The coupled system (2) $\langle \superscript{2}\text{Li} \rangle_{\text{88}} \langle \superscript{+}\text{Sr} \rangle$ and (1) $\langle \superscript{2}\text{Li} \rangle_{\text{7}} \langle \superscript{88}\text{Sr} \rangle$
The coupled system \( (2)^2\Sigma^+ \) and \((1)^2\Pi \) of \(^7\text{Li}^{88}\text{Sr}\)

**The coupled system \((2)^2\Sigma^+\) and \((1)^2\Pi\) of \(^7\text{Li}^{88}\text{Sr}\)**

Erik Schwanke,1,2, a Julia Gerschmann,1,2 Horst Knöckel,1,2 Silke Ospelkaus,1,2 and Eberhard Tiemann1,2, b)  
1) Leibniz Universität Hannover, Institute of Quantum Optics, Welfengarten 1, 30167 Hannover, Germany  
2) Leibniz Universität Hannover, Laboratory for Nano- and Quantum Engineering, Schneiderberg 39, 30167 Hannover, Germany  
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We analyze rovibrational transitions of the \((2)^2\Sigma^+ - X(1)^2\Sigma^+\) system of LiSr and find the energy levels of the \((2)^2\Sigma^+\) state to be perturbed by coupling between the \((2)^2\Sigma^+\) and \((1)^2\Pi\) states. We present an analysis of the coupled system yielding molecular parameters for the lowest vibrational levels of the \((2)^2\Sigma^+\) state and for higher vibrational levels of the \((1)^2\Pi\) state together with molecular coupling constants. Improved Dunham coefficients for the rovibrational levels of the \(X(1)^2\Sigma^+\) state are also obtained, where the correlation with the parameters of the excited states is removed completely.

Keywords: PACS 31.50.-x Potential energy surfaces, PACS 33.15.Mt Rotation, vibration, and vibration-rotation constants, PACS 33.20.-t Molecular spectra

**INTRODUCTION**

In the last decade, much research on the energy structure of molecules consisting of one alkali and one alkaline earth atom has been conducted with the aim to produce ultracold samples of such molecules (see e.g. References 1–5). There are proposals to use these open-shell molecules for precision measurements6, to study scattering phenomena of ultracold particles7 and dipolar interactions8.

There exist ab initio calculations1,9–14 of potential energy curves of several electronic states of LiSr and transition probabilities between its electronic ground state and excited states. These data are very useful guides for spectroscopic studies, which then will yield the desired accurate molecular parameters for the experiments with ultracold ensembles.

We previously investigated the near infrared spectrum of \(^7\text{Li}^{88}\text{Sr}\) and derived Dunham coefficients for the lowest vibrational levels of the \(X(1)^2\Sigma^+\) and \((2)^2\Sigma^+\) states15. Figure 1 shows the electronic states in the observed spectral range. We observed perturbations in the \((2)^2\Sigma^+\) energy levels and used a coupled-state model for the \((2)^2\Sigma^+\) and \((1)^2\Pi_{1/2}\) states to explain a strong perturbation in the \(0–0\) band of rotational angular momenta up to \(N = 65\), while neglecting the weaker coupling with the \((1)^2\Pi_{1/2}\) state.

Here, we present an extensive study of the perturbations in the first two vibrational levels of the \((2)^2\Sigma^+\) state for \(N \leq 119\). These perturbations allow to find a consistent description of the \(\Omega = 1/2\) and \(\Omega = 3/2\) components of six vibrational levels in the \((1)^2\Pi\) state. Our extended model also gives improved Dunham coefficients for the \(\Sigma^+\) states and molecular coupling constants compared to our previous work.

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1) Electronic mail: schwanke@iqo.uni-hannover.de  
2) Electronic mail: tiemann@iqo.uni-hannover.de

**FIG. 1.** Ab initio potential energy curves16 of the doublet manifold of LiSr corresponding to the singly excited atomic states. The hatched region indicates the energy range of the observed NIR spectrum of the \((2)^2\Sigma^+ - X(1)^2\Sigma^+\) system. The perturbation of \((2)^2\Sigma^+\) levels is explained by the coupling to the \((1)^2\Pi\) state.

**I. EXPERIMENTAL WORK AND QUANTUM NUMBER ASSIGNMENT**

For spectroscopy, the molecular sample was produced in a heat pipe in the same way as described in our previous work15. We recorded the thermal emission spectrum and laser-induced fluorescence (LIF) spectra. The thermal emission spectrum of LiSr is shown in Figure 2 with assignments of vibrational bands. These assignments are an extrapolation from our modelled system (see Section IV), which incorporates only \(v'' = 0, 1, 2\) for \(X(1)^2\Sigma^+\) and \(v' = 0, 1\) for \((2)^2\Sigma^+\). The shapes and positions of band structures simulated with our findings are in good agreement with the observed spectrum. We only assigned the spectrum of \(^7\text{Li}^{88}\text{Sr}\), the spectra of other isotopologues are too weak to be identified in our recorded spectrum. The LIF spectra show certain transition lines with a higher intensity as in the pure thermal emission spectrum. These exaggerations tell which lines in the
The coupled system \((2)^2\Sigma^+\) and \((1)^2\Pi\) of \(^7\text{Li}^{88}\text{Sr}\)

![Graph showing thermal emission spectrum of LiSr](image)

**FIG. 2.** Thermal emission spectrum of LiSr in the near-infrared spectrum. Vibrational bands with \(v\) up to 3 are indicated, where distinctly visible.

dense thermal emission spectrum have a common upper level. Rotational lines from the \(P(N'' = N' + 1)\) and \(R(N'' = N' - 1)\) branches with the same upper level \(N'\) were identified by this method. The frequency difference between a PR line pair together with the ground state rotational constant \(B''\) allow to unambiguously assign the rotational quantum numbers to the lines. The LIF spectra further highlight lines with the same rotational quantum numbers in other vibrational bands following a vibrational progression. For an illustration, Figure 3 shows laser-induced fluorescence lines of the \(F_1\) R branch of the \((1–1)\) band. \((F_1\) or \(F_2\) label the spin-rotation doublets of rotational state \(N''\) as defined at the end of this section.) They result from laser excitations in the P branch of the same band around 9300 cm\(^{-1}\).

The fluorescence lines (coloured) have a higher intensity than the corresponding ones in the thermal emission spectrum (black). We used the frequency difference of the \(P\) and \(R\) lines together with the rotational constant of the electronic ground state to find the quantum numbers of the excited and answering lines. The same excitations also yield fluorescence in the \((1–0)\) band, at frequencies from 9440 cm\(^{-1}\) to 9520 cm\(^{-1}\) (see Figure 2), as well as PR pairs in the \((1–2)\) band (not labelled in Figure 2). The \((1–1)\) band is mostly obscured by the more intense \((0–0)\) band, so that many \((1–1)\) lines cannot reliably be identified in the thermal emission spectrum. However, their frequencies can be determined from the LIF spectra despite the strong overlap of the \((0–0)\) band. For the lines \(F_1\) R 78 and 80, the fluorescence shows a visibly different central frequency than the line structure in the thermal emission spectrum.

Because of the shape and very similar equilibrium positions of the potentials of the \(X(1)^2\Sigma^+\) and \((2)^2\Sigma^+\) states in alkali-alkaline earth diatomics, vibrational bands with large differences in the vibrational quantum number have small Franck-Condon factors\(^{13}\). Therefore, only short progressions are expected and in the actual case only the \(v'' = 0, 1, 2\) levels of the \(X(1)^2\Sigma^+\) state could be detected by laser excitation of \(v' = 0, 1\) levels of the \((2)^2\Sigma^+\) state.

We observed that the frequencies of transition lines deviate from a regular rovibrational energy structure for many \((2)^2\Sigma^+\) levels. These deviations were too large to convincingly identify the quantum numbers from the rotational series in the thermal emission spectrum. Moreover, the intensity of the strongly perturbed lines dropped considerably, which further hindered the tracking of a rotational branch in the spectrum. Here, the LIF experiments in the perturbed parts of rotational branches helped to determine rotational quantum numbers of unknown spectral lines in small steps. By this approach, we were able to identify and assign lines with a large frequency shift and thus gather data about the perturbations in the \((2)^2\Sigma^+\) state. Figure 4 shows the \(F_1\) P and R branches of the \((1–1)\) band for \(N' \geq 99\). These lines are mostly discernible in the thermal emission spectrum but much weaker than the surrounding \((0–0)\) lines. Excitations at the frequencies indicated in Figure 4 (a) lead to
The coupled system \( (2)^2\Sigma^+ \) and \( (1)^2\Pi \) of \( ^7\text{Li}^{88}\text{Sr} \)

The rotational ladder for \( \nu' = 0 \) could be followed up to \( N' = 115 \) and that for \( \nu' = 1 \) from \( N' = 40 \) to \( N' = 109 \). Especially for \( \nu' = 1 \), we could not completely follow the series to the perturbation centres because the lines became too weak due to the strong coupling.

All observed transitions belong to the \( (2)^2\Sigma^+-X(1)^2\Sigma^+ \) spectrum and no transition to the neighboring state \( (1)^2\Pi \) (see Figure 1) was found. Our description of the \( (1)^2\Pi \) state, derived from the coupling to the \( (2)^2\Sigma^+ \) (see below), is confined to a few vibrational levels around \( \nu_1 = 14 \) according to ab initio calculations\(^{11,13} \). With those data, it is not possible to accurately extrapolate to lower \( (1)^2\Pi \) levels. We tried to directly observe the \( (1)^2\Pi-\Sigma^+ \) spectrum, which the ab initio calculations predict around 5000 cm\(^{-1} \), using an InGaAs avalanche photodiode (G8931-20, Hamamatsu). We did not observe any spectrum in the region from 5000 cm\(^{-1} \) to 9000 cm\(^{-1} \). Reasons are: (1) the InGaAs detector is less responsive and noisier than the Si-APD we used to observe the \( (2)^2\Sigma^+-X(1)^2\Sigma^+ \) spectrum (S11519-30, Hamamatsu) and (2) the molecular transitions are much weaker than those of the observed \( (2)^2\Sigma^+-X(1)^2\Sigma^+ \) spectrum according to the ab initio calculations\(^{11,13} \).

Our previous, simplified analysis\(^{15} \), which neglected the \( (1)^2\Pi \) spin-orbit-coupling, was performed in Hund’s coupling case (b). The basis vector in this case is \( |\Delta, (N, S), J \rangle \) with the quantum numbers of the projection of the orbital angular momentum onto the molecular axis, \( \Delta \), the total angular momentum without spins, \( N \), the electronic spin \( S \) and the total angular momentum without nuclear spins, \( J = N + S \). We used Dunham coefficients for the rovibrational energy description in both electronic states, so that the energies for \( \Sigma \) states can be expressed as

\[
E_{\text{Dun}}^{(b)}(v, N) = \sum_{m,n} \gamma_{m,n}^{(b)}[v + 1/2]^m[N(N + 1)]^n.
\]  

Due to the coupling between the electronic spin and the nuclear rotation, the rovibrational energy levels in a \( ^5\Sigma^+ \) state are split. The levels for which \( J = N + 1/2 \) are called \( F_1 \) levels and have \( e \)-type symmetry. The levels for which \( J = N - 1/2 \) are called \( F_2 \) levels and have \( f \)-type symmetry (see section 2.2.2 in Reference 17). The spin-rotation energy \( E_{SR} \) from \( \tilde{H}_{SR} = \gamma \hat{S} \hat{N} \) is simply added to the rovibrational energy because this operator is diagonal in the quantum numbers \( N \) and \( S \). It reads with the spin-rotation coupling constant \( \gamma \) as

\[
E_{SR} = \pm \frac{\gamma}{2} [N + 1/2 \mp 1/2] = -\frac{\gamma}{2} [1 \mp (J + 1/2)],
\]  

where the upper sign is for \( F_1 \) and the lower sign for \( F_2 \) levels. We expressed \( \gamma \) itself with a Dunham-like expansion to include a dependence on the rovibrational motion:

\[
\gamma(v, N) = \sum_{m,n} \gamma_{m,n}[v + 1/2]^m[N(N + 1)]^n
\]  

FIG. 4. Line assignments of \( F_1 \) (1–1) lines by fluorescence experiments for strongly perturbed spectral ranges in the thermal emission spectrum. For LIF experiments, the laser was tuned to the frequencies marked in (a) and fluorescence was seen at the frequencies indicated by the vertical lines in (b). Each excited line has a corresponding fluorescence line marked by the same quantum number \( N' \). The clear structure of the LIF spectra allowed us to quickly identify PR pairs from the lines in (a) and (b). The thermal emission lines in (a) and (b) are finally all assigned within our analysis, except some of the weak ones.

fluorescence at the frequencies marked in Figure 4 (b). The \( (2)^2\Sigma^+ \) levels corresponding to the excitations are strongly perturbed (see Figure 6 later). This perturbation heavily disorganizes the order of the lines. In Figure 4 (a), it describes a zigzag pattern when following \( N' \). Here again the LIF spectra allowed us to quickly identify the correct quantum numbers, which together with the line frequencies showed us the development of the perturbation. The lines with \( N' = 120 \) in Figure 4 could be assigned via their PR distance but the derived level energies could not be included in the fit, because the gap to the systematically recorded rotational ladder is too big and probably contains the next strongly perturbed region (see Figure 6).
The coupled system \((2)^2\Sigma^+ + (1)^2\Pi\) of \(7\text{Li}^{88}\text{Sr}\)

II. GROUND STATE ANALYSIS

The lines addressed with the LIF experiments covered ground state levels with \(\nu'' = 0, 1, 2\) and \(\nu'' \leq 115\). The frequency differences between fluorescence and/or laser lines in a LIF spectrum correspond to energy differences in the \((1)^2\Sigma^+\) state. These are independent of perturbations in the excited state and will suppress the correlations between upper and lower state parameters typically obtained from the fit of optical transition frequencies. We chose a minimum uncertainty of 0.02 cm\(^{-1}\) that is slightly less than the Doppler broadening of 0.024 cm\(^{-1}\).

Thus we take into account that we are limited by the observed thermal emission spectrum when setting the laser frequency to a transition line. We conducted a linear fit of Dunham and spin-rotation coupling parameters according to equations (1) to (3) to the frequency differences. This fit gave a weighted standard deviation of \(\sigma = 0.25\), with 99.64\% of data being modelled within their assumed uncertainty. The low value of \(\sigma\) indicates that we could have assumed a smaller minimum uncertainty which shows that setting the laser on resonance was typically better than within the Doppler width. The fitted coefficients and their standard deviations are given in Table I. Calculating ground state energies with these parameters will have an uncertainty of at least that of the absolute accuracy of the frequency scale, i.e., 0.01 cm\(^{-1}\).

We applied the derived molecular parameters of the ground state to construct the level system of the perturbed state \((2)^2\Sigma^+\) from the measured transition frequencies. Now the data for the \((1)^2\Sigma^+ + (2)^2\Sigma^+\) levels are independent of each other as long as the assignment is correct. The uncertainties of the constructed \((2)^2\Sigma^+\) levels are composed of the uncertainties of the ground state levels and the uncertainty of a spectral line. We chose a minimum uncertainty of 0.02 cm\(^{-1}\) for the constructed levels, but most levels of the state \((2)^2\Sigma^+\) have an uncertainty of 0.03 cm\(^{-1}\).

III. THE COUPLED SYSTEM \((2)^2\Sigma^+ + (1)^2\Pi\)

For the joint description of the \((2)^2\Sigma^+\) and \((1)^2\Pi\) level systems and their coupling, we apply Hund’s coupling case (a) with the state vector \(|\Lambda, \Sigma, \Omega, \nu', e/f\rangle\) because the \(2\Pi\)-state with its significant spin-orbit interaction\(^1\) is close to this coupling case. Here, \(\Lambda\) and \(\Sigma\) are the quantum numbers of the projections of the electronic orbital angular momentum and the spin onto the molecular axis and \(\Omega = \Lambda + \Sigma\) is defined as the projection of the total angular momentum \(J\) excluding nuclear spins. The terms in the Dunham energies have to be adapted for this coupling case. For \(\Sigma^+\) levels, they are given by

\[
E_{\text{Dun}}^{(a)}(v, J, \epsilon/f) = \sum_{m, n} Y_{m, n}^{(a)} \left(v + \frac{1}{2}\right)^m \left(J + \frac{1}{2}\right)^n P_{\nu, e/f}(J, \epsilon/f, 1)\]

(4)

with the upper sign for \(e\) (\(F_1\)) levels and the lower sign for \(f\) (\(F_2\)) levels. With this convention, equations (1) and (4) have equal parameters in Hund’s cases (a) and (b). For \(2\Pi\) levels, the energy is given by

\[
E_{\text{Dun}}^{(a)}(v, J, \epsilon/f) = \sum_{m, n} Y_{m, n}^{(a)} \left(v + \frac{1}{2}\right)^m \left[\frac{(J + 1)}{2}/2\right]^n\]

(5)

being the same for \(e\) and \(f\) levels. Similarly, the rotational constant of a vibrational level \(v\) is expressed as

\[
B_{\Pi}^{(v, e/f)} = \sum_m Y_{m, 1}^{(a)} \left(v + \frac{1}{2}\right)^m.

(6)

Table II shows the matrix for the coupling of a \(2\Sigma^+\) state and a \(2\Pi\) state with their vibrational levels \(\epsilon\) and \(f\) by the spin-orbit and rotational interaction with its respective molecular coupling parameters \(A\) (spin-orbit interaction) and \(B\) (rotational interaction)\(^1\). The rovibrational energies of the basis states are represented according to equations (4) and (5). The diagonal term for the state \((2)^2\Sigma^+\) in Hund’s case (a) is exactly the value calculated with equations (1) to (3) in Hund’s case (b), only expressed with the quantum numbers \(J\) and \(\epsilon/f\). The matrix includes the spin-rotation coupling (with constant \(\gamma\)) resulting from "distant" electronic states not considered in the basis system, i.e., those not included in our model. The spin-rotation from the "local" system results from the non-diagonal term of \(2\Sigma\) and \(2\Pi_{1/2}\), which also gives the \(e/f\)-splitting of the state \((1)^2\Pi\), normally called \(\Lambda\)- or \(\Omega\)-doubling. The subscripts \(\Sigma\) and \(\Pi\) at the parameters indicate to which electronic state(s) they belong. Because the matrix in Table II describes the restricted Hilbert space for selected vibrational levels, the primary coupling parameters are multiplied by overlap integrals of vibrational wave functions of the \(2\Sigma^+\) and the \(2\Pi\) state \((\Sigma_{\Pi} = (\Sigma/\epsilon)\)). Here, we neglect a dependence of the coupling parameters on the internuclear distance \(R\). The parameter \(p\) is a shorthand for the expectation value of the ladder operator of the electronic orbital angular momentum:

\[
p := \langle \Pi | L^+ | \Sigma \rangle

(7)

It would evaluate to \(\sqrt{2}\) if the molecular states could be traced back to \(p\)-electrons, as it is true asymptotically \((R \to \infty)\) for \(\text{LiSr}\). There are no couplings between levels with different \(J\) quantum numbers because we neglect hyperfine interactions and also \(e/f\) are constants of motion; so the matrix in Table II can be used for each observed \((2)^2\Sigma^+\) level independently.

In the non-diagonal matrix elements \(\langle \Pi | ... | \Sigma \rangle\), products of parameters appear which cannot be determined separately by the fit. Thus we introduce the effective
The coupled system \((2)^2\Sigma^+ + (1)^2\Pi\) of \(^7\text{Li}\)\(^{88}\text{Sr}\)

### TABLE I. Derived Dunham and spin-rotation parameters for levels with \(N \leq 115\) for \(v = 0, 1\) and \(40 \leq N \leq 63\) for \(v = 2\) of the ground state \((1)^2\Sigma^+\). The uncertainties are those derived in the linear fit (resulting in \(\sigma = 0.25\), see text). All values are given in \(\text{cm}^{-1}\) and are valid for \(^7\text{Li}\)\(^{88}\text{Sr}\).

<table>
<thead>
<tr>
<th>(\nu)</th>
<th>(Y_{1n})</th>
<th>(Y_{2n})</th>
<th>(\gamma_{1n})</th>
<th>(\gamma_{2n})</th>
<th>(\sigma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.001(1)</td>
<td>-3.1018(17)</td>
<td>10(^{-5})</td>
<td>8.18(37)</td>
<td>10(^{-2})</td>
</tr>
<tr>
<td>1</td>
<td>2.6740(244)</td>
<td>-5.724(78)</td>
<td>10(^{-5})</td>
<td>-5.04(11)</td>
<td>10(^{-2})</td>
</tr>
<tr>
<td>2</td>
<td>-1.0831(50)</td>
<td>-3.540(19)</td>
<td>10(^{-8})</td>
<td>-6.628(71)</td>
<td>10(^{-9})</td>
</tr>
<tr>
<td>3</td>
<td>-2.309(15)</td>
<td>10(^{-16})</td>
<td>-1.1953(51)</td>
<td>10(^{-16})</td>
<td>-</td>
</tr>
</tbody>
</table>

### TABLE II. Matrix representation of couplings between \(^2\Sigma^+, ^2\Pi_{1/2}\) and \(^2\Pi_{3/2}\) states in Hund’s case (a) for a total angular momentum \(J\). The upper and lower signs are for \(e\) and \(f\) levels, respectively. See the text for an explanation of the constants.

<table>
<thead>
<tr>
<th>(\langle v,^2\Sigma^+_1/2, e/f \rangle)</th>
<th>(\langle v,^2\Pi_{1/2}, e/f \rangle)</th>
<th>(\langle v,^2\Pi_{3/2}, e/f \rangle)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-v_{\Sigma}^2 \Sigma^+_1/2, e/f)</td>
<td>(-v_{\Sigma}^2 \Pi_{1/2}, e/f)</td>
<td>(-v_{\Sigma}^2 \Pi_{3/2}, e/f)</td>
</tr>
<tr>
<td>(\xi^v_{\Sigma}^2 \Sigma^+_1/2, e/f)</td>
<td>(\xi^v_{\Sigma}^2 \Pi_{1/2}, e/f)</td>
<td>(\xi^v_{\Sigma}^2 \Pi_{3/2}, e/f)</td>
</tr>
</tbody>
</table>

### parameters:

\[
\begin{align*}
V_{\Sigma}^H & := \tau_v^H \cdot A_{\Sigma} - \gamma_{\Sigma} \\
B_{\Sigma}^H & := B_{\Sigma}/(A_{\Sigma} - \gamma_{\Sigma})
\end{align*}
\]

The first of these is later on called simply overlap integral and the second the rotational constant. The considered matrix elements only depend on these parameters.

The value of \(\gamma_{\Pi}\) is generally small compared to \(A_{\Pi}\) and \(B_{\Pi}\), therefore we disregard it. As such, only expression (3) with \(\gamma_{\Sigma}\) is left for the spin-rotation interaction in our model. The supplementary explains the approximation of the effective rotational constant

\[
B_{\Pi}^{(n,v)} \approx \sqrt{\frac{4 \xi_{\Sigma}^2}{\xi_{\Sigma}}} \text{ for the non-diagonal matrix element }
\]

\[
\langle v_{\Pi},^2\Pi_{1/2}, e/f \rangle \| H \| v_{\Pi},^2\Pi_{3/2}, e/f \rangle \text{ for } v_{\Pi} \neq v_{\Pi}.
\]

When plotting the energy levels of the \((2)^2\Sigma^+\) state derived from the observations as function of \(J(J+1)\), the series of rotational energies appears as an almost straight line for each vibrational level. The slopes of these lines represent effective rotational constants. Figure 5 shows this level scheme for \(v = 0, 1\) (red and blue series of dots) and additionally the assumed rotational ladder of the \((1)^2\Pi\) state (black and green series of dots) where the crossings of both states become visible. If one redraws such plots for the directly observed \((2)^2\Sigma^+\) levels by shifting the rotational energy according to effective rotational constants, almost horizontal lines appear, where within narrow \(J\)-ranges deviations show up. These deviations indicate the perturbation and thus the crossing of the rotational ladders of the studied state (here \((2)^2\Sigma^+)\) with those of the perturbing state (here \((1)^2\Pi\)) more clearly. This is illustrated in Figure 6, where we plotted the energy shift introduced by the coupling compared to “unperturbed” levels calculated with the simple Dunham model. In such a plot, perturbed ranges are undoubtedly detected.

For the non-linear fit of the molecular parameters according to Table II, we compared each energy level with quantum numbers \((v_{\Sigma}, J, e/f)\) constructed from the observations with the eigenvalue of the appropriate coupling...
The coupled system \((2)^2\Sigma^+\) and \((1)^2\Pi\) of \(^7\text{Li}^{88}\text{Sr}\) matrix with the largest amplitude of the basis vector \(|v_{\Sigma}^2\Sigma_{1/2}\rangle / f\). The fit was conducted with the MINUIT software\(^\text{18}\).

As starting values for the Dunham coefficients of the state \((2)^2\Sigma^+\), we used the preliminarily obtained ones from a linear Dunham fit of those levels which could be identified as very weakly perturbed by its regular structure of the rotational energy. Four such crossings, strong deviations were observed for each of the rotational ladders. Initial energy values for the state \((1)^2\Pi\) were estimated from these crossing energies taken as approximate energies of the \(\Omega = 1/2\) component of state \((1)^2\Pi\). A vibrational quantum number was assigned simply by counting from \(n_{\Pi} = 0\) for the lowest included vibrational level.

The observed energy structure of the perturbation does not contain information to establish absolute \(v_{\Pi}\) quantum numbers and our model does not use vibrational wave functions explicitly. Thus the present choice of low \(v_{\Pi}\) has no drawback for the description of the energies.

In Figure 6, sharp small deviations are visible which we attribute to the rotational coupling between \((2)^2\Sigma^+\) and \((1)^2\Pi\) according the coupling matrix in Table II. These give us additional information on this electronic state.

For each \(v_{\Sigma}\) level, only the two closest levels for each \(\Pi\) component, with \(v_{\Pi}\), were selected, giving for each \(v_{\Sigma}\) and \(J \geq 3/2\) a \((5 \times 5)\)-matrix. The actual level scheme for \((2)^2\Sigma^+\) and \((1)^2\Pi\), shown in Figure 5, allowed to derive the level structure of the \(\Pi\) state for six adjacent vibrational levels, labelled as \(n_{\Pi} = 0\) to 5.

We varied the Dunham coefficients for \((2)^2\Sigma^+\) and \((1)^2\Pi\), the coupling constants defined by expressions (8) and (9) and \(A_{\Pi}\).

**IV. RESULTS**

![Figure 6](image_url)  
**FIG. 6.** Energy shift of \((2)^2\Sigma^+\) levels due to the coupling to the \((1)^2\Pi\) state. There are extended regions of strong perturbations around crossing points with the \(2\Pi_{1/2}\) levels and narrow ones of weaker perturbations around crossing points with the \(2\Pi_{3/2}\) levels. The symbols (red circles for \(e\) levels and blue diamonds for \(f\) levels) represent measured data (the experimental uncertainty lies within the symbol size) and the lines are data calculated with the coupled state model. These lines are mostly hidden by the symbols.

First we analysed the coupling of the upper \(2\Sigma^+\) state to the state \(2\Pi_{1/2}\) only, because of the significant magnitude of spin-orbit coupling to this component with \(\Omega = 1/2\). The perturbation for the presently assigned \(e\) levels appears at lower \(J\) than for the \(f\) levels. This is a clear confirmation that the assumed assignment of \(e/f\) (\(F_1/F_2\)) is correct because the rotational energy for the same \(J\) is higher for \(f\) than for \(e\) levels and the rotational constant of \((1)^2\Pi\) is larger than for \((2)^2\Sigma^+\).

For the remaining deviations, we found a regular series of localized (spanning 2 to 3 \(J\) levels) deviations, which could be undoubtedly assigned to the perturbation by the \(2\Pi_{3/2}\) state. Thus we extended the fit to the full size matrix \(5 \times 5\) and also fitted the spin-orbit splitting \(A_{\Pi}\) of \((1)^2\Pi\).

We cover a large interval of rotational states and thus consider a \(J\)-dependence of the overlap integrals – or more specifically, the product of overlap integral and spin-orbit coupling – according to

\[
V_{\Sigma\Pi}^f = V_{\Sigma\Pi}^{\text{const}} + V_{\Sigma\Pi}^J \cdot J(J + 1). \tag{10}
\]

We do not have a solid theoretical basis for this analytic representation. Only for strong perturbations, the deviations are spread over a large range of \(J\) levels, namely
The coupled system \((2)^2\Sigma^+\) and \((1)^2\Pi\) of \(^7\text{Li}^{88}\text{Sr}\)

TABLE III. Derived Dunham and spin-rotation parameters for the state \((2)^2\Sigma^+\) and the state \((1)^2\Pi\) of \(^7\text{Li}^{88}\text{Sr}\). They give an accurate description for levels with \(N \leq 119, v = 0\) for state \((2)^2\Sigma^+\). The \((1)^2\Pi\) parameters describe the vibrational structure around the observed \((2)^2\Sigma^+\) levels with the assumed vibrational assignment \(v_1 = 0\) to 5 for the deperturbation. Their absolute vibrational quantum numbers are not yet known. All values are given in \(\text{cm}^{-1}\).

<table>
<thead>
<tr>
<th>(Y_{0n})</th>
<th>(Y_{1n})</th>
<th>(Y_{2n})</th>
<th>(\gamma_{0n})</th>
<th>(\gamma_{1n})</th>
<th>(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>((2)^2\Sigma^+)</td>
<td>(\text{Hund’s case (a)})</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(9.391(8)(14) \times 10^5)</td>
<td>(1.815(27) \times 10^4)</td>
<td>(- \times 10^3)</td>
<td>(- 4.542(19) \times 10^{-2})</td>
<td>(8.8(40) \times 10^{-3})</td>
<td>(0)</td>
</tr>
<tr>
<td>(1.895(25) \times 10^{-1})</td>
<td>(- 7.14(44) \times 10^{-4})</td>
<td>(- \times 10^{-3})</td>
<td>(- 8.16(50) \times 10^{-8})</td>
<td>(- \times 10^{-3})</td>
<td>(1)</td>
</tr>
<tr>
<td>(- 8.186(11) \times 10^{-7})</td>
<td>(4.8(19) \times 10^{-9})</td>
<td>(- \times 10^{-3})</td>
<td>(- \times 10^{-3})</td>
<td>(- \times 10^{-3})</td>
<td>(2)</td>
</tr>
<tr>
<td>(4.69(13) \times 10^{-12})</td>
<td>(- \times 10^{-3})</td>
<td>(- \times 10^{-3})</td>
<td>(- \times 10^{-3})</td>
<td>(- \times 10^{-3})</td>
<td>(3)</td>
</tr>
<tr>
<td>(- 2.59(98) \times 10^{-17})</td>
<td>(- \times 10^{-3})</td>
<td>(- \times 10^{-3})</td>
<td>(- \times 10^{-3})</td>
<td>(- \times 10^{-3})</td>
<td>(4)</td>
</tr>
<tr>
<td>((1)^2\Pi)</td>
<td>(\text{Hund’s case (a)})</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2.2286(87) \times 10^{5})</td>
<td>(2.7713(23) \times 10^{4})</td>
<td>(- 4.53(20) \times 10^{0})</td>
<td>(- \times 10^{-3})</td>
<td>(- \times 10^{-3})</td>
<td>(0)</td>
</tr>
<tr>
<td>(2.621(35) \times 10^{-1})</td>
<td>(- 3.04(57) \times 10^{-3})</td>
<td>(- \times 10^{-3})</td>
<td>(- \times 10^{-3})</td>
<td>(- \times 10^{-3})</td>
<td>(1)</td>
</tr>
<tr>
<td>(- 1.43(45) \times 10^{-6})</td>
<td>(- 6.3(41) \times 10^{-8})</td>
<td>(- \times 10^{-3})</td>
<td>(- \times 10^{-3})</td>
<td>(- \times 10^{-3})</td>
<td>(2)</td>
</tr>
<tr>
<td>(3.8(24) \times 10^{-11})</td>
<td>(- \times 10^{-3})</td>
<td>(- \times 10^{-3})</td>
<td>(- \times 10^{-3})</td>
<td>(- \times 10^{-3})</td>
<td>(3)</td>
</tr>
<tr>
<td>(- 1.85(62) \times 10^{-15})</td>
<td>(- \times 10^{-3})</td>
<td>(- \times 10^{-3})</td>
<td>(- \times 10^{-3})</td>
<td>(- \times 10^{-3})</td>
<td>(4)</td>
</tr>
</tbody>
</table>

for the overlap integrals \((v_1|v_2)\) with \(|0\rangle, |1\rangle, |2\rangle\) and \(|3\rangle\) (compare Figure 6). Therefore, we introduced the J-dependence only for these perturbation pairs and got an improvement of the fit by almost a factor of two. The normalized standard deviation of the fit is \(\sigma = 1.11\).

Figure 6 plots besides the observed perturbation energies for the two observed vibrational levels of \((2)^2\Sigma^+\) also as solid lines the calculated values from the fit. On the shown scale, there is no deviation visible between observation and calculation. The regular structure in the appearance of \(2\Pi_1/2\) and \(2\Pi_3/2\) perturbations when plotted over \(J(J + 1)\) is convincing evidence of the presented assignment of \(2\Pi_1/2\) and \(2\Pi_3/2\) crossings. Thus the observations also allow from the differences between the perturbations of the \(\Omega = 1/2\) and \(\Omega = 3/2\) components the determination of the spin-orbit splitting within the state \((1)^2\Pi\).

Figure 7 shows the relative deviations of observed level energies from energies predicted by the coupling model. The dashed lines in Figure 7 mark the levels which were sufficiently unperturbed to be describable with a Dunham approach (equations (1) and (2)) within their experimental uncertainty. With the deperturbation model, most lines are well reproduced within their experimental uncertainty, however, the calculated lines near the perturbation centres do not fit the observed data similarly well. The stronger a level is perturbed, the lower is its \(^2\Sigma^+\) amplitude and the lower is the intensity of a spectral line, because the transition dipole moment for \((1)^2\Pi-X(1)^2\Sigma^+\) transitions is much smaller than for \((2)^2\Sigma^+\) transitions\(^{11,13}\). Due to this decreasing intensity and the expected large frequency shift (see Figure 6), lines are more difficult to find and to assign in the spectrum, the closer they are to a perturbation centre.

With the deperturbation model, we were able to identify approximately 100 new lines that are considerably perturbed. However, there are still data gaps left at the perturbation centres.

The fitted molecular parameters are given in Tables III, IV and V. In Table V, also the rotational quantum numbers for the crossing of the \(2\Sigma^+\) and \(2\Pi_1/2\) levels are given for each pair of \(v_2\) and \(v_1\) (compare Figure 5) for getting the right order of magnitude for the J-dependence of the overlap integrals.

TABLE IV. Fitted coupling parameters of the \((2)^2\Sigma^+-(1)^2\Pi\) system in \(\text{cm}^{-1}\). The magnitude of the \(^2\Sigma^+-^2\Pi\) -coupling is scaled with the overlap integrals given in Table V. See the text for details.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B_{\Sigma\Pi}/(A_{\Sigma\Pi} - \gamma_{\Sigma\Pi}))</td>
<td>0.000196(60)</td>
</tr>
<tr>
<td>(A_{\Pi})</td>
<td>155.68(78)</td>
</tr>
</tbody>
</table>

Since the actual quantum numbers \(v_1\) are not known from the deperturbation process, the Dunham coefficients give only a local description for the \((1)^2\Pi\) levels, with the assumed vibrational range \(v_1 \geq 0\) to 5. The otherwise unobserved \((1)^2\Pi\) energy ladders cannot be expected to be predicted as well as those of the \((2)^2\Sigma^+\) state.

The deperturbation was achieved with a nonlinear fit, thus it is difficult to estimate how closely we approached the absolute minimum of the squared sum of the deviations. For the uncertainties of the parameters given in Tables III and IV, we considered the development of these parameters during the course of our analysis period while adding more and more data. In this way, we consider the variances of the fit parameters under different sets of starting parameters so that the change of a given parameter over time gives a range of confidence. The relative uncertainties of the \((2)^2\Sigma^+\) Dunham parameters are generally one order of magnitude higher than of those from the linear fit in Table I. This is expected, since we constructed the \((2)^2\Sigma^+\) levels with the parameters from Table I and the observed line frequencies with an as-
TABLE V. Overlap integrals $V_{\Sigma \Pi}^{\text{fit}}$ between the $(2)^2\Sigma^+$ and $(1)^2\Pi$ states from the deperturbation. The overlap integrals have units of [cm$^{-1}$], because the contain the spin-orbit interaction. The left and right values are $V_{\Sigma \Pi}^{\text{const}}$ and $V_{\Sigma \Pi}^{f}$ according to equation (10). See the text for an estimate of the uncertainties. We also give the quantum numbers $J$ closest to the crossing of $(2)^2\Pi_j/2$ levels with the $e$ and $f$ $(2)^2\Sigma^+$ levels. “x” indicates that the crossing is beyond our data range (compare Figure 5).

<table>
<thead>
<tr>
<th>$v_{\Pi}$</th>
<th>$V_{\Sigma \Pi}^{\text{const}}$</th>
<th>$V_{\Sigma \Pi}^{f}$</th>
<th>$e$</th>
<th>$f$</th>
<th>$V_{\Sigma \Pi}^{\text{fit}}$</th>
<th>$V_{\Sigma \Pi}^{f}$</th>
<th>$e$</th>
<th>$f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.5869$^a$</td>
<td>-</td>
<td>x</td>
<td>x</td>
<td>3.5869$^a$</td>
<td>-</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>1</td>
<td>$1.8498 \times 10^1$</td>
<td>-</td>
<td>$117^{1/2}$</td>
<td>$119^{1/2}$</td>
<td>3.5869$^a$</td>
<td>-</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>2</td>
<td>$6.3015$</td>
<td>$4.1233 \times 10^{-4}$</td>
<td>$97^{1/2}$</td>
<td>$100^{1/2}$</td>
<td>2.8782$ \times 10^1$</td>
<td>$9.8436 \times 10^{-4}$</td>
<td>$118^{1/2}$</td>
<td>$116^{1/2}$</td>
</tr>
<tr>
<td>3</td>
<td>$5.2871$</td>
<td>-</td>
<td>$74^{1/2}$</td>
<td>$76^{1/2}$</td>
<td>2.0611$ \times 10^1$</td>
<td>$5.3282 \times 10^{-4}$</td>
<td>$93^{1/2}$</td>
<td>$96^{1/2}$</td>
</tr>
<tr>
<td>4</td>
<td>$2.5188$</td>
<td>-</td>
<td>$39^{1/2}$</td>
<td>$42^{1/2}$</td>
<td>1.4468$ \times 10^1$</td>
<td>-</td>
<td>$68^{1/2}$</td>
<td>$72^{1/2}$</td>
</tr>
<tr>
<td>5</td>
<td>$7.3432 \times 10^{-3}$</td>
<td>-</td>
<td>x</td>
<td>x</td>
<td>7.4308</td>
<td>-</td>
<td>$27^{1/2}$</td>
<td>$30^{1/2}$</td>
</tr>
</tbody>
</table>

$^a$ These parameters were kept fixed during the fit.

The effective overlap integrals in Table V have a numerical sensitivity of within the fourth digit shown in the table, which was checked by varying them by hand. But physically they are probably only good up to 1% because we have to consider significant correlation between them. We give their values with five significant figures, with which our calculations can safely be reproduced.

V. DISCUSSION AND OUTLOOK

The effective overlap integrals $V_{\Sigma \Pi}^{\text{fit}}$ scale the perturbation strength for individual crossings of $v_\Sigma$ and $v_{\Pi}$ level systems. These overlap integrals were treated as simple fit parameters in our model but they are effectively products of overlap integrals and the spin-orbit interaction. To untangle the different factors in the off-diagonal matrix elements, we took the opportunity that we derived several such products which have one constant, namely the spin-orbit parameter, in common. Therefore, the variation from vibrational pair to vibrational pair reflects the variation of the true overlap integrals. For the comparison between observation and theory, we applied the potentials from ab initio calculations from Pototschnig et al.\textsuperscript{16}, calculated the overlap integrals $\langle 0|5\rangle$ to $\langle 0|20\rangle$ and $\langle 1|5\rangle$ to $\langle 1|20\rangle$ and compared the distribution with that of the fitted overlap integrals $V_{\Sigma \Pi}^{\text{fit}}$ in Table V. We found a good agreement of the two distributions when shifting our assumed vibrational quantum number by +12. This offset of vibrational quantum numbers also fits well to

the energy range of vibrational $(1)^2\Pi$ levels that cross with the lowest two vibrational $(2)^2\Sigma^+$ levels according to the calculations by Pototschnig et al. and shows that the calculated energy difference between the two ab initio potentials is in no conflict with our observations. It should be noted that the $J$-dependent contributions to the overlap integrals in Table V increase with $J$, whereas the corresponding ab initio overlap integrals get slightly smaller with increasing $J$. The fitted values of the $J$-dependent overlap integrals (equation 10) therefore likely contain contributions of other $J$-dependencies, e.g., the spin-orbit coupling itself.

The comparison with the ab initio work leads to a scaling factor for the overlap integrals of

$$
\frac{V_{\Sigma \Pi}^{\text{fit}}}{V_{\Sigma \Pi}^{\text{calc}}} \approx 302,
$$

assuming the shift of 12 units in $v_{\Pi}$. With this, we can estimate now the spin-orbit coupling with equation (8). We assumed $p = \sqrt{2}$ as an approximation for a $sp$-atom pair from the atomic asymptote of the electronic molecular states in question (see Figure 1) and $V_{\Sigma \Pi}^{\text{calc}} = V_{\Sigma \Pi}^{\text{fit}}$ for obtaining

$$
A_{\Sigma \Pi} \approx \frac{V_{\Sigma \Pi}^{\text{fit}}}{V_{\Sigma \Pi}^{\text{calc}}} \frac{1}{\sqrt{2}} = (214 \pm 38) \text{ cm}^{-1}.
$$

We neglect here the influence of the spin-rotation contribution in the non-diagonal matrix element. The value of $A_{\Sigma \Pi}$ is close to the magnitude of the molecular splitting constant $A_{\Pi}$ (see Table IV) and the spin-orbit constant $A_{\Sigma}$ from the atomic asymptote with Sr 5p$^3$P (see Reference 19). This might indicate that the value of $p$ is slightly smaller than $\sqrt{2}$. Analogously, we get with equations (8) and (9)

$$
B_{\Sigma \Pi} = 0.0428(51) \text{ cm}^{-1},
$$

while the ab initio potentials yields for $B_{\Sigma \Pi} = \langle v_{\Sigma}^2 \rangle / (2\mu r^2) |v_{\Pi}|$, with relevant $v_{\Sigma}$ and $v_{\Pi}$, values from 0.018 cm$^{-1}$ to 0.066 cm$^{-1}$. Our approach to factor out the overlap integral by assuming the other factors are independent of $v$ seems to be a sufficient approximation.
The coupled system $(2)^2Σ^+ + (1)^2Π$ of $^7Li^{88}Sr$

for getting consistent coupling parameters and it indicates that the potential scheme from ab initio results is well applicable for this detailed analysis.

In Table VI, we compare derived molecular constants with those of Pototschnig et al., whose work we already used for the vibrational analysis of the $(1)^2Π$ state. The comparison in Table VI shows a good general agreement between the calculations and our observations, with deviations of a few percent. For the $(1)^2Π$ constants, there are larger differences than for the $(2)^2Σ^+$ constants. We could model the $(1)^2Π$ levels only indirectly, whereas the $(2)^2Σ^+$ levels could be observed directly. The other ab initio calculations$^{1,9–12,14}$ come to similar results. The value of $A_Π$ is higher by 30% compared to the ab initio result by Gopakumar et al.$^{11}$ and is about $3/4$ of the atomic value $A_Σ$, which indicates that the $(1)^2Π$ state has a large contribution from the $5p$ electron of Sr. For such a comparison, it would be good to obtain a deeper insight into the eigenvectors derived in the ab initio calculation.

In the thermal emission spectrum, other vibrational bands than (0–0), (1–1), (1–0) and (1–2) are visible but too weak for rotational analysis (see Figure 2). The fitted Dunham coefficients in Tables I and III reproduce the $(2)^2Σ^+−X(1)^2Σ^+$ band heads with $v ≤ 3$ of both $2^Σ^+$ states within 3 cm$^{-1}$. This means an extrapolation of two vibrational quanta. We identified the bands in Figure 2 with these parameters. Since the $(2)^2Σ^+$ coefficients come from a fit of coupled electronic states, the coupling model has to be extended to describe the spectrum reliably over a wider range.

Higher vibrational $X(1)^2Σ^+$ levels are needed for a more complete description of the $X(1)^2Σ^+$ potential energy curve with the final goal to allow for the description of cold atomic collisions. The thermal emission spectrum, recorded around 1000°C, shows distinct bands only for the four lowest vibrational levels of both the $2^Σ^+$ and the $X(1)^2Σ^+$ states. This, together with the predicted distribution of Franck-Condon factors$^{10}$, leads us to conclude that the experimental methods deployed here will not allow us to observe much higher $X(1)^2Σ^+$ levels. To achieve a full description of the electronic ground state, other electronic transitions (e.g. $(3)^2Σ^+−X(1)^2Σ^+$) have to be used. Such transitions, with different Franck-Condon factors, could allow to address intermediate or high vibrational $X(1)^2Σ^+$ levels which could be used to interpolate the energy structure. But one encounters a significant problem, namely the overlap of the desired spectrum with strong spectra of the diatomic alkali metals also formed in the heat pipe.

A description of the $(2)^2Σ^+−(1)^2Π$ coupling with fitted potential energy curves would allow to greatly reduce the number of fit parameters since these curves provide both energy levels and overlap integrals. Such curves represent a global description of the electronic states. Because we only incorporated a few high vibrational $(1)^2Π$ levels, there is too much ambiguity regarding the shape of the $(1)^2Π$ potential energy curve to describe the perturbation with such a model. Like for the ground state, this problem can be overcome by extending the energy level data via observations of transitions to other electronic states which might fluoresce to the manifold of the $(1)^2Π$ state.

SUPPLEMENTARY MATERIAL

See the supplementary material for a derivation of the approximation of the value $B(v,v')$ and a list of assigned thermal emission lines.

ACKNOWLEDGMENTS

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REFERENCES

TABLE VI. Comparison of measured spectroscopic constants of $^7$Li$^{88}$Sr with results of the Graz group$^{13,16}$ (MRCI), converted to $^7$Li$^{88}$Sr. For (1)$^2$Π state, we calculated the parameters for $v_{11} = 12$ from the ab initio potential and compare these with $v = 0$ of our local description. All values are given in cm$^{-1}$ except $R_e$, which is given in Å.

<table>
<thead>
<tr>
<th>State</th>
<th>$R_e$</th>
<th>$\omega_e \approx Y_{10}$</th>
<th>$B_e \approx Y_{01}$</th>
<th>$T_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)$^2$Σ$^+$</td>
<td>3.574</td>
<td>179.1</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3.5405$^a$</td>
<td>183.07</td>
<td>0.2074</td>
<td>0</td>
</tr>
<tr>
<td>(2)$^2$Σ$^+$</td>
<td>3.728</td>
<td>183.0</td>
<td>-</td>
<td>9375</td>
</tr>
<tr>
<td></td>
<td>3.704$^a$</td>
<td>181.51</td>
<td>0.1895</td>
<td>9391.98</td>
</tr>
<tr>
<td>(1)$^2$Π</td>
<td>3.130</td>
<td>241</td>
<td>0.32</td>
<td>8258</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>277.12</td>
<td>0.2621</td>
<td>8366.0</td>
</tr>
</tbody>
</table>

$^a$ Calculated from Y$_{01}$

$^b$ Taken from Reference 11 (SO-MS-CASPT2). The molecular constants of References 13 and 11 agree within ca. 10%.