Journal of Physics B: Atomic,
Molecular and Optical Physics

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To cite this article: H B Ambalampitiya *et al* 2024 *J. Phys. B: At. Mol. Opt. Phys.* **57** 10LT01

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IOP Publishing

Letters

Journal of Physics B: Atomic, Molecular and Optical Physics

J. Phys. B: At. Mol. Opt. Phys. **57** (2024) 10LT01 (5pp)

https://doi.org/10.1088/1361-6455/ad3f5a

Letter

Dipolar focusing in laser-assisted positronium formation

H B Ambalampitiya*, M K Matfunjwa and I I Fabrikant

Department of Physics and Astronomy, University of Nebraska, Lincoln, NE 68588-0299, United States of America

E-mail: hambalampitiya2@unl.edu

Received 7 February 2024, revised 3 April 2024
Accepted for publication 16 April 2024
Published 24 April 2024

Abstract

We consider positronium formation in collisions of positrons with excited hydrogen atoms H(n) in an infrared laser field theoretically. This process is assisted by the dipolar focusing effect: a positron moving in a superposition of a laser field and the dipolar field can approach the atomic target even if its trajectory starts with a very large impact parameter, leading to a significant enhancement of the Ps formation cross section. The classical trajectory Monte Carlo method, which is justified for $n \geqslant 3$, allows efficient calculation of this enhancement. A similar effect can occur in collisions of positrons with other atoms in excited states, which can lead to improvements in the efficiency of positronium formation.

Keywords: positronium, antihydrogen, laser-assisted process, classical trajectory Monte Carlo

Collisions involving Ps are important for the study and testing of the fundamental symmetry laws of the Universe. Ps–antiproton collisions can be used to produce antihydrogen (the smallest antimatter atom) via the charge-transfer process [1–5]. In support of these endeavors, beams of Ps atoms need to be first produced via an efficient mechanism. The charge-exchange reaction,

$$e^+ + A \to \mathrm{Ps} + A^+,$$

where A is a neutral atom or molecule which are often used to produce Ps beams. In particular, Ps beams for Ps collision studies [6–8] are formed by the process of Ps formation in collisions of positrons with neutral targets. In antihydrogen ATRAP experiments [1, 2] laser-excited Cs atoms were used in a charge exchange reaction with a cloud of positrons to produce excited state Ps.

The problem of Ps formation in positron collisions with hydrogen in its ground state is well studied theoretically [9–13]. The process of Ps formation in collisions of e^+ with excited hydrogen atoms,

$$e^+ + \mathrm{H}\,(n) \to \mathrm{Ps} + p, \qquad (1)$$

where $n \geqslant 2$ is the principal quantum number of the hydrogen atom, is less investigated [14], although it is of a fundamental interest. The reverse charge-conjugated process of reaction (1) can also be employed to produce antihydrogen as a different approach, see, for example, the works reported in [15–19]. In the present letter, we show that when an external laser field is introduced to the process (1), an effect known as dipolar focusing will be in action, resulting in a significant enhancement of the Ps formation cross section. A hydrogen atom in an excited state possesses an effective dipole moment leading to a long-range interaction between e^+ and H(n) [20]. A combination of the dipolar field and a monochromatic laser field can produce the dipolar focusing effect similar to the Coulomb focusing effect in the processes of double photoionization [21, 22], electron bremsstrahlung [23], radiative recombination [24], and dissociative recombination [25]. The dipolar focusing effect can be expected to play a role in similar laser-assisted positron

* Author to whom any correspondence should be addressed.

scattering processes, such as the process of photorecombination involving positrons and atoms [26].

The motion of low-energy electrons in a combined laser and Coulomb field can be treated classically because the long-range feature of the Coulomb force, the de Broglie wavelength, is small compared to the radius of the Coulomb interaction if the Coulomb parameter $e^2/(\hbar v)$ is large [27]. Here e is the elementary charge and v is the electron velocity. Similarly, the classical treatment of $e^- - \mathrm{H}(n)$ or $e^+ - \mathrm{H}(n)$ interaction is valid if the de Broglie wavelength λ is small compared to the effective radius of the dipolar interaction, $r_c \sim (ed/E)^{1/2}$ where d is the effective dipole moment of the target and E is the projectile energy. Estimating $d \sim ea_0 n^2$, where a_0 is the Bohr radius and n is the principal quantum number, we obtain $n^2 \gg 1$, i.e. the classical treatment is valid if the target is in a highly excited state. For higher-lying orbits the dipole moment grows and the effect of the dipolar interaction increases accordingly. Numerical classical trajectory Monte Carlo (CTMC) simulations and their comparison with quantum convergent close coupling calculations [14, 15] showed that the classical approach works in the low-energy region if $n \geqslant 3$. In the present paper we investigate the dipolar focusing and the cross section enhancement in the laser-assisted reaction (1) for $n = 3$. Experimentally, the $n = 3$ excited level of H can be achieved via photoexcitation, as was done in the experiment of Fleurbaey *et al* [28] where the 3s state of H is achieved in a two-photon excitation by a 205 nm cw laser.

From now on we are using atomic units. We use the dipole approximation to describe the positron-laser interaction. Consider a linearly polarized monochromatic electromagnetic wave with the electric field oscillating according to

$$\mathbf{F}(t) = \mathbf{e}F_0 \cos(\omega t + \phi_0),$$

where $\mathbf{e}$ is the polarization vector and F_0 the electric field amplitude. Consider a positron beam whose initial velocity $\mathbf{v}_0$ is parallel to the laser polarization vector. This relative orientation of $\mathbf{v}_0$ and $\mathbf{e}$ is the most efficient for the Coulomb focusing effect [24, 25]; therefore, we expect that this geometry will be efficient for the dipolar focusing as well. In spite of this geometry, our simulations are three-dimensional since they take into account the full dimensionality of the hydrogen atom. Far away from the target, the positron performs one-dimensional motion with the mean velocity

$$\bar{v} = v_0 - \frac{F}{\omega}\sin\phi_0, \qquad (2)$$

where $v_0 > 0$ is the velocity at $t = 0$. If the positron enters the field region at time τ, this results in replacing ϕ_0 by $\phi_0 + \omega\tau$, therefore averaging over τ is equivalent to averaging over ϕ_0. The positron will approach the target if $\bar{v} > 0$, and from equation (2) we have $0 < \phi_0 < \phi_1$, and $\pi - \phi_1 < \phi_0 < 2\pi$ where

$$\sin\phi_1 = \frac{\omega v_0}{F_0}, \quad 0 \leqslant \phi_1 \leqslant \pi/2 \qquad (3)$$

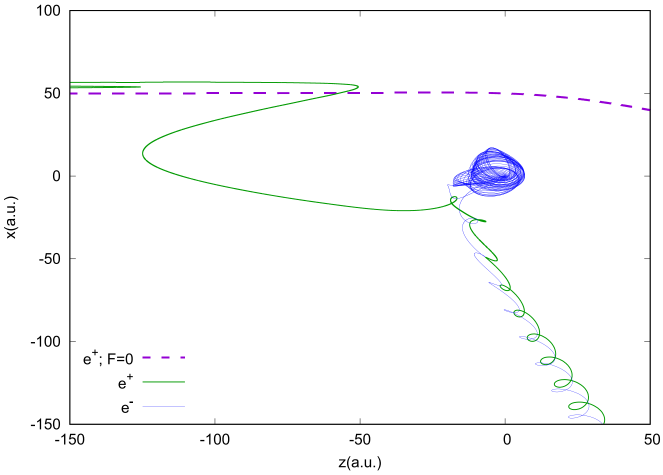

Figure 1. xz projection of positron and electron trajectories in $e^+ - \mathrm{H}(n=3)$ collisions for $b = 50$ a.u. Purple dashed line: e^+, $F = 0$; green and blue lines: e^+ and e^- trajectories for $F = 0.000\,244$ a.u, $\omega = 0.002$ a.u., $\phi_0 = 0.8$ and a random choice of the parameters of the electron orbit; the positron trajectory is shown by the thick line, and electron trajectory by the thin line.

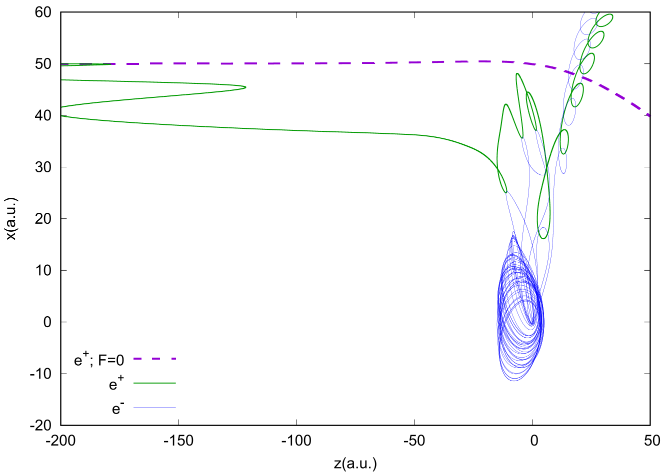

Figure 2. The same as in figure 1 for a different choice of parameters of the electron orbit.

therefore the probability of Ps formation will be nonzero in this range of ϕ_0. The probability peaks at values of ϕ_0 close to ϕ_1 and $\pi - \phi_1$. As an illustrative example we choose such field parameters that $\omega v_0/F_0$ is close to 1. This example is typical for a broad parameter subspace where dipolar focusing is important.

In figures 1 and 2 we present xz projections of electron and positron trajectories for collisions of e^+ with $\mathrm{H}(n=3)$ with the impact parameter $b = 50$ a.u. in the field with the amplitude $F_0 = 0.000\,244$ a.u. (intensity $I = 2.09$ GW cm^{-2}) and frequency $\omega = 0.002$ a.u. (wavelength $\lambda = 22.8$ μm), positron energy 0.14 eV corresponding to a velocity 0.1015 a.u., so that $\omega v_0/F_0 = 0.832$. Parameters of the initial electron orbit are chosen randomly. The impact parameter vector is directed along the x axis, and the initial velocity $\mathbf{v}_0$ along the z axis. Note that for these field parameters the oscillatory positron

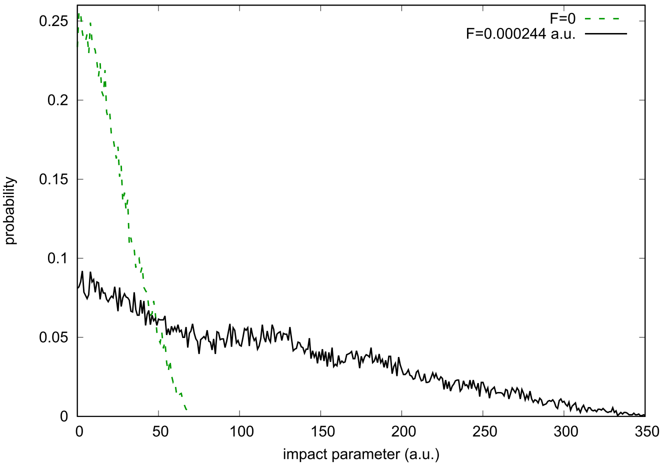

Figure 3. Probability of Ps formation in $e^+ - \mathrm{H}(n=3)$ collisions as a function of the impact parameter at $E = 0.14$ eV, comparison of zero-field probability with the probability for $F_0 = 0.000\,244$ a.u., $\omega = 0.002$ a.u., $\phi_0 = 0.8$.

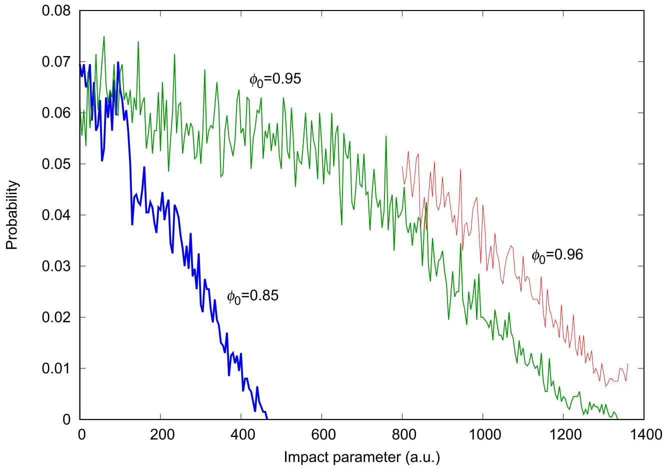

Figure 4. Probability of Ps formation in $e^+ - \mathrm{H}(n=3)$ collisions as a function of the impact parameter at $E = 0.14$ eV, $F_0 = 0.000\,244$ a.u., $\omega = 0.002$ a.u. comparison of probabilities for $\phi_0 = 0.85$, 0.95, and 0.96. For $\phi_0 = 0.96$ the probability is shown starting from $b = 800$ a.u.

motion in the absence of the $e^+ - \mathrm{H}$ interaction is characterized by a displacement amplitude 61 a.u., and a velocity amplitude 0.122 a.u. For the zero field the positron trajectory is very slightly affected by the presence of the hydrogen atom whereas in the presence of the field it is distorted enough for the positron to be captured by the electron and form Ps. This is clearly an example of dipolar focusing. For ϕ_0 very close to ϕ_1 and $\pi - \phi_1$ this dipolar focusing effect occurs for any impact parameter, resulting in a theoretically infinite Ps formation cross section.

This is illustrated in figures 3 and 4 where we present the probability of charge transfer in collisions of e^+ with $\mathrm{H}(n=3)$ in a field with the same field parameters as in figure 1, and several values of phase ϕ_0. It is important that for the chosen field parameters the multiphoton ionization rate is very small, about $0.5 \times 10^{-6}\ \mathrm{s}^{-1}$ [24], therefore the $n = 3$ state is not ionized by the laser field during the charge transfer process.

For low impact parameters the probability of Ps formation in a nonzero field is not necessarily greater than that for a zero field. However, in the former case it extends to much greater impact parameters resulting in cross sections 8.61×10^3, 1.59×10^4 and 1.54×10^5 a.u. for $\phi_0 = 0.80$, 0.85 and 0.95, respectively, whereas the corresponding zero-field cross section is 1.12×10^3 a.u. The range of impact parameters contributing to the cross section grows with ϕ_0, and is diverging when ϕ_0 approaches ϕ_1. For $\phi_1 < \phi_0 < \pi - \phi_1$ the mean positron velocity in the electric field becomes negative resulting in a zero cross section. However, it turns positive again for $\phi_0 > \pi - \phi_1$, and a similar divergence of the cross section is observed when ϕ_0 approaches $\pi - \phi_1$ from above.

Another interesting feature of the present problem is the absence of chaos: the probability as a function of the impact parameter does not exhibit fractal structures; irregularities in the $P(b)$ dependence simply represent statistical uncertainties in the CTMC calculations. This is in contrast to the problem of the motion of a charged particle in a combined laser and Coulomb field whereby the function $P(b)$ is chaotic [24, 25, 29, 30].

In figure 5 we present the Ps formation cross section as a function of ϕ_0. For comparison, the field-free cross section in this case is 1120 a.u. The cross section divergence when ϕ_0 approaches ϕ_1 or $\pi - \phi_1$ is strong, approximately as $(\phi_1 - \phi_0)^{-1.33}$ in the first case, and similar in the second, therefore averaging over ϕ_0 leads to an infinite cross section. In real experimental conditions it might become finite due to instrumental functions. For example, if the duration of the laser pulse t is finite, for large enough impact parameters the positron would not reach the target during the time period t, and this would provide the cross section cutoff. This possibility is discussed in detail in the treatment of laser-assisted radiative recombination [24]. Another possibility is the angular spread of the positron beam. If the initial positron velocity is not parallel to the laser polarization, the focusing effect is not as efficient, and the averaging over the angular divergence would provide another cutoff. As an example, we have calculated the averaged cross sections for the pulse durations $t = 5$ and 10 ps for the parameters indicated in the figure caption. The calculated cross sections are 3917 and 4650 a.u. which are factors of 3.50 and 4.15 higher than the field-free cross section, respectively.

The discussed focusing effect can be of practical importance for collisions with targets other than hydrogen atoms. For example, in antihydrogen ATRAP experiments [1, 2] laser-excited Cs atoms were used in a charge exchange reaction with a cloud of positrons to produce excited state Ps with very high cross sections [31]. Although alkali-metal atoms do not exhibit the same level of degeneracy as hydrogen, some of their excited levels of opposite parity, for example $6p$ and $5d$ in Cs, are close enough to induce a strong long-range interaction with e^+. For highly-excited Rydberg states the energy levels become hydrogen-like, and the dipolar

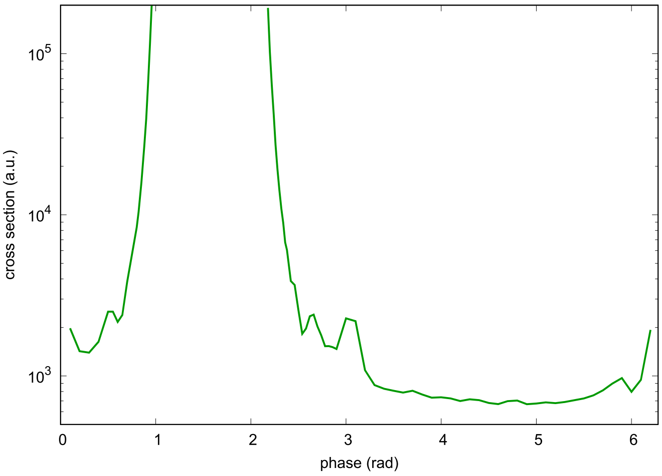

Figure 5. Cross section for Ps formation in $e^+ - \mathrm{H}(n=3)$ collisions as a function of the phase ϕ_0. For $E = 0.14$ eV, $F_0 = 0.000\,244$ a.u., $\omega = 0.002$ a.u.

focusing effect in collisions of e^+ with atoms in these states increases.

In conclusion, we have predicted a strong dipolar focusing effect in collisions of positrons with hydrogen atoms in an excited state leading to Ps formation. To demonstrate this effect we have employed the CTMC method, which has been shown to be valid for the description of $e^+ + \mathrm{H}(n)$ collisions if $n \geqslant 3$. The predicted effect can increase the efficiency of Ps production used for fundamental research and applications. Collisions involving Rydberg states with higher n can lead to an efficient production of Ps atoms with long lifetimes.

Data availability statement

The data cannot be made publicly available upon publication because no suitable repository exists for hosting data in this field of study. The data that support the findings of this study are available upon reasonable request from the authors.

Acknowledgments

The authors are grateful to Marcel Ngoko Djiokap for useful comments. This work has been supported by the National Science Foundation, Grant Nos. PHY-1803744 and PHY-2309261. H B A was supported by the US Department of Energy Office of Science, Basic Energy Sciences, under Grant No. DE-SC0021054.

ORCID iD

H B Ambalampitiya https://orcid.org/0000-0002-5843-4798

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