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Scattering cross-section of a transformation optics-based metamaterial cloak

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Abstract. We present experimental quantitative scattering cross-section (SCS) measurements for a metamaterial cloak. The cloak is nearly identical to that reported in 2006; however, quantitative experimental measurements have not yet been reported for such a structure. This cylindrically symmetric cloak is designed to operate at a frequency of 10 GHz and to reduce the SCS of a cylinder 50 mm in diameter. Despite being only a crude approximation of the ideal transformation optical design, the fabricated metamaterial cloak is shown to reduce the SCS of the cylinder over the frequency range from 9.91 to 10.14 GHz, a span of 230 MHz or a 2.3% bandwidth. The maximum reduction in the SCS is 24%. This result provides a useful experimental, quantitative benchmark that can form the basis for comparison of the performances of future improved cloaking structures.

Contents

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1. Introduction

Transformation optics is a general technique that makes use of coordinate transformations to design a functional electromagnetic device or structure. Although the transformation optical approach can be utilized to design an enormous range of optical and quasi-optical structures, such as lenses [1, 2], field rotators [3], concentrators and waveguide bends [4], one of the most intriguing examples of a transformation optical device is that of the 'invisibility cloak'. The prospect of cloaking was introduced in 2006 in the wave-optic [5] and ray-optic [6] limits. Pendry *et al* [5] suggested a radial transform that would squeeze a volume of space into an infinitesimal region, effectively rendering the interior volume electromagnetically undetectable. Using a cloaking transformation in conjunction with Maxwell's equations, a prescription for a spherically (or cylindrically) symmetric material that would exhibit exactly the same scattering solution as empty space was developed. At the time, it was suggested that although the material specification was extremely challenging, the targeted material properties might be achievable by employing artificially structured metamaterials.

The introduction of the coordinate transformation technique to electromagnetic design and the experimental demonstration of a metamaterial cloak [7] and other compelling structures have stimulated fervent interest in all aspects of transformation optics [1–4], [6], [8–11] and cloaking [12–20]. As new techniques in the design of artificial materials emerge, the cloak will likely serve as a useful canonical example for the application of these nascent tools. It is thus useful to seek one or more figures of merit by which progress in the implementation of cloaking designs can be experimentally assessed.

The scattering cross-section (SCS), sometimes referred to as the scattering area (or scattering width in two dimensions), is a measure of the total energy scattered by an object normalized to the incident energy density. The SCS includes the effects of both reflections and shadowing, making it a natural metric by which to judge the efficacy of a cloak. The SCS is often used as the metric of interest in theoretical cloaking designs [21–24] and experimental results of the SCS have been reported for a plasmonic cloak [18]. This important experimental evidence has, however, not been reported for transformation optics-based cloaking.

The SCS measurement is of particular importance in assessing the effective bandwidth of a cloaking structure. The typical material properties of transformation optical cloaking structures are extreme, requiring values of the relative permittivity or permeability components that range between zero and values much greater than unity. If artificial magnetic metamaterials with a paramagnetic response are used as the building blocks for transformation optical media or if materials with an effective refractive index below unity are used, then the underlying materials must exhibit frequency dispersion. Since the frequency dispersion associated with metamaterials is often severe, the usable bandwidth of a transformation optical metamaterial is not obvious.

An implementation of the full transformation optical cloaking parameters would reduce the SCS of an enclosed object to zero [20]. Any physical cloak implementation will be an approximation that will diminish the SCS to some nonzero value. A minimal SCS is a key figure of merit for the metamaterial cloak.

2. Cloak design

Subsequent to the introduction of the transformation optical technique, an approximate metamaterial cloak was designed and fabricated, demonstrating the underlying concept [7]. Transformation optical media, including cloaking structures, are generally complex, requiring independent gradients in one or more of the constitutive tensor parameters. Although there exists an extraordinary degree of flexibility in the design of a given structure—there are an infinite number of transformations for a given device—the resulting material is nevertheless more demanding than would be deemed feasible using conventional materials. In fact, an experimental cloak exhibiting the full transformation is yet to be demonstrated.

To alleviate the design and fabrication burden, a 'reduced-parameter' cloak with simplified requirements was proposed and fabricated to illustrate the potential of metamaterials as a means of realizing transformation optical media. The origin of the reduced parameter cloak is that in the ray limit, the propagation of waves is governed by an anisotropic index, $n_\rho = \sqrt{\varepsilon_z \mu_\theta}$ and $n_\theta = \sqrt{\varepsilon_z \mu_\rho}$. Since only combinations of parameters enter into the anisotropic index, it is possible to remove the need for a magnetic response in one of the propagation directions, arriving at

$$\mu_\rho = \left(\frac{\rho - R_1}{\rho}\right)^2, \quad \mu_\theta = 1 \quad \text{and} \quad \varepsilon_z = \left(\frac{R_2}{R_2 - R_1}\right)^2. \tag{1}$$

In equation (1), R_1 is the inner radius of the cloak and R_2 is the outer radius. The reduced parameter cloak exhibits an impedance mismatch at the outer interface and also imposes limitations on the efficacy of the cloak in reducing scattering from cylindrical waves [25]. Furthermore, the size of the cloak is of the order of a wavelength, and so errors are expected from the eikonal approximation due to the gradients in the effective properties. Finally, the finite size of the metamaterial elements used in the realization of the cloak leads to a discrete stepping of the constitutive parameter profile rather than a continuous one, which can also be expected to contribute to scattering. In this case, rather than a continuous medium, ten metamaterial layers are used. The reduced cloak was expected to exhibit both reflection and shadowing, both of which will contribute to the SCS. Although far from ideal, the reduced-parameter cloak operated as was qualitatively expected from simulations, diverting the incident fields around the central region and partially restoring the planar waves on the opposite side.

To better quantify the operation of the reduced parameter cloak, we perform measurements of the SCS on a 50 mm diameter metal cylinder with and without the cloak. The cloak consists of ten concentric rings of patterned circuit board materials, held together by circuit board spokes. The metamaterial elements patterned on the circuit board are designed to achieve a gradient in the radial permeability from roughly zero to a value of 0.25, with the permittivity held constant at a value of 3.6. The distribution of material parameters is designed to approximate the reduced parameter cloak, described in more detail in [12]. These results are compared with an analytic model of a perfectly homogenized realization of the cloaking material that was used.

3. Measurements

In two dimensions, electromagnetic scattering measurements are somewhat easier than in three dimensions, because the polarization is restricted; thus cross-polarization concerns can be ignored and the scattering problem becomes scalar in character. In fact, the same type of field mapping data reported in the original metamaterial cloaking results [7] can be used to obtain the SCS of a cloaked object.

To experimentally determine the SCS of the metamaterial cloak, we perform scattering experiments using a planar waveguide apparatus [26]. The planar waveguide supports a single transverse-electric mode with the electric field polarized normal to the plates over the X-band range of frequencies (8–12 GHz). Antennas in the upper plate acquire phase-sensitive, two-dimensional (2D) field maps over the region interior to and exterior to the cloak structure.

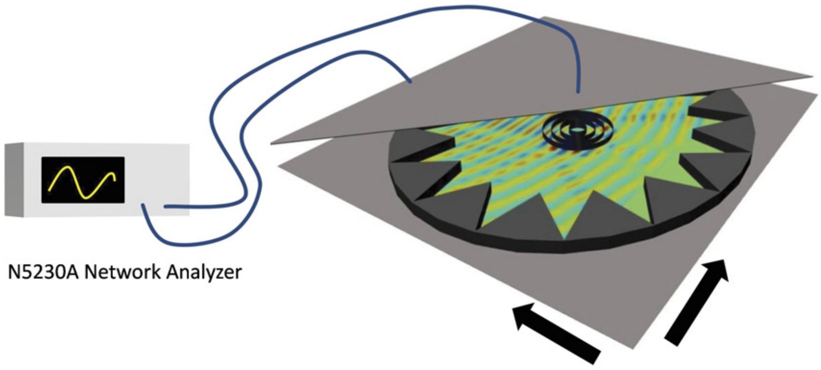

Figure 1. A 3D cutaway model of the waveguide apparatus with a cloak inside.

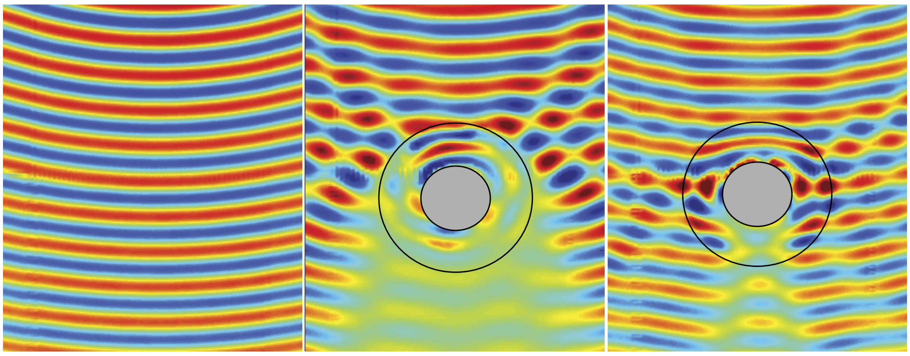

Figure 2. A snapshot of the electric field mapping is shown (a) with no scatterer present and for a cylinder and cloak (10 GHz) (b) away from the cloak's frequency of operation (9 GHz) and (c) near the cloak's frequency of operation (10 GHz). Only the fields in a ring around the scattering object are needed to find its SCS. The scale is arbitrary but consistent across the images, with red indicating the most positive electric field and blue the most negative.

The significantly subwavelength antennas are weakly coupled to the waveguide and do not introduce significant additional scattering. An Agilent N5230A phase network analyzer is used to measure the coupling between the waves launched into the waveguide and the antennas, expressed in terms of scattering (S-) parameters (shown schematically in figure 1). The bottom plate translates in the lateral directions as measurements are taken, such that a spatial field map can be generated. The amplitude and phase of the transmitted scattering parameter (S_{12}) are a measure of the electric field at that location. Linear steppers attached to the lower plate enable the measurement process to be automated.

To make an SCS measurement, both the incoming and scattered fields must be measured separately. The incoming field, E_{inc}, is determined by performing a measurement without introducing any scatterer into the waveguide. A second measurement, in which a scattering object is introduced into the waveguide, provides E_{tot}, defined by $E_{\text{tot}} = E_{\text{s}} + E_{\text{inc}}$. These two measurements are subtracted from one another to find the scattered field, E_{s}. Example field mapping data for an empty chamber and a cloak are shown in figure 2.

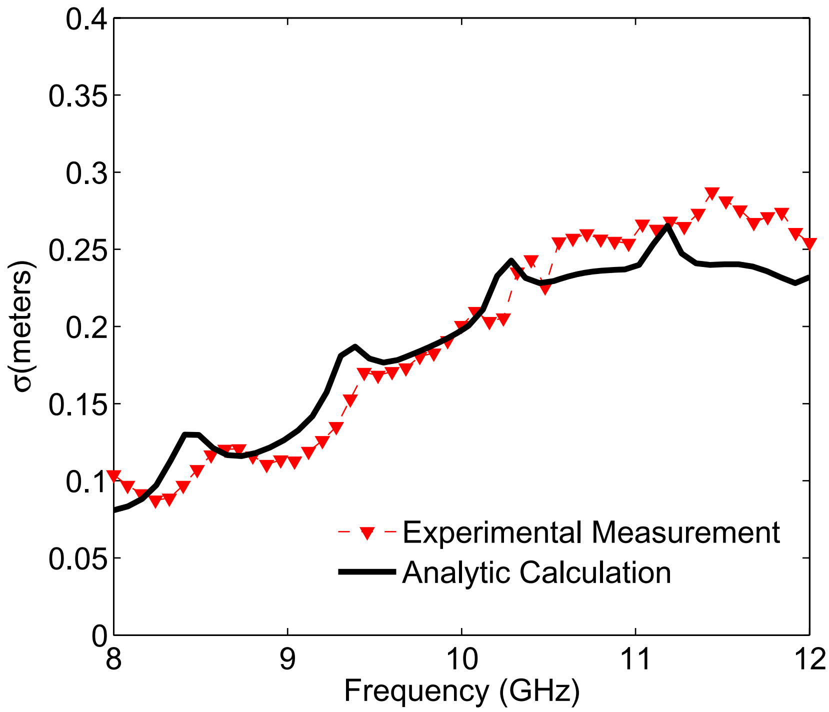

Figure 3. Measurements of the SCS for a dielectric cylinder 76 mm in diameter are compared with an analytic calculation. The cylinder has a dielectric constant of 2.45, which was determined experimentally by retrieval in a closed waveguide.

As an alternative to the direct integration of the Poynting vector described in [18], which requires experimentally finding the gradients in the electric field, the SCS was found with only a knowledge of the fields on a ring that encompasses the cloak. Measurements of the scattered field were expanded as

$$E_s(\rho,\theta)=\sum_{m=0}^{\infty} H_m^{(1)}(k_0\rho)\,(A_m\sin(m\theta)+B_m\cos(m\theta))\,. \tag{2}$$

The amplitudes of each term in this expansion, A_m and B_m, were easily found using the orthogonality of the sin and cos functions and the fields at a radius, ρ_0.

$$A_m=\frac{1}{\pi\,(1+\delta_{m0})\,H_m^{(1)}(k_0\rho_0)}\int_0^{2\pi} E_s(\rho_0,\theta)\sin(m\theta)\,\mathrm{d}\theta, \tag{3}$$

$$B_m=\frac{1}{\pi\,(1+\delta_{m0})\,H_m^{(1)}(k_0\rho_0)}\int_0^{2\pi} E_s(\rho_0,\theta)\cos(m\theta)\,\mathrm{d}\theta. \tag{4}$$

With the Bessel amplitudes, A_m and B_m, being known, the SCS, σ, was calculated using

$$\sigma=\frac{2}{k_0\,|E_{\mathrm{inc}}|^2}\sum_{m=0}^{\infty}\left(|A_m|^2+|B_m|^2\right)(1+\delta_{m0}). \tag{5}$$

To verify the accuracy of this process for obtaining the SCS, experimental results for the SCS of a polycarbonate dielectric cylinder were first compared with analytic calculations. The exact dielectric constant of the cylinder material was determined through an experimental waveguide retrieval process. The resulting comparison of the calculated and experimental SCS results shows good agreement, as can be seen in figure 3.

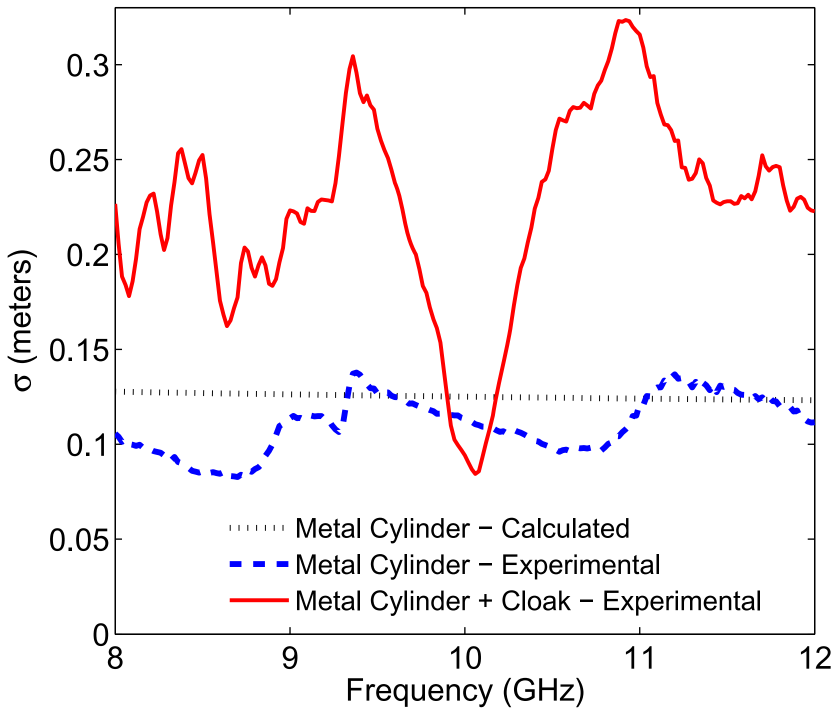

Figure 4. SCS measurements for a cloak 117 mm in diameter and an aluminum cylinder 50 mm in diameter are shown. The cross section of the cloaked cylinder drops below that of the inner ring near 10 GHz.

4. Results

The SCS versus frequency of a bare metallic cylinder and the cylinder surrounded by the cloak are shown in figure 4. The SCS of the cloaked cylinder drops below the uncloaked value, reaching a minimum at 10.06 GHz, indicating that cloaking occurs in this frequency range. The cloak, which is 117 mm in diameter, actually reduces the SCS of the metal cylinder, which is 50 mm in diameter. At the maximum cloaking frequency, the cloak reduces the SCS of this cylinder from 110 mm without the cloak to 84 mm: a reduction in σ of 24%.

A comparison of the expected SCS for a cylinder (treated as a perfect electric conductor (PEC)) with our experimental result shows some deviations. This disagreement is associated with the fact that the cylinder does not make electrical contact with the top plate of the waveguide used in these measurements. This is a necessity in the experimental setup as the plates must be free to move relative to one another. A simple PEC model for this boundary is thus insufficient to characterize its scattering behavior. The measured SCS is nonetheless expected to be an accurate characterization of the scatterer employed under the experimental conditions.

While a reduction in the total SCS from a device that is more than double the size of the initial scattering body is an intriguing result, a perfect cloak would reduce the SCS to zero. A reasonable starting point in understanding the errors introduced in the implementation of this device is a simple boundary matching model of the employed cloak [21]. There are several approximations made in the realization of this cloak that can be taken into account using this model. These include the 'reduced parameter' design and the discrete number of layers used to

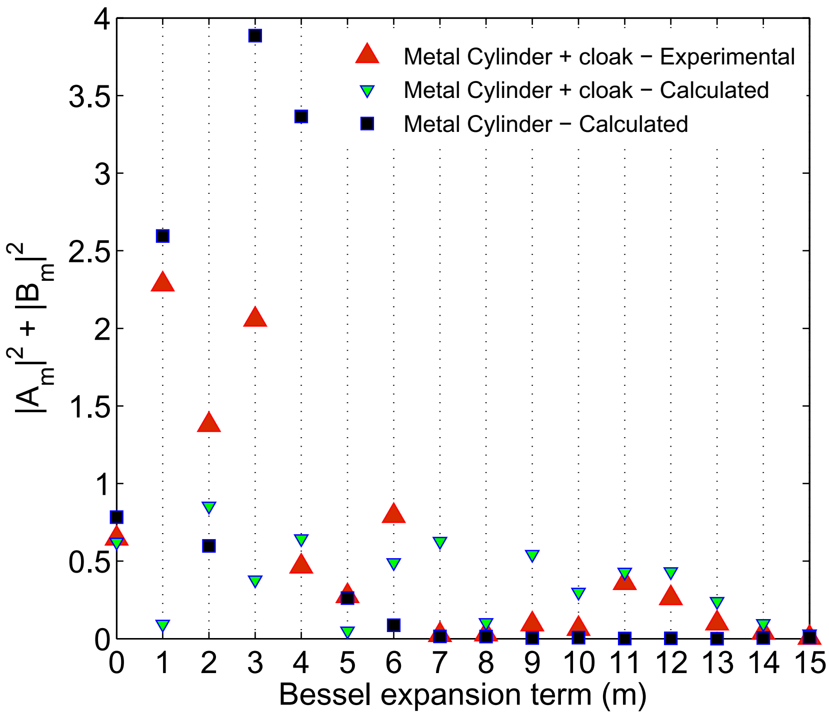

Figure 5. Bessel Decomposition of the measurement of the cloak at 10.06 GHz, the modeled PEC at 10.06 GHz and the modeled cloak at 10.06 GHz.

realize it. At the lossless, effective medium level the extent to which these errors will perturb the cloaking properties from a perfect cloak can be semi-analytically analyzed and compared with the experimental result. This comparison can be performed for the SCS as well as each term in the Bessel decomposition for the scattered wave.

Figure 5 shows the Bessel decomposition at the maximal cloaking frequency of 10.06 GHz of the scattered wave, which was experimentally found for the cloak and the metal cylinder. The coefficients are compared to the calculated decomposition of the lossless, homogeneous, layered cloak described in [21]. This model predicts an SCS of 34 mm for the cloak, compared with a minimum measured SCS of 84.4 mm. The error in measurement is associated with the loss in our metamaterial cloak and errors in assuming that the metamaterial realization of the cloak may be treated as homogenized rings. In addition to this, our incident fields show some curvature over the width of the cloak, whereas the analytical calculation assumes that the incoming waves are planar.

The data presented in figure 5 suggest that the coefficients of the scattered waves from the physical and analytical cloaks drop below those of the cylinder in the lower order terms while they are greater in the higher order terms; this is attributed to the overall increase in size of the device.

5. Conclusion

While only a modest reduction in SCS is observed for the reduced-parameter cloak, the fact that the SCS drops at the design frequency—and to a value below that of the bare cylinder—provides additional validation of the metamaterial approach. The cloak, like any transformation optical structure, represents a complex medium requiring precisely controlled material parameters. The resonant metamaterials employed make the actual implementation very challenging, so that if the target properties are not realized exactly, a significant degradation in performance can be expected. A variety of factors could influence how closely the unit cell design achieves the desired constitutive parameter set. While fabrication tolerances and precision will undoubtedly play a role, the underlying effective medium approach in the design stage must be considered. For both the original and present metamaterial cloak designs, cubic unit cells were considered in the simulation stages, with periodic boundary conditions assumed. The actual structure is non-periodic, with non-cubic unit cells. By properly taking actual details such as these into consideration, it is conceivable that considerable improvements in cloaking and transformation optical structures will be achieved.

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