we shall treat as a constant.

Statistical fluctuations of the RA result in a coherent magnetic anisotropy on the scale $R(\lambda)$. As we shall see its strength is proportional to the square root of the number of spins on that scale, which is linear on $R(\lambda)$. The



corresponding pinning energy is given by the statistical dispersion of the anisotropy energy in the region of size $\int d^2r$ matching the skyrmion size $R(\lambda)$,

$$E_A = -\frac{\beta_R}{2} \sqrt{\left\langle \left[\int d^2r (\mathbf{n} \cdot \mathbf{S})^2 \right]^2 \right\rangle - \left\langle \int d^2r (\mathbf{n} \cdot \mathbf{S})^2 \right\rangle^2}. \quad (6)$$

To calculate this, it is convenient to switch from the fixed-length anisotropy vector \mathbf{n} to a Gaussian distributed vector $\mathbf{h}(\mathbf{r}) \propto \mathbf{n}(\mathbf{r})$ of the average strength $h = \sqrt{\beta_R/2}$, having

$$\langle h_\alpha(\mathbf{r}') h_\beta(\mathbf{r}'') \rangle = \frac{1}{3} h^2 \delta_{\alpha\beta} \Gamma(|\mathbf{r}' - \mathbf{r}''|/a) \quad (7)$$

with $\Gamma(0) = 1$ and $\Gamma_{a \rightarrow 0} \rightarrow \delta[|\mathbf{r}' - \mathbf{r}''|/a]$ defined as a nascent delta function. Writing $\langle h'_\alpha h'_\beta h''_\delta h''_\gamma \rangle = \langle h'_\alpha h'_\beta \rangle \langle h''_\delta h''_\gamma \rangle + \langle h'_\alpha h'_\delta \rangle \langle h'_\beta h''_\gamma \rangle + \langle h'_\alpha h''_\gamma \rangle \langle h'_\beta h''_\delta \rangle$, where $\mathbf{h}' = \mathbf{h}(\mathbf{r}')$, $\mathbf{h}'' = \mathbf{h}(\mathbf{r}'')$, one obtains from equation (6) $E_A = -\frac{1}{3} \beta_R a S^2 \sqrt{\int d^2r}$ for the average fluctuation of the RA in the area of size $\int d^2r$. We use the fact that $\mathbf{S}(\mathbf{r})$ in a skyrmion changes on a scale $R(\lambda)$ that is much greater than the scale, a , of the RA change, which decouples the averaging over $\mathbf{h}(\mathbf{r})$ from $\mathbf{S}(\mathbf{r})$. Replacing $\int d^2r$ with $\pi R^2(\lambda)$, we have

$$E_A = -\frac{2}{3} \sqrt{\pi l} \beta_R S^2 a \lambda = -\frac{4}{3} \sqrt{\pi l} D_R s^2 \frac{\lambda}{a}, \quad (8)$$

in accordance with our expectation that fluctuation of the RA energy is linear on the skyrmion size.

The discreteness of the lattice breaks the scale invariance of the exchange interaction. The corresponding energy,

$$E_l = -(2\pi J s^2/3)(a/\lambda)^2, \quad (9)$$

has been computed in [20].

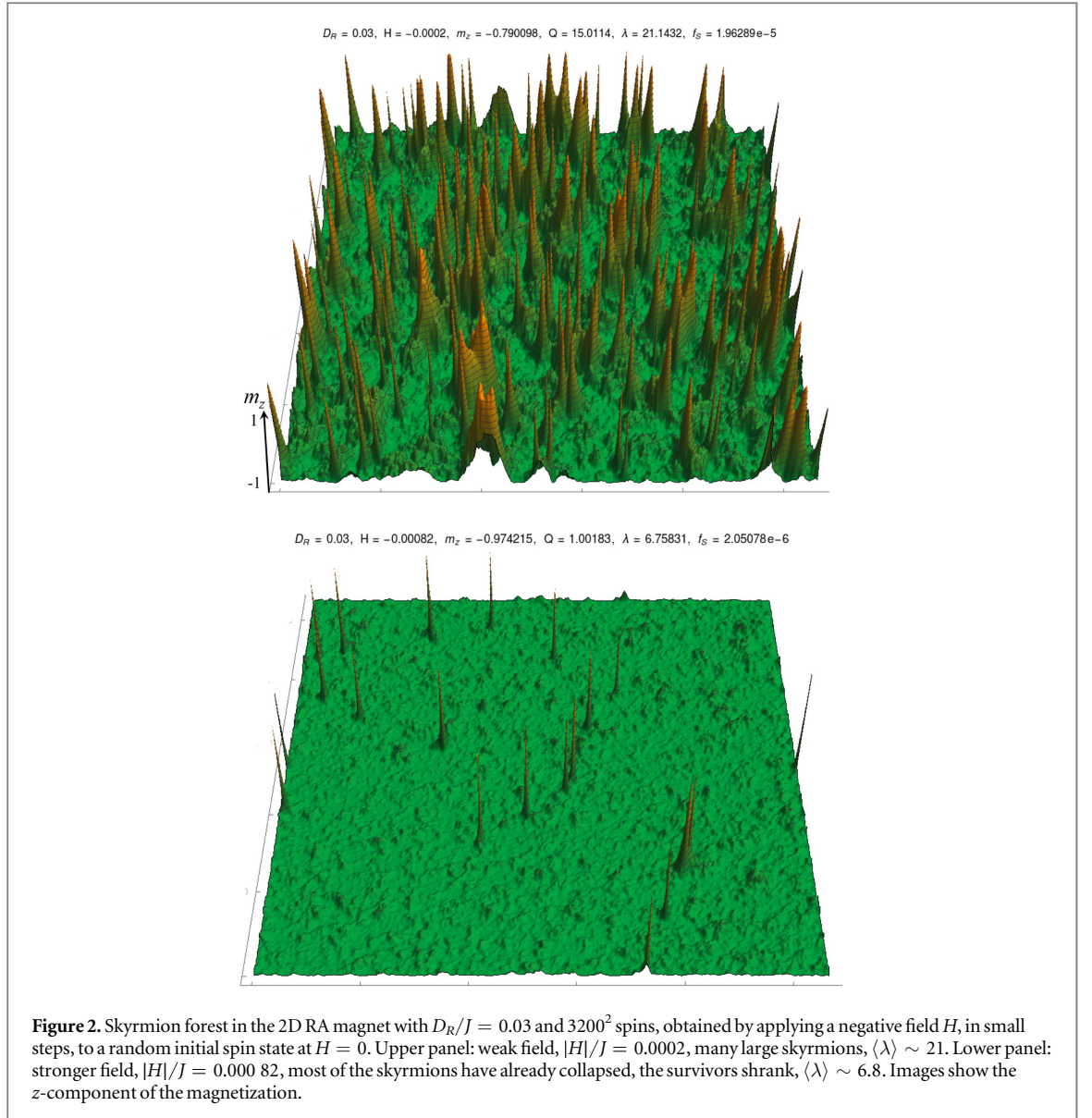
Introducing

$$H_c = \frac{J s}{8(4\pi^2 l)^{1/3}} \left(\frac{D_R}{J} \right)^{4/3}, \quad (10)$$

$$\lambda_c = \left(\frac{16\pi}{l} \right)^{1/6} \left(\frac{J}{D_R} \right)^{1/3} a \quad (11)$$

it is convenient to write the total energy in the form

$$E = E_R \left(\frac{3}{2} \bar{H} \bar{\lambda}^2 - 4\bar{\lambda} - \frac{1}{2\bar{\lambda}^2} \right), \quad (12)$$



where $\bar{H} = H/H_c$, $\bar{\lambda} = \lambda/\lambda_c$, and

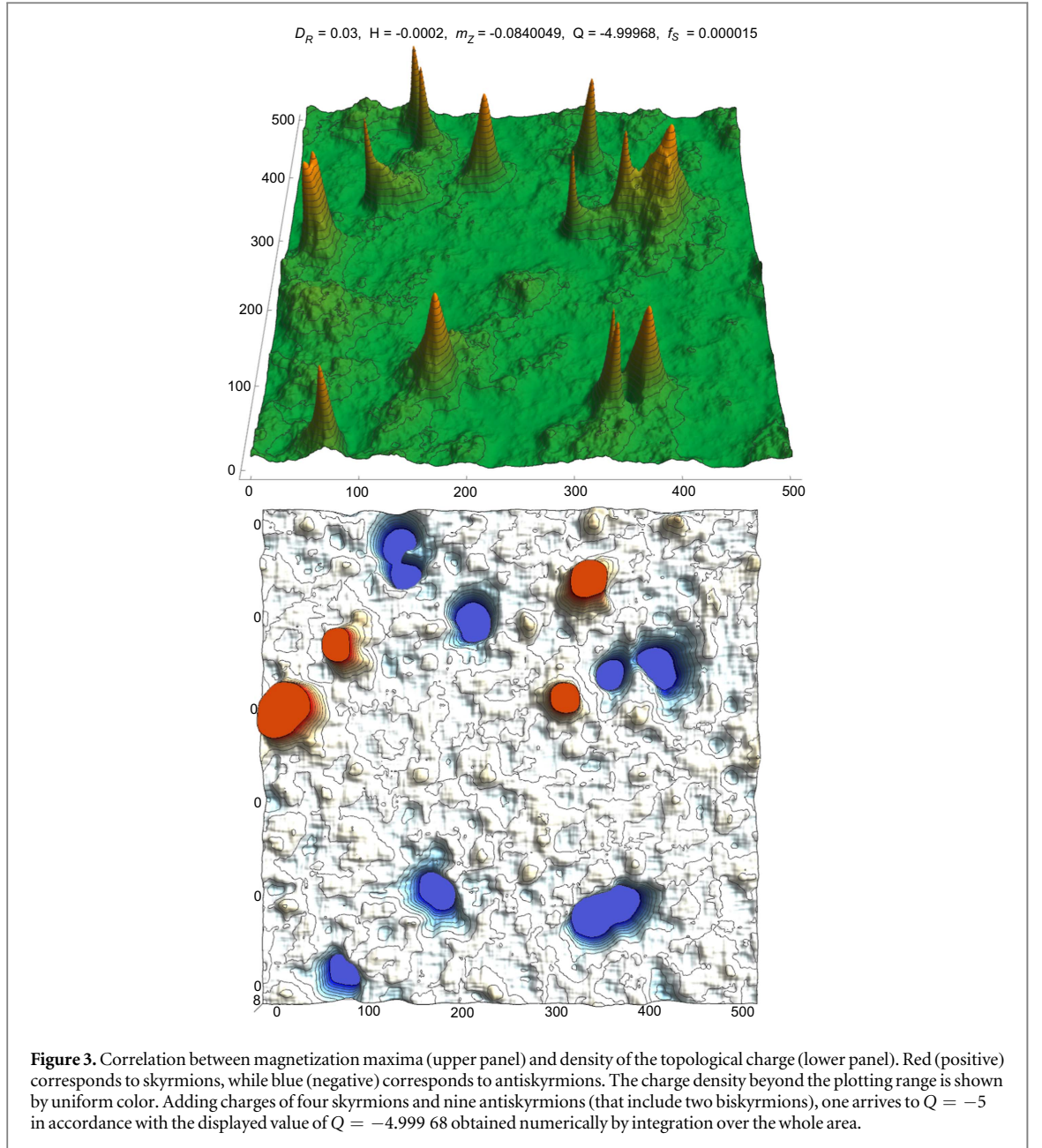
$$E_R = \frac{2\pi}{3} J s^2 \left(\frac{l}{2\pi} \right)^{1/3} \left(\frac{D_R}{J} \right)^{2/3}. \quad (13)$$

Equation (12) describes three competing effects. The first term is a positive Zeeman energy that tends to shrink the skyrmion because its magnetic moment is opposite to the field. The second term is a negative pinning energy due to the fluctuations of RA. It tends to expand the skyrmion. The third term is a negative lattice energy that tends to collapse the skyrmion. The dependence of the total energy on λ is shown in figure 1. It has a maximum and a minimum at $\bar{H} < 1$, and an inflection point at $\bar{H} = 1$ when $\bar{\lambda} = 1$. At $\bar{H} \ll 1$ the minimum is dominated by the first two terms in equation (12), leading to

$$\bar{\lambda} = \frac{4}{3\bar{H}}, \quad \frac{\lambda}{a} = \frac{1}{6\sqrt{\pi}l} \left(\frac{sD_R}{H} \right). \quad (14)$$

Note that the skyrmion size given by this equation is bound by the length of order R_f at small H . The energy minimum disappears at $H = H_c$, which corresponds to skyrmion collapse at $H > H_c$ and $\lambda < \lambda_c$.

Equation (8) defines the average pinning energy for a skyrmion of size λ that we can denote as $\langle E_A \rangle$. In fact, this pinning energy is fluctuating. It can be shown that in our model the distribution of E_A is Gaussian, described by the function



$$F_{E_A}(E_A) = \frac{1}{\sqrt{2\pi \langle E_A \rangle^2}} \exp\left(-\frac{E_A^2}{2\langle E_A \rangle^2}\right). \quad (15)$$

Introducing $p \equiv -E_A / \langle E_A \rangle$, one can write the normalized distribution for the fluctuating factor p as $F_p(p) = (2\pi)^{-1/2} \exp(-p^2/2)$. With account of fluctuations, the skyrmion-collapse field H_c given by equation (10) acquires the fluctuating factor $p^{4/3}$ for $p > 0$, whereas $H_c = 0$ (no pinning) for $p < 0$. That is, $H_c = p^{4/3} H_0$, where H_0 is the old H_c given by equation (10). We shall see that H_0 differs from $\langle H_c \rangle$ by a numerical factor. The normalized distribution for $\zeta \equiv H_c / H_0 = p^{4/3}$ with $p > 0$ can be written as $F_\zeta(\zeta) = F_p(p(\zeta))(dp(\zeta)/d\zeta)$. With $p = \zeta^{3/4}$ and $dp/d\zeta = (3/4)\zeta^{-1/4}$ one obtains

$$F_\zeta(\zeta) = \sqrt{\frac{2}{\pi}} \frac{3}{4\zeta^{1/4}} \exp\left(-\frac{\zeta^{3/2}}{2}\right), \quad \zeta > 0. \quad (16)$$

From this formula $\langle \zeta \rangle = 2^{2/3} \Gamma(7/6) / \sqrt{\pi} \simeq 0.831$, i.e., $\langle H_c \rangle \simeq 0.831 H_0$.

When the magnetic field H increases, skyrmions with $H_c < H$ collapse. If at $H = 0$ the concentration of skyrmions is $f_S(0)$, then their concentration at nonzero field is given by

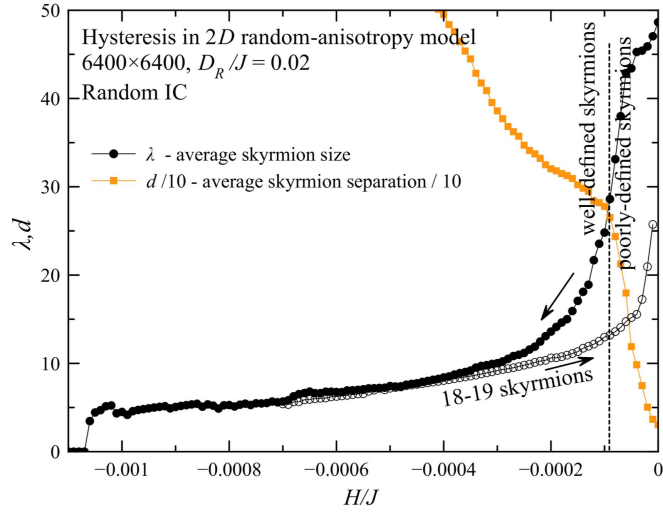


Figure 4. Evolution of the average skyrmion size and the average distance between the skyrmions on changing the field.

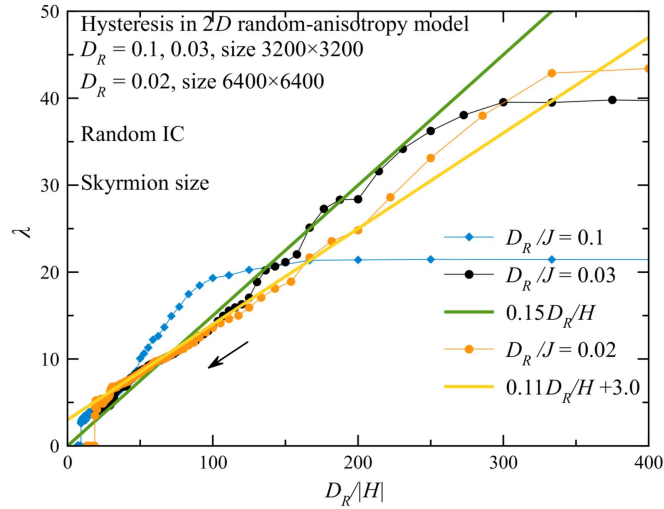


Figure 5. Dependence of the average skyrmion size on $D_R/|H|$.

$$\frac{f_s(H)}{f_s(0)} = 1 - \int_0^{\zeta_H} F_\zeta(\zeta) = \int_{\zeta_H}^{\infty} F_\zeta(\zeta) = \text{Erfc}\left(\frac{\zeta_H^{3/4}}{\sqrt{2}}\right), \quad (17)$$

where $\zeta_H \equiv H/H_0$. For strong fields, $\zeta_H \gg 1$, equation (17) simplifies to

$$\frac{f_s(H)}{f_s(0)} \cong \sqrt{\frac{2}{\pi}} \frac{1}{\zeta_H^{3/4}} \exp\left(-\frac{\zeta_H^{3/2}}{2}\right). \quad (18)$$

Notice that $f_s(H)$ depends on $f_s(0)$, that is, it depends on the initial conditions.

These results, including the D_R/H scaling of the average skyrmion size at intermediate fields, equation (14), are supported by the numerical experiment on spin lattices, see below. In comparing our theory with numerical results we are after functional dependences of physical quantities on the strength of quenched disorder. We find good agreement between theory and numerical experiment, thus confirming our picture with a high degree of confidence.

3. Methods

We minimize the energy of the spin system numerically in the presence of the exchange, the RA which direction is chosen randomly at each lattice site, and the external field on 2D lattices of size $L \gg R_f$. Our numerical

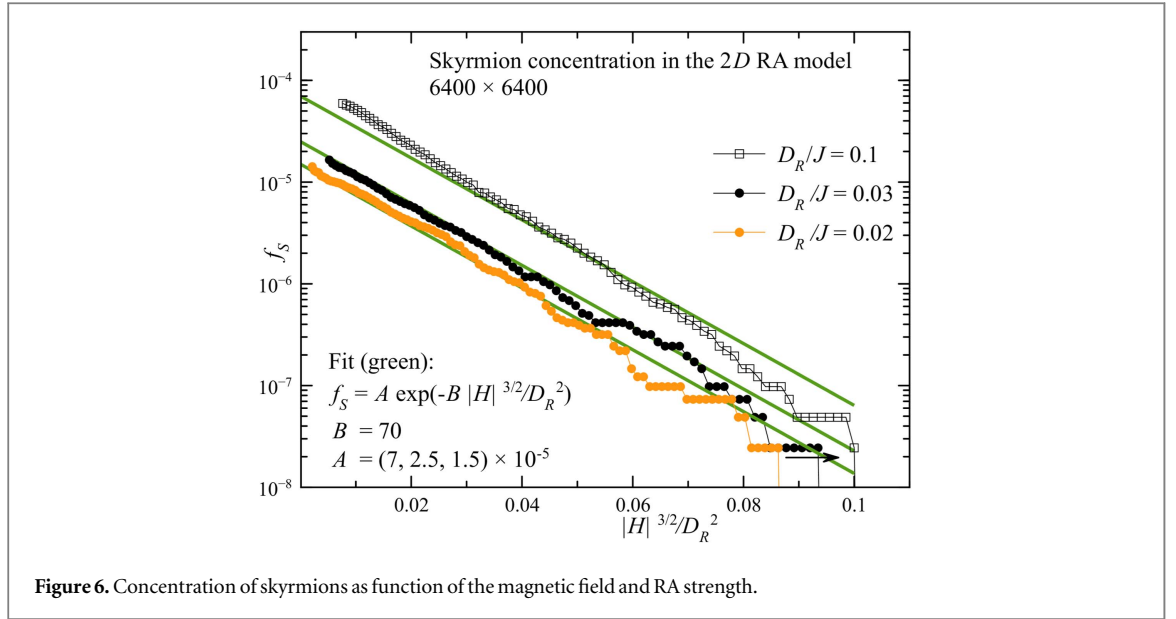


Figure 6. Concentration of skyrmions as function of the magnetic field and RA strength.

method combines sequential rotations of spins towards the direction of the local effective field, $\mathbf{H}_{i,\text{eff}} = \sum_j J_{ij} \mathbf{s}_j + \mathbf{h}_j + \mathbf{H}$, with energy-conserving spin flips: $\mathbf{s}_i \rightarrow 2(\mathbf{s}_i \cdot \mathbf{H}_{i,\text{eff}}) \mathbf{H}_{i,\text{eff}} / H_{i,\text{eff}}^2 - \mathbf{s}_i$, applied with probabilities γ and $1 - \gamma$ respectively; γ playing the role of the relaxation constant. High efficiency of this method for glassy systems under the condition $\gamma \ll 1$ has been demonstrated by us in the past, see, e.g., [13]. The software used was Wolfram Mathematica that allows compilation (including usage of an external C compiler that doubles the speed) and parallelization. The main operating computer was a 20-core Dell Precision T7610 Workstation. The largest-scale computation, that lasted a few days, has been done on a square lattice containing 6400×6400 spins. Skyrmions have been counted by detecting spots where m_z reaches the value 1 and by computing topological charge around such spots.

4. Numerical results

We begin at $H = 0$ and apply the field in the negative direction, in small steps, each time minimizing the energy of the system. Topological charge, Q , computed on the lattice by using discretized form of equation (1), is very close to the integer. It is typically small and is due to the statistical difference between the numbers of skyrmions and antiskyrmions. In counting them we do not distinguish between the two. The total number of skyrmions and antiskyrmions is found by looking at the maxima of the z -component of the magnetization, m_z . At $H = 0$ skyrmions are comparable in size to the IM domains and are difficult to identify. When the field is applied, the well-separated compact skyrmions with the magnetization opposite to the background emerge, see figure 2. Most of the observed topological defects are simple skyrmions and antiskyrmions with $Q = \pm 1$, while a small fraction are biskyrmions. At small fields concentration of skyrmions is high. As the absolute value of the field increases, skyrmions shrink and begin to collapse. Correlation between the magnetization maxima and the density of the topological charge, confirming that we are dealing with skyrmions, is illustrated in figure 3.

The average skyrmion size in a weak field is independent of H , see figure 4, in accordance with our expectation that it is determined by $R_f \sim (J/D_R)a$ at $H = 0$. As the field continues to increase in the negative z -direction, the skyrmions go down in size and become less abundant. The average skyrmion separation d shown in figure 4 is defined as $1/\sqrt{f_S}$, where f_S is concentration of skyrmions, that is, their number per spin. On decreasing the field back to zero no new skyrmions are formed but the existing skyrmions grow in size to adjust to the pinning potential. The sizes of skyrmions in the numerically obtained skyrmion forest were evaluated by computing second derivatives of m_z at the maxima and comparing it with the known profile of the BP skyrmion.

The dependence of the average skyrmion size on $D_R/|H|$ on increasing the field is shown in figure 5. In accordance with analytical results, it is close to linear in the intermediate field range, saturates at small fields, and shows collapse of the smallest skyrmions at high fields. Linear part of the graph allows one to extract the parameter l . In accordance with expectation, it has a weak (apparently logarithmic) dependence on D_R . Comparing the slope in figure 5 with equation (14) one obtains $l \approx 0.4$. Note that at very small D/J (large R_f) the finite size of the system begins to affect numerical results.

Figure 6 shows concentration of skyrmions, f_s , as function of H and D_R . The proportionality of the fitting exponent to $|H|^{3/2}/D_R^2$ is in a remarkable agreement with the distribution given by equation (18), where $\frac{1}{2}\zeta_H^{3/2} = \frac{1}{2}(|H|/H_0)^{3/2} = 16\pi\sqrt{2l}|H|^{3/2}/D_R^2$. Substituting here $l = 0.4$ one obtains $16\pi\sqrt{2l} \approx 50$. The error in our evaluation of the numerical factor B in the fitting exponent in figure 6 corresponds to a difference by a factor 0.8 between H_0 of equation (10) derived within continuous theory and H_0 that follows from the numerical experiment on a lattice.

5. Conclusion

We have studied properties of a skyrmion glass created by RA and the magnetic field in a two-dimensional Heisenberg ferromagnet. Good understanding of the field dependence of the concentration of skyrmions and their average size has been developed by analytical and numerical methods that agree with each other. Our findings suggest that a significant concentration of stable skyrmions can be achieved in 2D ferromagnets with quenched randomness. They can be tested in experiments on weakly disordered magnetic films.

Acknowledgments

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References

- [1] Imry Y and Ma S-k 1975 Random-field instability of the ordered state of continuous symmetry *Phys. Rev. Lett.* **35** 1399–401
- [2] Chudnovsky E M, Saslow W M and Serota R A 1986 Ordering in ferromagnets with random anisotropy *Phys. Rev. B* **33** 251–61
- [3] Seshadri R and Westervelt R M 1992 Statistical mechanics of magnetic bubble arrays: I. Topology and thermalization *Phys. Rev. B* **46** 5142–9
Seshadri R and Westervelt R M 1992 Statistical mechanics of magnetic bubble arrays: II. Observations of two-dimensional melting *Phys. Rev. B* **46** 5150–61
- [4] See, e.g., Blatter G, Feigel'man M V, Geshkenbein V B, Larkin A I and Vinokur V M 1994 Vortices in high temperature superconductors *Rev. Mod. Phys.* **66** 1125–388 and references therein.
- [5] Volovik G E 2008 On Larkin–Imry–Ma state of $^3\text{He-A}$ in aerogel *J. Low Temp. Phys.* **150** 453–63
- [6] Chudnovsky E M 2009 Instanton glass generated by noise in a Josephson-junction array *Phys. Rev. Lett.* **103** 137001
- [7] Giamarchi T and Doussal P Le 1994 Elastic theory of pinned flux lattices *Phys. Rev. Lett.* **72** 1530–3
Giamarchi T and Doussal P Le 1995 Elastic theory of flux lattices in the presence of weak disorder *Phys. Rev. B* **52** 1242–70
- [8] Nattermann T and Scheidl S 2000 Vortex-glass phases in type-II superconductors *Adv. Phys.* **49** 607–704 and references therein.
- [9] Le Doussal P and Wiese K J 2006 Random-field spin models beyond 1 loop: a mechanism for decreasing the lower critical dimension *Phys. Rev. Lett.* **96** 197202
Le Doussal P 2006 Chaos and residual correlations in pinned disordered systems *Phys. Rev. Lett.* **96** 235702
Middleton A A, Le Doussal P and Wiese K J 2007 Measuring functional renormalization group fixed-point functions for pinned manifolds *Phys. Rev. Lett.* **98** 155701
- [10] Dieny B and Barbara B 1990 XY model with weak random anisotropy in a symmetry-breaking magnetic field *Phys. Rev. B* **41** 11549–56
- [11] Dickman R and Chudnovsky E M 1991 XY chain with random anisotropy: magnetization law, susceptibility, and correlation functions at $T = 0$ *Phys. Rev. B* **44** 4397–405
- [12] Gingras M J P and Huse D A 1996 Topological defects in the random-field XY model and the pinned vortex lattice to vortex glass transition in type-II superconductors *Phys. Rev. B* **53** 15193–200
- [13] Proctor T C, Garanin D A and Chudnovsky E M 2014 Random fields, topology, and the Imry–Ma argument *Phys. Rev. Lett.* **112** 097201
- [14] Nagaosa N and Tokura Y 2013 Topological properties and dynamics of magnetic skyrmions *Nat. Nanotechnol.* **8** 899–911
- [15] Finocchio G, Büttner F, Tomasello R, Carpentieri M and Klauui M 2016 Magnetic skyrmions: from fundamental to applications *J. Phys. D: Appl. Phys.* **49** 423001
- [16] Fert A, Reyren N and Cros V 2017 Magnetic skyrmions: advances in physics and potential applications *Nat. Rev. Mater.* **2** 17031
- [17] Skyrme T H R 1958 A nonlinear theory of strong interactions *Proc. R. Soc. A* **247** 260–78
- [18] Belavin A A and Polyakov A M 1975 Metastable states of two-dimensional isotropic ferromagnets *Pis'ma Zh. Eksp. Teor. Fiz.* **22** 503–6
Belavin A A and Polyakov A M 1975 Metastable states of two-dimensional isotropic ferromagnets *JETP Lett.* **22** 245–8 (http://www.jetpletters.ac.ru/ps/1529/article_23383.pdf)
- [19] Chudnovsky E M and Tejada J 2006 *Lectures on Magnetism* (Princeton, NJ: Rinton Press)
- [20] Cai L, Chudnovsky E M and Garanin D A 2012 Collapse of skyrmions in two-dimensional ferromagnets and antiferromagnets *Phys. Rev. B* **86** 024429
- [21] Moutafis C, Komineas S and Bland J A C 2009 Dynamics and switching processes for magnetic bubbles in nanoelements *Phys. Rev. B* **79** 224429
- [22] Ezawa M 2010 Giant skyrmions stabilized by dipole–dipole interactions in thin ferromagnetic films *Phys. Rev. Lett.* **105** 197202
- [23] Makhfudz I, Krüger B and Tchernyshyov O 2012 Inertia and chiral edge modes of a skyrmion magnetic bubble *Phys. Rev. Lett.* **109** 217201
- [24] Rößler U K, Bogdanov N and Pfléiderer C 2006 Spontaneous skyrmion ground states in magnetic metals *Nature* **442** 797–801
- [25] Heinze S, von Bergmann K, Menzel M, Brede J, Kubetzka A, Wiesendanger R, Bihlmayer G and Blugel S 2011 Spontaneous atomic-scale magnetic skyrmion lattice in two dimensions *Nat. Phys.* **7** 713–8
- [26] Leonov A O and Mostovoy M 2015 Multiply periodic states and isolated skyrmions in an anisotropic frustrated magnet *Nat. Commun.* **6** 8275

- [27] Lin S-Z and Hayami S 2016 Ginzburg–Landau theory for skyrmions in inversion-symmetric magnets with competing interactions *Phys. Rev. B* **93** 064430
- [28] Leonov A O, Monchesky T L, Romming N, Kubetzka A, Bogdanov A N and Wiesendanger R 2016 The properties of isolated chiral skyrmions in thin magnetic films *New J. Phys.* **18** 065003
- [29] Roldán-Molina A, Santander M J, Nunez A S and Fernández-Rossier J 2015 Quantum fluctuations stabilize skyrmion textures *Phys. Rev. B* **92** 245236
- [30] Derrick G H 1964 Comments on nonlinear wave equations as models for elementary particles *J. Math. Phys.* **5** 1252–4