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Do three dimensions tell us anything about a theory of everything?

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Abstract. It has been conjectured that four-dimensional $\mathcal{N} = 8$ supergravity may provide a suitable framework for a ‘theory of everything’, if its composite $SU(8)$ gauge fields become dynamical. We illustrate that supersymmetric three-dimensional coset field theories, motivated by lattice models, provide toy laboratories for aspects of this conjecture. They feature dynamical composite supermultiplets made of constituent holons and spinons. We show how these models may be extended to include $\mathcal{N} = 1$ and $\mathcal{N} = 2$ supersymmetry, enabling dynamical conjectures to be verified more rigorously. We highlight some special features of these three-dimensional models and mention open questions about their relevance to the dynamics of $\mathcal{N} = 8$ supergravity.

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1. Introduction

Once upon a time, four-dimensional $\mathcal{N} = 8$ supergravity was touted as a possible ‘theory of everything’ [1]. This suggestion was sparked by the observation that $\mathcal{N} = 8$ supergravity has a hidden, composite SU(8) gauge structure [2]. The interacting scalar fields of $\mathcal{N} = 8$ supergravity have a non-compact $E_{7(7)}$ structure with 133 components that are reduced by the 63 generators of the SU(8) symmetry to the 70 physical, on-shell scalar fields. Attention was drawn by the realization that SU(8) is a gauge symmetry large enough to include a flavour symmetry as well as an SU(5) grand-unification group. This motivated the suggestion that the composite SU(8) gauge fields might become physical particles [1], as a result of some unspecified dynamical mechanism. Already in the first papers on this subject, attention was drawn to two-dimensional coset models in which the composite gauge fields became dynamical as a result of infrared singularities [2].

However, the suggestion that $\mathcal{N} = 8$ supergravity was a promising framework for a ‘theory of everything’ rapidly became unfashionable, for several reasons. One was that no convincing mechanism for making the gauge fields dynamical came to mind, and another was that the SU(8) gauge symmetry was thought to be anomalous [3]. However, the big blow was the realization that ten-dimensional $E_8 \times E_8$ and SO(32) heterotic string theories had no anomalies, and could provide suitable frameworks for grand unification, after compactification to four dimensions [4]. Subsequently, many promising four-dimensional models were derived from string theory as understanding of string dynamics deepened. This deepening understanding led to the realization that all string theories are linked and to the discovery that they are related to 11-dimensional supergravity [5]. In turn, this theory may be related to $\mathcal{N} = 8$ supergravity by compactification to four dimensions [6]. However, there is no simple limit in which string theory can be compactified to four-dimensional $\mathcal{N} = 8$ supergravity without additional low-mass fields [7].

Interest in $\mathcal{N} = 8$ supergravity has recently revived, motivated by the realization that it is very well behaved in the ultraviolet, and may well be finite in perturbation theory [8]. This development has brought back to the collective consciousness a ‘forgotten’ paper by Marcus [9], in which he showed that the SU(8) gauge symmetry of $\mathcal{N} = 8$ supergravity is, in fact, anomaly-free. This paper had been overlooked in the euphoria surrounding string theory but now, when coupled with the good ultraviolet behaviour of $\mathcal{N} = 8$ supergravity, it motivates a re-examination of the possibility that this might be a promising road towards a ‘theory of everything’. This possibility is not necessarily in conflict with the candidacy of string theory as the ‘theory of everything’, at least as long as the relation of string theory to $\mathcal{N} = 8$ supergravity remains to be elucidated.

In this paper, we review the relevant aspects of three-dimensional lattice models that may serve as an inspiration for modelling the dynamics of $\mathcal{N} = 8$ supergravity. In particular, we display the role of dynamical composite supermultiplets made of constituent holons and spinons. As we show explicitly, supersymmetry is an inessential complication, in the sense that it does not alter the nature of the infrared behaviour. On the other hand, the elevation of $\mathcal{N} = 1$ supersymmetry to $\mathcal{N} = 2$ does enable dynamical results to be placed on a more rigorous basis. We finish by recalling some of the limitations of the three-dimensional models, and highlighting some of the questions that arise before the existence of a similar mechanism in $\mathcal{N} = 8$ supergravity could be addressed.

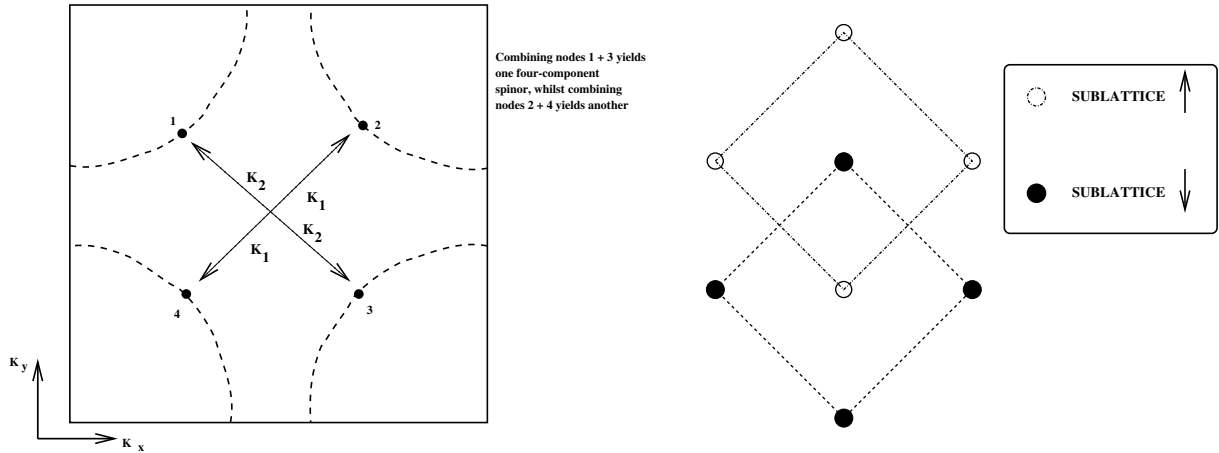


Figure 1. Left: the Fermi surface of underdoped cuprates consists of four nodes, as indicated by the dashed lines. The continuum effective theory may be obtained in a standard way by linearization about such nodes, which leads, at the constituent level, to two flavours of four-component Dirac spinors for the holon degrees of freedom (d.o.f.). These flavours are obtained by combining the nodes along the diagonal lines, as indicated in the figure: nodes 1 + 3 yield one flavour, and nodes 2 + 4 yield the other. Right: a simple antiferromagnetic sublattice structure, which can be used to define local (spin) SU(2) gauge groups, playing the role of ‘colour’. There are two coloured four-component spinors in this construction, each obtained by taking the continuum limit in a sublattice. The dynamical breaking of the local SU(2) with mass generation for fermions implies suppression of inter-sublattice hopping in such models.

2. 2 + 1-dimensional condensed-matter models with dynamical gauge bosons

The understanding of field theories in three space–time dimensions has largely been driven by the impetus to study condensed-matter systems formulated on a lattice. These field theories become relativistic near nodes of the Fermi surface, see figure 1, and may exhibit dynamical gauge bosons and supersymmetry, as we discuss below.

A classic model studied with antiferromagnetism and high-temperature superconductivity has been the $t - J$ Hamiltonian [13, 15], which is expressed in terms of electron creation operators $c_{a,i}^\dagger$ and their conjugates. As shown in [11, 12], when intersublattice hopping is included, this Hamiltonian can be expressed in terms of field operators χ , whose structure we discuss below. For example, the hopping term reads

$$H_{\text{hop}} = - \sum_{\langle ij \rangle} t_{ij} c_{\alpha,i}^\dagger c_{\alpha,j} = - \sum_{\langle ij \rangle} t_{ij} [\chi_{i,\alpha\gamma}^\dagger \chi_{j,\gamma\alpha} + \chi_{i,\alpha\gamma}^\dagger (\sigma_3)_{\gamma\beta} \chi_{j,\beta\alpha}], \quad (1)$$

where i denotes a lattice site, σ_3 is a 2×2 Pauli matrix, and summation over the spin indices is implied. Heisenberg interaction terms may be added to this, which can be written in the following convenient form [13]:

$$H_{\text{HI}} = -\frac{1}{8} J \sum_{\langle ij \rangle} \text{tr}[\chi_i \chi_j^\dagger \chi_j \chi_i^\dagger]. \quad (2)$$

The antiferromagnetic nature of such systems translates into the existence of more than one sublattice (see figure 1), associated with the spin states of the hole excitations, with intersublattice hopping nonzero but suppressed relative to the hopping inside a single sublattice. As we review below, this results in internal gauge d.o.f. for the hole excitations, which becomes a fully fledged dynamical non-Abelian gauge symmetry in the effective field theory. There is an even number of sublattices or nodes in the Fermi surface of such systems, and hence an even number of spinor d.o.f., which results in parity remaining unbroken. (We remind the reader that in $2+1$ dimensions a single fermion mass term breaks parity, but this can be restored if there is an even number of fermionic species, with mass terms in pairs of opposite signs.)

To see this, as discussed in [11, 12], one may represent the field χ via a spin-charge separation Ansatz in the case of a planar antiferromagnetic lattice with intersublattice hopping for a particle–hole symmetric formulation away from half-filling:

$$\chi_i \equiv \begin{pmatrix} \psi_1 & \psi_2 \\ -\psi_2^\dagger & \psi_1^\dagger \end{pmatrix}_i \begin{pmatrix} z_1 & -\bar{z}_2 \\ z_2 & \bar{z}_1 \end{pmatrix}_i. \quad (3)$$

The fields z_a obey canonical *bosonic* commutation relations, and are associated with the *spin* d.o.f. (‘spinons’), while the fields ψ_a are Grassmann variables on the lattice, which obey *fermionic* statistics, and are associated with the electric charge d.o.f. (‘holons’).

The ansatz (3) admits a *hidden* non-Abelian local SU(2) spin symmetry at the constituent level, as discussed in [11, 12]. This may readily be seen by considering the invariance of the χ variable under simultaneous local SU(2) rotations of the spinon and holon components:

$$\begin{aligned} \Psi_i &\longrightarrow \Psi_i h_i, & \text{where } \Psi &\equiv \begin{pmatrix} \psi_1 & \psi_2 \\ \psi_2^\dagger & -\psi_1^\dagger \end{pmatrix}, \\ Z_i &\longrightarrow h_i^\dagger Z_i, & \text{where } Z &\equiv \begin{pmatrix} z_1 & -\bar{z}_2 \\ z_2 & \bar{z}_1 \end{pmatrix}, \end{aligned} \quad (4)$$

where $h_i \in \text{SU}(2)$. In three space–time dimensions, due to the fractional statistics of the planar excitations, one has an additional Abelian phase rotation, forming a $U_S(1)$ ‘statistical’ Abelian gauge group (which should not be confused with the $U(1)$ of electromagnetism). Hence, the full local gauge group of the spin-separation ansatz is $\text{SU}(2) \otimes U_S(1)$. In terms of the spin and charge excitations appearing in (3), the hopping term in the Hamiltonian (1) may be written as

$$H_{\text{hop}} = - \sum_{\langle ij \rangle} t_{ij} [\bar{z}_{i,b} \psi_{i,b,\alpha}^\dagger \psi_{j,c,\alpha} z_{j,c} + \bar{z}_{i,b} \psi_{i,b,\alpha}^\dagger (\sigma_3)_{\alpha\beta} \psi_{j,c,\beta} z_{j,c}], \quad (5)$$

which has a trivial local SU(2) symmetry.

The Heisenberg interaction term (2) can be linearized in terms of the fermion bilinears if one introduces in the path integral a Hubbard–Stratonovich field Δ_{ij} , in a standard fashion. The result of the linearization for the combined hopping and interaction Hamiltonian is of the form [11, 12]:

$$\begin{aligned} H_{\text{HF}} = \sum_{\langle ij \rangle} \text{Tr} &\left[\frac{8}{J} \Delta_{ij}^\dagger \Delta_{ji} + (-t_{ij}(1 + \sigma_3) + \Delta_{ij}) \Psi_j^\dagger \langle Z_j \bar{Z}_i \rangle \Psi_i \right] \\ &+ \sum_{\langle ij \rangle} \text{Tr} \left[\bar{Z}_i \langle \Psi_i^\dagger (-t_{ij}(1 + \sigma_3) + \Delta_{ij}) \Psi_j \rangle Z_j + \text{h.c.} \right]. \end{aligned} \quad (6)$$

This Hamiltonian determines the nature of any spontaneous symmetry breaking that occurs and the associated phase structure of the low-energy theory near the nodes of the Fermi surface. Using the gauge symmetries of (3), we can write

$$\langle Z_j \bar{Z}_i \rangle \equiv |A_1| V_{ij} U_{ij}, \quad \langle \Psi_i^\dagger (-t_{ij}(1 + \sigma_3) + \Delta_{ij}) \Psi_j \rangle \equiv |A_2| V_{ij} U_{ij}, \quad (7)$$

where $V \in \text{SU}(2)$ and $U \in \text{U}_S(1)$ are group elements. The group $\text{U}_S(1)$ expresses the fractional statistics of the three-dimensional excitations of spinons Z and holons Ψ . The fact that apparently gauge non-invariant correlators are nonzero on the lattice is standard in gauge theories, and does not violate Elitzur's theorem [14], due to the fact that in order to evaluate the physical correlators, one must follow a gauge-fixing procedure, which should be done prior to any computation. The amplitudes $|A_1|$ and $|A_2| \equiv K > 0$ are considered frozen, which is a standard assumption in the gauge theory approach to strongly correlated electron systems [13, 15]. The group elements V and U are phases of the above field bilinears and, to a first (mean-field) approximation, can be considered as *composites* of the constituent spinons z and holons ψ . Fluctuations about such ground states can then be considered by integrating over the constituent fields.

The above lattice action does not contain kinetic terms for the $\text{SU}(2)$ and $\text{U}_S(1)$ groups, which would take the form

$$\sum_p [\beta_{\text{SU}(2)}(1 - \text{Tr} V_p) + \beta_{\text{U}_S(1)}(1 - \text{Tr} U_p)], \quad (8)$$

where the sum is over the plaquettes p of the lattice, and the coefficients

$$\beta_{\text{U}_S(1)} \equiv \beta_1, \quad \beta_{\text{SU}(2)} \equiv \beta_2 = 4\beta_1 \quad (9)$$

are the inverse square couplings of the $\text{U}_S(1)$ and $\text{SU}(2)$ groups, respectively. The specific relation between the coefficients of the $\text{SU}(2)$ and $\text{U}_S(1)$ terms is a consequence of the appropriate normalizations of the generators of the groups. The absence of gauge kinetic terms in our case implies that we are in the limit of infinitely strong coupling for both gauge groups:

$$\beta_{\text{U}_S(1)} = \beta_{\text{SU}(2)} = 0 \leftrightarrow g_{\text{U}_S(1)}, g_{\text{SU}(2)} \rightarrow \infty. \quad (10)$$

However, as discussed in detail in [11], integration of the Z -spinon fields results in kinetic terms for gauge fields, which couple to the spinons Ψ only. The analysis of [11] assumes that the Z -spinons have a mass gap that is larger than the dynamical mass gap generated by the holon fields through their interactions with the Abelian gauge group, so that one can define the appropriate effective theory of the light d.o.f. by integrating out the heavy ones. (We recall that the kinetic terms are the lowest terms in a derivative expansion, in which terms with more than two derivatives are irrelevant operators in the infrared and hence can be ignored when one considers the low-energy continuum limit of interest here.) In such a case, one obtains [11]

$$\beta_1 = \frac{1}{J\eta a}, \quad (11)$$

where a is the (sub)lattice spacing, η denotes the doping concentration in the sample and J is the Heisenberg interaction in the condensed-matter model (6), and group theory maintains the relationship (9) between the $\text{SU}(2)$ and $\text{U}_S(1)$ couplings. The strong- $\text{U}_S(1)$ coupling corresponds formally to $J\eta a \rightarrow \infty$.

We now turn to the dynamical breaking of the $\text{SU}(2)$ group [11], using a Schwinger–Dyson treatment of dynamical symmetry breaking based on a large- N approximation, in which the

SU(2) spin group is replaced by SU(N) with N large. In this case, the non-Abelian coupling is related to the Abelian one through

$$\beta_{\text{SU}(N)} = 2N\beta_{\text{U}_S(1)} = 2N\beta_1. \quad (12)$$

This implies that, even in the case of strong $\text{U}_S(1)$ coupling, $\beta_1 \rightarrow 0$, the large- N (large-spin) limit may be implemented in such a way so that $\beta_{\text{SU}(N)}$ is finite, and may be assumed weak.

As discussed in detail in [11] and references therein, the lattice Hamiltonian (6) after integrating out the massive spinon Z fields can be expressed in the infrared limit as a lattice spinor Hamiltonian for the holon fields Ψ coupled to the $\text{U}_S(1)$ and SU(2) gauge fields:

$$S = \frac{1}{2} K \sum_{i,\mu} [\bar{\Psi}_i(-\gamma_\mu) U_{i,\mu} V_{i,\mu} \Psi_{i+\mu} + \bar{\Psi}_{i+\mu}(\gamma_\mu) U_{i,\mu}^\dagger V_{i,\mu}^\dagger \Psi_i] \\ + \beta_1 \sum_p (1 - \text{tr} U_p) + \beta_{\text{SU}(N)} \sum_p (1 - \text{tr} V_p), \quad (13)$$

where $\mu = 0, 1$ and 2 , $U_{i,\mu} = \exp(i\theta_{i,\mu})$ represents the statistical $\text{U}_S(1)$ gauge field, $V_{i,\mu} = \exp(i\sigma^a B_a)$ is the SU(2) gauge field, and the plaquette terms appear as a result of the z (spinon) integration, as mentioned previously. Working in the large N -fermion flavour limit (N even sublattices, $N \rightarrow \infty$) and keeping the SU(N) coupling $\beta_{\text{SU}(N)}$ finite, according to (12), the SU(2) gauge field becomes dynamical. The $\text{U}_S(1)$ field, on the other hand, is assumed strongly coupled: $\beta_1 \rightarrow \infty$, and hence its kinetic term is absent. The fact that kinetic terms for the SU(2) gauge fields can be induced in the effective action by quantum corrections reflects the existence of a nontrivial infrared fixed point in the $(2+1)$ -dimensional Abelian gauge theory, after the dynamical breaking of the SU(2). This was discussed in detail in [16], and has also been discussed in the $\mathcal{N} = 1$ supersymmetric case [17].

After integrating the effective spinon–gauge-field Lagrangian over the strongly coupled statistical $\text{U}_S(1)$ dynamical gauge group, one finds the following effective partition function:

$$\int [dV d\bar{\Psi} d\Psi] \exp(-S_{\text{eff}}), \quad (14)$$

where

$$S_{\text{eff}} = \beta_2 \sum_p (1 - \text{tr} V_p) + \sum_{i,\mu} \ln I_0(\sqrt{y_{i\mu}}), \quad (15) \\ y_{i\mu} \equiv K^2 \bar{\Psi}_i(-\gamma_\mu) V_{i\mu} \Psi_{i+\mu} \bar{\Psi}_{i+\mu}(\gamma_\mu) V_{i\mu}^\dagger \Psi_i.$$

Here, K is the amplitude $|A_2|$ of the fermionic bilinears in the Hartree–Fock lattice action (6) and I_0 is the zeroth-order Bessel function. The quantity $y_{i\mu}$ may be written in terms of the bilinears

$$M_{ab,\alpha\beta}^{(i)} \equiv \Psi_{i,b,\beta} \bar{\Psi}_{i,a,\alpha}, \quad a, b = \text{colour}, \quad \alpha, \beta = \text{Dirac}, \quad i = \text{lattice site}, \quad (16)$$

with the result

$$y_{i\mu} = -K^2 \text{tr}[M^{(i)}(-\gamma_\mu) V_{i\mu} M^{(i+\mu)}(\gamma_\mu) V_{i\mu}^\dagger]. \quad (17)$$

In the language of particle physics, quantities analogous to the $M^{(i)}$ would represent physical *meson* states. Converting from fermionic to bosonic variables, the low-energy (long-wavelength) effective action may be written as a path integral in terms of gauge-boson and meson fields [11]

$$Z = \int [dV dM] \exp(-S_{\text{eff}} + \sum_i \text{tr} \ln M^{(i)}), \quad (18)$$

where the meson-dependent term in (18) comes from the Jacobian that arises when converting the integral from fermion to meson variables.

A method was presented in [11] for identifying the symmetry-breaking patterns of the gauge theory (18), by studying the dynamically generated mass spectrum. The method consists of expanding $\sum_{i,\mu} \ln I_0(\sqrt{y_{i,\mu}})$ in powers of $y_{i,\mu}$ and concentrating on the lowest orders, which yield the gauge boson masses, while higher orders describe interactions. Keeping only the linear term in the expansion yields

$$\ln I_0(\sqrt{y_{i,\mu}}) \simeq \frac{1}{4} y_{i,\mu} = -\frac{1}{4} K^2 \text{tr}[M^{(i)}(-\gamma_\mu) V_{i\mu} M^{(i+\mu)}(\gamma_\mu) V_{i\mu}^\dagger]. \quad (19)$$

It is evident that the pattern of SU(2) breaking is determined by nonzero vacuum expectation values (VEVs) for the meson matrices $M^{(i)}$. One has the following expansion for the meson states in terms of SU(2) bilinears [11]:

$$M^{(i)} = \mathcal{A}_3(i)\sigma_3 + \mathcal{A}_1(i)\sigma_1 + \mathcal{A}_2(i)\sigma_2 + \mathcal{A}_4(i)\mathbf{1} \\ + i[B_{4,\mu}\gamma^\mu + B_{1,\mu}(i)\gamma^\mu\sigma_1 + B_{2,\mu}\gamma^\mu\sigma_2 + B_{3,\mu}\gamma^\mu\sigma_3], \quad (20)$$

with $\mu = 0, 1, 2$, and

$$\mathcal{A}_1 \equiv -i[\bar{\Psi}_1\Psi_2 - \bar{\Psi}_2\Psi_1], \quad \mathcal{A}_2 \equiv \bar{\Psi}_1\Psi_2 + \bar{\Psi}_2\Psi_1, \quad \mathcal{A}_3 \equiv \bar{\Psi}_1\Psi_1 - \bar{\Psi}_2\Psi_2, \\ B_{1\mu} \equiv \bar{\Psi}_1\sigma_\mu\Psi_2 + \bar{\Psi}_2\sigma_\mu\Psi_1 \quad B_{2\mu} \equiv i[\bar{\Psi}_1\sigma_\mu\Psi_2 - \bar{\Psi}_2\sigma_\mu\Psi_1], \quad B_{3\mu} \equiv \bar{\Psi}_1\sigma_\mu\Psi_1 - \bar{\Psi}_2\sigma_\mu\Psi_2, \\ \mathcal{A}_4 \equiv \bar{\Psi}_1\Psi_1 + \bar{\Psi}_2\Psi_2, \quad B_{4,\mu} \equiv \bar{\Psi}_1\sigma_\mu\Psi_1 + \bar{\Psi}_2\sigma_\mu\Psi_2, \quad \mu = 0, 1, 2. \quad (21)$$

Here, A_1, A_2, A_3, B_1, B_2 and B_3 transform as triplets under SU(2) and A_4 and B_4 transform as SU(2) singlets, the γ_μ are (anti-Hermitian) Dirac (space-time) 2×2 matrices and the σ_i , $i = 1, 2, 3$, are the (Hermitian) 2×2 SU(2) Pauli matrices.

Naively, the composite meson fields A_i and B_i transform under a global SU(2) group [11, 18]. However, because of the Hartree–Fock form of the lattice Hamiltonian (6), these composites realize a strongly coupled local (gauged) SU(2) $\otimes$ U_S(1) group, which (following the above discussion) enables the representation of the SU(2) gauge link variables in terms of the B_i composites [11]:

$$V_{i\mu} = \exp(i\sigma^a B_a) = \cos(|\mathbf{B}_{i\mu}|) + i\sigma \cdot \mathbf{B}_{i\mu} \sin(|\mathbf{B}_{i\mu}|)/|\mathbf{B}_{i\mu}|, \quad (22)$$

where $a = 1, 2, 3$ are SU(2) indices, and boldface letters indicate vectors in SU(2) space. (The reader should bear in mind the trick (12), of working with large SU(N) instead of the SU(2) group, so as to guarantee a finite $\beta_{\text{SU}(N)}$ coupling while working with a strong U_S(1) group.)

The VEV of the matrix $\langle M^{(i)} \rangle = u\sigma_3$ is proportional to the chiral condensate u , which is responsible for generating dynamically a (parity-conserving) mass gap for the holons [11]. Substituting (20) into (19) and performing a naive perturbative expansion over the fields $\mathbf{B}$, one finds

$$\ln I_0(\sqrt{y_{i,\mu}}) \propto K^2 u^2 [(B_{i\mu}^1)^2 + (B_{i\mu}^2)^2] + \text{interaction terms}. \quad (23)$$

From this it follows that two of the SU(2) gauge bosons, namely the B^1 and B^2 , become massive, with masses proportional to Ku , whereas the gauge boson B^3 remains massless. These mass terms break SU(2) down to a U(1) subgroup.

We draw the reader's attention to the similarity of the above mechanism for symmetry breaking with the situation in the adjoint gauge–Higgs model [19]. There, the SU(2) symmetry is also broken down to a U(1) subgroup whenever the constant multiplying the Higgs–gauge interaction is larger than a critical value. In our case the role of this constant is played by K^2 , as

can be seen by the formal analogy between the adjoint-Higgs–gauge interaction terms and (19). In our approach, symmetry breaking was achieved via the infinitely strong $U_5(1)$ coupling. In view of the above analogy with the adjoint-Higgs model, however, one may speculate that an interesting phase diagram for the symmetry breaking of $SU(2)$ could also emerge due to the K^2 coupling, whatever the $U_5(1)$ coupling is.

At this point we would like to clarify certain points concerning the above-mentioned models: the lattice systems we examined above are simplifications of the actual situation encountered in realistic condensed-matter systems, where in some regions of their parameter space, the Fermi surface is characterized by pockets of finite size rather than points. In such models, the spin-charge separation ansatz is more subtle than the one presented here. The interested reader can find more details of such issues in the recent literature [20], where detailed phase diagrams are presented. However, for our purposes here, we are simply interested in the compositeness aspects of the above decomposition of spin and charge d.o.f., which are merely used here as a motivation for our particle-physics-oriented analysis rather than a rigorous study of realistic condensed-matter systems and their properties. In this respect, in what follows we shall make use of the above formalism in order to draw some useful lessons and thus be able to discuss the notion of composite operators and their relevance to $N = 8$ supergravity phenomenology.

We conclude this section by mentioning the possibility of extending the above results to other gauge groups, provided ansatz (3) for spin-charge separation in the simple $SU(2)$ case that represents the spin d.o.f. in an antiferromagnetic system is modified accordingly.

3. Supersymmetric 2 + 1-dimensional condensed-matter models with dynamical gauge bosons

3.1. $\mathcal{N} = 1$ supersymmetry

A further step was taken in [21, 22], where conditions were derived under which the continuum low-energy limit of the above lattice Hamiltonian becomes supersymmetric. First, it was observed, on the basis of an appropriate power counting of the fundamental spinon and holon d.o.f. and taking into account the gauge freedom, that there are regions in the parameter space of the underlying $t - J$ model which exhibit dynamical $\mathcal{N} = 1$ supersymmetry between the fundamental constituents of the model, namely the spinons and holons z_a and ψ_a , $a = 1$ and 2 , in ansatz (3).

Mavromatos and Sarkar [22] considered the lattice $t - J$ models discussed in the previous section, but also included non-nearest-neighbour hopping and interaction terms in the antiferromagnetic sublattices. Such models have been used in theoretical modelling of d -wave high-temperature superconductors [23]. The analysis led to the following $\mathcal{N} = 1$ supersymmetric continuum Lagrangian in terms of component spinon ψ and holon z fields:

$$L = g^2 \sum_{a=1}^2 \left[D_\mu \bar{z}_a D^\mu z_a + i \bar{\psi}_a \not{D} \psi_a + \bar{f}_a f_a + 2i(\bar{\eta} \psi_a \bar{z}_a - \bar{\psi}_a \eta z_a) \right], \quad (24)$$

where D_μ denotes the gauge-covariant derivative with respect to the $U_5(1)$ gauge field, and f_a is an auxiliary field. The field η is a Majorana fermion, which is viewed as the supersymmetric partner of the Abelian $U_5(1)$ gauge field, needed to reproduce (as an appropriate Lagrange

multiplier field [22]), the CP^1 σ -model constraint, which in superfield notation reads

$$\sum_{a=1}^2 \bar{\phi}_a \phi_a = 1, \quad (25)$$

with the superfield ϕ_a being given by

$$\phi_a = z_a + \theta \psi_a + \frac{1}{2} \theta^2 f_a. \quad (26)$$

This contains, for each colour, a complex scalar z_a , a Dirac spinor ψ_a and a complex auxiliary field f_a . We refer the reader to [22] for a detailed discussion on how this formalism can actually describe realistic models of doped antiferromagnets in certain regions of parameter space that guarantee $\mathcal{N} = 1$ supersymmetry and under what circumstances one can have $\mathcal{N} = 2$ supersymmetric extensions.

For our purposes we note that, in component form, the constraint (25) reads for physical fields:

$$\sum_{a=1}^2 |z_a|^2 = 1 \quad \text{and} \quad \sum_{a=1}^2 (\bar{z}_a \psi_\alpha + z_a \bar{\psi}_\alpha) = 0, \quad (27)$$

and these results can be generalized to non-Abelian cases, such as the broken $\text{SU}(2)$ case discussed previously.

In addition to supersymmetry between the constituent spinon and holons, it was suggested in [24], motivated by the Hartree–Fock form (6) and the associated spinon and holon bilinears (7) forming gauged group-link variables, that composite fields made out of the fundamental constituents z and Ψ exhibit an on-shell $\mathcal{N} = 2$ supersymmetry, in the above-mentioned regions of the parameter space where the constituent $\mathcal{N} = 1$ supersymmetry exists. Since these composite fields could constitute observable excitations of these materials, this kind of composite supersymmetry could provide a way to obtain some exact results in the phase diagrams of doped antiferromagnets and hence high-temperature superconductors.

This induced $\mathcal{N} = 2$ supersymmetric structure in the low-energy composite theory provides exact results on the phase structure of doped antiferromagnets in this regime of the corresponding parameter space because the effective field theory is [24] $\mathcal{N} = 2$ scalar quantum electrodynamics, in which extra $\mathcal{N} = 1$ matter multiplets may be present, with nontrivial superpotentials. The moduli spaces of such theories have been studied extensively in [25], following the pioneering work of Seiberg and Witten [26] in four dimensions, with the result that there is a nontrivial non-perturbative infrared fixed point, induced by the matter multiplets. We recall that a $(2+1)$ -dimensional gauge theory is super-renormalizable, without ultraviolet divergences, by elementary power counting. However, it has a nontrivial low-energy (infrared) structure. There are increasing hints that four-dimensional $\mathcal{N} = 8$ supergravity theory might also lack the expected ultraviolet divergences. However, the low-energy (infrared) structure of the four-dimensional $\mathcal{N} = 8$ theory remains to be elucidated.

The nontrivial infrared structure of the $(2+1)$ -dimensional theory was interpreted in [24] as indicating a deviation from the trivial Landau-liquid fixed point, implying several observable properties in the normal phase of the superconductor. In addition, these theories also exhibited a superconducting phase [10, 24], associated with a phase in which the $\text{SU}(2)$ non-Abelian symmetry is broken down to an Abelian subgroup. The superconducting phase corresponds to the Coulomb phase of the Abelian subgroup (not to be confused with electromagnetic gauge symmetry), in which the corresponding gauge boson is massless. This leads to an

anomalous current–current one loop diagram with a massless pole that, according to the anomaly mechanism of [10], results in superconductivity (according to the Landau criterion) upon coupling to a real external electromagnetic field. The Higgs phase of the Abelian subgroup of the original SU(2) gauge group, in which the gauge field is massive, corresponds to the pseudogap phase of the underdoped cuprates (in condensed-matter parlance), which occurs for low doping. In addition, non-perturbative (monopole) configurations have also been discussed in [24], in various supersymmetric effective gauge theories, following [25], in an attempt to discuss the formation of domain-wall structures, such as the stripe phase of high-temperature superconductors, in which the spin and charge are separated in spatial domains in the underdoped (non-superconducting) regions of the material. Thus, the emergence of dynamical composite gauge fields in supersymmetric 2 + 1 lattice models and nontrivial aspects of its spontaneous breaking by composite scalar fields can, in principle, be subjected to experimental tests.

This completes our review of relevant known exact results in condensed-matter systems in which a composite gauge symmetry is realized dynamically.

3.2. $\mathcal{N} = 2$ supersymmetry

Having in mind the $\mathcal{N} = 8$ supergravity theory of interest, we now describe an $\mathcal{N} = 2$ supersymmetric SU(2) coset field theory in 2 + 1 dimensions, in which the superpartners of the theory are composite fields generalizing the previous construction out of fundamental spinon and holon d.o.f. The latter are assumed to belong to $\mathcal{N} = 1$ supermultiplets, but the supersymmetry may be extended to $\mathcal{N} = 2$, if the gauge condition $\partial^\mu A_\mu = 0$ is satisfied. Hlousek and Spector [27] needed this gauge condition in the Abelian–Higgs model in order to identify the gauge field with a topologically conserved current, which was at the origin of their elevation of the supersymmetry. The same condition was found for the CP¹ model in [28], and was generalized to Yang–Mills theory in [29]. Its recurrence suggests that this gauge condition is necessary, model independently, for supersymmetry elevation in 2 + 1 dimensions.

Our starting point is to assume the existence of two complex $\mathcal{N} = 1$ scalar supermultiplets (i is now a flavour index)

$$\phi_i = z_i + \theta \psi_i + \frac{1}{2} \theta^2 f_i, \quad i = 1, 2, \quad (28)$$

where the two-component Grassman coordinate θ is real, and the complex field components z_i , ψ_i and f_i are, respectively, the scalar, the two-component fermion and the auxiliary field. Note that, in 2 + 1 dimensions, a real two-component fermion represents one d.o.f., such that, in the supermultiplet (28), the complex scalar d.o.f. compensates for the complex fermionic d.o.f., and the role of the auxiliary field is to balance the numbers of bosonic and fermionic components in the superfield. We also remind the reader that in 2 + 1 dimensions there is no chirality condition, as this would restrict the space–time dependence of the field components.

Based on these fundamental d.o.f., the following $\mathcal{N} = 1$ composite supermultiplets were considered in [24], where $D_\alpha = \partial/\partial\theta^\alpha + i(\not{\partial}\theta)_\alpha$ is the supercovariant derivative:

- Scalar supermultiplets, forming an SU(2) triplet,

$$\begin{aligned} \Phi^1 &= D^\alpha \phi_1 D_\alpha \phi_1, \\ \Phi^2 &= D^\alpha \phi_2 D_\alpha \phi_2, \\ \Phi^3 &= D^\alpha \phi_1 D_\alpha \phi_2. \end{aligned} \quad (29)$$

- Parity-conserving vector supermultiplets, forming an SU(2) triplet,

$$\begin{aligned} V_\alpha^1 &= \text{Re}\{\phi_1 D_\alpha \phi_2 + \phi_2 D_\alpha \phi_1\}, \\ V_\alpha^2 &= \text{Im}\{\phi_1 D_\alpha \phi_2 + \phi_2 D_\alpha \phi_1\}, \\ V_\alpha^3 &= \text{Re}\{\phi_1 D_\alpha \phi_1 - \phi_2 D_\alpha \phi_2\}. \end{aligned} \quad (30)$$

- A parity-violating composite vector supermultiplet, forming an SU(2) singlet,

$$V_\alpha^4 = \text{Re}\{\phi_1 D_\alpha \phi_1 + \phi_2 D_\alpha \phi_2\}. \quad (31)$$

The next step is to gather the $\mathcal{N} = 2$ supermultiplet d.o.f. into the following complex superfields:

$$G^a = \text{Re}\{\Phi^a\} + iD^\alpha V_\alpha^a \quad a = 1, 2, 3. \quad (32)$$

Each of these contains a real scalar field, a gauge field, a complex two-dimensional gaugino and a real auxiliary field. Let us denote by g the gauge coupling and by f^{abc} the SU(2) structure constants. Using the Wess–Zumino gauge for V_α^a , it was shown in [29] that the following Lagrangian describes the resulting $\mathcal{N} = 2$ supersymmetric SU(2) gauge theory:

$$\mathcal{L} = \int d^2\theta \left| D^\alpha G^a + g f^{abc} \left(G^b V^{c\alpha} + \frac{i}{2} D^\alpha (V^{b\beta} V_\beta^c) \right) \right|^2, \quad (33)$$

provided the gauge condition

$$\partial^\mu A_\mu^a = 0 \quad (34)$$

holds for any gauge index a . This gauge condition was found in [29] to be necessary to cancel unwanted contributions proportional to $\partial^\mu A_\mu^a$, when $\mathcal{L}$ is expanded in terms of the field components. Although we exhibit here an SU(2) model, it is clear that this construction may be extended to any SU(n) symmetry group and is not restricted to the ‘= 2’ case presented here.

An important question is how the above-described composite (‘meson-like’) fields become dynamical. In section 2, we have seen that the SU(2) holon composites, which are obtained by integrating out a non-dynamical (strongly coupled) U(1), can become dynamical, as a result of their gauging, which is a direct consequence of the Hartree–Fock lattice Hamiltonian (6) and the associated spinon and holon bilinears (7). A similar situation characterizes the supersymmetric composites, where again there are lattice Hamiltonians of Hartree–Fock type, as we discussed extensively in section 3. However, there are a few subtleties, which we now outline. Unlike the non-supersymmetric case, where the mass gaps of the spinon z and holons Ψ were different, in the supersymmetric case one is not allowed to integrate out the spinons, which now are degenerate in mass with the holons. Hence both the groups $U_5(1)$ and SU(2) appear strongly coupled at the constituent level. However, we can apply here again the trick of working with large- N SU(N), instead of the SU(2) group. This implies that, as in the non-supersymmetric case, one can consider finite couplings for the non-Abelian part, in which case one may demonstrate that the above-described $\mathcal{N} = 2$ SU(N) supersymmetric composites become dynamical.

Finally, the addition of matter in the fundamental representation necessitates two new $\mathcal{N} = 1$ scalar supermultiplets, made of composite field components, and containing different d.o.f. from those considered so far

$$Q_1 = \phi_1^2, \quad Q_2 = \phi_2^2, \quad (35)$$

which we represent as a two-component superfield $Q = (Q_1, Q_2)$. We note that the matter superfield components must be complex, since they transform according to the fundamental representation of SU(2).

It is easy to see that the gauge–matter interaction then reads

$$\int d^2\theta \left\{ \frac{1}{2} ((D^\alpha Q)^\dagger - ig Q^\dagger V^{a\alpha} T^a) (D_\alpha Q + ig V_\alpha^a T^a Q) + g Q^\dagger \Phi^a T^a Q \right\}, \quad (36)$$

where the T^a are the SU(2) generators in the fundamental representation.

In this respect, we note that, from the point of view of condensed-matter models, these matter fields describe interactions external to the two sublattice structures depicted in figure 1, e.g. by coupling the superconducting planes, etc. For our purposes however, such matter fields may be integrated out, which renders the gauge multiplets dynamical, by inducing kinetic terms for the gauge fields, in analogy with the z -spinon integration in the non-supersymmetric models of section 2. In this way one still has formally a strongly coupled SU(N) gauge theory, but the quantum corrections renormalize the effective coupling.

To summarize, starting from the two complex $\mathcal{N} = 1$ scalar supermultiplets (28), we have the following construction:

- Build three complex $\mathcal{N} = 1$ scalar supermultiplets (29) and three $\mathcal{N} = 1$ vector supermultiplets (30), all composed of more elementary d.o.f.
- Gather these d.o.f. into three $\mathcal{N} = 2$ supermultiplets (32), so as to obtain an $\mathcal{N} = 2$ supersymmetric SU(2) gauge theory, which is possible if the gauge condition (34) is imposed.
- Add the composite matter $\mathcal{N} = 1$ supermultiplets (35) so as to generate an $\mathcal{N} = 2$ supersymmetric SU(2) theory interacting with matter in the fundamental representation.

The theory obtained in the Lagrangians (33) and (36) can of course be expressed in a more compact way using the $\mathcal{N} = 2$ superspace formalism, but the point of the present derivation is to start from $\mathcal{N} = 1$ supersymmetry, using the $\mathcal{N} = 1$ formalism, to arrive at a theory invariant under $\mathcal{N} = 2$ supersymmetry.

As mentioned in the previous section, one may use exact results on $\mathcal{N} = 2$ gauge theories to understand the dynamical behaviour of this model.

4. Summary and prospects

We have shown that supersymmetric models in $2 + 1$ dimensions, formulated either at the lattice level or as coset field theories, exhibit the emergence of dynamical gauge bosons realizing an SU(N) gauge group. There are also composite scalar fields whose expectation values may break this composite gauge symmetry spontaneously to a subgroup that is realized in the Coulomb phase. Specifically, in the lattice models discussed in sections 2 and 3, the underlying constituents are spinons and holons, and the composite scalars are in the adjoint representation of the gauge group. In the coset field theories discussed in section 3, there may also appear scalar fields in the fundamental representation of the SU(N) gauge group. By formulating $\mathcal{N} = 2$ supersymmetric versions of these lattice and coset field theories, the derivation of these results may be placed on a rigorous basis.

An important feature of our models was that the gauge fields were strongly coupled, in the sense of not having kinetic terms (i.e. plaquette terms in lattice models). However, we have argued that, by working in the large N limit, where N is the number of species (i.e. sublattices

or nodes) that must be even for reasons of parity conservation, one could formally reinstate a finite value of the $SU(N)$ gauge coupling, thus making the gauge fields dynamical.

Moreover, by using arguments based on a Hartree–Fock approximation in the microscopic models, we have supported the idea that composite gauge fields made out of spinons and holons can, also in the supersymmetric regions of the parameter space, become dynamical, leading to full-fledged $\mathcal{N} = 2$ supersymmetric $SU(N)$ gauge theories. In $(2+1)$ -dimensions these are characterized by a nontrivial infrared fixed-point structure, which constitutes an additional, more rigorous one for inferring the dynamical nature of the gauge fields from the effects of the quantum corrections. Even if the bare action is characterized by infinite gauge couplings, quantum loop corrections can generate such terms, thereby generating the dynamics of the gauge fields. This feature has been seen explicitly in non-supersymmetric models, by the integration of the spinon (magnon) fields z . In supersymmetric theories, formally one works with additional matter multiplets, which can be integrated out, thereby inducing finite (renormalized) gauge kinetic terms and hence rendering the strongly coupled gauge theory dynamical.

Although in this work we have dealt explicitly with unitary $SU(N)$ gauge theories, in which N must be even because of arguments based on the parity invariance that characterizes the antiferromagnetic case, nevertheless other gauge groups are expected to lead to qualitatively similar conclusions.

These features are exactly what one might like to see [1] in $\mathcal{N} = 8$ supergravity in $3+1$ dimensions [2], which has a non-anomalous $SU(8)$ gauge symmetry [9]. If this gauge symmetry would become dynamical, an adjoint scalar multiplet could break it spontaneously into a rank-7 subgroup such as $SU(5) \times U(1)^3$ or $SU(3) \times SU(2) \times U(1)^3$, and scalars in fundamental representations could, in principle, reduce the rank to the $SU(2) \times U(1)$ gauge group of the standard model. However, infrared behaviour in $3+1$ dimensions is different from that in $2+1$ dimensions, and we have no real understanding of how the composite gauge fields might become dynamical in this case. In this connection, we recall that $\mathcal{N} = 8$ supergravity cannot be obtained simply as some limit of string theory without the appearance of additional low-mass fields, and it is possible that these might play an essential role.

On the other hand, the results displayed here may be directly applicable to the $2+1$ -dimensional exceptional supergravity theories that can be obtained by truncations of $\mathcal{N} = 8$ supergravity in $3+1$ dimensions, which have the following non-compact coset structures: $F_{4(4)}/USp(6) \times SU(2)$, $E_{6(2)}/SU(6) \times SU(2)$, $E_{7(-5)}/SO(12) \times SU(2)$ and $E_{8(-24)}/E_7 \times SU(2)$. Since gravity in $2+1$ dimensions is topological, it might be just an ‘inessential complication’ in the understanding of the dynamics of these theories. They could be more complete laboratories for probing the dynamics of the $3+1$ -dimensional $E_{7(7)}/SU(8)$ theory, though they still evade the tough issue of the infrared behaviour in $3+1$ dimensions.

Our work is complementary to the mechanism for making dynamical the gauge fields of three-dimensional $\mathcal{N} = 8$ supergravity based on the maximal supersymmetric effective low-energy theory of multiple M2 branes that was presented in [30] (and references therein). There, the relevant non-Abelian gauge fields have no Maxwell term, but only mixed (parity-conserving) Chern–Simons structures. Our approach is to suggest that a kinetic term for the gauge field may be generated through renormalization-group effects in the infrared region of the effective theory, in direct analogy to what happens in planar condensed-matter models on the lattice and their continuum effective theories. It remains to be seen whether such an approach, which works so well in three space–time dimensions, can describe correctly the properties of the four-dimensional $\mathcal{N} = 8$ supergravity theory.

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