



## OPEN ACCESS

# An acoustic metafluid: realizing a broadband acoustic cloak

To cite this article: J B Pendry and Jensen Li 2008 *New J. Phys.* **10** 115032

View the [article online](#) for updates and enhancements.

## You may also like

- [Nonlinear dynamic modeling of ultrathin conducting polymer actuators including inertial effects](#)  
Ngoc Tan Nguyen, Yuta Dobashi, Caroline Soyer et al.
- [Theoretical and experimental analysis of a piezoelectric plate connected to a negative capacitance at MHz frequencies](#)  
S A Mansoura, P Benard, B Morvan et al.
- [Research on attitude compensated algorithm for shipborne dynamic weighing](#)  
Meifeng Xie, Ping Zhang, Kundong Wang et al.

## An acoustic metafluid: realizing a broadband acoustic cloak

J B Pendry<sup>1,3</sup> and Jensen Li<sup>2</sup>

<sup>1</sup> The Blackett Laboratory, Department of Physics, Imperial College, London SW7 2AZ, UK

<sup>2</sup> NSF Nanoscale Science and Engineering Center (NSEC) 5130 Etcheverry Hall, University of California, Berkeley, CA 94720–1740, USA

E-mail: [j.pendry@imperial.ac.uk](mailto:j.pendry@imperial.ac.uk)

*New Journal of Physics* **10** (2008) 115032 (9pp)

Received 13 June 2008

Published 27 November 2008

Online at <http://www.njp.org/>

doi:10.1088/1367-2630/10/11/115032

**Abstract.** Recent theory shows that sound can be controlled and directed almost at will provided that suitable materials can be found. Here, we propose a structure permeated by a fluid and designed so that the composite medium, the metafluid, has an anisotropic density tensor, and a compressibility of choice.

### Contents

1. Introduction	2
2. Anisotropic mass density	4
3. Adjusting the bulk modulus	6
4. Realizing a broadband acoustic cloak	7
Acknowledgments	8
References	9

<sup>3</sup> Author to whom any correspondence should be addressed.

## 1. Introduction

Controlling the flow of energy is a prized goal whether the energy be in the form of sound, ocean waves, or electromagnetic radiation. The recently introduced technique of transformation optics has given us the power to design control systems for electromagnetic waves and the concept of a metamaterial [1] enables realization of many of these schemes. Perhaps the most quoted example is the ability to build a cloak that hides a given volume of space from radiation but at the same time remains invisible [2]–[5]. The trick is to guide light smoothly around the hidden region emerging on the far side travelling in the same direction as if no obstacle were present. Obviously this requires exquisite control of the materials from which the cloak is built. Milton *et al* [6] analysed the full equations of motion for a general elastic medium under coordinate transformations and found that in general the equations of motion are not coordinate invariant. For the simpler case of acoustic waves in fluids coordinate invariance is obeyed and theoretical schemes closely related to the electromagnetic case have been proposed [7]–[11] but their implementation has been hampered by limited availability of suitable materials. This paper is devoted to defining an acoustic metamaterial, a material whose acoustic properties are dictated by its internal subwavelength structure rather than by the substances from which it is manufactured. Li and Chan [12] and Torrent and Sanchez-Dehesa [13] have also suggested candidate metamaterials.

The equations of motion for a continuous compressible fluid are as follows:

$$\rho' \ddot{\mathbf{s}}' = -i\omega \rho' \dot{\mathbf{s}}' = -\nabla p', \quad (1)$$

$$\dot{p}' = -i\omega p' = -\Lambda' \nabla \cdot \dot{\mathbf{s}}', \quad (2)$$

where  $\omega$  is the frequency,  $p'$  the pressure,  $\rho'$  the mass density,  $\mathbf{s}'$  is the local displacement of the fluid and  $\dot{\mathbf{s}}'$  the velocity.  $\Lambda'$  is the bulk modulus. The two equations can be reduced to a single scalar equation:

$$\ddot{p}' = \Lambda' \nabla \cdot (\rho'^{-1} \nabla p'). \quad (3)$$

An isotropic and homogeneous fluid supports waves,

$$\mathbf{s}'(r', t) = \mathbf{s}'_0 \exp i(k'x' - \omega t) \quad (4)$$

with the following dispersion relationship,

$$\omega^2 = k'^2 \Lambda' / \rho'. \quad (5)$$

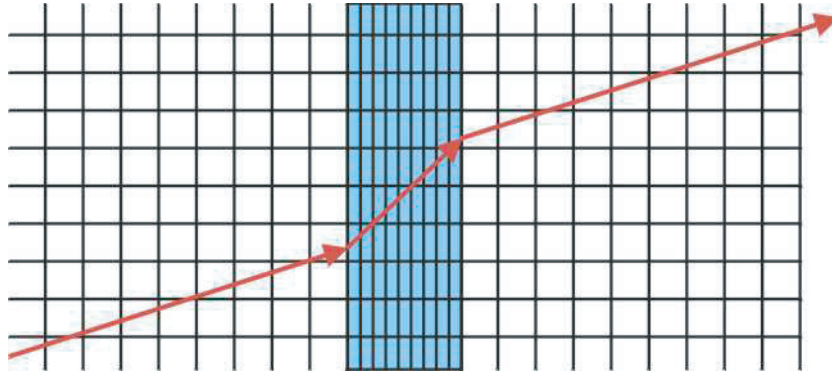
Recently several papers have appeared following the philosophy of the transformation optics approach used in electromagnetism [2, 14], rewriting (1) and (2) in a general coordinate system [7, 8]. For a fluid the new equations have the same form but with revised values for the bulk modulus and density the latter of which is now a tensor. The point of the exercise is that if a material could be found corresponding to the revised values of  $\Lambda'$  and  $\rho'$ , then energy flow would follow contours of the new coordinate system. In this way coordinate transformations give a prescription for redirecting the energy flow.

Applying the techniques of transformation optics to equation (3) gives,

$$\Lambda^{-1}(\mathbf{r}') \ddot{p} = \nabla \cdot ([\rho(\mathbf{r}')]^{-1} \nabla p), \quad (6)$$

where (see [7]),

$$[\rho(\mathbf{r})]^{-1} = \mathbf{A}[\rho'(\mathbf{r}')]^{-1} \mathbf{A}^T / \det \mathbf{A}, \quad \Lambda(\mathbf{r}) = \Lambda'(\mathbf{r}') \det \mathbf{A}, \quad (7)$$



**Figure 1.** Rays of sound travel in a straight line in a uniform homogeneous medium but we can distort their trajectories by notionally squashing the coordinate system then calculating what densities and bulk moduli would direct the sound along this path. In this instance, we show a simple compression of the coordinates along one axis.

$$A_{ij} = \partial x_i / \partial x'_j. \quad (8)$$

Here, we give an alternative intuitive derivation of (7) by considering the simplest example: compression of a region of space by a factor of  $\alpha$  so that rays of sound follow the trajectories shown in figure 1 and do this without any reflection at the interfaces. The corresponding transformations are given below,

$$\begin{aligned} x &= x', & y &= y', \\ z &= z', & -\infty < z' < 0, \\ z &= \alpha z', & 0 < z' < d, \\ z &= z' + (\alpha - 1)d, & d < z' < +\infty. \end{aligned} \quad (9)$$

We wish to find values for  $\Lambda$  and  $\rho$  in the compressed region that allow the sound to emerge as if it had passed through an uncompressed medium. Consider a sound wave propagating along the  $z$ -axis. In the compressed region the wave vector must increase so that the phase change across the compressed region,  $k\alpha d$ , is the same as across the uncompressed region,  $k'd$ . Therefore the wave vector is increased by a factor  $\alpha^{-1}$

$$k = \alpha^{-1} k', \quad (10)$$

which from (5) implies that,

$$\Lambda / \rho = \alpha^2 \Lambda' / \rho'. \quad (11)$$

At the boundary both the pressure and displacement must match across the interface so from (2) it follows that,

$$-ik\Lambda s = p = p' = -ik'\Lambda' s'. \quad (12)$$

Hence,

$$\frac{\Lambda}{\Lambda'} = \alpha, \quad \frac{\rho_{zz}}{\rho'} = \frac{1}{\alpha}. \quad (13)$$

Finally, consider a wave propagating normal to the axis of compression, say along the  $x$ -axis. Since there is no compression along this axis, the wave vector must take the uncompressed value. From (5),

$$\frac{\Lambda'}{\rho'} = \frac{\omega^2}{k'^2} = \frac{\omega^2}{k^2} = \frac{\Lambda}{\rho_{xx}} \quad (14)$$

and therefore

$$\frac{\rho_{xx}}{\rho'} = \frac{\Lambda}{\Lambda'} = \alpha, \quad (15)$$

i.e. if we *compress* a region we *increase* the density and *decrease* the bulk modulus along the axis of compression. The bulk modulus perpendicular to the axis of compression is decreased.

Any arbitrary distortion of a coordinate system can be represented as a sequence of compressions or stretchings as described above and therefore we have a complete prescription for the new material parameters.

Our strategy is to find a simple metafluid that reproduces the required values of  $\rho$  and  $\Lambda$ . If possible the metafluid should allow the embedding fluid to percolate freely through its structures thus eliminating the need for separating the two fluids. Or if that is not possible to have the whole system at constant pressure so that regions can be separated by thin membranes. The structures we present will be capable of adaptation to many situations more complex than the simple compression considered here. They are also independent of the frequency of the sound and therefore can be used for broad band control.

## 2. Anisotropic mass density

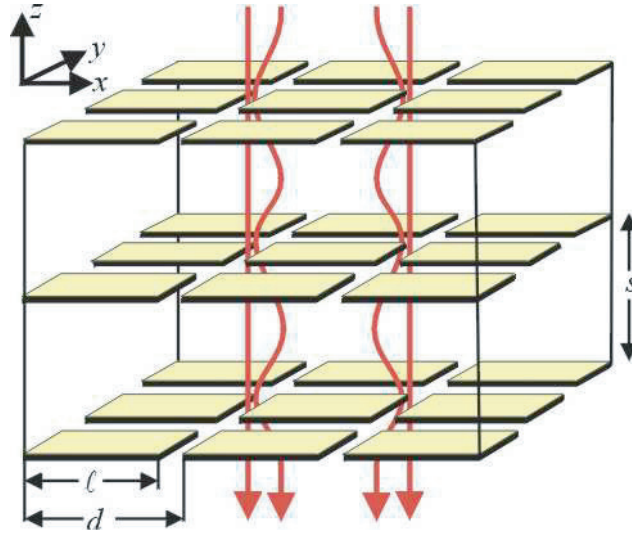
Any normal fluid has an isotropic density tensor but we can exploit the concept of metamaterials to change this. We plan to alter the flow pattern of the liquid depending on the direction of flow. If we force fluid moving along the  $z$ -direction to follow a long winding path then more force must be applied to effect the net movement along the  $z$ -axis and the fluid will appear to be more dense when moving in this direction. Here, we make an analogy with electromagnetism. In an earlier paper, we showed how to decrease the magnetic permeability along the  $z$ -axis by introducing a set of thin superconducting plates oriented normal to the axis in question. This requires the magnetic lines of force never to penetrate the plates and always to lie parallel to the surface. As a result the lines of force wind around the plates thus decreasing the permeability. In the present instance, we introduce a set of rigid plates of infinite mass around which the fluid must flow altering the response to pressure gradients along the  $z$ -axis, but leaving the response to gradients in the  $xy$ -plane much the same as for a free fluid (figure 2). Like the magnetic field, the fluid flow can never penetrate the plates and must lie parallel to the plates at the surface.

The flow pattern around the plates is not susceptible to simple analytic solution. Fortunately in an earlier paper [15] we calculated the effect of a similar structure on magnetic fields lines. In both cases, the scale of the structure is much smaller than the wavelength of radiation,

$$\ell \approx d \approx s \ll \lambda = 2\pi \sqrt{\Lambda' / \rho' \omega^2} \quad (16)$$

and hence in the immediate locality of the plates we can approximate (3) by,

$$\nabla^2 p' = 0, \quad (17)$$



**Figure 2.** Rigid plates of infinite mass oriented normal to the  $z$ -axis increase the pressure gradient needed for a given  $z$ -oriented flow of fluid whilst leaving flow in the  $xy$ -plane unaffected. Hence, the system can be described by an anisotropic density tensor.

**Table 1.** Enhancement  $\rho_{zz}/\rho_{xx}$  for the structure shown in figure 1, as a function of  $\ell$  for fixed  $d = 167 \mu\text{m}$  and  $s = 100 \mu\text{m}$ . The results are scale invariant provided that condition (16) is met.

$\ell(\mu\text{m})$	$\rho_{zz}/\rho_{xx}$
133	1.56
153	2.08
163	4.35

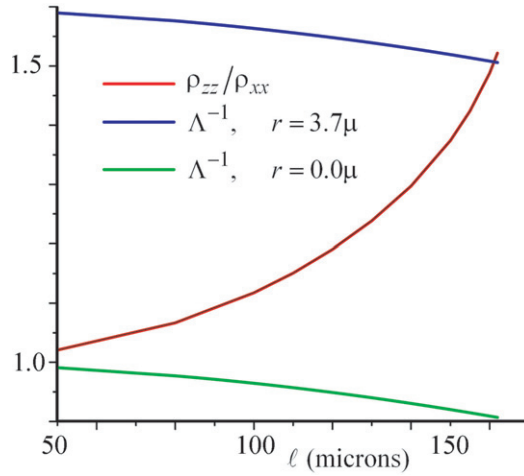
where we have assumed that the density of the filling fluid,  $\rho'$ , is locally constant. The boundary conditions on  $p'$  ensures that the velocity, given by (1), is parallel to the surface. Under these conditions the flow lines of velocity map onto the lines of magnetic  $\mathbf{B}$  field of our earlier calculation and  $-\nabla p'$  maps onto the  $\mathbf{H}$  field. Since  $\mathbf{B} = \mu\mu_0\mathbf{H}$  and from (1),

$$\dot{\mathbf{s}}' = \frac{1}{i\omega\rho'}\nabla p', \quad (18)$$

it follows that  $\rho'^{-1}$  and  $\mu$  scale in the same fashion. Therefore enhancement of the pressure differential across a single cell is the same as enhancement of the magnetic potential in our previous work

Using this analogy, we can immediately write down the corresponding enhancements to  $\rho_{zz}/\rho_{xx}$  for the above acoustic metamaterial as shown in table 1. It is evident from table 1 that a wide range of anisotropy is easily obtainable using this structure. In this simple example, we have tuned the density tensor only along one axis but it is obvious that the trick of redirecting fluid along a complex path can be repeated for all three axes.

In the example given above we assumed that the confining plates were perfectly rigid which might be a good approximation if the fluid between the plates were a gas but not if a



**Figure 3.** The enhancement  $\rho_{zz}/\rho_{xx}$  (red curve) as a function of  $\ell$  for fixed  $d = 167 \mu\text{m}$  and  $s = 100 \mu\text{m}$ . In contrast to table 1 the plates are assumed to be steel of thickness  $10 \mu\text{m}$  with density of  $7860 \text{ kg m}^{-3}$  and bulk modulus of  $1.6 \times 10^{11} \text{ Pa}$ . The fluid is water with density of  $1000 \text{ kg m}^{-3}$  and bulk modulus of  $2.2 \times 10^9 \text{ Pa}$ . Also shown is the effect of including a small hemisphere of air, radius  $r = 3.7 \mu\text{m}$ , which is attached to each of the steel sheets. The hemisphere has negligible impact on  $\rho_{zz}/\rho_{xx}$  while the relative bulk modulus (with respect to water) with/without the hemisphere of air is shown in green/blue colour.

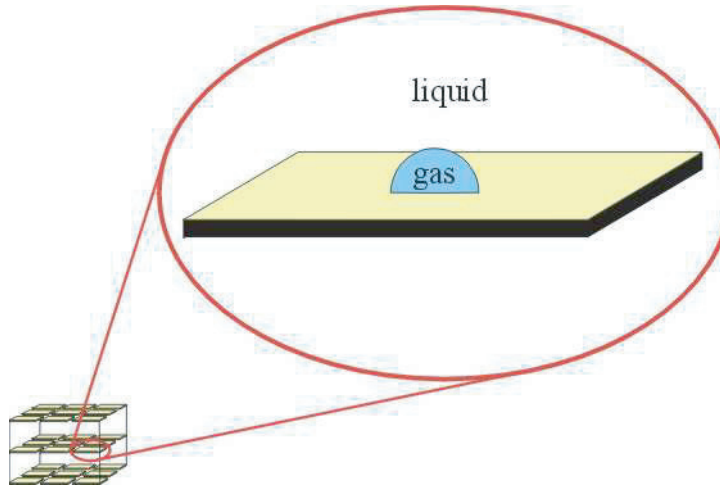
liquid. In this case, we simulate a more realistic example in which the fluid is water (density  $1000 \text{ kg m}^{-3}$ , bulk modulus  $2.2 \times 10^9 \text{ Pa}$ ) and the plates are made of steel (density  $7860 \text{ kg m}^{-3}$ , bulk modulus  $1.6 \times 10^{11} \text{ Pa}$ ). We used COMSOL Multiphysics (with the Acoustic Module) to calculate the transmission and reflection coefficients of a section of acoustic metafluid from which the effective density and effective bulk modulus can be extracted. Again, we vary the size of the square steel plate ( $\ell$ ) of thickness  $10 \mu\text{m}$  for fixed  $d = 167 \mu\text{m}$  and  $s = 100 \mu\text{m}$ . The enhancements to  $\rho_{zz}/\rho_{xx}$  are shown in figure 3.  $\rho_{xx}$  is approximately one for thin plates as the fluid can move freely in the  $x$ -direction. As expected the effect is less than for the previous case of perfectly rigid plate but nevertheless significant enhancements are found.

Now that we have the anisotropic density required for realizing an acoustic cloak, it remains to find a means of changing  $\Lambda$ , the bulk modulus.

### 3. Adjusting the bulk modulus

Our solution to reducing the bulk modulus is a simple one illustrated in figure 4: we attach a sack of gas to each of the plates. The effective modulus is then a weighted average of those of the gas and the liquid. Changing the local pressure in the system by  $\delta p$  changes the volume of gas by  $\delta p \Lambda_{\text{gas}}^{-1} \Omega_{\text{gas}}$ , where  $\Omega_{\text{gas}}$  is the volume of gas in the sack, and changes the volume of liquid in the unit cell by  $\delta p \Lambda_{\text{liq}}^{-1} \Omega_{\text{liq}}$ . Therefore the effective modulus is,

$$\Lambda^{-1} = \frac{\Lambda_{\text{liq}}^{-1} \Omega_{\text{liq}} + \Lambda_{\text{gas}}^{-1} \Omega_{\text{gas}}}{\Omega_{\text{liq}} + \Omega_{\text{gas}}} \approx \frac{\Lambda_{\text{gas}}^{-1} \Omega_{\text{gas}}}{\Omega_{\text{liq}} + \Omega_{\text{gas}}}. \quad (19)$$



**Figure 4.** To each of the plates shown in figure 2, we attach a hemispherical sack containing a gas. The effective bulk modulus is then a weighted average of those of the gas and the liquid.

Since a gas is so very much more compressible than a liquid, only a small volume fraction of gas would be required: water has a bulk modulus  $10^4$  greater than that of air.

Of course in some circumstances where the coordinate system has been expanded rather than compressed it would be necessary to increase the bulk modulus. However, the obvious solution is to include some incompressible material so that (19) becomes,

$$\Lambda^{-1} = \frac{\Lambda_{\text{liq}}^{-1} \Omega_{\text{liq}} + \Lambda_{\text{inc}}^{-1} \Omega_{\text{inc}}}{\Omega_{\text{liq}} + \Omega_{\text{inc}}} \approx \frac{\Lambda_{\text{liq}}^{-1} \Omega_{\text{liq}}}{\Omega_{\text{liq}} + \Omega_{\text{inc}}}. \quad (20)$$

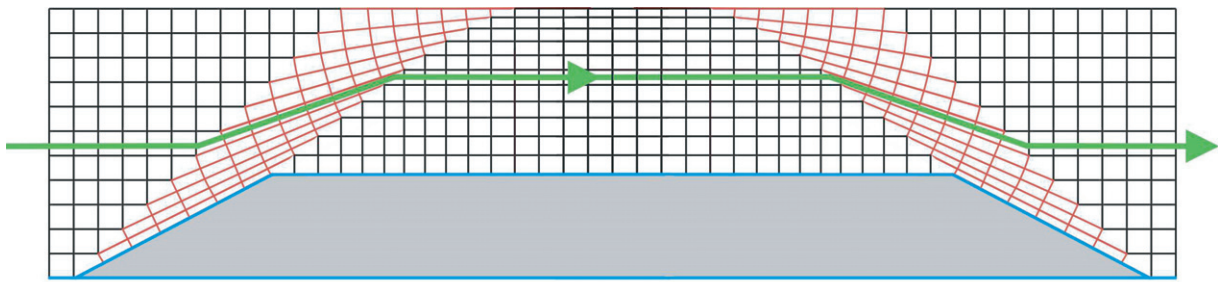
The difficult case is *reduction* of the bulk modulus of a gas. Since a gas is already highly compressible, finding a suitable filling for the sack in figure 4 is difficult and it would probably be necessary to abandon the requirement that we use the same gas to fill the metamaterial as that which surrounds it. Reduction of the gas pressure is the most effective way of reducing the modulus but brings with it the difficulty of separating the filling gas from the embedding gas.

As an example, for the metamaterial (steel plates in water) we have discussed, the bulk modulus is around that of water as shown in the green curve in figure 3. The bulk modulus decreases for a thicker steel plate since steel is less compressible than water. On the other hand, if we attach hemispheres of air with radius  $r = 3.7 \mu\text{m}$  to every steel plate, the bulk modulus can be decreased enormously. In this case, it is decreased to nearly  $1.7 \times$  the bulk modulus of water.

#### 4. Realizing a broadband acoustic cloak

Several authors have given prescriptions for an acoustic cloak [2]–[4] and we use it here as one example of the possible applications of acoustic metamaterials. There are three topologically distinct ways of making an object invisible: by making it vanish to an infinitely dense incompressible point, line, or plane. The first two options necessarily result in singular values of the density tensor but the third option has the advantage of not being singular. It also





**Figure 5.** A 2D coordinate transformation that directs sound (green arrows) around a cloaked tunnel (grey). The transformation is chosen to be everywhere orthogonal: Cartesian to the far left and right, and in the central region (black lines), and cylindrical polar in the transition regions (red lines). Blue lines represent a dense incompressible interface. To the far left and right and above the system lies the uncompressed region filled with normal fluid. The red regions and the central portion are filled with a suitable acoustic metamaterial.

has a disadvantage, namely that a plane of material is not invisible unless it sits on an infinite background of similarly dense material. A similar scheme has been proposed in the electromagnetic context [16]. Figure 5 shows a schematic representation of such a cloak designed to be sited on an incompressible infinitely dense surface, and creating a hidden tunnel shown in grey in the figure.

Starting from a Cartesian coordinate system embedded in a uniform fluid, we distort the coordinates as shown in the figure to create our hidden tunnel. The transformation is chosen to be everywhere orthogonal as described in the figure caption so that by examining each cell we can discover the principal axes of the anisotropic density tensor. Furthermore the local bulk modulus is proportional to the area of the cell from (13). Finally by recognising the effect of compression along successive axes we can calculate the values of each component of the local density tensor from (13) and (15).

By inspection we note that each distorted cell has a reduced area and therefore the bulk modulus is decreased. The density tensor is anisotropic: large along the directions of flattening, small perpendicular to this direction. Each cell is to be filled with a fluid whose density corresponds to the least dense component of the tensor and the cell isolated from its neighbours by a thin light membrane to prevent mixing. The other components are then to be enhanced by inserting an appropriate set of dense plates as described in previous sections.

It will be noted that the cell distortions are everywhere rather modest and therefore undemanding of metamaterial parameters which should be easily met by the recipes given in this paper.

## Acknowledgments

JBP thanks the EPSRC for a Senior Fellowship, and financial support of the Future and Emerging Technologies (FET) programme within the Seventh Framework Programme for Research of the European Commission, under FET-Open grant number 213390.

## References

- [1] Pendry J B, Holden A J, Robbins D J and Stewart W J 1999 *IEEE Trans. Microw. Theory Tech.* **47** 2075
- [2] Pendry J B, Schurig D and Smith D R 2006 *Science* **312** 1780
- [3] Leonhardt U 2006 *Science* **312** 1777
- [4] Schurig D, Mock J J, Justice B J, Cummer S A, Pendry J B, Starr A F and Smith D R 2006 *Science* **314** 977–80
- [5] Cai W, Chettiar U K, Kildishev A and Shalaev V M 2007 *Nat. Photonics* **1** 224
- [6] Milton G W, Briane M and Willis J R 2006 *New J. Phys.* **8** 248
- [7] Cummer S A and Schurig D 2007 *New J. Phys.* **9** 45
- [8] Chen H and Chan C T 2007 *Appl. Phys. Lett.* **91** 183518
- [9] Cheng Y, Yang F, Xu J Y and Liu X J 2008 *Appl. Phys. Lett.* **92** 151913
- [10] Greenleaf A, Lassas M and Uhlmann G 2003 *Math. Res. Lett.* **10** 685
- [11] Greenleaf A, Lassas M and Uhlmann G 2003 *Physiol. Meas.* **24** 413
- [12] Li J and Chan C T 2004 *Phys. Rev. E* **70** 055602
- [13] Torrent D and Sanchez-Dehesa J 2008 *New J. Phys.* **10** 023004
- [14] Ward A J and Pendry J B 1996 *J. Mod. Opt.* **43** 773
- [15] Wood B and Pendry J B 2007 *J. Phys.: Condens. Matter* **19** 7
- [16] Rahm M, Schurig D, Roberts D A, Cummer S A, Smith D R and Pendry J B 2007 arXiv:0706.2452