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Using freeware planetarium software to simulate the astronomical measurements of ancient Greeks

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Abstract

The ancient Greek astronomers devised ingenious methods for measuring the size and distances of the Earth, Moon and Sun. The concepts were beautifully simple and although in most cases could only yield approximate results, they have the advantage that they can be understood by anyone with a basic mathematical background. The emergence of affordable planetarium software enables educators to reproduce these pioneering measurements in the classroom. In this paper, several activities are presented that are based on observations and experiments performed over 2000 years ago. By using freeware software students are introduced to key milestones in the history of astronomy in an immersive and interactive way.

1. Introduction

Observational astronomy is one of the oldest sciences and continues to this day to inspire many pupils and students. In recent years the development of low cost and in some cases, free planetarium software has enabled educators to reproduce the magic of the sky in the classroom. With this type of software the position and motion of celestial bodies, such as the stars, Moon and Sun, are simulated. What is more, one can choose the observation location, date and time to obtain a view of the sky from anywhere in the world in the present, past or in the future.

Planetarium software can be used to develop a variety of educational tasks and several educators have come up with interesting ideas (for example [1]). In this paper, a set of activities will be presented inspired from historical measurements made in antiquity. In this way, the students are not only exposed to the relevant scientific methodology but are also introduced in an immersive way to the history of astronomy thus obtaining a more holistic learning experience.

The activities presented are based on measurements made by the ancient Greeks whose contribution to the understanding of astronomical phenomena was immense. More than 2000 years ago, although lacking in observational technology such as telescopes, those scientific pioneers were able to use simple but ingenious scientific calculations to provide estimates for the size of the Earth, Moon and Sun and the distance of the Sun and Moon from our planet.

The main idea is to combine the scientific calculations of the ancient Greeks with the capabilities of a freeware planetarium software package, to encourage students to replicate some of those landmark steps of astronomical development. The theoretical background to the measurements has primarily been sourced from [2–4].

The idea presented offers the following advantages to educators: using planetarium software rather than conducting actual observations enables the activities to be pursued without the need of expensive observation instrumentation such as telescopes and without teaching being affected by weather conditions. The use of a freeware package means that no software costs are incurred. The visual effects offered by the simulations add to the realism of the activities. Finally, including a component of the history of science in the teaching of astronomy will increase student interest and motivation.

2. Astronomical measurements

All the activities presented are implemented in the *Stellarium* software [5] (version 0.19.2) which can be downloaded as freeware and is easy to use in a number of languages. For students completing the activities, the following learning outcomes can be achieved: become familiar with the sizes and distances of the Sun and Moon relative to the Earth; learn to use simple geometrical and mathematical processes to obtain estimates of astronomical sizes and distances; appreciate a series of important landmarks in the history of astronomical development; become familiar with the use of planetarium software for obtaining basic astronomical parameters.

2.1. Measuring the size of the Earth (Eratosthenes)

Eratosthenes (276 BCE—194 BCE) was a Greek polymath born in Cyrene (modern Libya). He served as the librarian at the famous library of Alexandria in Egypt and is often referred to as the 'Father of Geography'.

Eratosthenes used the following method to calculate the circumference of the Earth (see figure 1). At noon on the summer solstice (when the Sun is highest in the sky and daylight time is longest), he measured the length of the shadow cast by a column of known height at Alexandria. He then used his measurements to estimate the angle between the rays of the Sun and the vertical. Eratosthenes also knew that on the same date and time, at Syene (modern day Aswan) located (approximately) on the same meridian line as Alexandria and at a distance of $S = 5000$ stadia (a unit used for measuring distance at his time), the rays of the Sun fell directly vertically on to a well. By using the simple proportion that:

$$\frac{L}{S} = \frac{360}{\theta}$$

where L is the circumference of the Earth, S is the distance between Alexandria and Syene and θ is the angle the rays of the Sun make with the vertical measured in degrees, he was able to provide an estimate for the size of our planet, more than 2200 years ago.

2.1.1. Implementation in Stellarium. Eratosthenes reportedly performed his experiment on the summer solstice of 240 BCE. We therefore set the date to: −239/6/26 (−239 is equivalent to 240 BCE).

This was the date of the summer solstice at that year. Note that for dates in antiquity Stellarium uses the Julian calendar, so any dates presented here will refer to this calendar system.

Next, the location must be set to Alexandria in Egypt. To perform the experiment correctly we must find the *local noon* in Alexandria, i.e. the exact time when the Sun's *altitude* (i.e. its angular distance from the horizon) is at its greatest, which for the Northern Hemisphere is when it is located exactly South. This is when the Sun's *azimuth* is 180°. The azimuth and the altitude of a selected body are both provided as part of the information displayed by the software. Note that for a variety of reasons, the local noon time is not exactly at midday (this can be discussed with students).

The angle θ measured by Eratosthenes is related to the Sun's altitude by:

$$\theta = 90 - \Phi$$

where Φ is the altitude of the Sun measured in degrees. The angle θ is also known as the *zenith distance*.

It is therefore possible to estimate the angle θ using the results produced by Stellarium and to subsequently use the simple formula employed by Eratosthenes to calculate the circumference of the Earth in stadia. To obtain the results in kilometers rather than stadia, we multiply by a factor of 0.185 (although the exact conversion factor is still disputed by scholars). Students can thus obtain the circumference of the Earth in kilometers and compare it to the accepted value in modern times.

2.2. Measuring the size of the Earth (Posidonius)

Posidonius (135 BCE—51 BCE) was a Greek polymath born in the city of Apamea, in modern day Syria.

Posidonius estimated the Earth's circumference by observing the position of the bright star Canopus (see figures 2–4). He observed that the star could be viewed on, but never above, the horizon at Rhodes, while at Alexandria it could be seen to emerge clearly above the horizon. By assuming (as he thought) that Rhodes was 5000 stadia due north of Alexandria (on the same meridian), he was able to estimate the circumference of the Earth as follows.

The difference in the maximum altitude of Canopus as viewed from two locations on the same meridian, is equal to the difference in the latitudes of the two locations. Once the difference in latitudes is obtained, a formula similar to the one employed by Eratosthenes can be used to find the circumference of the Earth:

$$\frac{L}{S} = \frac{360}{\Delta\varphi}$$

where in this case $\Delta\varphi$ is the difference in latitudes measured in degrees.

2.2.1. Implementation in Stellarium. We can set the date to: −100/1/1 (any other day when Canopus is visible by night in Alexandria will do).

Next, we must set the location to Alexandria in Egypt and note the latitude of our observation point. The time should then be varied to find the maximum altitude of Canopus (i.e. when it is exactly South with an azimuth of 180 degrees).

We then set the location to Rodos (Rhodes) in Greece and the same measurement is performed. The difference in the two altitudes (and subsequently the two latitudes) can be used with the relevant formula to estimate the circumference of the Earth in stadia and in kilometers.

2.3. Measuring the distance to the Sun (Aristarchus)

Aristarchus (310 BCE—230 BCE) was an ancient Greek astronomer and mathematician born on the Greek island of Samos. He is famous for being the first scientist to support the heliocentric model of the solar system, i.e. that the Earth and planets revolve around the Sun. In his work 'On the Sizes and Distances of the Sun and Moon', he also presented a series of calculations which, although due to his lack of accurate scientific instruments were prone to relatively large errors, were impressive in their theoretical make up. In this and the next section, we will replicate two of his brilliant experiments.

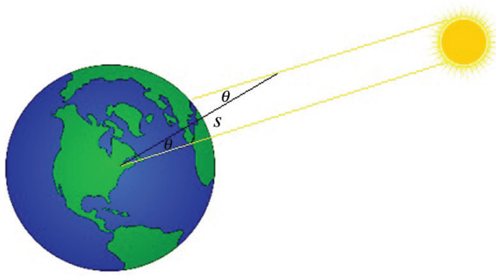

Figure 1. Method of Eratosthenes for measuring the circumference of the Earth.

Aristarchus estimated the distance between the Earth and the Sun in terms of the distance between the Earth and Moon. His idea was beautifully simple. As seen in the figure 5, when the Moon is at the quarter Moon phase (i.e. when it seems half lit to us), the Sun–Moon–Earth angle is a right angle. So, by determining the Moon–Earth–Sun angle (let us call it ψ), we can find the distance of the Sun from the Earth from:

$$\cos(\psi) = \frac{L}{S}$$

where L is the distance between the Moon and Earth and S is the distance between the Sun and Earth.

2.3.1. Implementation in Stellarium. Initially we must set the date to: −280/10/15 (any other day when the Moon is half illuminated will do).

We can then set the location to Athens in Greece (Aristarchus spent some of his time there). We must find and freeze the exact moment when the Moon is illuminated by 50%; this is part of the information provided by Stellarium.The Angle Measure plugin can then be used to measure the angular distance between the Moon and the Sun at this moment, this is the angle ψ. The cosine formula provided above can be used to find the distance of the Sun from the Earth in terms of the distance of the Moon from the Earth.

Note that this experiment provides us with an estimate of the distance of the Sun in terms of the distance to the Moon. In the next two activities we will calculate the actual distance to the Moon and therefore it is then possible for the students to produce an absolute value for the distance to the Sun.

2.4. Measuring the size of the Moon (Aristarchus)

Aristarchus also used his geometrical and astronomical knowledge to estimate the relative size of the Moon compared to that of the Earth. To do this he observed a lunar eclipse (see figures 6 and 7).

When the Sun–Earth–Moon are aligned, the Moon will move through the dark shadow of the Earth. If the alignment is such that the Moon becomes totally submerged in the dark shadow (known as umbra) then we witness a total lunar eclipse.

Aristarchus realised that by observing the Moon in the Earth's shadow he could estimate its relative size. One way to do this would be to measure the duration of time between the moment when the edge of the Moon first became eclipsed and the moment when the Moon was first totally obscured (let us call this t_1). You can then measure the time duration for which the Moon was totally eclipsed (i.e. before it starts to emerge from the Earth's shadow), let us call this t_2.

By using the formula:

$$Rs = \frac{t_2}{t_1} + 1$$

where Rs is the relative size of the breadth of the Earth's shadow with respect to the diameter of the Moon.

We are then able to estimate how many times larger than the Moon the Earth's shadow is. So, for example, if $t_2 = t_1$ (in actual fact t_2 is greater than t_1), then the breadth of the Earth's shadow is double the diameter of the Moon. If the breadth of the shadow of the Earth was constant then we could also deduce that the Earth is double the size of the Moon. In fact the shadow tapers (narrows) as we move away from the Earth. So for a better approximation we must multiply our result by a certain factor. Using geometrical calculations, this factor turns out to be roughly 1.35. So in our example the diameter of the Earth would have actually been estimated as 2.7 times the diameter of the Moon.

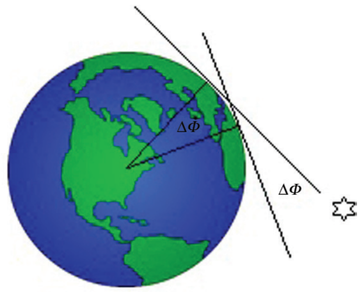

Figure 2. Method of Posidonius for measuring the circumference of the Earth.

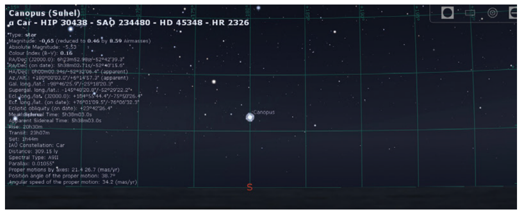

Figure 3. Screen dump from *Stellarium* with view of Canopus from Alexandria.

Figure 4. Screen dump from *Stellarium* with view of Canopus from Rhodes.

2.4.1. Implementation in Stellarium.

We set the date to: −286/5/19. This is when a central lunar eclipse occurred during Aristarchus's time. The more central the lunar eclipse, i.e. the closer the Moon passes to the centre of the Earth's shadow, the better the Sun, Earth and Moon are aligned, and the more accurate the measurements that can be made.

Next, we set to Athens in Greece and the local time to around 11:10 pm. We must now zoom in to the Moon and progress the time until we observe the beginning of the lunar eclipse. During a total lunar eclipse the Moon does not vanish (like the Sun does in a total solar eclipse) but noticeably changes colour.

We can then measure the times t_1 and t_2 and the expression

$$\text{Ratio} = 1.35\left[\frac{t_2}{t_1} + 1\right]$$

to estimate the ratio of the radius (or diameter) of the Earth to that of the Moon. By assuming that the radius of the Earth is 6371 km, we can estimate the actual radius of the Moon.

Finally, as viewed from the Earth, the Moon's angular size is approximately 0.5°. As an extra exercise, students can use this fact to find the absolute distance of the Moon from the Earth. This result can also be combined with that of the previous experiment to obtain the distance of the Sun from the Earth in kilometers.

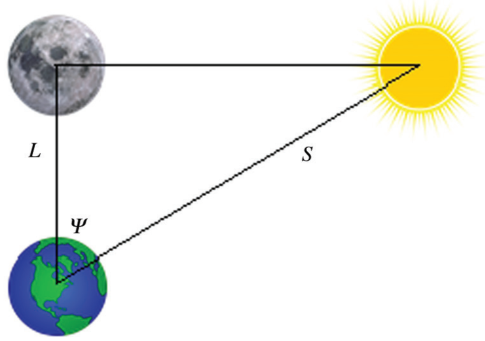

Figure 5. Method of Aristarchus for measuring the distance to the Sun.

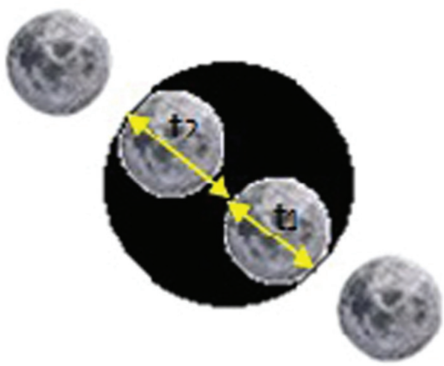

Figure 6. Moon passing through the shadow of the Earth.

Figure 7. Screen dump from *Stellarium* with view of lunar eclipse of 19/5/287 BCE.

2.5. Measuring the distance to the Moon (Hipparchus)

Hipparchus (190 BCE—120 BCE) was a Greek astronomer, geographer, and mathematician, born in the ancient city of Nicaea (modern day Turkey). In astronomy, he is known as the greatest star observer of antiquity as he cataloged the position and brightness of hundreds of stars.

Hipparchus used observations from a total eclipse of the Sun to estimate the distance of the Moon from the Earth (see figures 8–10). The eclipse was total at the Hellespont (the narrow strait that separates the European and Asian parts of Turkey) but only part of the Sun was seen covered from Alexandria, in Egypt.

For a total eclipse of the Sun–Moon–Earth are aligned. To gain an understanding of the principle governing the method used by Hipparchus (for more details see [6]), we must view the following, simplifying, diagram.

The point C at the edge of the Moon, during totality, when viewed from the Hellespont (point A) just overlapped point D on the edge of the Sun. Viewed from Alexandria (point B), the point C only overlapped point E on the Sun, about x solar diameters short of the edge, which was why the eclipse there was not total. The number x (smaller than 1), denotes the fraction of the Sun that remains visible. So, for example if 20% of the Sun remains visible then x is 0.2.

A fraction x of the diameter of the Sun covers an angle of $0.5x$ in the sky (since the angular size of the Sun as viewed from the Earth is approximately 0.5°, the same as that of the Moon). So the small angle α between the two directions is approximately $0.5x$ degrees. Note that in astronomy this angle is called the parallax of the edge of the Moon as viewed from the above two locations.

Hipparchus knew the latitudes of Hellespont and Alexandria. He also assumed that Alexandria is exactly due south. Furthermore, from simple geometry:

$$AB = (2\pi r)\,\frac{\Delta\varphi}{360}$$

where $\Delta\varphi$ is the difference in latitude between the Hellespont and Alexandria and r is the radius of the Earth.

The points A, B are also located on another circle which has the Moon at its centre. The radius in that case is the distance R to the Moon, and because the arc AB covers $0.5x$ degrees, we get:

$$AB = (2\pi R)\,\frac{0.5\,x}{360}.$$

Of course, each of the two arcs AB expressed in the above equations is measured along a different circle, with a different radius (and the two circles curve in opposite ways). Since, however, in both cases AB covers only a small part of the circle as an approximation we may regard each of the arcs as equal to the chord AB. This assumption allows us to make the two expressions equal:

$$(2\pi r)\,\frac{\Delta\varphi}{360} = (2\pi R)\,\frac{0.5\,x}{360}.$$

This in turn gives,

$$\frac{R}{r} = \frac{\Delta\varphi}{(0.5\,x)}.$$

Therefore, by knowing the difference in latitude $\Delta\varphi$ and also the fraction x of the Sun that is still visible, we can estimate the distance to the Moon in terms of the radius of the Earth.

It is important to note that the approximate geometric method described here is a very simplified version of the geometric calculations that Hipparchus himself used. His more complex calculations obviously yielded more accurate results as described in [7].

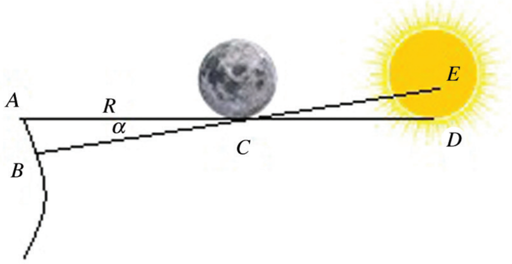

Figure 8. Method of Hipparchus for measuring the distance to the Moon.

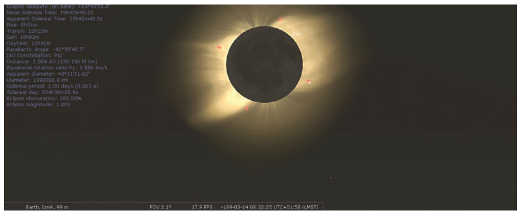

Figure 9. Screen dump from *Stellarium* with view of total solar eclipse of 14/3/190 BCE as seen from Iznik.

Figure 10. Screen dump from *Stellarium* with view of partial solar eclipse of 14/3/190 BCE as seen from Alexandria.

2.5.1. Implementation in Stellarium. According to most scholars, Hipparchus used the solar eclipse which took place on the 14th of March, 190 BCE, for his estimation (he must have learned of the measurements from earlier sources).

Initially, we therefore need to set the date to: −189/3/14.

Next, we can set the location to Iznik in modern day Turkey. This is where the birthplace of Hipparchus (the ancient city of Nicaea) stood on the Hellespont strait. We must also note the latitude of the location. The local time is then set to around 7:25 am. By zooming in and letting the time run we can verify that a total solar eclipse did occur on this day and was indeed visible from the Hellespont.

We then change the location to Alexandria in Egypt, note the latitude and set the local time to 7:00 am. We can observe the partial solar eclipse and try to make an estimate of the maximum percentage of the Sun that is covered by the shadow of the Moon as viewed from this location, i.e. the number x described in the calculations above.

We can use the procedure described above to estimate the absolute distance to the Moon assuming that the radius of the Earth is 6371 km.

Students can also combine this result with that of the first experiment of Aristarchus to obtain the actual distance to the Sun in kilometers.

3. Discussion and conclusions

Five different activities implemented in the Stellarium planetarium software have been presented that are based on astronomical measurements made by ancient Greek astronomers.

Initially, the famous measurement made by Eratosthenes was replicated by obtaining a value from Stellarium for the altitude of the Sun at the Summer solstice in 240 BCE. When Eratosthenes performed his experiment he actually measured the zenith distance of the Sun to be 7.2° yielding an answer of approximately 250 000 stadia for the circumference of the Earth. Students should be able to obtain a more accurate value from Stellarium. It is also worth asking students to change the observation location to Aswan (called Syene in Eratosthenes's time) and repeat the above procedure. They can then use the software to obtain the altitude of the Sun at local noon as viewed from this location. They should see that it is not exactly 90° as expected from Eratosthenes's assumptions and it can be pointed out that this is one of the sources of error. Other sources of error, such that the fact that Alexandria and Syene do not in fact lie on exactly the same meridian, can also be discussed.

A second estimate of the size of the Earth is obtained from the measurement made by Posidonius using the altitude of the star Canopus. Once again, Stellarium should be able to provide more accurate results than those obtained from the ancient Greek who estimated the altitude difference at 7.5° (it is in fact a bit smaller). This can be pointed out to students and discussions can be made on observational errors encountered due, for example, to the phenomenon of refraction.

In the third experiment, the distance of the Sun is estimated based on a method conceived by Aristarchus of Samos. This offers an excellent chance to demonstrate to students the limitations of ancient Greek astronomy imposed from the lack of sophisticated scientific instrumentation. Students can vary their angular measurement by 0.5 of a degree and see what effect it has on their final result. This will provide them with an understanding of how difficult the measurement was for Aristarchus (he in fact measured the angle to be 87°, more than 2° off) and why his final result was prone to significant error.

In the fourth experiment, the relative size of the Moon with respect to the Earth is estimated using the geometry of a lunar eclipse. Aristarchus devised a sophisticated geometrical solution to the problem. At his time, trigonometry had not yet been invented so cumbersome geometrical solutions had to be used instead (the same applies to the previous experiment). This is another point that can be made to the students. The instruments used for measuring time in antiquity, such as water clocks, can also be discussed. For most students it is probably prudent to skip the details of a geometrical solution. Instead a conversion factor estimate of the ratio of the breadth of the Earth's shadow to the diameter of the Earth itself can be provided to simplify the procedure.

Finally, the lunar parallax method implemented by Hipparchus is replicated to provide an estimate for the distance of the moon. Once again, the ancient astronomer would have used a more sophisticated and accurate geometrical approach (see, [7]). In our case, we have presented a very approximate but simple method of calculation which will produce an overestimation of the actual distance.

Overall, it should be evident that the aim of the activities is not to produce accurate results but to simulate the basic ideas of the simple but ingenious measurements made by the ancient Greek astronomers. By replicating the measurements, students are introduced to key milestones in the development of astronomy in an interactive way. It is hoped this will enhance both their interest and their understanding of the subject.

Appendix

The following is a list of basic instructions useful for completing the activities when using the Stellarium software.

Setting the location: this can be done by pressing the F6 key or clicking on the appropriate icon in the left toolbar to open the Location window. You can then either search for a city or country, use the provided map, or enter your coordinates manually.

Setting the date and time: you can select the Date/Time window from the side bar or by pressing the F5 key. You can also control time by using the J key to slow down or reverse the time increment speed, the L key to increase the speed and the K key to return to normal speed. You can also use the time control buttons in the lower right of the screen.

Navigating the sky: you can press the F3 key to open the Search window and search for a specific object such as the Moon and Sun. You must press the Return key to select the object. You can also use the mouse or arrow keys to look around and the Page Up and Page Down keys to zoom in and out of objects. You can use the left mouse button to select an object, the right button to deselect the object and middle mouse button or the spacebar to center on the selected object.

Measuring angles: the Angle Measure tool can be used to measure the angular distance between objects. To enable the Angle Measure tool, you must first go to the Configuration Window by pressing F2. You then select Plugins, the Angle Measure and Load at Startup. You must restart Stellarium in order to have this plugin activated.

Opening the help menu: you can press the F1 key for help.

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