New Journal of Physics

The open access journal at the forefront of physics

Deutsche Physikalische Gesellschaft DPG IOP Institute of Physics

OPEN ACCESS

Generation and detection of NOON states in superconducting circuits

To cite this article: Seth T Merkel and Frank K Wilhelm 2010 New J. Phys. 12 093036

View the article online for updates and enhancements.

You may also like

- Efficient entanglement concentration for arbitrary less-entangled NOON state assisted by single photons Lan Zhou, , Yu-Bo Sheng et al.
- Heralded path-entangled NOON states generation from a reconfigurable photonic chip Xinyao Yu, , Pingyu Zhu et al.
- One-step engineering many-atom NOON state Chuang Li, Elijah M Sampuli, Jie Song et al.

New Journal of Physics

The open-access journal for physics

Generation and detection of NOON states in superconducting circuits

Seth T Merkel $^{\rm 1}$ and Frank K Wilhelm

Institute for Quantum Computing and Department of Physics and Astronomy, University of Waterloo, Waterloo, ON, N2L 3G1, Canada E-mail: smerkel@uwaterloo.ca

New Journal of Physics **12** (2010) 093036 (11pp) Received 9 June 2010 Published 23 September 2010 Online at http://www.njp.org/ doi:10.1088/1367-2630/12/9/093036

Abstract. NOON states, states between two modes of light of the form $|N, 0\rangle + e^{i\phi}|0, N\rangle$, are highly nonclassical entangled states with applications in super-resolution interferometry. We show how NOON states can be efficiently produced in circuit quantum electrodynamics using superconducting phase qubits and resonators. We propose a protocol where only one interaction between the two modes is required, creating all the necessary entanglement at the start of the procedure. This protocol makes active use of the first three states of the phase qubits. Additionally, we show how to efficiently verify the success of such an experiment, even for large NOON states, using randomly sampled measurements and semidefinite programming techniques.

¹ Author to whom any correspondence should be addressed.

Contents

1.	Introduction		
2.	The system and the Hamiltonian		3
3. NOON state preparation		ON state preparation	4
	3.1.	Coupling and decoupling	4
	3.2.	Bell pairs	5
	3.3.	Shelving	6
	3.4.	Climbing the Jaynes–Cummings (JC) ladder	6
	3.5.	Unshelving	6
	3.6.	Disentangling	6
4.	Impact of decoherence, maximal size attainable		7
5.	State verification		7
6.	. Conclusions		10
Ac	Acknowledgments		
Re	References		

1. Introduction

The particle–wave duality of quantum physics has striking consequences on the physics of electromagnetic radiation, necessitating a photonics description of electromagnetic radiation, spanning frequencies from γ quanta via visible light to microwave fields. The phenomena of classical optics are fully characterized by only a tiny fraction of the wealth of states attainable in the quantum world. Creating and quantifying these others, the *nonclassical states* of light, has defined the field of quantum optics [1, 2].

Recently, this quantum-optical paradigm has been extended to the field of circuit quantum electrodynamics (cQED) [3, 4] where nonlinear superconducting circuits [5] are used to manipulate the quantum state of microwaves in waveguides in much the same way as atoms are used to manipulate optical fields in traditional quantum optics. Unlike atoms, however, superconducting circuits can be tuned over a broader range of parameters, allowing for highly flexible control. The current state of the art allows for the full control of quantum states up to 12 photons [6] as well as violations of locality in a Bell experiment [7, 8]. Small quantum information algorithms have also been successfully demonstrated [9].

A new frontier in this program is the realization of nonclassical states that are entangled between modes in distinct waveguides. A prominent example is the NOON state [10]–[12]

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|N, 0\rangle + e^{i\phi} |0, N\rangle \right), \tag{1}$$

i.e. an equal coherent superposition of N photons in the first mode and vacuum in the second mode with the reverse, vacuum in the first mode and N photons in the second mode. NOON states were introduced in quantum metrology as a means of surpassing the shot noise limit for phase sensitivity achievable by classical light by a factor of $1/\sqrt{N}$, which instead saturates the Heisenberg bound. At optical frequencies, NOON states up to N = 4 have been realized to date, relying on cascaded pair sources and postselection [13, 14]. Schrödinger cat states interpreted



Figure 1. Schematic diagram of an experimental setup for generating NOON states consisting of two phase qubits, a coupling cavity and two data cavities. The NOON state will be between the data cavities, which in the end are disentangled from the rest of the system. The boxed \times 's label Josephson junctions and the small circles label external ports to which driving fields can be applied.

as NOON states up to N = 10 have been realized in spin systems, along with an enhanced sensitivity to the magnetic field [15]. Such states can be viewed as a benchmark for multi-mode state preparation and coherent control capabilities.

We propose a scheme to create NOON sates in cQED. In cQED, sources that create pairs of photons *independent* of the number of photons already in the mode are currently not available. These would be needed for direct implementation of schemes developed at optical frequencies. Recently, Strauch *et al* [16] proposed a method to deal with this by utilizing state-dependent Stark shifts. We will circumvent this restriction by utilizing an additional energy level in the superconducting qubit we are considering for our artificial atom. We also place bounds on the size of NOON state that can be achieved and propose a scheme on how to verify the preparation of a NOON state using single-qubit measurements. A related protocol has been presented in [16].

2. The system and the Hamiltonian

The system under consideration consists of two microwave cavities, each coupled to its respective phase qubit. The qubits are also coupled to each other, either directly [17]–[19] or by a coupling cavity [20, 21]. This being a nonessential difference, we are going to focus on the case of cavity-mediated interaction, where by programming the frequency in phase qubits one achieves tunability. A schematic diagram of this system is given in figure 1 and is described by the Hamiltonian

$$\hat{H}(t) = \hat{H}_0 + \hat{H}_{01}^{c}(t) + \hat{H}_{02}^{c}(t) + \hat{H}_{C1}^{c}(t) + \hat{H}_{C2}^{c}(t).$$
(2)

The static (or 'drift') part of the Hamiltonian can be decomposed as

$$\hat{H}_0 = \sum_{i=1,2} \hat{H}_{Qi} + \sum_{j=1,2,C} \hat{H}_{Cj} + \sum_{i,j} \hat{H}_{QiCj}.$$
(3)

Here, $\hat{H}_{Qi} = E_{1i}|1\rangle_i \langle 1|_i + E_{2i}|2\rangle_i \langle 2|_i$ is the Hamiltonian for the phase qubit truncated to its three lowest energy levels $|0\rangle$, $|1\rangle$ and $|2\rangle$. Higher states will never be resonant during the whole protocol. The relevant cavity modes are described by $\hat{H}_{Cj} = \omega_j \hat{a}_j^{\dagger} \hat{a}_j$ with j = 1, 2, C and \hat{a}_j^{\dagger} and \hat{a}_j being the photon-raising and -lowering operators, respectively. With the same index space, the qubit–cavity couplings can be written in rotating wave approximation (RWA), assuming $g_{ij}^{(k)} \ll \omega_j$, E_{1i} as

$$\hat{H}_{QiCj} = \left(g_{ij}^{(1)} |1\rangle_i \langle 0|_i + g_{ij}^{(2)} |2\rangle_i \langle 1|_i\right) \hat{a}_j + \text{h.c.},$$
(4)

which is a straightforward three-level generalization of the Jaynes–Cummings (JC) interaction [22].

The system is primarily manipulated by controlling the phase qubits. Driving one of the qubits with an external field results in a control of the form, $\hat{H}_{Qi}^{c}(t) = \frac{\Omega_{i}(t)}{2}(|0\rangle_{i}\langle 1|_{i} + \lambda |1\rangle_{i}\langle 2|_{i} + h.c.)$. Here, $\lambda \approx \sqrt{2}$ is a constant whose precise value depends on the details of the qubit. Generally, $\Omega(t)$ will be a resonant microwave drive $\Omega(t) = \Omega_{0} \cos(\omega_{d} t)$. Additionally, changing the bias current gives a time-dependent E_{1i} and E_{2i} approximately following $E_{1i} = pE_{2i}$ with a fixed p < 2. Changing the energy of the qubit will primarily be used as a switch to move interactions in and out of resonance. Finally, the cavities themselves can be driven externally by microwave sources $H_{Ci}^{(c)} = \lambda_{i}(t)(\hat{a}_{i}^{\dagger} + \hat{a}_{i})$.

3. NOON state preparation

3.1. Coupling and decoupling

In order to understand the coupling Hamiltonian, it is helpful to transform it to the interaction representation with respect to the \hat{H}_{Qi} and \hat{H}_{Cj} . In that frame

$$\hat{H}^{I}_{QiCj} = \left(g^{(1)}_{ij} |1\rangle_{i} \langle 0|_{i} e^{i\Delta^{(1)}_{ij}t} + g^{(2)}_{ij} |2\rangle_{i} \langle 1|_{i} e^{i\Delta^{(2)}_{ij}t}\right) \hat{a}_{j} + \text{h.c.}$$
(5)

with the detuning $\Delta_{ij}^{(k)} = E_i^{(k)}(t) - E_i^{(k-1)}(t) - \omega_j$. If $\Delta_{ij}^{(k)}(t) \gg g_{ij}^{(k)}\sqrt{n_j+1}$, a standard rotating wave argument allows us to neglect the terms that couple $|k\rangle_i |n\rangle_j \leftrightarrow |k-1\rangle_i |n+1\rangle_j$. The implication is that by rapidly tuning $E_i^{(k)}$, one can switch the coupling between the cavity and the qubit on or off. This has been successfully demonstrated in a multitude of experiments [6, 20, 23]. The two regimes of interest for the following protocol are the case when both detunings are large, leading to uncoupled systems, and the case when $\Delta_{ij}^{(1)} = 0$ and $\Delta_{ij}^{(2)} \gg g_{ij}^{(2)}\sqrt{2}$, which reduces to the standard resonant JC interaction between states $|0\rangle_i$, $|1\rangle_i$ and the qubit only; see figure 2 (lower right).

$$\hat{H}_{QiCj}^{I} = g_{ij}^{(1)} \left(|1\rangle_{i} \langle 0|_{i} \, \hat{a}_{j} + |0\rangle_{i} \, \langle 1|_{i} \, \hat{a}_{j}^{\dagger} \right).$$
(6)



Figure 2. The NOON-state preparation protocol. Top: overview of the steps, highlighting qubits 1 and 2 and cavities 1, 2 and C. Parties 1 and 2 only interact during the initial Bell state preparation. Bottom left: Bell-state preparation by interacting qubits 1 and 2 with the cavity. Qubit 1 has already undergone a π -rotation and is in the state $|1\rangle$. The time axis shows interaction times τ_1 and τ_2 necessary for performing a \sqrt{iSWAP} operation with the first and an iSWAP with the second qubit. The frequency axis indicates that the interaction is controlled by moving the qubits in and out of resonance with the coupling cavity. Bottom right: energy scheme indicating the cavity energy ladders for the three different phase qubit states, assuming resonance between the cavity and the $0 \leftrightarrow 1$ transition of the qubit. In the last 'unshelving' step of the protocol, $|0, n\rangle$ and $|1, n - 1\rangle$ are swapped.

3.2. Bell pairs

Preparation of Bell states in this setting has already been demonstrated in [7, 8]. It works as outlined in figure 2 (lower left) using the notation $|q_1, n_C, q_2\rangle$ for qubit 1, cavity C and qubit 2. Starting from all elements decoupled by detuning, we let the system relax into its ground state $|0, 0, 0\rangle$. A resonant π -pulse on Ω_1 generates the state $|1, 0, 0\rangle$. Then, we bring qubit 1 into resonance with the coupling cavity, $\Delta_{1C}^{(1)} = 0$, for a time τ_1 such that $g_{1C}^{(1)}\tau_1/2 = \pi/2$ to create $(|1, 0, 0\rangle - i|0, 1, 0\rangle)/\sqrt{2}$. Next, we move qubit 1 out of resonance and qubit 2 into resonance for $g_{2C}^{(1)}\tau_2/2 = \pi$, leading to the state $(|1, 0, 0\rangle + i|0, 0, 1\rangle)/\sqrt{2}$, which is a Bell state between qubits 1 and 2, decoupled and disentangled from the cavity; see figure 2. After this step, there is no requirement to couple the qubits again throughout the whole protocol, and so we have no further use for the coupling cavity. All that remains is to separately generate a map on each subsystem, (qubit + cavity)_i denoted by $|q_i, n_i\rangle$, which transforms $|0, 0\rangle_i \rightarrow |0, N\rangle_i$

NOON state has exactly the same amount of entanglement as a Bell pair. This is entanglement between the *subsystems* (i.e. the two cavity/qubit subsystems), as opposed to an entanglement measure that treats the photons themselves as particles that could be made distinguishable by later splitting them into different modes [10, 11].

3.3. Shelving

In order to selectively increase the photon number of the qubit ground-state branch of the superposition while keeping the excited-qubit branch in vacuum, we shelve the excited state $|1, 0\rangle_i$ in the second excited state $|2, 0\rangle_i$ by selectively driving the qubits at frequency $\hbar\omega_d = E_{2,i} - E_{1,i}$. The $|2, 0\rangle_i$ state is considered 'shelved' in the sense that the JC interaction that is resonant with the transition $|0, 1\rangle_i \leftrightarrow |1, 0\rangle_i$ is far detuned from the transition $|1, 1\rangle_i \leftrightarrow |2, 0\rangle_i$ as long as $E_{2,i} - 2E_{1,i} \gg g_{ii}^{(2)}$; see figure 2. The $|2, 0\rangle_i$ state is also left invariant by rotations on the first two qubit levels assuming that the nonlinearity leaves it far off-resonant, or through the use of pulse shaping techniques such as DRAG [24].

3.4. Climbing the Jaynes-Cummings (JC) ladder

As a next step, the nonshelved state $|0, 0\rangle_i$ is transformed into the state $|0, N - 1\rangle_i$. This is a special case of the Eberly–Law protocol [25] that has been demonstrated in [23, 26]. Each step from $|0, n - 1\rangle_i$ to $|0, n\rangle_i$ is realized by first exciting the qubit by a resonant π -pulse, generating $|1, n - 1\rangle_i$. Next, the qubit is moved into resonance with its cavity for time $g_{ii}^{(1)}\sqrt{nt/2} = \pi$, moving the excitation from the qubit to the resonator; see figure 2. This is repeated until $|0, N - 1\rangle_i$ is reached. At the same time, due to efficient shelving, the other branch of the superposition stays in $|2, 0\rangle_i$ as the $|2, 0\rangle_i \leftrightarrow |1, 1\rangle_i$ transition is off-resonant with the cavity; see figure 2 (lower right).

3.5. Unshelving

The procedure so far brings the total state, written in notation $|n_1, n_2, q_1, q_2, n_C\rangle$ in state $(|N - 1, 0, 0, 2, 0\rangle - |0, N - 1, 2, 0, 0\rangle)/\sqrt{2}$. At this point, it may be counter-intuitive as to why we stopped at N - 1 photons in the previous section, but the simple explanation is that to unentangle the qubits from the cavities we must use the JC-interaction, which will inject an additional photon into our state. The unshelving procedure is as follows. Applying a π -pulse to both qubits on the $|2\rangle_i \leftrightarrow |1\rangle_i$ transition, followed by π -pulses on the transitions $|1\rangle_i \leftrightarrow |0\rangle_i$, permutes the qubit states such that $|2\rangle_i \rightarrow |0\rangle_i$ and $|0\rangle_i \rightarrow |1\rangle_i$. This results in the state $(|N - 1, 0, 1, 0, 0\rangle - |0, N - 1, 0, 1, 0\rangle)/\sqrt{2}$.

3.6. Disentangling

Now we couple both qubits to their respective cavities, $\Delta_{ii}^{(1)} = 0$, for time $g_{ii}^{(1)}\sqrt{N}t/2 = \pi$. If both the cavity and the qubit are in their ground states, they are left invariant by the JC interaction. On the other hand, the branch of the wavefunction that contains $|1, N - 1\rangle_i$ becomes $|0, N\rangle_i$. This step disentangles the cavities from the qubit and leads to the desired NOON state

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|N, 0\rangle - |0, N\rangle) \otimes |0, 0, 0\rangle.$$
 (7)

4. Impact of decoherence, maximal size attainable

While there is no principal bound on the maximum size N of NOON state this protocol can achieve, there are practical limitations. One will be related to the precision of the control pulses that can be applied, which in principle can be very high using pulse shaping [24, 27, 28]. More fundamentally, decoherence limits the size of the state attainable. There are two crucial limitations: one is that the qubit state has to be shelved in the second excited state for a long time and the other is that the superposition should not dephase. Given that this happens if any of the two qubits decays, we can go up to $T_2/2$. The total duration of our protocol can be simply calculated as

$$T_{\rm tot} = \frac{1}{g} \sum_{n=1}^{N} \sqrt{\frac{1}{n}} + N T_{\rm Rabi},$$
(8)

where T_{Rabi} is the time for a single qubit Rabi oscillation. Assuming $T_2 \simeq 200 \text{ ns}$ [29] and $T_{\text{Rabi}} \simeq 10 \text{ ns}$ and $1/g \simeq 20 \text{ ns}$, this allows NOON states up to N = 4. These numbers fall well below the number of photons already stored and investigated in such resonators, so photon loss will not be a limiting factor [6, 30]. In transmons, which also offer an accessible third level [31], coherence times are longer [32] and larger NOON states may be attainable.

5. State verification

Decoherence and imperfect control ensure that in the laboratory setting, the prepared state described by a density matrix $\hat{\rho}_{exp}$ is not a perfect NOON state. The fidelity between the desired and actual state can be obtained by measuring the overlap

$$\mathcal{F} = \langle \psi_{\text{NOON}} | \, \rho_{\text{exp}} \, | \psi_{\text{NOON}} \rangle. \tag{9}$$

This is not a trivial task with the measurement tools at our disposal, which consist of driving the system through some simple dynamics followed by qubit measurements. In this section, we will show that we can create a set of measurement operators that is informationally complete, that is, they span the operator space $\mathfrak{su}(d)$ where d is the dimension of the problem under consideration. Here $d = (N + 1)^2$ when we consider just the microwave cavities. The outcomes from an informationally complete set of measurements can be used to perform tomography on the quantum state, but we will also present a method of bounding the fidelity directly.

Restricting our attention to a single cavity, the types of measurements we consider consist of driving the cavity by a coherent displacement α , switching on the JC Hamiltonian for a time $g_{ii}^{(1)}t/2 = \tau$ and then measuring the qubit in the z-basis by measuring the switching probability out of state $|1\rangle$ [5, 29]. We can workout the corresponding operator by solving the timedependent Schrödinger equation for the Hamiltonian $\hat{H}_{Q_i} + \hat{H}_{C_i} + \hat{H}_{QiCi} + \hat{H}_{Ci}^{(c)}$ as defined in equations (2) and (3) for this protocol starting from the simplifying assumption that the qubit is initially uncoupled from the cavity and in its ground state. We can express this observable on the microwave cavity as

$$\hat{\mathcal{M}}(\alpha,\tau) = \sum_{n} \cos(2\sqrt{n}\tau) \hat{D}^{\dagger}(\alpha) |n\rangle \langle n| \hat{D}(\alpha).$$
(10)

By sampling different values of the dimensionless time τ , with the amplitude set to $\alpha = 0$, one can only infer the diagonal elements of the density operator $\langle n|\rho|n\rangle$. The injection of α

is crucial because diagonal measurements alone do not distinguish a NOON state from the mixed state $\hat{\rho} = \frac{1}{2}(|N, 0\rangle\langle N, 0| + |0, N\rangle\langle 0, N|)$, the latter being not entangled and therefore useless for quantum metrology. In [6], these types of measurements were used to construct the Wigner function of the cavity, which is a complete description of the quantum state of the cavity mode. This implies that at least to some finite truncation of the cavity, the set of measurements $\hat{\mathcal{M}}(\alpha, \tau)$ are informationally complete in that one can construct a set of $(N+1)^2 - 1$ linearly independent observables spanning the space $\mathfrak{su}(N+1)$. To generate such a set it is sufficient to simply sample pairs (α_j, τ_j) from some random distribution. This will almost surely generate a linearly independent set of measurement operators. It is not impossible that there are optimized choices of operators that are close to orthonormal, thus optimizing the sampling of phase space, but in practice it is most often easier to instead just oversample from our distribution on (α, τ) .

Correlating the results of measurements from the two individual cavities yields the set of operators $\hat{\mathcal{M}}_2(\alpha, \tau, \alpha', \tau') = \hat{\mathcal{M}}(\alpha, \tau) \otimes \hat{\mathcal{M}}(\alpha', \tau')$. This is a complete set of operators on the joint space; however, since the Hilbert space of the two-cavity system has dimension $(N+1)^2$, reconstructing the entire density matrix requires measuring at least $(N+1)^4 - 1$ of these operators. To perform full tomography we repeat each measurement a number of times to obtain measurement statistics

$$M_j = \operatorname{Tr}\left(\hat{\mathcal{M}}_2(\alpha_j, \tau_j, \alpha'_j, \tau'_j)\rho_{\exp}\right) + \sigma_j W_j, \tag{11}$$

where the final term is a Gaussian white noise variable with variance σ_j . In principle, one can arbitrarily decrease the strength of this noise by repeated measurements; however, it is generally more efficient to fix σ_j and instead sample more measurement operators from our distribution on $\hat{\mathcal{M}}(\alpha_j, \tau_j, \alpha'_j, \tau'_j)$. From such a measurement record, one can obtain a maximum likelihood estimate of the quantum state through a least squares fit [33]. The resulting state may have unphysical, negative eigenvalues, which need to be compensated for either by simply setting them to zero or by using slightly more involved semidefinite programming techniques [34, 35] to find the physical state that is most consistent with the measurement outcomes. The assumption that the qubits were initially in their ground states was not essential and can be circumvented by adding to our set of measurements by additionally performing some unitary operations to the superconducting qubit immediately prior to measurement. This increases the number of independent measurements necessary for tomography to $16(N+1)^4 - 1$ for qubits, or $81(N+1)^4 - 1$ when we also account for the second qubit excited state.

Placing bounds on the experimental value of \mathcal{F} can be accomplished with a smaller set of measurement outcomes. We can lower bound \mathcal{F} , defined in equation (9), by directly solving the optimization problem

$$\min \mathcal{F}$$
(12)

subject to

$$M_j = \operatorname{Tr}\left(\hat{\mathcal{M}}_2(\alpha_j, \tau_j, \alpha'_j, \tau'_j)\rho_{\exp}\right), \quad \rho \ge 0,$$
(13)

where we have neglected the noise on the measurements. The second constraint ensures that ρ is a physical density matrix, and thus we will find the state with the lowest overlap with the NOON state that nevertheless agrees with the outcomes from our measurements. This optimization is of the form of a semidefinite program and can be solved very efficiently numerically [34]. We used the Matlab program yalmip running the solver SeDuMi. Additionally, since the objective



Figure 3. Numerical analysis of bounding the fidelity of NOON state construction with an incomplete set of measurement outcomes. All simulations are for noiseless measurements and assume that the qubits are initially in their ground states. (a) Upper and lower bounds for the overlap between the four-photon NOON state and various mixtures of the NOON state and the maximally mixed state, plotted versus the number of randomly sampled measurements performed. (b) Lower bounds on the NOON state fidelity for mixtures of different dimensions. The *x*-axis of this plot represents the number of measurement results divided by the dimension of $\mathfrak{su}(d)$ for the different NOON states. (c) The difference between the upper and lower bounds for mixtures of the six-photon NOON state and the maximally mixed state, plotted versus the number of measurements performed.

of this optimization (\mathcal{F} , equation (9)) is linear, solving the maximization problem (i.e. finding an upper bound) is of the same difficulty.

To determine the utility of this bounding scheme, we ran some simple numerical simulations, the results of which are displayed in figure 3. We place upper and lower bounds on states that are convex sums of the NOON state and the maximally mixed state, or $\rho_{\text{exp}} = p\rho_{\text{NOON}} + (1-p)\mathbb{I}/d$. For a given overlap with the NOON state, these mixtures are

generally the states that are hardest to bound and so represent a worst case test of the protocol. The measurements were chosen by sampling the τ_i uniformly from the interval $[0, \pi]$ and the α 's uniformly from the interior of the circle on phase space with radius N. For a simple test we looked at NOON states with photon numbers going up to N = 6, where even the longest semidefinite optimizations were solved in about a minute on a laptop running Matlab.

In the case of randomly sampled measurements, the lower bound has a rapid initial rise with respect to the fraction of the possible independent operators measured. The time for convergence of the upper and lower bounds is highly dependent on the fidelity of the experimental state, with high-fidelity states converging very quickly. In order to get a fairly exact estimate of the fidelity for a density matrix far from the desired state, this method may not be much more efficient than full tomography; however, it is often useful to quickly generate bounds out of small data sets in order to diagnose state generation protocols. Also, we suspect that this protocol is more amenable to optimizing the measurement operators than full tomography, and so it should be possible to get faster convergence than can be achieved by random sampling.

6. Conclusions

In conclusion, we have shown how to deterministically create large NOON states of microwave photons in superconducting resonators and how to verify the success of this experiment in a more efficient way than full state tomography. It can be shown that this reconstruction protocol can be extended to account for detector noise.

NOON states in the optical domain have been proposed for super-resolution in quantum lithography and interferometry [10, 11, 15]. With the wavelengths of microwave photons in our case being of the order of centimetres, lithography is an unlikely application. Microwave interferometry has applications in plasma diagnostics [36] and astrophysics relating to the cosmic microwave background. The application of NOON states to these systems remains speculative and will be explored in future work.

Acknowledgments

We acknowledge useful discussions with J M Martinis and A Cleland, who independently developed a similar protocol for NOON state creation [37], as well as discussions with M Mariantoni, H Wang, F Motzoi and B McNamara. This work was supported by NSERC through QuantumWorks and the Discovery Grants program and in part by the DARPA/MTO QuEST program through a grant from AFOSR and by IARPA through the MQCO program.

References

- [1] Glauber R 1963 Phys. Rev. 131 2766
- [2] Cahill K and Glauber R 1969 Phys. Rev. 177 1882
- [3] Blais A, Huang R-S, Wallraff A, Girvin S and Schoelkopf R 2004 Phys. Rev. A 69 062320
- [4] Schoelkopf R and Girvin S 2008 Nature 451 664
- [5] Clarke J and Wilhelm F 2008 Nature 453 1031
- [6] Hofheinz M et al 2009 Nature 459 546
- [7] Ansmann M et al 2009 Nature 461 504

- [8] Chow J M, DiCarlo L, Gambetta J M, Nunnenkamp A, Bishop L S, Frunzio L, Devoret M H, Girvin S M and Schoelkopf R J 2010 Phys. Rev. A 81 062325
- [9] DiCarlo L *et al* 2009 *Nature* **460** 240
- [10] Boto A, Kok P, Abrams D, Brownstein S, Williams C and Dowling J 2000 Phys. Rev. Lett. 85 2733
- [11] Kok P, Lee H and Dowling J 2002 Phys. Rev. A 65 052104
- [12] Sanders B 1989 Phys. Rev. A 40 2417
- [13] Mitchell M, Lundeen J and Steinberg A 2004 Nature 429 161
- [14] Nagata T, Okamoto R, O'Brien J, Sasaki K and Takeuchi S 2006 Science 316 726
- [15] Jones J, Karlen S, Fitzsimons J, Adravan A, Benjamin S, Briggs G and Morton J 2009 Science 324 1166
- [16] Strauch F W, Jacobs K and Simmonds R W 2010 Phys. Rev. Lett. 105 050501
- [17] Berkley A, Xu H, Ramos R, Gubrud M, Strauch F, Johnson P, Anderson J, Dragt A, Lobb C and Wellstood F 2003 Science 300 1548
- [18] McDermott R, Simmonds R, Steffen M, Cooper K, Cicak K, Osborn K, Oh S, Pappas D and Martinis J 2005 Science 307 1299
- [19] Steffen M, Ansmann M, Bialczak R C, Katz N, Lucero E, McDermott R, Neeley M, Weig E M, Cleland A N and Martinis J 2006 Science 313 1423
- [20] Sillanpää M, Park J and Simmonds R 2007 Nature 449 438
- [21] Majer J et al 2007 Nature 449 443
- [22] Jaynes E and Cummings F 1963 Proc. IEEE 51 89
- [23] Hofheinz M, Weig E, Ansmann M, Bialczak R, Lucero E, Neeley M, O'Connell A, Wang H and Martinis J 2008 Nature 454 310
- [24] Motzoi F, Gambetta J, Rebentrost P and Wilhelm F 2009 Phys. Rev. Lett 103 110501
- [25] Law C K and Eberly J H 1996 Phys. Rev. Lett. 76 1055
- [26] Fink J, Göppl M, Baur M, Bianchetti R, Leek P, Blais A and Wallraff A 2008 Nature 454 315
- [27] Khaneja N, Reiss T, Kehlet C, Schulte-Herbrüggen T and Glaser S 2005 J. Magn. Reson. 172 296
- [28] Spörl A, Schulte-Herbrueggen T, Glaser S, Bergholm V, Storcz M, Ferber J and Wilhelm F 2007 Phys. Rev. A 75 012302
- [29] Martinis J 2009 Quantum Inf. Process. 8 81
- [30] Wang H et al 2009 Phys. Rev. Lett. 103 200404
- [31] Koch J, Yu T, Gambetta J, Houck A, Schuster D, Majer J, Blais A, Devoret M, Girvin S and Schoelkopf R 2007 Phys. Rev. A 76 042319
- [32] Houck A, Koch J, Devoret M, Girvin S and Schoelkopf R 2009 Quantum Inf. Process. 8 105
- [33] Paris M and Rehacek J (ed) 2004 Quantum State Estimation (Berlin: Springer)
- [34] Vandenberghe L and Boyd S 1996 SIAM Rev. 38 49
- [35] Merkel S T, Riofrío C A, Flammia S T and Deutsch I H 2010 Phys. Rev. A 81 032126
- [36] Laroussi M 1999 Int. J. Infrared Millim. Waves 8 1501
- [37] Martinis J and Cleland A 2010 personal communication

11