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Dynamics of baryons from string theory and vector dominance

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Abstract: We consider a holographic model of QCD from string theory, *à la* Sakai and Sugimoto, and study baryons. In this model, mesons are collectively realized as a five-dimensional $\mathrm{U}(N_F) = \mathrm{U}(1)\times\mathrm{SU}(N_F)$ Yang-Mills field and baryons are classically identified as $\mathrm{SU}(N_F)$ solitons with a unit Pontryagin number and N_c electric charges. The soliton is shown to be very small in the large 't Hooft coupling limit, allowing us to introduce an effective field $\mathcal{B}$. Its coupling to the mesons are dictated by the soliton structure, and consists of a direct magnetic coupling to the $\mathrm{SU}(N_F)$ field strength as well as a minimal coupling to the $\mathrm{U}(N_F)$ gauge field. Upon the dimensional reduction, this effective action reproduces all interaction terms between nucleons and an infinite tower of mesons in a manner consistent with the large N_c expansion. We further find that all electromagnetic interactions, as inferred from the same effective action via a holographic prescription, are mediated by an infinite tower of vector mesons, rendering the baryon electromagnetic form factors *completely* vector-dominated as well. We estimate nucleon-meson couplings and also the anomalous magnetic moments, which compare well with nature.

Keywords: Gauge-gravity correspondence, AdS-CFT Correspondence, QCD.

Contents

1. Introduction

The recent development in applying the concept and the methodology of AdS/CFT duality [1] to low-energy hadron dynamics, referred to as the holographic QCD or AdS/QCD, brings out two related issues from opposite directions, one top-down from string theory [2, 3] and the other bottom-up from low-energy chiral effective field theory of mesons and baryons [4 − 6].

From the string theory point of view, what one is interested in is to assess to what extent the gravity theory in the bulk sector in a controlled weak coupling limit can address, via duality, the strongly coupled dynamics of QCD and if so, how well and how far. In this respect, the aim there is to "post-dict" what is established in low-energy hadron dynamics, and try to reproduce what has been well understood in low-energy effective theories. The principal goal here is to establish its raison-d'être in the strong interaction sector. On the other hand, from the low-energy effective theory perspective on which we will elaborate in some detail below, the aim is, if it is firmly established that the holographic QCD has definite connection to real QCD, whether it can make clear-cut and falsifiable predictions on processes which are difficult to access by QCD proper.

A notable example of this sort is the prediction by AdS approaches of low viscosity-entropy ratio [7] and also of low elliptic flow in matter at high temperature above the chiral restoration point [8], which is presumed to be observed at RHIC. Given the complete inability of the QCD proper to handle this regime, this development gives a hope that the holographic approach could provide a powerful tool going beyond perturbative QCD and elucidate strongly interacting matter under extreme conditions that are otherwise inaccessible, such as the phenomenon of jet-quenching [9]. Another outstanding immediate challenge to AdS approaches is to identify and elucidate the degrees of freedom figuring just below (in the Nambu-Goldstone phase) and just above (in the Wigner-Weyl phase) T_c, the chiral transition temperature presumed to have been probed at RHIC [10]. At present, however, in the paucity of better understanding, it is not clear whether the current "explanation" of the properties of quark-gluon plasma at RHIC reflects directly certain specific properties of nonperturbative QCD or whether they are simply in a same universality class unspecific to dynamics. For instance, recent works suggest that the prediction of viscosity-entropy ratio could be common to all AdS-based models regardless of details [11].

Another example of this sort is the exploitation of the conformal structure of AdS/CFT to deduce the analytic form of the frame-independent light-front wavefunctions of hadrons which could allow the computation of various observables that are found to be difficult to obtain in QCD itself [12].

In this paper, we would like to zero in on a more specific set of problems that are typically of strong-coupling QCD and are very difficult to access by established QCD techniques, namely chiral dynamics of hadrons, in particular baryons, at low energy. Unlike pions, which are relatively well-understood from the chiral Lagrangian approach to QCD, baryons remain more difficult to pin down. This may account for the reason why in the chiral lagrangian approach, baryons are either put in by hand as point-like objects or

JHEP09(2007)063

built up as solitons (i.e., Skyrmions) from mesons. The former suffers from the lack of theoretical justification as a local field when the energy scale reaches the inverse of its Compton wavelength as evidenced in the growing number of unknown parameters, while the latter in its simplest approximation does not fare well in phenomenology. Attempts to marry the two pictures are often difficult, given the relatively large size of the Skyrmion.

This work was motivated by an astute modelling of chiral dynamics within the framework of AdS/CFT by Sakai and Sugimoto [2] that correctly describes the spontaneous breaking of chiral $\mathrm{U}(N_F)_L \times \mathrm{U}(N_F)_R$ to the diagonal subgroup $\mathrm{U}(N_F)_{L+R}$. For our purpose, the most salient feature of the holographic model of Sakai and Sugimoto (SS for short) is that the entire tower of vector mesons plus the pions are built into a single $\mathrm{U}(N_F)$ gauge field in five-dimensions, immensely simplifying possible interaction structures among mesons, and eventually with baryons as well. This also implies that the low-energy chiral dynamics incorporating the "hidden local gauge symmetry" (HLS) is manifest in five dimensions. The $\mathrm{U}(N_F)$ gauge field is supported by N_F D8 branes compactified on S^4 while the strongly coupled $\mathrm{SU}(N_c)$ dynamics is hidden in the background AdS-like geometry.

The effective chiral theory, defined at a KK scale, M_{KK} (commensurate with the chiral scale $\Lambda_\chi \sim 4\pi f_\pi$) is valid and justified in the limit of large 't Hooft coupling constant $\lambda = g_{\mathrm{YM}}^2 N_c$ and large N_c. Surprisingly even in this limit, the SS model has been shown to possess the power to reproduce rather well most of the low-energy hadron properties in the meson sector that are highly non-perturbative, such as, for example, soft-pion theorems, KSRF relations, various sum rules etc. Most notable among what has been obtained is that *all* hadron processes involving mesons, both normal (e.g., π-π scattering) and anomalous (e.g., $\pi^0 \to 2\gamma$, $\omega \to 3\pi$ etc), are *vector-dominated* with all the members of the infinite tower participating non-trivially in the process, including the well-established vector dominance of the pion EM form factor.

Now given an effective theory that captures the physics of the meson sector, one immediate question is how the baryons figure in the story and how well the picture approximates static and dynamic properties of baryons. A related question is whether or not the vector dominance which holds *naturally* in the meson sector also holds with baryons. This question has a bearing on the concept of "universality" that has played an important role in the history of vector dominance in hadron physics. As described in detail in what follows, a baryon in the SS model is a soliton with instanton-like configuration in a five-dimensional Yang-Mills action, which encodes the winding number of the four-dimensional Skyrmion made up not only of the pion field but also of an infinite tower of vector mesons.

In fact, perhaps the most appealing possibility for the holographic QCD to unravel something truly novel in low-energy hadron dynamics is in the baryon structure — which is the principal subject of this article. In the past, several authors [13] studied Skyrmions with the HLS Lagrangian containing the lowest vector mesons, ρ and ω, of [14][1] as merely an *alternative or improved* description of the same soliton given by the Skyrme model with the pion hedgehog [15, 16]. The essential idea was that vector mesons, in particular the

[1]Some works include the a_1 meson as well, but the idea is essentially the same as without it.

ρ meson, could replace the Skyrme quartic term in the role of stabilizing the soliton.[2] It was only recently suggested that hidden local fields bring a drastically different or novel aspect to the soliton structure of baryons [18 − 20]. Indeed what we have found is that the instanton baryon, which is a Skyrmion with an infinite tower of hidden local fields, presents an aspect of baryons hitherto left largely unexplored.

A major part of this paper will be devoted to understanding the simplest of static properties, and subsequently the chiral dynamics of the baryons realized as five-dimensional solitons. One consequence of the fully five-dimensional picture is that, in the large 't Hooft coupling, the instanton size is so small to be amenable to a simple effective field theory approach. Our strategy in uncovering the dynamics of baryons relies on an effective field theory of the small instanton in the five dimensional setting. The quantum numbers of the small instanton are commensurate with those of the Skyrmion, except that it naturally and minimally couples to five-dimensional $\mathrm{U}(N_F)$ gauge fields instead of to four-dimensional $\mathrm{SU}(N_F)$ pion fields. This results in a simple five-dimensional Dirac field representing baryons, minimally coupled to the $\mathrm{U}(N_F)$ gauge field. While the instanton size is small, on the other hand, the long distance power-like tail cannot be ignored and leads to a higher-dimensional coupling between the $\mathrm{SU}(N_F)$ field strength and the Dirac field whose coupling strength is determined by the size of the instanton. It is plausible that this picture can justify the long-standing tradition — recently given a support in terms of chiral perturbation theory — in nuclear physics where the nucleon is considered as a point-like object and its finite size effects are taken into account via "meson cloud."

At the end of the day, what will have transpired is that these two simple and explicitly computable five-dimensional interaction terms in the baryon effective action encode all the four-dimensional meson-baryon interactions, up to quadratic order in the baryon field. This includes the pions, the entire tower of vector mesons and axial vector mesons, once and for all, and also incorporates iso-scalar and iso-vector mesons on equal footing. Needless to say, this will result in a large number of predictions on various meson-baryon couplings, and more indirectly, various electromagnetic interactions.

While the photon field is not present among the degrees of freedom in this model, the electromagnetic current can also be extracted following the general prescription of AdS/CFT. An interesting outcome of this investigation is that, although the effective action approach in five dimensions predicts a minimal coupling between photon field and the baryon, a mixing between massive vector mesons and the photon field effectively replaces this with an infinite number of vector mesons coupling to the baryons. The resulting electromagnetic form factors show a complete vector dominance in the sense that all electromagnetic interactions are mediated by exchange of vector mesons, generalizing old notion of vector dominance by the lightest vector mesons. In particular, full vector dominance is recovered in the electromagnetic form factors of the nucleon in the same fashion as in the pion.

We will also discuss subleading $1/N_c$ corrections and compare these findings against experimental values. Throughout the derivation of the effective action, we stay in the

[2]We will argue in section 8 that this idea is not correct.

JHEP09(2007)063

regime of large N_c and large 't Hooft coupling $g_{\rm YM}^2 N_c$ where the size of baryon is small enough to justify this approach. On the other hand, the realistic regime of $N_c = 3$ QCD with the pion decay constant $f_\pi \sim 93\,{\rm MeV}$ demands $g_{\rm YM}^2 N_c \sim 17$, which is not large enough. The baryon size is difficult to estimate in this regime but is clearly of the same order as $1/M_{\rm KK}$. To avoid the difficulties associated with the latter, we take the route of doing most of computation in large 't Hooft coupling limit and extrapolate only at the end of the day. We expect that this strategy works best when the quantities in question are not sensitive to the 't Hooft coupling in the large N_c limit, such as the chiral coupling between pions and the baryon and the anomalous magnetic moment of the nucleon.

Section 2 will review the holographic QCD model of Sakai and Sugimoto but also serve as the reference section for many of the computations. We will in particular introduce a conformal coordinate system which is useful for dealing with the instanton soliton, which upon quantization will be identified with baryons. In section 3, we consider basic static properties of baryons and include a careful derivation of the size and the energetics. The instanton must then be quantized to become physical baryon, and when the size is small, namely when the 't Hooft coupling is very large, it can be treated as a point-like object but with long range gauge field tails. The resulting five-dimensional effective action with a novel and essential magnetic coupling is derived in section 4.

Beginning with section 5, we start to discuss the chiral dynamics of nucleons in four dimensions. We first describe how to reduce the five-dimensional effective action of nucleons to four dimensions, whose only nontrivial feature is a single magnetic coupling, and produce a four-dimensional effective action of nucleons coupled to the infinite tower of mesons. Some of the simplest predictions on Yukawa coupling constants will be given as examples and compared to experimental values. Section 6 will delve into numerical estimates and extrapolation to realistic regimes, and points out subtleties and potential problem in doing so.

Beginning with section 7, we consider electromagnetic coupling of nucleons. We review how the vector dominance in the meson sector came about and show how this generalizes to nucleon sector rather nontrivially. While the vector dominance here involves the entire tower of vector mesons, we will show that truncating down to the first four vector mesons, in both iso-scalar and iso-vector sectors, respectively, provides a very good approximation to the complete form factors of the model. As a bonus, one can also compute the magnetic dipole moment of nucleons in section 8 which also compares favorably with experimental values. In section 9, we perform numeric analysis of electromagnetic form factors (Sachs form factors) and also extract various nucleon charge radii. We close with discussions.

An abbreviated version of this work has been reported elsewhere [21] with emphasis on the derivation of the effective action. The present paper expands upon the previous paper by including more detailed derivation leading to the effective action and exploring the implications comprehensively.

2. A string theory model of holographic QCD

Among the holographic models of QCD proposed recently, one most interesting and realistic model is the one by Sakai and Sugimoto (SS) [2], who considered N_c ($\gg 1$) stack of D4 branes and N_F D8 branes in the background of Type II A superstring. The key point of the model[3] is that the flavor symmetries of the quark sector are embedded into a $\mathrm{U}(N_F)$ gauge symmetry in $R^{1+3} \times I$. The fifth direction is topologically an interval, and the four-dimensional low energy physics is found by restricting to the modes that are localized near the "origin" of this fifth direction.

The stack of D4 branes at low energy carries $\mathrm{SU}(N_c)$ Yang-Mills theory. In the large N_c limit, the dynamics of D4 is dual to a closed string theory in some curved background with flux in accordance with the general AdS/CFT idea. In the large 't Hooft coupling limit, $\lambda \equiv g_{\rm YM}^2 N_c \gg 1$, and neglecting the gravitational back-reaction from the D8 branes, the metric is [17]

$$ds^2 = \left(\frac{U}{R}\right)^{3/2} \left(\eta_{\mu\nu} dx^\mu dx^\nu + f(U) d\tau^2\right) + \left(\frac{R}{U}\right)^{3/2} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right) \tag{2.1}$$

with $R^3 = \pi g_s N_c l_s^3$ and $f(U) = 1 - U_{\rm KK}^3/U^3$. The coordinate τ is compactified as $\tau = \tau + \delta\tau$ with $\delta\tau = 4\pi R^{3/2}/(3U_{\rm KK}^{1/2})$.

2.1 Five-dimensional $\mathrm{U}(N_F)$ theory on D8-branes

The D8 branes, which share the coordinates x^1, x^2, x^3 with the D4 branes, admit the massless quark degrees of freedom as open strings attached to both the D4 and D8 branes. The effective action on the D8 brane, embedded in the D4 background, is the DBI action

$$S_{D8} = -\mu_8 \int \mathrm{d}^9 x\, e^{-\phi} \sqrt{-\det\left(g_{MN} + 2\pi\alpha' F_{MN}\right)} + \mu_8 \int \sum C_{p+1} \wedge \mathrm{Tr}\, e^{2\pi\alpha' F}\,, \tag{2.2}$$

with

$$\mu_p = \frac{2\pi}{(2\pi l_s)^{p+1}}\,, \tag{2.3}$$

where $l_s^2 = \alpha'$. $\sum C_{p+1}$ is a formal sum of the antisymmetric Ramond-Ramond fields of odd-ranks, C_1, C_3, C_5, C_7, C_9. These fields couple, respectively, to D0, D2, D4, D6, and D8 branes.

The D8 brane in this set-up occupies a $5D$ curved spacetime times S^4 whose radius is position-dependent along $5D$. The induced metric on D8 is

$$g_{8+1} = \left(\frac{U}{R}\right)^{3/2} \left(\eta_{\mu\nu} dx^\mu dx^\nu\right) + \left(\frac{R}{U}\right)^{3/2} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)\,. \tag{2.4}$$

We transform the coordinates so that the noncompact 5D part of the metric is conformally flat,

$$g_{4+1} = H(w)\left(dw^2 + \eta_{\mu\nu} dx^\mu dx^\nu\right)\,, \tag{2.5}$$

where

$$w = \int_{U_{\rm KK}}^{U} \frac{R^{3/2} dU'}{\sqrt{U'^3 - U_{\rm KK}^3}}\,. \tag{2.6}$$

[3] Unless otherwise stated, we follow the notations of SS.

Note that the parameters of dual QCD are mapped to the parameters here as

$$R^3 = \frac{g_{\rm YM}^2 N_c l_s^2}{2M_{\rm KK}}\,, \qquad U_{\rm KK} = \frac{2g_{\rm YM}^2 N_c M_{\rm KK} l_s^2}{9}\,, \tag{2.7}$$

where the KK mass $M_{\rm KK}$ is the dimensionful free-parameter of the theory. Note that

$$M_{\rm KK} \equiv 3U_{\rm KK}^{1/2}/2R^{3/2}\,. \tag{2.8}$$

Another dimensionful quantity that appears in the chiral Lagrangian formulation of QCD is f_π which determines the scale of chiral symmetry breaking. In terms of the above, we have [2, 22][4]

$$f_\pi^2 = \frac{1}{54\pi^4}(g_{\rm YM}^2 N_c) N_c M_{\rm KK}^2\,. \tag{2.9}$$

As was shown in detail by Sakai and Sugimoto, it is $M_{\rm KK}$ that enters the mass spectra of mesons. For real QCD, $M_{\rm KK}$ would be roughly $M_{\rm KK} \sim m_N \sim 0.94\,$GeV, while $f_\pi \sim 93\,$MeV, and this requires

$$(g_{\rm YM}^2 N_c) N_c \sim 50\,. \tag{2.10}$$

For $N_c = 3$, this gives

$$g_{\rm YM}^2 N_c \sim 17. \tag{2.11}$$

This certainly is not big enough for truncating at the leading order, indicating that it might be difficult to naively apply this model to the realistic QCD regime. For this reason, the best we can do is to look at dimensionless quantities in which the limiting constants cancel out, such as ratio of masses of the mesons, and hope that such quantities are insensitive to the precise values of these physical parameters.

Note that this fifth coordinate is of finite range since

$$w_{\rm max} = \int_0^\infty \frac{R^{3/2} dU}{\sqrt{U^3 - U_{\rm KK}^3}} = \frac{1}{M_{\rm KK}} \frac{3}{2} \int_1^\infty \frac{d\tilde{U}}{\sqrt{\tilde{U}^3 - 1}} \simeq \frac{3.64}{M_{\rm KK}} < \infty\,. \tag{2.12}$$

Thus, the 5D spacetime part of D8 brane is conformally equivalent to an interval $[-w_{\rm max}, w_{\rm max}]$ times R^{3+1}. This makes the search for smooth instanton solution rather subtle. This matter will be discussed later in this paper. Another choice of coordinate convenient for us is z defined as

$$U^3 = U_{\rm KK}^3 + U_{\rm KK} z^2\,, \tag{2.13}$$

which is related to w as

$$dw = \frac{R^{3/2} dU}{\sqrt{U^3 - U_{\rm KK}^3}} = \frac{2R^{3/2} U_{\rm KK}^{1/2}\, dz}{3(U_{\rm KK}^3 + U_{\rm KK} z^2)^{2/3}}\,. \tag{2.14}$$

Near origin $w \simeq 0$, we have the approximate relation,

$$M_{\rm KK} w \simeq \frac{2}{3}\left(\frac{R}{U_{\rm KK}}\right)^{3/2} \times (M_{\rm KK} z) = \frac{z}{U_{\rm KK}}\,, \tag{2.15}$$

[4]See section 2.2.

which implies

$$U^3 \simeq U_{\rm KK}^3(1+M_{\rm KK}^2 w^2) \tag{2.16}$$

for the conformally flat coordinate. This shows that the deviation of the metric from the flat one is dictated entirely by the mass scale $M_{\rm KK}$. In fact, the same is true of the full 10-dimensional spacetime metric, and thus from this we can see that $M_{\rm KK}$ is the only mass scale of the theory in the low energy limit.

In the low energy limit, the worldvolume dynamics of the D8 brane is well-described in terms of a derivative expansion of the full stringy effective action, and the result of this gives the Yang-Mills action with a Chern-Simons term. The Yang-Mills part of this effective action is

$$\frac{1}{4}\int_{8+1}\sqrt{-g_{8+1}}\,\frac{e^{-\Phi}}{2\pi(2\pi l_s)^5}\,{\rm tr}F_{MN}F^{MN} = \frac{1}{4}\int_{4+1}\sqrt{-g_{4+1}}\,\frac{e^{-\Phi}V_{S^4}}{2\pi(2\pi l_s)^5}\,{\rm tr}F_{\hat m\hat n}F^{\hat m\hat n}\,. \tag{2.17}$$

Here V_{S^4} is the position-dependent volume of the compact part with

$$V_{S^4} = \frac{8\pi^2}{3}R^3 U\,, \tag{2.18}$$

while the dilaton is

$$e^{-\Phi} = \frac{1}{g_s}\left(\frac{R}{U}\right)^{3/4}\,. \tag{2.19}$$

The Chern-Simons coupling arises from the second set of terms because $\int_{S^4} dC_3 \neq 0$ takes a quantized value, and was worked out by Sakai and Sugimoto in some detail. The answer after integration over the four-sphere is

$$\frac{N_c}{24\pi^2}\int_{4+1}\omega_5(A) \tag{2.20}$$

with $d\omega_5(A) = {\rm tr}F^3$.

2.2 Chiral lagrangian and Hidden Local Symmetry (HLS)

The main point of this model is that the D8 comes with two asymptotic regions (corresponding to UV) at $w \to \pm w_{\rm max}$ which are continuously connected via the infrared region near $w=0$. The usual chiral symmetry ${\rm U}(N_F)_L \times {\rm U}(N_F)_R$ is implicitly embedded into the ${\rm U}(N_F)$ gauge symmetry of D8 branes [2]. The ${\rm U}(N_F)_{L,R}$ are the remnant of the five-dimensional gauge symmetry; those on the left-end and the right-end are each interpreted as ${\rm U}(N_F)_{L,R}$, respectively. While the gauge symmetry is broken, its global counterpart survives as ${\rm U}(N_F)_{L+R}$.

The five-dimensional gauge field has three polarizations. Thus the generic KK modes become massive vector fields in four dimensions, namely massive vector mesons whose parity is decided by the shape of the KK eigenfunction, while there is a single massless adjoint multiplet which arises from the Wilson line degrees of freedom, which are the pions. This can be seen more clearly when one expands A_μ in terms of eigenmodes along w directions, decomposing it into infinite towers of KK states as seen by 4D observers. The

JHEP09(2007)063

lowest modes are then interpreted as the low-lying vector mesons of the chiral Lagrangian formulation. In the gauge $A_5 = 0$, this expansion was worked out by Sakai and Sugimoto. Introducing a gauge function $\xi(x)$ at $w = 0$, which is related to the pion field in unitary gauge as

$$\xi^2(x) = U(x), \qquad U(x) = e^{2i\pi(x)/f_\pi}\,, \tag{2.21}$$

we have the following expansion,[5]

$$A_\mu(x;w) = i\alpha_\mu(x)\psi_0(w) + i\beta_\mu(x) + \sum_n a_\mu^{(n)}(x)\psi_{(n)}(w) \tag{2.22}$$

with

$$\alpha_\mu(x) \equiv \{\xi^{-1}, \partial_\mu\xi\} \simeq \frac{2i}{f_\pi}\partial_\mu\pi, \qquad \beta_\mu(x) \equiv \frac{1}{2}[\xi^{-1}, \partial_\mu\xi] \simeq \frac{1}{2f_\pi^2}[\pi, \partial_\mu\pi]\,, \tag{2.23}$$

where $\psi_0(w) = \psi_0(w(z)) = \frac{1}{\pi}\arctan\left(\frac{z}{U_{\rm KK}}\right)$. Inserting this into the DBI-action (2.17), we can obtain a low-energy Lagrangian for the pions as well as massive vector/axial-vector mesons.

As for the pions, this reproduces the Skyrme Lagrangian[6]

$$\mathcal{L}_{\rm pion} = \frac{f_\pi^2}{4}{\rm tr}\left(U^{-1}\partial_\mu U\right)^2 + \frac{1}{32e^2_{Skyrme}}{\rm tr}\left[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U\right]^2 \tag{2.24}$$

with

$$f_\pi^2 = \frac{1}{54\pi^4}(g_{\rm YM}^2 N_c)M_{\rm KK}^2 N_c\,, \quad e^2_{\rm Skyrme} \simeq \frac{54\pi^7}{61}\frac{1}{(g_{\rm YM}^2 N_c)N_c}\,. \tag{2.25}$$

For the massive tower of (axial) vector mesons, we have the standard kinetic term

$$\mathcal{L}_{\rm massive} = \sum_n \left\{\frac{1}{4}F_{\mu\nu}^{(n)}F^{\mu\nu(n)} + \frac{1}{2}m_n^2 a_\mu^{(n)}a^{\mu(n)}\right\}\,, \tag{2.26}$$

with $F_{\mu\nu}^{(n)} = \partial_\mu a_\nu^{(n)} - \partial_\nu a_\mu^{(n)}$, plus various interactions between them as well as with pions.

The interesting point in this theory is that the gauge symmetry localized in the fifth direction can be identified as an infinite number of "hidden local symmetries" (HLS) in four dimensions, and each massive vector meson plays a role as a gauge field for some part of them. A hidden gauge symmetry theory with the (ρ, ω), the lowest members of the tower, was introduced into hadron physics two decades ago by Bando et al [14] and revived recently by Harada and Yamawaki [23]. The key observation that led to the formulation of [14] was that the chiral field U which figures in the low-energy dynamics of the Goldstone

[5]Our gauge field is defined by $D = \partial - iA$, which differs from $D = \partial + A^{\rm SS}$ of SS.

[6]After this paper has appeared, we learned of a factor two error in ref. [2]. We thank S. Sugimoto for informing us [22]. In the present paper, all quantities are derived from the D-brane physics and did not rely on the computations in ref. [2]. The only exception is the chiral Lagrangian here, which affects the two coefficients f_π^2 for the kinetic term and $1/e^2_{\rm Skyrme}$ for the Skyrme term. This enters physical quantities considered only indirectly via the determination of $\lambda \sim 17$ for the realistic QCD regime, which can be seen to affect slightly the subleading corrections for quantities we consider.

JHEP09(2007)063

pions possesses a hidden local symmetry that can be exploited to bring the energy scale to $\sim 4\pi m_V/g$ (where m_V is the vector meson mass and g is the hidden gauge coupling). In the modern terminology, one can consider the hidden gauge field so obtained as an emergent field as in other areas of physics [24, 25]. (See section 8 for more on this.) In the current holographic model, this idea finds a natural home simply because HLS arises automatically from the five dimensional description which incorporates not only (ρ, ω) but the entire tower of vector mesons. In our formulation we took a definite gauge choice (i.e., unitary gauge) so that $\xi = \xi_R = \xi_L^\dagger$. One can think of the SS model descending top-down from string theory to the hidden local symmetry of QCD. Indeed when restricted to the lowest member of the tower, the SS action reduces to the HLS action of [14] with $a = 4/3$.

3. Baryons as small and hairy instantons

Conventional chiral Lagrangian approaches realize baryons as Skyrmions, usually made of the pion field U only. As we couple higher massive vector mesons to the Skyrme action, the size-stabilizing mechanism for topological solitons is significantly affected by massive vector mesons. If we approach this problem from the above five-dimensional viewpoint, however, it is natural to consider the problem as a five-dimensional one. It has been known for some time that what replaces the Skyrmion is the instanton soliton since the two share the same topological winding number [26]. However, what has not been clear is whether and how much of the instanton is born out of the Skyrmion. As we will begin to see from this section, the instanton interpretation of the baryon will give a very different route to the low energy effective dynamics of the baryons.

We know that a D4 brane wrapping the compact S^4 will correspond to a baryon vertex on the $5D$ spacetime, which follows from an argument originally given by Witten [27]. On the D4-brane we have a Chern-Simons coupling of the form,

$$\mu_4 \int C_3 \wedge 2\pi\alpha' d\tilde{A} = 2\pi\alpha'\mu_4 \int dC_3 \wedge \tilde{A} \tag{3.1}$$

for D4 gauge field $\tilde{A}$. Since D4 wraps the S^4 which has a quantized N_c flux of dC_3, one finds that this term induces N_c unit of electric charge on the wrapped D4. The Gauss constraint for $\tilde{A}$ demands that the net charge should be zero, however, and the D4 can exist only if N_c fundamental strings end on it. In turn, the other end of fundamental strings must go somewhere, and the only place it can go is D8 branes. Thus a D4 wrapping S^4 looks like an object with electric charge with respect to the gauge field on D8. With respect to the overall U(1) of the latter, whose charge is the baryon number, the electric charge is N_c. Thus, we may identify the baryon as wrapped D4 with N_c fundamental strings sticking onto it.

Of course, things are more complicated than this since D4 can dissolve into D8 branes and become an instanton soliton on the latter. From D8's viewpoint, a D4 wrapped on S^4 once is interchangeable with the unit instanton

$$\frac{1}{8\pi^2} \int_{R^3 \times I} \mathrm{tr} F \wedge F = 1\,, \tag{3.2}$$

as far as the conserved charge goes. This follows from a Chern-Simons term on D8,[7]

$$\mu_8 \int_{R^{3+1}\times I\times S^4} C_5\wedge 2\pi^2(\alpha')^2 \mathrm{tr} F\wedge F = \mu_4 \int_{R^{0+1}\times S^4} C_5\wedge \frac{1}{8\pi^2}\int_{R^3\times I} \mathrm{tr} F\wedge F\,, \tag{3.3}$$

which shows that a unit instanton couples to C_5 minimally, and carries exactly one unit of D4 charge. When the size of the instanton becomes infinitesimal, it can be freed from D8's, and this is precisely D4. From the viewpoint of D4, this corresponds to going from the Higgs phase into the Coulomb phase.

In flat background geometry and no flux, the moduli space of D4 contains both the Coulomb branch where D4 maintains its identity separated from D8, and the Higgs branch where D4 is turned into a finite size Yang-Mills instanton on D8. With the present curved geometry, this is no longer a matter of choice. The energy of the D4 will differ depending on the configurations. As we will see shortly, to the leading approximation, the D4 will settle at the border of the two branches, both of which disappear apart from basic translational degrees of freedom along R^{3+1}. The reason for why D4 cannot dissociate away from D8 is obvious. The D4 has N_c fundamental strings attached, whose other ends are tied to D8. Moving away from D8 by distance L means acquiring extra mass of order N_cL/l_s^2 due to the increased length of the strings, so the D4 would stay on top of D8 for a simple energetics reason. The question is then how small or big will a D4 spread inside D8 as an instanton. Consider the kinetic part of $D8$ brane action, compactified on S^4, in the Yang-Mills approximation,

$$-\frac{1}{4}\int\sqrt{-g_{4+1}}\,\frac{e^{-\Phi}V_{S^4}}{2\pi(2\pi l_s)^5}\,\mathrm{tr}F_{\hat m\hat n}F^{\hat m\hat n}\,. \tag{3.4}$$

After taking the volume of S^4, the dilaton, and the conformally flat metric, this reduces to

$$-\int dx^4dw\,\frac{1}{4e^2(w)}\,\mathrm{tr}F_{mn}F^{mn}\,, \tag{3.5}$$

where the contraction is with respect to the flat metric $dx_\mu dx^\mu + dw^2$ and the position-dependent electric coupling $e(w)$ of this five dimensional Yang-Mills is such that

$$\frac{1}{e^2(w)} \equiv \frac{8\pi^2R^3U(w)}{3(2\pi l_s)^5(2\pi g_s)}\,. \tag{3.6}$$

In the SS model, the string coupling g_s is related to the dimensionful parameters and four-dimensional Yang-Mills coupling of the QCD as,

$$2\pi g_s = \frac{g_{\rm YM}^2}{M_{\rm KK}l_s}\,, \tag{3.7}$$

so we find

$$\frac{1}{e^2(w)} = \frac{(g_{\rm YM}^2N_c)N_c}{108\pi^3}M_{\rm KK}\frac{U(w)}{U_{\rm KK}} \tag{3.8}$$

[7]Recently, this term was shown to play an interesting role in a different aspect of baryonic physics with finite baryon density [28].

Since an instanton has

$$\int \mathrm{tr} F_{mn}F^{mn} = 2\int \mathrm{tr} F\wedge F = 16\pi^2\,, \tag{3.9}$$

a point-like instanton that is localized at $w=0$ would have the energy

$$m_B^{(0)} \equiv \frac{4\pi^2}{e^2(0)} = \frac{(g_{\rm YM}^2 N_c)N_c}{27\pi} M_{\rm KK}\,. \tag{3.10}$$

This mass also equals that of an S^4 wrapped D4 located at $w=0$, in accordance with the string theory picture of the instanton [2]. If the instanton gets bigger, on the other hand, the configuration costs more and more energy, since $1/e^2(w)$ is an increasing function of $|w|$, thus the leading behavior of the instanton is to collapse to a point-like instanton.

However, the N_c fundamental strings attached to D4 manifest themselves as N_c units of electric charge on D8's. There will be in general Coulomb repulsion among these electric charges, and this would favor spreading of instanton to a finite size. So it is the competition of the two effects, mass of instanton vs. Coulomb energy of fundamental strings. For very small instanton of size ρ, the energy picks up a size-dependent piece from the action of Yang-Mills field which goes as

$$\sim \frac{1}{6}\, m_B^{(0)} M_{\rm KK}^2 \rho^2\,, \tag{3.11}$$

while the five dimensional Coulomb energy goes as

$$\sim \frac{1}{2}\times \frac{e(0)^2 N_c^2}{10\pi^2\rho^2}\,, \tag{3.12}$$

provided that $\rho M_{\rm KK}\ll 1$. The estimate of energy here takes into account the spread of the instanton density $D(x^i,w)\sim \rho^4/(r^2+w^2+\rho^2)^4$, but ignores the deviation from the flat geometry along the four spatial directions.

We kept an overall factor of 1/2 in the Coulomb energy separated from the rest because it deserves a further explanation. The rest of the term is the five dimensional U(1) (with electric coupling constant $e(0)$) Coulomb energy for charge N_c whose distribution follows the instanton density $D(x^i,w)$. To see the origin of the additional factor of 1/2, recall that the Chern-Simons term responsible for this charge is

$$\frac{N_c}{24\pi^2}\int \mathrm{tr}\,(A\wedge F\wedge F+\cdots)\,, \tag{3.13}$$

from which we obtain the coupling between instanton $\bar F$ and the rest of the gauge field as

$$\frac{N_c}{8\pi^2}\int \mathrm{tr}\left(A\wedge \bar F\wedge \bar F\right)\,. \tag{3.14}$$

Gauge rotating a single instanton into the form

$$\frac{N_c}{8\pi^2}\,\bar F\wedge\bar F = \frac{N_c D(x^i,w)}{2}\begin{pmatrix} 1 & 0 & 0 & \cdots & 0\\ 0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 0 & \cdots & 0\\ \cdots & \cdots & \cdots & \cdots & \cdots\\ 0 & 0 & 0 & \cdots & 0\end{pmatrix} dx^3\wedge dw\,, \tag{3.15}$$

we have a minimal coupling to the instanton worldline

$$N_c \sum_a \int A^a \,\mathrm{tr}\,(T^a I_2/2) \ , \tag{3.16}$$

where I_2 denotes the matrix in eq. (3.15).

For $N_F = 2$, only the trace part of A can couple to I_2, and $A_{\mathrm{U}(1)}$ in

$$A = A_{\mathrm{U}(1)} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{3.17}$$

sees charge N_c on top of a single instanton. However, the kinetic term for $A_{\mathrm{U}(1)}$ would have the coefficient $1/2e^2$ instead of $1/4e^2$, which changes the effective electric coupling constant and introduces a factor of 1/2 to the Coulomb energy. For $N_F > 2$, the same factor of 1/2 arises for more complicated reasons. There are now $N_F - 2$ vector fields in $\mathrm{SU}(N_F)$, as well as the U(1) vector field from the trace part, that couple to this charge under the above Chern-Simons term. Each of them contributes some fraction of the above U(1) Coulomb energy. But the sum can be seen to be always 1/2.

More simply, this reduction can be seen from the fact that the total electric charge N_c on the instanton is shared, evenly split, by a pair of mutually orthogonal U(1)'s of $\mathrm{U}(N_F)$, which is evident in the form of I_2. In each sector the electric charge generates the Coulomb energy, proportional to $(N_c/2)^2$. Since the total Coulomb energy is obtained by a sum, we find $2 \times (N_c/2)^2 = N_c^2/2$ in place of N_c^2.

The size of the small instanton is determined where the combined energy is minimized [21, 29][8]

$$\rho^2_{\mathrm{baryon}} \simeq \frac{1}{M_{\mathrm{KK}}} \sqrt{\frac{3e(0)^2 N_c^2}{10 \cdot \pi^2 m_B^{(0)}}} = \frac{\sqrt{2 \cdot 3^7 \cdot \pi^2/5}}{M^2_{\mathrm{KK}}(g^2_{\mathrm{YM}} N_c)} \ , \tag{3.18}$$

and

$$\rho_{\mathrm{baryon}} \sim \frac{9.6}{M_{\mathrm{KK}} \sqrt{g^2_{\mathrm{YM}} N}} \ . \tag{3.19}$$

For an arbitrarily large 't Hooft coupling limit, the size of baryon is then significantly smaller than the scale of the dual QCD. Subsequently the mass correction to the baryon due to its 5-dimensional electric coupling

$$m_0^e \simeq \frac{1}{3} m_B^{(0)} (M_{\mathrm{KK}} \rho_{\mathrm{baryon}})^2 \simeq \frac{31}{g^2_{\mathrm{YM}} N_c} m_B^{(0)} \ll m_B^{(0)} \tag{3.20}$$

is also small if the 't Hooft coupling is arbitrarily large.

In the next section, we will thus assume a point-like baryon as a leading approximation and incorporate baryons into the chiral Lagrangian formulation. While this is a meaningful computation in holographic QCD setting, matching the scales and couplings to the realistic QCD requires a further refinement, since as mentioned, the scales M_{KK} and f_π are actually too low to insist on very large value of $g^2_{\mathrm{YM}} N_c$.

[8]The derivation of soliton size in this paper is an expanded version of that in ref. [21]. Note that an independent derivation was given in ref. [29] which appeared simultaneously with the former.

In making the estimate above, we ignored so far details of the geometry away from the origin. For instance, the spatial part of the geometry is conformally equivalent to $R^3 \times I$, instead of R^4. It is unlikely that the lowest energy configuration is a self-dual instanton solution based on R^4, yet we use it as a trial configuration to estimate the potential. We believe that this will not affect the asymptotic estimate in this section, when the size of the instanton is very small compared to the effective length of the fifth direction $\sim 1/M_{\rm KK}$. This subtlety would be more important for larger instanton size, as we will discuss in section. 6.

4. A 5D effective field theory of the baryon

We saw in the previous section that in the large 't Hooft coupling limit, the underlying instanton configuration for the baryon is rather small. Since the instanton is a small object in 5D sense, we may treat it as a point-like quantum field in 5D in a natural way. Upon quantizing the collective coordinates of the solitonic configuration, there are a variety of baryonic excitations with different spins and flavor charges. From the gauge theory point of view, baryons are composed of N_c elementary quarks forming a color singlet through a total anti-symmetrization of their color indices. The remaining spin and flavor indices together must then form a totally symmetric combination. It is always possible to have one such combination via totally anti-symmetrizing both spin and flavors, giving us the minimal spin and flavor quantum numbers. For even N_c we would have a spin 0 baryon, while a fermionic spin $\frac{1}{2}$ baryon would occur when N_c is odd. Having in mind an extrapolation to the real QCD, we restrict ourselves to the case of fermionic baryons, and the effective field $\mathcal{B}$ would mean a 5D Dirac spinor field. For simplicity we will consider $N_F = 2$ and consider the lowest baryons which form the proton-neutron doublet under $\mathrm{SU}(N_F = 2)$. We are thus lead to introduce an isospin 1/2 Dirac field $\mathcal{B}$ for the five-dimensional baryon.

From the invariance under local coordinate as well as local gauge symmetries on the D8 branes reduced along internal S^4, the leading 5D kinetic term for $\mathcal{B}$ is simply the standard Dirac kinetic term in the curved space in addition to a position dependent mass term that we will specify shortly,

$$-\int dz \int dx^4 \left[i\bar{\mathcal{B}}\Gamma^{\hat{m}} D_{\hat{m}}\mathcal{B} + i m_b(z)\bar{\mathcal{B}}\mathcal{B} \right] , \tag{4.1}$$

where $D_m = \partial_{\hat{m}} + \frac{1}{4}\Gamma_{\hat{n}\hat{p}}\omega_{\hat{m}}^{\hat{n}\hat{p}} + iA_{\hat{m}}^a T^a$ with T^a a representation matrix for $\mathcal{B}$. To determine $m_b(z)$, it is convenient to work in the conformally flat coordinate (w, x^μ) where the spin connection piece can be removed upon suitable rescaling of the $\mathcal{B}$ field [6]. Thus we have

$$-\int dw \int d^4x \left[i\bar{\mathcal{B}}\gamma^\mu(\partial_\mu - iA_\mu^a T^a)\mathcal{B} + i\bar{\mathcal{B}}\gamma^5\partial_w\mathcal{B} + i m_b(w)\bar{\mathcal{B}}\mathcal{B} \right] , \tag{4.2}$$

in the conformal coordinate system and with the $A_5 = 0$ gauge. Here, γ^μ and γ^5 are the standard gamma matrices in the flat space.

The position-dependent mass term requires a further clarification. An elementary excitation approximately localized at the position w would have an energy $m_b(w)$, which must be identified as the energy of an S^4-wrapped D4 brane localized at the position w. From the DBI action of D4 brane, this mass is found to be

$$m_B^{(0)} \cdot \left(\frac{U}{U_{\rm KK}}\right) , \tag{4.3}$$

where U should be considered as an implicit function of w upon the coordinate change from U to w, and

$$m_B^{(0)} = \frac{(g_{\rm YM}^2 N_c)\cdot N_c}{27\pi}\cdot M_{\rm KK} = \frac{\lambda N_c}{27\pi}\cdot M_{\rm KK} , \tag{4.4}$$

with $M_{\rm KK} = \frac{3}{2}\left(\frac{U_{\rm KK}}{R}\right)^{\frac{3}{2}} U_{\rm KK}^{-1}$. In addition, there is a self-energy m_0^e coming from the 5D U(1) field which stabilizes the instanton at some small but finite size. Since this self-energy is a local effect as the baryon size is negligible, this effect should be, at least approximately, independent of the position w and the resulting $m_b(w)$ will be

$$m_b(w) = m_B^{(0)} \cdot \left(\frac{U}{U_{\rm KK}}\right) + m_0^e . \tag{4.5}$$

In large 't Hooft coupling limit, the estimate (3.20) shows that the Coulomb energy m_0^e is negligible compared to the first piece. But, we will keep it in our later numerical analysis for completeness.[9]

However, this cannot be the complete form of the baryon action. As we saw above the baryon is represented by a small instanton soliton, which comes with a long range tail of the gauge field of type $F \sim \rho_{\rm baryon}^2/r^4$. Since we are effectively replacing the baryon by a point-like field $\mathcal{B}$, there should be a coupling between a $\mathcal{B}$ bilinear and the five-dimensional gauge field such that each $\mathcal{B}$-particle generates such a long range tail on F.[10] The minimal coupling originates from fundamental strings attached to D4, and reflects the fact that the instanton carries additional electric charge. This coupling cannot generate a self-dual or anti-self-dual configuration.

As we will see shortly, there is only one vertex that can reproduce the right long-range tail. In our conformal coordinate (x^μ, w), the action including the gauge field and the baryon field must read as[11]

$$\int d^4x dw \left[-i\bar{\mathcal{B}}\gamma^m D_m \mathcal{B} - i m_b(w)\bar{\mathcal{B}}\mathcal{B} + g_5(w)\frac{\rho_{\rm baryon}^2}{e^2(w)}\bar{\mathcal{B}}\gamma^{mn}F_{mn}\mathcal{B}\right] - \int d^4x dw \frac{1}{4e^2(w)}\,{\rm tr}\, F_{mn}F^{mn} , \tag{4.6}$$

where $\rho_{\rm baryon}$ is the stabilized size of the 5D instanton representing baryon, and $g_5(w)$ is an unknown function whose value at $w=0$ can be determined as follows. Throughout this

[9] One may also worry about self-energy from SU($N_F = 2$) gauge field on D4 branes. However, m_0^e scales linearly with N_c because the baryon has charge N_c with respect to U(1)$_V$, while there is no such scaling for SU($N_F = 2$). In the present model, m_0^e is suppressed due to the further requirement of large 't Hooft coupling.

[10] The same type of consideration was employed by Adkins, Nappi and Witten (ANW) [16] to compute $g_{\pi NN}$ which is related to g_A by Goldberger-Treiman relationship.

[11] As usual, we define $\gamma^{mn} = [\gamma^m, \gamma^n]/2$.

article we will refer to the last term in the first line as the magnetic coupling. Its coefficient function is displayed in a particular form with the known function $\rho^2_{\rm baryon}/e(w)^2$ factored out. This is done for the sake of later convenience, where we compute $g_5(0)$ which turns out to be of a purely geometrical origin.

The uniqueness of the operator can be seen from the long range behavior of the instanton. The field strength decays as $1/r^4$, which in five dimensions is one power higher than the Coulomb field. This requires dimension six operators (i.e. one dimension higher than the kinetic term) which contain a cubic term with a baryon bilinear current and the SU(N_F) gauge field. These requirements, together with the approximate Lorentz symmetry, pick out the above form of the operator uniquely. The only other choice would be $\bar{\mathcal{B}}(*F)_{mnk}\gamma^{mnk}\mathcal{B}$, but this is actually equivalent to the above, thanks to the five-dimensional Clifford algebra. Given that this operator is the unique possibility, the remaining question is whether this operator is really capable of the task in hand and if so how to derive the coupling strength g_5, to which we devote the rest of the section.

The instanton must be located at $w = 0$ along the fifth direction, and generates a source term to Yang-Mills field F_{MN}. Provided that the instanton size, $\rho_{\rm baryon}$ is small enough, we only need to consider the immediate vicinity of $w = 0$ where the geometry is R^{4+1} approximately. Take the 5-dimensional Dirac matrices of the form

$$\gamma^0 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{4.7}$$

The on-shell condition of the baryon field is then

$$\begin{pmatrix} i\partial_5 & -i\partial_t + i\sigma^i\partial_i \\ i\partial_t + i\sigma^i\partial_i & -i\partial_5 \end{pmatrix} \mathcal{B} = -im_b\mathcal{B}\,, \tag{4.8}$$

which can be solved by writing the upper 2-component part of $\mathcal{B}$ as $\mathcal{U}\,e^{-iEt+i\vec{p}\cdot\vec{x}}$, and approximating m_b by its central value,

$$\mathcal{B} = \begin{pmatrix} \mathcal{U} \\ \frac{E-\sigma\cdot p}{-im_b-p_5}\mathcal{U} \end{pmatrix} e^{-iEt+i\vec{p}\cdot\vec{x}} \quad \rightarrow \quad \mathcal{B} = \begin{pmatrix} \mathcal{U} \\ \pm i\mathcal{U} \end{pmatrix} e^{\mp im_b t} \tag{4.9}$$

for general plane-wave and for the $p = 0$ limit. The two signs originate from the sign of E/m_b and thus correspond to the baryon and the anti-baryon, respectively.

This spinor configuration sources the Yang-Mills field since[12]

$$\bar{\mathcal{B}}\gamma^{mn}F_{mn}\mathcal{B} \quad \rightarrow \quad \pm F^a_{jk}\left[\mathcal{U}^\dagger\tau^a\epsilon^{jki}\sigma_i\mathcal{U}\right] + 2F^a_{5i}\left[\mathcal{U}^\dagger\tau^a\sigma_i\mathcal{U}\right]\,, \tag{4.11}$$

where we assumed a gauge-doublet under SU($N_F = 2$) with 2×2 generators $(\tau^a/2)_{AB}$. Terms linear in F_{0M} vanish identically when $p = 0 = p_5$, thanks to the on-shell condition.

[12]Note that

$$\gamma^0\gamma^{jk} = \begin{pmatrix} 0 & -i\epsilon^{jki}\sigma_i \\ i\epsilon^{jki}\sigma_i & 0 \end{pmatrix}, \quad \gamma^0\gamma^{5i} = \begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix}, \quad \gamma^0\gamma^{0m} = -\gamma^m\,. \tag{4.10}$$

Note that the proper normalization of Dirac spinor demands $2\mathcal{U}^*_{\alpha A}\mathcal{U}^{\alpha A} = 1$. Defining the bilinear

$$\langle\sigma_i\tau^a\rangle_{\mathcal{B}} = 2\left[\mathcal{U}^\dagger\sigma_i\tau^a\mathcal{U}\right] , \tag{4.12}$$

we thus find

$$\bar{\mathcal{B}}\gamma^{mn}F_{mn}\mathcal{B} \quad \rightarrow \quad \pm\frac{1}{2}F^a_{jk}\epsilon^{jki}\langle\sigma_i\tau^a\rangle_{\mathcal{B}} + F^a_{5i}\langle\sigma_i\tau^a\rangle_{\mathcal{B}} . \tag{4.13}$$

Clearly the spinor bilinear couples to self-dual or an anti-self-dual part of the gauge field strength, regardless of the detailed values of $\langle\sigma_i\tau^a\rangle_{\mathcal{B}}$. Thus, if we relate the effect of the latter to the smearing of the classical long-range field due to quantization of the instanton, identification of $\mathcal{B}$ as the effective field for isospin 1/2 baryons would be complete and this would give information on the coupling strength g_5.

However, for clarity, let us first try to search for an instanton-like long-range field. For instance, one choice for $\mathcal{U}$ that generates instanton-like field is a spin-isospin locked state of the form,

$$\mathcal{U}_{\alpha A} = \frac{i}{2}\epsilon_{\alpha A} , \tag{4.14}$$

in which case $\langle\sigma_i\tau^a\rangle_{\mathcal{B}} = -\delta^a_i$ so that the source term (with the upper sign) is $-F^a_{mn}\bar{\eta}^a_{mn}/2$ with the anti-self-dual 't Hooft symbol $\bar{\eta}$ ($m,n = 1,2,3,5$ and $a = 1,2,3$) [30]. Now assume that such a source appears in a localized form at the origin. The gauge field far away from the source obeys (in an appropriate gauge)

$$\nabla^2 A^a_m = 2g_5(0)\rho^2_{\text{baryon}}\bar{\eta}^a_{mn}\partial_n\delta^{(4)}(x) , \tag{4.15}$$

whose solution goes as

$$A^a_m = -\frac{g_5(0)\rho^2_{\text{baryon}}}{2\pi^2}\bar{\eta}^a_{mn}\partial_n\frac{1}{r^2+w^2} . \tag{4.16}$$

The general shape of the long-range field is consistent with the identification of the baryon as the instanton. Since the actual instanton solution in 't Hooft ansatz has [31]

$$A^a_m = -\bar{\eta}^a_{mn}\partial_n \log\left(1+\frac{\rho^2}{r^2+w^2}\right) \simeq -\rho^2\bar{\eta}^a_{mn}\partial_n\frac{1}{r^2+w^2} , \tag{4.17}$$

one may be tempted to fix $g_5(0)$ as $2\pi^2$.

However, the right prescription is to match the states in $\mathcal{B}$ with *quantized* instanton. An SU(2) instanton of a fixed size has three gauge collective coordinates, spanning SU(2)/Z_2, which can be represented by a special unitary matrix S of size 2×2. Quantization of S can lead to spin 1/2 and isospin 1/2 states with proper choice of boundary condition on $S^3/Z_2 = \text{SU}(2)/Z_2$. This part of story proceeds identically with that of Skyrmions in 4 dimensions, which was explained in much detail by Adkins, Nappi and Witten [16].

One consequence of this quantization procedure is that the long range field of the instanton is modified due to quantum fluctuation of instanton along different global gauge directions. While the classical solution has

$$S^\dagger A^a_M\frac{\tau^a}{2}S = \sum_b A^a_M\frac{\tau^b}{2}\left(\text{tr}\left[S^\dagger\frac{\tau^a}{2}S\tau^b\right]\right) \tag{4.18}$$

for some arbitrary but fixed choice of the unitary matrix S, the quantum consideration replaces the classical coefficients by expectation values

$$\left(\mathrm{tr}\left[S^\dagger \frac{\tau^a}{2} S\tau^b\right]\right) \quad \Rightarrow \quad \left\langle \mathrm{tr}\left[S^\dagger \frac{\tau^a}{2} S\tau^b\right]\right\rangle , \tag{4.19}$$

which effectively lessen the strength of long-range gauge field. We may identify the states contained in $\mathcal{B}$ as spin 1/2 and isospin 1/2 wavefunctions of the instanton, in which case there is an identity,

$$\left\langle \mathrm{tr}\left[S^\dagger \frac{\tau^a}{2} S\tau^b\right]\right\rangle_{\mathcal{B}} = -\frac{1}{3}\langle\sigma_b\tau^a\rangle_{\mathcal{B}} . \tag{4.20}$$

This can be seen by an explicit quantization, which is mathematically identical to the one used by ANW [16] on the Skyrmion case.

Specializing back to the case of $\mathcal{U} = i\epsilon$, where $\langle\sigma_i\tau^a\rangle_{\mathcal{B}} = -\delta_i^a$, note that the classical counterpart would have corresponded to the choice $S = 1$ so that

$$\left.\left(\mathrm{tr}\left[S^\dagger \frac{\tau^a}{2} S\tau^b\right]\right)\right|_{S=1} = \delta_a^b , \tag{4.21}$$

while the actual comparison has to be made with its quantum counterpart

$$\left\langle \mathrm{tr}\left[S^\dagger \frac{\tau^a}{2} S\tau^b\right]\right\rangle_{\mathcal{B}} = -\frac{1}{3}\langle\sigma_b\tau^a\rangle_{\mathcal{B}} = \frac{1}{3}\delta_a^b . \tag{4.22}$$

This tells us that when making comparison between the long range part of quantized instanton solution, and the long range field generated by the baryon source, we must include a factor of 1/3 on the instanton size. Thus, we conclude that

$$g_5(0) = \frac{2\pi^2}{3} . \tag{4.23}$$

This fixes the value of $g_5(w)$ at origin of the fifth direction. Finding the form of the function $g_5(w)$ for general value of w seems very difficult from the present approach. However, for very small size of baryon/instanton, which is guaranteed by a large 't Hooft coupling, $\lambda = g_{\rm YM}^2 N_c$, only the central value will enter the physics and corrections are suppressed by inverse powers of λ.

In the above, we have extracted $g_5(0)$ by comparing the quantized instanton and the spinor state for a particular spin-isospin locked state. For a complete check, we must consider more general states with spin 1/2 and isospin 1/2, for which it suffices to rewrite eq. (4.13), say, with the upper sign choice, as

$$\begin{aligned}\frac{1}{2}F_{jk}^a\epsilon^{jki}\langle\sigma_i\tau^a\rangle_{\mathcal{B}} + F_{5i}^a\langle\sigma_i\tau^a\rangle_{\mathcal{B}} &= \frac{1}{2}F_{jk}^a\bar\eta_{jk}^b\langle\sigma_b\tau^a\rangle_{\mathcal{B}} + F_{5k}^a\bar\eta_{5k}^b\langle\sigma_b\tau^a\rangle_{\mathcal{B}} \\ &= \frac{1}{2}F_{mn}^a\bar\eta_{mn}^b\langle\sigma_b\tau^a\rangle_{\mathcal{B}} \\ &= -\frac{3}{2}F_{mn}^a\bar\eta_{mn}^b\left\langle \mathrm{tr}\left[S^\dagger \frac{\tau^a}{2} S\tau^b\right]\right\rangle_{\mathcal{B}} ,\end{aligned} \tag{4.24}$$

where the last step used the identity eq. (4.20) between expectation values in two different description. Since S represents $\mathrm{SU}(N_F = 2)$ rotation on the soliton side of the picture, the long range field generated by such a source would mimic that of the instanton field expectation value, evaluated on arbitrary quantized instanton state with spin 1/2 and isospin 1/2. We can follow a similar procedure above for the spin-isospin locked state, which shows that the on-shell degrees of freedom of $\mathcal{B}$ can be matched with the spin 1/2 isospin 1/2 sector states of the quantized instanton, given the effective action for $\mathcal{B}$ and $g_5(0) = 2\pi^2/3$.

5. The chiral dynamics of the nucleons in four dimensions

In the current effective theory approach, the physical 4D nucleons would arise as the lowest eigenmodes of this 5D baryon along w coordinate, which should be a mode localized near $w = 0$. From the string theory picture where the solitonic configuration for a baryon comes from a melted D4 brane inside $N_F = 2$ D8 branes, there are N_c fundamental strings ending on the D8 branes out of the D4 brane, which are nothing but the elementary quarks in the gauge theory view point. In the limit of large λ, this consideration leads us to treat the five-dimensional baryon as a point-like object in the doublet representation under $\mathrm{SU}(N_F = 2)$ with the effective action in (4.6). While the generalization to excited baryons, such as Δ's, should be straightforward, we will consider isospin doublets in this work. In particular, the lowest-mass eigenstates in 4D sense are nothing but the nucleons (protons and neutrons), whose low energy dynamics will be explored for the rest of the paper.

What we need now is to reduce this five-dimensional action down to four dimensions and extract the couplings between the nucleon and the infinite tower of mesons. In the usual chiral Lagrangian approach of QCD, the nucleon is often treated as a point-like Dirac field B, just as in our five-dimensional approach. In doing so, the form factors of the nucleons would be then encoded in how the nucleons couples to pions and all the massive vector mesons. The leading quadratic part of the nucleon effective action one usually writes down is

$$\int dtdx^3\ \mathcal{L}_4 = -\int dtdx^3\ \bar{B}(i\gamma^\mu D_\mu + im_B + g_A\gamma^\mu\gamma^5 A_\mu)B + \cdots\ , \tag{5.1}$$

where the covariant derivative

$$D_\mu = \partial_\mu - iV_\mu \tag{5.2}$$

encodes the coupling to the massive vector meson v_μ

$$V_\mu = v_\mu + i\beta_\mu(x) = v_\mu + \frac{i}{2}[\xi^{-1}, \partial_\mu\xi] \tag{5.3}$$

in a manner consistent with the hidden local gauge symmetry. The axial coupling provides the simplest vertex of this theory whereby nucleon emits a single pion. In terms of ξ, we have

$$A_\mu = \frac{i}{2}\,\alpha_\mu \simeq -\frac{1}{f_\pi}\,\partial_\mu\pi + O(\pi^3)\,. \tag{5.4}$$

The goal of this section is to reproduce this structure and more from our five-dimensional effective action.

5.1 4D nucleons and dimensional reduction

To make the preceding discussion concrete, let us perform KK-mode expansion for the action (4.2) to obtain the spectrum of spin-$\frac{1}{2}$ baryons in the large $\lambda N_c = (g_{\rm YM}^2 N_c)N_c$ limit. The lowest state is identified as the nucleon. The gauge field A_μ on the $N_F = 2$ D8 branes also has a mode expansion, including pions and ρ mesons, that is discussed in the preceding sections. From these we can read off the couplings of nucleons to mesons via numerical analysis.

We mode expand $\mathcal{B}_{L,R}(x^\mu, w) = B_{L,R}(x^\mu) f_{L,R}(w)$ where $\gamma^5 B_{L,R} = \pm B_{L,R}$ are 4D chiral components, with the profile functions $f_{L,R}(w)$ satisfying

$$\begin{aligned} \partial_w f_L(w) + m_b(w) f_L(w) &= m_B f_R(w) \,, \\ -\partial_w f_R(w) + m_b(w) f_R(w) &= m_B f_L(w) \,, \end{aligned} \tag{5.5}$$

in the range $w \in [-w_{\rm max}, w_{\rm max}]$. The 4D Dirac field for the nucleon is then reconstructed as

$$B = \begin{pmatrix} B_L \\ B_R \end{pmatrix} , \tag{5.6}$$

and the eigenvalue m_B is the mass of the nucleon mode $B(x)$.

The eigenfunctions $f_{L,R}(w)$ are also normalized to unit norm

$$\int_{-w_{\rm max}}^{w_{\rm max}} dw\ |f_L(w)|^2 = \int_{-w_{\rm max}}^{w_{\rm max}} dw\ |f_R(w)|^2 = 1 \,, \tag{5.7}$$

for $B(x)$ to have the standard 4D kinetic term. As we approach $w \to \pm w_{\rm max}$, $m_b(w)$ diverges as,

$$m_b(w) \sim \frac{1}{(w \mp w_{\rm max})^2} \tag{5.8}$$

and the above equations have normalizable eigen-functions with a discrete spectrum of m_B. It is more convenient to consider a second-order equation for $f_{L,R}(w)$

$$\begin{aligned} \left[-\partial_w^2 - \partial_w m_b(w) + (m_b(w))^2\right] f_L(w) = m_B^2 f_L(w) \,, \\ \left[-\partial_w^2 + \partial_w m_b(w) + (m_b(w))^2\right] f_R(w) = m_B^2 f_R(w) \,. \end{aligned} \tag{5.9}$$

Note that there is a 1-1 mapping of eigenmodes with $f_R(w) = \pm f_L(-w)$. Due to the asymmetry under $w \to -w$ in the term $-\partial_w m_b(w)$ above, $f_L(w)$ tends to shift to the positive w side, and the opposite happens for $f_R(w)$. This will then give us a non-vanishing contribution to the axial coupling of the nucleon to the pions, as we will see shortly.

The gauge field A_μ, in the $A_5 = 0$ gauge, has a mode expansion

$$A_\mu(x, w) = i\alpha_\mu(x)\psi_0(w) + i\beta_\mu(x) + \sum_n a_\mu^{(n)}(x)\psi_{(n)}(w) \,, \tag{5.10}$$

where $\hat{\Psi}_0(z) \equiv \psi_0(w(z)) = \frac{1}{\pi} \arctan\left(\frac{z}{U_{\rm KK}}\right)$ which is odd under $w \to -w$, and

$$\begin{aligned} \alpha_\mu &= \{\xi^{-1}, \partial_\mu \xi\} = \frac{2i}{f_\pi}\partial_\mu \pi + \cdots \,, \\ \beta_\mu &= \frac{1}{2}[\xi^{-1}, \partial_\mu \xi] = \frac{1}{2f_\pi^2}[\pi, \partial_\mu \pi] + \cdots \,. \end{aligned} \tag{5.11}$$

JHEP09(2007)063

We recall from the previous eigenmode analysis by SS that $\psi_{(2k+1)}(w)$ is even, while $\psi_{(2k)}(w)$ is odd under $w \to -w$, corresponding to vector and axial-vector mesons respectively.

Inserting this expansion into the action (4.6), and using the properties $f_L(w) = \pm f_R(-w)$ as well as the properties of $\hat{\Psi}_0$ and $\psi_{(n)}$ under $w \to -w$, we obtain a 4D nucleon action

$$\mathcal{L}_4 = -i\bar{B}\gamma^\mu\partial_\mu B - im_B\bar{B}B + \mathcal{L}_{\text{vector}} + \mathcal{L}_{\text{axial}}\,, \tag{5.12}$$

with the four-dimensional nucleon mass m_B. This nucleon mass will generally differ from the five-dimensional mass, due to spread of the wavefunction $f_{L,R}$ along the fifth direction. However, this difference arises only as a subleading correction in large N_c and large λ. Writing out the interaction terms explicitly, we have

$$\mathcal{L}_{\text{vector}} = -i\bar{B}\gamma^\mu\beta_\mu B - \sum_{k\geq 0} g_V^{(k)}\bar{B}\gamma^\mu a_\mu^{(2k+1)}B\,, \tag{5.13}$$

and the nucleon couplings to axial mesons, including pions, as

$$\mathcal{L}_{\text{axial}} = -\frac{ig_A}{2}\bar{B}\gamma^\mu\gamma^5\alpha_\mu B - \sum_{k\geq 1} g_A^{(k)}\bar{B}\gamma^\mu\gamma^5 a_\mu^{(2k)}B\,, \tag{5.14}$$

where various couplings constants $g_{V,A}^{(k)}$ as well as the pion-nucleon axial coupling g_A are calculated by suitable wave-function overlap integrals. In the above expression, the meson fields should be understood as being written in the nucleon isospin representation.

The nucleon-meson interaction terms arise from two sources, namely the magnetic-type direct coupling to the 5D gauge field strength and the more conventional minimal coupling in the kinetic term. The former comes with a coefficient $\rho^2_{\text{baryon}}/e^2$ in five dimensions, which scales linearly with N_c.

The minimal coupling contributions are summarized as

$$\begin{aligned}
g_{V,min}^{(k)} &= \int_{-w_{\max}}^{w_{\max}} dw\;|f_L(w)|^2\,\psi_{(2k+1)}(w)\,,\\
g_{A,min}^{(k)} &= \int_{-w_{\max}}^{w_{\max}} dw\;|f_L(w)|^2\,\psi_{(2k)}(w)\,,\\
g_{A,min} &= 2\int_{-w_{\max}}^{w_{\max}} dw\;|f_L(w)|^2\,\psi_0(w)\,.
\end{aligned} \tag{5.15}$$

Since in the large λN_c-limit, the nucleon wave-function $f_L(w)$ tends to be symmetric under $w \to -w$, we see that g_A and $g_A^{(k)}$ receive small contributions from the minimal 5D gauge interaction, in the large λN_c limit. On the contrary, due to the even nature of $\psi_{(2k+1)}$, the vector couplings $g_V^{(k)}$ receive an order one contribution from the minimal interaction.

To isolate similar interaction terms from the 5D magnetic coupling, we take the case of $(m,n) = (5,\mu)$, which becomes

$$-\frac{\lambda N_c(\rho_{\text{baryon}}M_{\text{KK}})^2}{108\pi^3}\int d^4x\int dw\left[\left(\frac{2g_5(w)U(w)}{U_{\text{KK}}M_{\text{KK}}}\right)\bar{\mathcal{B}}\gamma^\mu\gamma^5(\partial_w A_\mu)\mathcal{B}\right], \tag{5.16}$$

JHEP09(2007)063

where we have used

$$\frac{1}{e^2(w)} = \frac{\lambda N_c}{108\pi^3} M_{\rm KK} \frac{U(w)}{U_{\rm KK}} \,. \tag{5.17}$$

Defining the dimensionless number $C = \left(2\pi^2/3\right) \lambda N_c (\rho_{\rm baryon} M_{\rm KK})^2/108\pi^3$, we have contributions to $g^{(k)}_{V,A}$ and g_A as follows,

$$\begin{aligned}
g^{(k)}_{V,mag} &= 2C \int_{-w_{\rm max}}^{w_{\rm max}} dw \left(\frac{g_5(w)U(w)}{g_5(0)U_{\rm KK}M_{\rm KK}}\right) |f_L(w)|^2\, \partial_w \psi_{(2k+1)}(w)\,, \\
g^{(k)}_{A,mag} &= 2C \int_{-w_{\rm max}}^{w_{\rm max}} dw \left(\frac{g_5(w)U(w)}{g_5(0)U_{\rm KK}M_{\rm KK}}\right) |f_L(w)|^2\, \partial_w \psi_{(2k)}(w)\,, \\
g_{A,mag} &= 4C \int_{-w_{\rm max}}^{w_{\rm max}} dw \left(\frac{g_5(w)U(w)}{g_5(0)U_{\rm KK}M_{\rm KK}}\right) |f_L(w)|^2\, \partial_w \psi_0(w)\,.
\end{aligned} \tag{5.18}$$

Note that the sizes of this integral behave oppositely when compared to the similar overlap integrals for the minimal coupling term. Again since in the large λN_c-limit, the nucleon wave-function $f_L(w)$ tends to be symmetric under $w \to -w$, and since $\psi_{(2k+1)}$ ($\psi_{(2k)}$) is an even (odd) function of w, the vector coupling contributions become relatively suppressed in the large λN_c-limit, while the axial couplings remain order one times the large constant C. Using the estimate of $\rho_{\rm baryon}$ in eq. (3.18), we find

$$C \simeq 0.18 N_c\,. \tag{5.19}$$

With the present 5D effective theory approach, we have all mesons encoded in a single $\mathrm{U}(N_F = 2)$ gauge field in five dimensions. In particular, the iso-scalar mesons and iso-vector mesons arise from a single 2×2 gauge field. However, of these, only the traceless part appears in the magnetic coupling since instanton carries only non-Abelian field strength. Therefore, the iso-scalar mesons and iso-vector mesons couple to nucleons differently. For the iso-scalar mesons, such as for instance the ω meson in the vector channel, only the minimal term contributes

$$\begin{aligned}
g^{(k)}_A\Big|_{\rm iso-scalar} &= g^{(k)}_{A,min}\Big|_{\rm iso-scalar}\,, \\
g^{(k)}_V\Big|_{\rm iso-scalar} &= g^{(k)}_{V,min}\Big|_{\rm iso-scalar}\,.
\end{aligned} \tag{5.20}$$

But, for iso-vectors, we have contributions from both minimal and magnetic terms. Thus, we must add

$$g_A = g_{A,mag} + g_{A,min}\,. \tag{5.21}$$

and similarly for the vector and the axial vector in the iso-vector

$$\begin{aligned}
g^{(k)}_A\Big|_{\rm iso-vector} &= \left[g^{(k)}_{A,mag} + g^{(k)}_{A,min}\right]\Big|_{\rm iso-vector}\,, \\
g^{(k)}_V\Big|_{\rm iso-vector} &= \left[g^{(k)}_{V,mag} + g^{(k)}_{V,min}\right]\Big|_{\rm iso-vector}\,.
\end{aligned} \tag{5.22}$$

Naively, since the coefficient for the magnetic term grows linearly with N_c, one might be tempted to throw away the minimal coupling contribution. However, the vector-like couplings and axial-vector-like couplings behave quite differently. Note that the part of the 5D magnetic term we employed above has an explicit γ^5 while the minimal term (in the gauge $A_5 = 0$) cannot, yet both axial and vector-like coupling arise from each. It is only because of the asymmetry between f_L and f_R that we can find the axial interaction terms from the minimal coupling and the vector-like terms from the magnetic coupling. This asymmetry is strong when λN_c is small but diminishes as $\lambda N_c \to \infty$. In other words, the wavefunction overlap integral would be suppressed by $1/\lambda N_c$ for these interactions with the "wrong" number of γ_5.

For this reason, all axial couplings, g_A and $g_A^{(k)}$ are dominated, as expected, by the contribution from the magnetic terms, whereas all vector-like couplings, $g_V^{(k)}$, will be dominated by the contribution from the minimal couplings: at least in the large λ limit,

$$g_V^{(k)}\Big|_{\text{iso}-\text{vector}} \simeq g_{V,min}^{(k)}\Big|_{\text{iso}-\text{vector}} \quad ; \quad g_A^{(k)}\Big|_{\text{iso}-\text{vector}} \simeq g_{A,mag}^{(k)}\Big|_{\text{iso}-\text{vector}}, g_A \simeq g_{A,mag}. \tag{5.23}$$

Finally let us note that we have neglected part of the magnetic coupling in our discussion of the four-dimensional effective action, namely those with two 4D indices on the Dirac matrices,

$$\bar{\mathcal{B}}\gamma^{\mu\nu}F_{\mu\nu}\mathcal{B}\,. \tag{5.24}$$

When we reduce this dimensionally, we will find more couplings between the nucleons and the infinite tower of mesons but with one more derivative than the above Yukawa terms. Although they are higher power in usual power counting, the suppressing mass scale would be at most M_{KK}, so we expect these couplings to be very relevant to physical processes which are measured up to several GeV. We hope to come back to this aspect of holographic QCD in a later work.

5.2 Vector couplings: iso-scalar vs. iso-vector

As we mentioned earlier, the couplings between massive vectors $a_\mu^{(2k+1)}$ and nucleons arise primarily from the minimal coupling in the large λ limit. The leading coupling is then,

$$-\sum_{k\geq 0} g_V^{(k)}\bar{B}\gamma^\mu a_\mu^{(2k+1)}B\,, \tag{5.25}$$

where

$$g_V^{(k)} = g_{V,min}^{(k)} = \int_{-w_{\max}}^{w_{\max}} dw\ |f_L(w)|^2\,\psi_{(2k+1)}(w) \tag{5.26}$$

for $a_\mu^{(2k+1)}$ in the iso-scalar, while

$$g_V^{(k)} \simeq g_{V,min}^{(k)} \tag{5.27}$$

for $a_\mu^{(2k+1)}$ in the iso-vector in the large λ approximation.

Since the iso-scalar and iso-vector couplings here have the same origin in the five-dimensional dynamics, this immediately implies a simple algebraic relations between the two classes of couplings. Let us decompose the massive vectors as

$$a_\mu^{(2k+1)} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}\omega_\mu^{(k)} + \rho_\mu^{(k)} \tag{5.28}$$

into the trace part and the rest, where we wrote the gauge field in the fundamental representation. This is how individual massless quark doublet would see the vector mesons. However the baryon is made out of N_c product quark doublets, and we are considering the case of the doublet as the smallest irreducible representation under $\mathrm{SU}(N_F = 2)$. In the process, while the SU(2) representation is kept small as such, the trace part of the charge are simply added so that the above decomposition actually appears for nucleons as

$$a_\mu^{(2k+1)} = \begin{pmatrix} N_c/2 & 0 \\ 0 & N_c/2 \end{pmatrix} \omega_\mu^{(k)} + \rho_\mu^{(k)} . \tag{5.29}$$

We have been using the normalization of SU(2) generators consistently as $\mathrm{tr}\, T^a T^b = \delta^{ab}/2$, so the eigenvalues for doublets are $\pm 1/2$. Therefore, between the iso-scalar and the iso-vector, there is an overall factor of N_c. In other words, we have the universal relation, again in the large λ limit

$$|g_{\omega^{(k)}NN}| \simeq N_c \times |g_{\rho^{(k)}NN}| \tag{5.30}$$

between the Yukawa couplings involving iso-scalar and iso-vector vector mesons. Here g_{vNN} denotes the Yukawa coupling between the nucleon vector current and the canonically normalized vector field v. Note that the relation (5.30) is the same as what one obtains in CQM. We will see how the relation (5.30) fares with nature in section 6.1 below.

5.3 Pseudo-vector couplings

An important observation to keep in mind here is that the normalization condition of the eigenmode $\psi_{(n)}$ for $n \geq 1$ contains a factor of f_π, so that of all quantities above, only $g_{A,mag}$ grows linearly with N_c. Despite large C value for large N_c, all other g's are order $(N_c)^0$ at most, and in fact suppressed further by $1/\lambda$. Nevertheless, it remains true that the contribution from the magnetic coupling is dominant whenever present. Thus, depending on whether the pseudo-vector is in the iso-scalar or in the iso-vector, we have the following coupling

$$-\sum_{k\geq 1} g_A^{(k)} \bar{B}\gamma^\mu\gamma^5 a_\mu^{(2k)} B , \tag{5.31}$$

where

$$g_A^{(k)} = g_{A,min}^{(k)} = \int_{-w_{\max}}^{w_{\max}} dw\ |f_L(w)|^2\, \psi_{(2k)}(w) \tag{5.32}$$

for iso-scalar part of the pseudo-vector $a_\mu^{(2k)}$ while

$$g_A^{(k)} \simeq g_{A,mag}^{(k)} = 2C \int_{-w_{\max}}^{w_{\max}} dw \left(\frac{U(w)}{U_{\mathrm{KK}} M_{\mathrm{KK}}}\right) |f_L(w)|^2\, \partial_w \psi_{(2k)}(w) \tag{5.33}$$

for iso-vector part of the pseudo-vector $a_\mu^{(2k)}$.

5.4 Axial coupling to pions and an $O(1)$ correction

5.4.1 The leading $O(N_C)$ term

For g_A, the leading contribution is $g_{A,mag}$, for which the corresponding integral can be done exactly by using the explicit form of $\psi_0(w)$ and also by approximating $g_5(w) \simeq g_5(0)$. The latter approximation is harmless if λN_c is sufficiently large. Since

$$\left(\frac{U(w)}{U_{\rm KK}M_{\rm KK}}\right)\partial_w\psi_0(w)=\frac{1}{\pi}\,, \tag{5.34}$$

we have

$$g_{A,mag}=\frac{4C}{\pi}\simeq 0.7\times\frac{N_c}{3}\,. \tag{5.35}$$

While this depends on the substitution of $g_5(w)\to g_5(0)$, the result is robust as long as $f_L(w)$ is sufficiently localized at $w=0$. In turn, this is guaranteed by arbitrarily large λN_c.

The subleading contribution, $g_{A,min}$ is at most order $1/\lambda N_c$, and thus is negligible in the present AdS/CFT limit.

5.4.2 The $O(1)$ correction

As mentioned in section 4, the collective quantization that led to (5.35) was based on the mathematical manipulation of ANW that consisted of performing the isospin rotation $A=a_4+ia_i\tau_i$ (with $\sum_i a_i^2=1$) of the soliton and evaluating the corresponding element of the orthogonal space rotation group given by

$$R_{ij}=\frac{1}{2}{\rm Tr}\tau_i A\tau_j A^\dagger. \tag{5.36}$$

We can exploit the equivalence of the constituent quark model (CQM) and the Skyrmion in the large N_c limit [32] to obtain an $1/N_C$ correction to the leading term while fermion loops are kept suppressed. Briefly, the reasoning goes as follows.

(i) We first note that the collective quantization of the instanton we are dealing with involves, among various collective coordinates, the same isospin rotation (5.36) as in the Skyrme model. This can be seen in the collective quantization of the instanton by Hata et al. [29]. Now the ANW quantization is known to give the $\mathcal{O}(N_c)$ term to g_A which is identical to what is obtained in the large N_c limit of CQM [32].

(ii) A general large-N_c QCD analysis shows that g_A has the large N_c expansion [33]

$$g_A=\alpha\left(\frac{N_c+\beta}{3}\right)+\gamma\frac{1}{N_c}+\cdots \tag{5.37}$$

where α, β and γ are constants independent of N_c and the ellipsis stands for higher $1/N_c$ terms. An important point to note here is that fermion (quark) loop corrections first appear at $\mathcal{O}(1/N_c)$ and not at $\mathcal{O}(1)$. This means that the constant β survives "quenching," that is, it has *no dynamical loop effects.*

(iii) While general considerations leave the coefficient β undetermined, the CQM, however, gives a simple result coming from a simple (group-theoretic) book-keeping,

$$\beta=2. \tag{5.38}$$

One might a priori think that the Skyrmion model needs not give the same value. However it has been shown by a detailed group structure of the spin-isospin operator involved in the Skyrmion – and likewise in the instanton baryon — that the result (5.38) *does* hold [34].[13] Exactly the same argument holds for the iso-vector dipole magnetic moment operator and will be applied later in the next section.

If one shifts N_c to N_c+2 as argued above, we can include the $\mathcal{O}(1)$ correction to (5.35) and obtain

$$g_A \approx 0.7\left(\frac{N_c+2}{3}\right) \approx 1.17\,, \tag{5.39}$$

which is expected to be reliable up to $\mathcal{O}(1/N_c^2) \approx 10\%$. An interesting observation to make at this point is that the instanton baryon predicts $\alpha \approx 0.7$ in (5.37) which is close to the chiral perturbation theory prediction $\alpha_{\chi PT} \approx 0.75$ [35]. Another observation is that the "probe approximation" involved in the SS model appears to be equivalent to the quenched approximation in lattice calculations. The quenched lattice calculation contains no fermion loops while containing all orders of λ and $1/N_c$ pertaining to gluons. We conjecture that the quenched lattice result differs from the instanton result (5.39) only at the next order, i.e., $\mathcal{O}(1/N_c^2)$ relative to the leading order. This conjecture is numerically supported in that the quenched lattice result [36, 37] is quite close to (5.39), and furthermore the unquenched calculation [37] with dynamical quarks agrees closely with the quenched result indicating that the higher order $1/N_c$ corrections are not big.

6. Numerical estimates and extrapolations

6.1 Numerics and subleading corrections

In this effective theory approach, we consider the five-dimensional baryon as point-like, which is justified by the large 't Hooft coupling $\lambda = \mathrm{g}_{\mathrm{YM}}^2 N_c$. However, if we wish to extrapolate the result to finite $\lambda \sim 17$ regime, we cannot neglect the size of the instanton. Thus, we cannot say that the above can be extrapolated to a realistic QCD regime with justification. We will come back to the size issue in the last part of this section. However, with this caveat in mind, we wish to extrapolate the effective theory to the realistic regime and try to see how leading corrections would behave qualitatively. Typical quantities we must know to compare with nature are the trilinear couplings, namely $g_{V,min}^{(k)}$, $g_{V,mag}^{(k)}$, $g_{A,min}^{(k)}$, $g_{A,mag}^{(k)}$, as well as $g_{A,mag}$ and $g_{A,min}$. In the previous section, we outlined the large N_c and large λ behavior of these couplings which must be corrected as we approach realistic regimes.

[13] Briefly the argument is as follows [34]. The spin-isospin structure of the hedgehog ansatz adopted for the instanton (Skyrmion) suggests that the soliton is a U(4) coherent state in the large N_c limit. By realizing the soliton algebra in terms of N "interacting bosons" (with $N = N_c$) familiar in nuclear and molecular physics, projecting out the good spin and isospin of the nucleon in the matrix element of the axial-current operator is made both direct and simple. This permits us to calculate the leading $1/N_c$ correction based solely on symmetry consideration without involving any dynamical calculations.

λN_c	$m_B/M_{\rm KK}$	m_B/f_π	$g_{A,min}$	$g^{(0)}_{V,min}$	$g^{(1)}_{V,min}$	$g^{(0)}_{V,mag}(N_c=3)$	$g^{(1)}_{V,mag}(N_c=3)$
10	1.37	22.2	0.171	8.21	2.74	-1.28	-1.95
20	1.52	17.5	0.161	6.15	2.55	-0.95	-1.80
30	1.66	15.6	0.152	5.20	2.44	-0.78	-1.66
40	1.80	14.6	0.143	4.61	2.35	-0.66	-1.53
50	1.93	14.0	0.135	4.19	2.28	-0.58	-1.40
60	2.06	13.7	0.129	3.88	2.21	-0.51	-1.32
80	2.32	13.3	0.117	3.43	2.11	-0.42	-1.15
120	2.82	13.2	0.099	2.88	1.94	-0.30	-0.90
160	3.30	13.4	0.086	2.54	1.81	-0.23	-0.73
200	3.79	13.7	0.076	2.29	1.70	-0.19	-0.61

Table 1: Numerical result for $g_{A,min}$, the axial pion-nucleon-nucleon coupling, and the couplings to the lowest two vector mesons. Here we used $C \simeq 0.18N_c$ with $N_c = 3$ for the evaluation in the last two column. The realistic regime should be chosen so that $f_\pi/M_{\rm KK} = \sqrt{\lambda N_c/54\pi^4}$ fits with experimental values for these two scales. The resulting λN_c lies somewhere around 50. For $g_{V,mag}$'s, we approximated $g_5(w)/e(w)^2 = g_5(0)/e(0)^2$ which may not be justifiable in the present range of λN_c values.

The main object we need to understand in order to compute these couplings is the wavefunction of the nucleon $f_{L,R}(w)$. For efficient numerical estimates, we scale out dimensionful parameters from the spinor equations by introducing dimensionless variables $\tilde{w} = M_{\rm KK}w$, $\tilde{U} = U/U_{\rm KK}$, and $\tilde{z} = z/U_{\rm KK}$. These are related as

$$\tilde{w} = \int_0^{\tilde{z}} \frac{d\tilde{z}}{[1+\tilde{z}^2]^{\frac{2}{3}}} = \frac{3}{2}\int_1^{\tilde{U}} \frac{d\tilde{U}}{\sqrt{\tilde{U}^3 - 1}} \,. \tag{6.1}$$

In terms of these variables, we have

$$m_b(z) = m_B^{(0)} \cdot \tilde{U} + m_0^e = M_{\rm KK} \cdot \left(\frac{\lambda N_c}{27\pi}\tilde{U}(\tilde{w}) + \epsilon N_c\right) \tag{6.2}$$

with $\epsilon \equiv \sqrt{2/15} \simeq 0.37$. After dividing the eigenvalue equation (5.9) by $M_{\rm KK}^2$, we arrive at

$$\left[-\partial_{\tilde{w}}^2 - \frac{\lambda N_c}{27\pi}\,\partial_{\tilde{w}}\tilde{U}(\tilde{w}) + \left(\frac{\lambda N_c}{27\pi}\,\tilde{U}(\tilde{w}) + \epsilon N_c\right)^2\right] f_L(\tilde{w}) = \left(\frac{m_B}{M_{\rm KK}}\right)^2 f_L(\tilde{w})\,, \tag{6.3}$$

and the wave-function $f_L(\tilde{w})$ does not depend on the scales. Since $\psi_0(w)$ is also a universal function in terms of our dimensionless variables, the two axial coupling contributions, $g_{A,min}$ and $g_{A,mag}$, are indeed functions of λN_c and N_c. Specifically, the previous formula tells us that $g_{A,min}$ is a function of λN_c and $g_{A,mag}$ depends on N_c only.

We solve $f_L(\tilde{w})$ and its eigenvalue $m_B/M_{\rm KK}$ numerically for a given value of λN_c using shooting method. As mentioned before, the Coulomb energy part, C_0N_c, is subleading and negligible in the 't Hooft limit. For large λN_c, the effective potential in (6.3) is very steep and the wave-function would tend to localize at the minimum point which scales as $\tilde{w}_{\rm min} \sim \mathcal{O}((\lambda N_c)^{-1})$.

JHEP09(2007)063

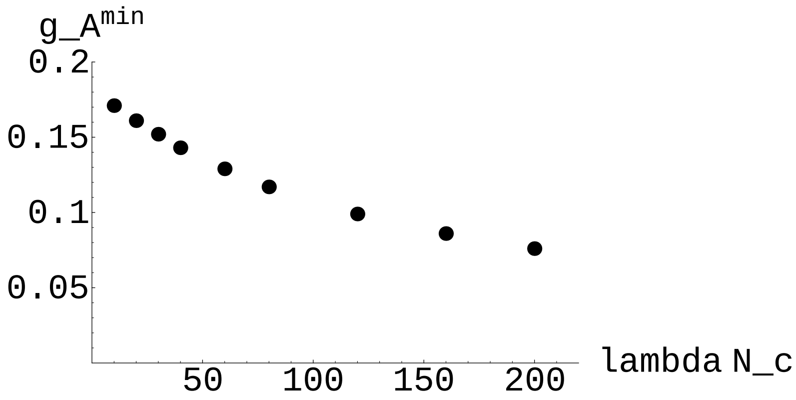

Figure 1: Plot of $g_{A,min}$ versus λN_c.

For instance, we can see that $g_{A,min}$ (and $g^{(k)}_{A,min}$) are proportional to the asymmetry of $|f_L(\tilde{w})|^2$ in $\tilde{w}$ for small $\tilde{w}_{\rm min}$, we conclude that $g_{A,min}$ also scales as $\mathcal{O}((\lambda N_c)^{-1})$ for large λN_c. Our numerical result is shown in table 1, and the values of $g_{A,min}$ for large λN_c confirm this expectation. The same is true of $g^{(k)}_{V,mag}$, relative to $g^{(k)}_{V,min}$. Table 1 provides some numerical values for $g_{A,min}$, $g^{(k)}_{V,min}$, and $g^{(k)}_{V,mag}$. The first represents a subleading correction to the axial coupling between pions and nucleons, whereas $g^{(k)}_V$ are quantities which are also well-measured via scattering processes of nucleons.

Before proceeding further, however, we must warn the readers of another approximation we took which goes beyond the usual large N_c and large λ limit. Note that our computation in section 4 revealed the value of coupling g_5 at $w = 0$. Extending this to a bona-fide function of $g_5(w)$ has so far proven very difficult. While some quantities, such as $g_{A,mag}$, is insensitive to the detailed form of this function, generic numerical estimate requires its precise form. Roughly speaking, this problem will become more and more severe for large values of k since its wavefunction would be spread more and more away from the origin $w = 0$. Also the smaller the value of λN_c, the less reliable will be our estimate since $f_{L,R}(w)$ will be also spread more and more away from the origin. This is a technical problem that affects all terms arising from the magnetic terms. For numerical estimates of $g^{(k)}_{V,mag}$ here and later in section 7, we chose to sidestep the issue by replacing $g_5(w)/e(w)^2$ by its value at the origin $g_5(0)/e(0)^2$.

Using table 1, in conjunction with the results of previous section on leading large N_c behaviors, we can make semi-quantitative estimates of g_A and g_{VNN} for $V = \rho, \omega$ and compare with nature. To do this, we adopt the parameters $M_{\rm KK}$ and λN_c fixed by the pion decay constant $f_\pi \approx 86 \sim 93\,{\rm MeV}$,[14] and the ρ-meson mass in the meson sector [2] i.e., $M_{\rm KK} \approx .94\,{\rm GeV}$ and $\lambda N_c = 50$.

[14] f_π is 86 MeV for $m_\pi = 0$ and 93 MeV for $m_\pi \approx 140\,{\rm MeV}$.

- **The axial coupling constant**:

  Adding the subleading contribution $g_{A,min}$ of table 1 to the leading term (5.39), we have for $\lambda N_c = 50$,

$$g_A \approx 1.30 - 1.31 \tag{6.4}$$

  which compares well with the experimental value $g_A^{\mathrm{exp}} = 1.2670 \pm 0.0035$. As discussed in section 5.4.1, there are indications from lattice calculations that higher-order $1/N_c$ corrections or "unquenching" are suppressed. The same suppression seems to be taking place in our calculation.

- **The ρNN and ωNN coupling constants:**

  Consider the lowest members of the tower $V = \rho, \omega$ that correspond to $k = 0$ in eq. (5.13). The leading order relation (5.30) will be spoiled at the subleading order since the magnetic term contributes only to the ρNN coupling. From table 1, we have for $\lambda N_c = 50$

$$\begin{aligned} g_{\rho NN} &\approx 3.6 \\ g_{\omega NN} &\approx 12.6 \end{aligned} \tag{6.5}$$

  Thus the relation (5.30) is modified to

$$\mathcal{R} \equiv \frac{g_{\omega NN}}{3 g_{\rho NN}} \approx 1.2 \tag{6.6}$$

  roughly independently of λN_c. We should stress that while the sign of $g_{V,mag}$ is robust, the approximation that goes into the estimate of $g_{V,mag}$ is uncertain, so we cannot take the numerical values too seriously. *However considering that there are no theoretical estimates — instead of fits to experiments — of the above quantities, we offer (6.5) and (6.6) as the first theoretical prediction of those quantities.*

  There are no direct experimental determinations of these constants. However indirect "empirical" values have been extracted from various sources including precision fits to nucleon-nucleon scattering phase shifts up to lab energy $\sim 350\,\mathrm{MeV}$ using one-boson-exchange potentials. In addition, purely phenomenological potentials parameterized with a large number of parameters fit to phase shifts can be translated into the form of boson-exchange potentials and provide information on the effective constants [38]. Unfortunately since no direct determination from experimental data are feasible, the numbers extracted from such analysis are far from unique and in fact they can vary quite widely.[15] With this caveat in mind, let us quote the ranges of values found in the literature. They are

$$g_{\rho NN}^{\mathrm{emp}} \approx 4.2 - 6.5, \quad \mathcal{R} \approx 1.1 - 1.5. \tag{6.7}$$

[15] Over the three decades from the early efforts in 1970's [39] through extensive studies in 1980's and 1990's [40] to the most recent ones [41, 42], there seems to be little convergence on both $g_{\rho NN}$ and $g_{\omega NN}$ except that $g_{\omega NN} > 3 g_{\rho NN}$.

Although the individual values for g_{VNN} extracted empirically, e.g., (6.7), are subject to uncertainties mentioned above, it has been a mystery why NN phase shifts *invariably* required that $g_{\omega NN}$ be larger than the CQM prediction $3g_{\rho NN}$. Remarkably, this observation is naturally explained in the holographic QCD model as one can see in (6.6) although the quantitative comparison may not be meaningful as mentioned above. One should also note that (6.5) violates what is referred to as "universality," namely, $g_{\rho\pi\pi} = g_{\rho NN}$, as empirically $g_{\rho\pi\pi} \approx 6$ which is closer to $g_{\rho,min}$ in table 1 for the relevant range of λN_c. The source for this violation is in the magnetic contribution $g_{\rho,mag}$ which is also responsible for the ratio $\mathcal{R}$ to deviate from 1.

6.2 An issue with extrapolation: size of the baryon

So far, we studied static and dynamical behaviors of baryons by starting with small instantons with fundamental string hairs, in the very large 't Hooft coupling limit. However, for intermediate values of 't Hooft coupling, the story has to change qualitatively. Recall that the size of the instanton

$$\frac{9.6}{M_{\mathrm{KK}}\sqrt{\lambda}} \tag{6.8}$$

can be fairly large for the 't Hooft coupling of order 10. As we consider larger and larger instanton size, however, the computation leading to this estimate loses the validity. In particular, the instanton energy from Yang-Mills action is affected drastically. The effective mass from the instanton density scales with $1/e^2(w) \propto (U/U_{\mathrm{KK}})$. While we used the leading behavior $U/U_{\mathrm{KK}} \sim 1 + \frac{1}{3}M_{\mathrm{KK}}^2 w^2$ for small w, $1/e^2(w)$ is in fact divergent as $w \to \pm w_{\max} \simeq 3.64/M_{\mathrm{KK}}$. With $\lambda \sim 17$, the diameter of the instanton according to the above estimate is about $2/M_{KK}$, which immediately shows that we are well out of region of validity. The extra energy in eq. (3.11) is a gross underestimate.

Also the Coulomb energy eq. (3.12) can be seen to be modified. It treats the five-dimensional gauge field as a massless gauge field living in flat R^{4+1}. In reality, for configurations of size comparable to $1/M_{\mathrm{KK}}$, this is not the right picture. In particular, the increasing value of $1/e(w)^2$ outward along w effectively makes the physics four-dimensional, where the five-dimensional vector field should be replaced by an infinite tower of massive vector mesons. The lightest has the mass $\sim 0.8M_{\mathrm{KK}}$, so the Coulomb energy estimated in section. 3 must be augmented by an exponential suppression as well, changing to the power-exponent,

$$\sim \frac{M_{\mathrm{KK}}e(0)^2N_c^2}{\rho}e^{-0.8\rho M_{\mathrm{KK}}}\,. \tag{6.9}$$

Thus, eq. (3.12) is a bit of overestimate for large sizes.

Making these estimates more precise requires further effort. The main difficulty comes from the fact that we cannot use the usual self-dual instanton on R^4 to estimate the potential which is to be minimized. The problem is that the latter does not satisfy physical boundary condition at $w = \pm w_{\max}$ and that, even if we wish to use the usual instanton only as an approximate trial configuration, the divergent $1/e^2(w)$ at the boundary makes

JHEP09(2007)063

the energy of such configuration always infinite. What we need is a reasonable trial configuration whose gauge field strength vanishes very fast as $w \to \pm w_{\rm max}$. These difficulties were in fact also present in the estimate of section. 3 as well, which we ignored without justification, but it is unlikely that this detail would change the large 't Hooft coupling behaviors since the instanton involved is very small. Here we need to correct it since we are now talking about instantons whose size is comparable to the length scale of the fifth direction and since the order one factors are more important.

At the end of the day, however, the combined effect has to be that the instanton gets stabilized at much smaller size than predicted by the naive extrapolation of the size estimate we used. We anticipate that the size would be stabilized to be no larger than $1/M_{\rm KK}$. Once we are in this regime, on the other hand, the strategy we followed loses all of its validity, and it would be misleading to proceed in the same manner, only with the size of the instanton modified. As long as we are interested in interactions of the baryons with other fields in this theory, we propose that the right thing to do is to set up the effective field theory in the large λ limit, where all computations we carried out are well-justified, and extrapolate only at the end of the day when comparing scattering amplitudes. When we consider four-dimensional processes which are not very sensitive to the 't Hooft coupling in the large N_c expansion, this strategy is most likely to be successful. We believe this is the reason why our approach produced reasonable numbers even when compared to experimental values.

7. Electromagnetic interaction and vector dominance

A prominent feature of the holographic dual QCD is that its interaction with electromagnetic field is vector dominated. Let us first consider the situation with pions in the SS holographic model of QCD. There, the electromagnetic form factor of the pion is given by the entire tower of the vector mesons [2],

$$F_1^\pi(q^2) = \sum_{k=0}^{\infty} \frac{g_{v^{(k)}} g_{v^{(k)}\pi\pi}}{q^2 + m^2_{v^{(k)}}} \; . \tag{7.1}$$

The quantities $g_{v^{(k)}\pi\pi}$ are the trilinear couplings between pions and the vector mesons. The vector meson $v^{(k)}$ are defined as linear combinations of $a^{(2k+1)}$ and $\mathcal{V}$, as will be shown explicitly below. Accordingly its mass $m_{v^{(k)}}$ is m_{2k+1} in our notation. The parameters $\zeta_k \equiv g_{v^{(k)}}/m^2_{2k+1}$, which will be introduced shortly, encode how the photon field mixes with the massive vector mesons.

This form factor shows that there is no direct contact charge and arises because all electromagnetic interaction of pions necessarily goes through intermediate vector mesons. The charge form factor evaluated at $p^2 = 0$ is the charge of the particle, and thus we must have the normalization

$$F_1^\pi(0) = \sum_{k=0}^{\infty} \frac{g_{v^{(k)}} g_{v^{(k)}\pi\pi}}{m^2_{v^{(k)}}} = 1 \; . \tag{7.2}$$

In the SS model of QCD, this sum rule is a mathematical consequence of the completeness of the normalizable eigenmodes along the fifth direction. This sum rule has been also checked numerically, and for pions, it has been shown that the sum rule (7.2) is saturated within less than 1% by the first four low-lying vector mesons in the ρ quantum number. However the lowest member ρ exceeds the sum rule by $\sim 30\%$, so the next three are important in the sum rule.

In the following we will sketch how this vector dominance arises for pions and how this generalizes *naturally* to nucleons in our current effective action approach. Several analog of the above sum rule will also appear naturally from the completeness of eigenmodes, and we will see that again truncation down to the first four massive modes saturates these sum rules for nucleons within 1%.

7.1 The vector dominance for the nucleons

As we saw before, vector mesons, $a_\mu^{(2k+1)}$, and axial vector mesons, $a_\mu^{(2k)}$, arise as massive KK modes and exhaust all normalizable eigenmodes of the vector field which can be used upon dimensional reduction. Among the normalizable degrees of freedom, there is no room for photon field. Instead, the coupling to the photon field must be read out via the usual AdS/CFT prescription by computing an appropriate current to be matched with an external $\mathrm{U}(1)_{\mathrm{em}}$ field, $\mathcal{V}$. The latter, in our language, shows up as non-normalizable term added to the $i\beta_\mu(x)$ term[16]

$$A_\mu(x;w) = i\alpha_\mu(x)\psi_0(w) + \mathcal{V}_\mu(x) + i\beta_\mu(x) + \sum_n a_\mu^{(n)}(x)\psi_{(n)}(w)\,. \tag{7.3}$$

Upon integrating over the fifth direction, this generates a term of the type

$$\int dx^4\, \mathcal{V}_\mu J^\mu \tag{7.4}$$

giving us the vertex J. After specializing $\mathcal{V}$ to a $\mathrm{U}(1)$ subgroup, appropriately chosen to be consistent with four dimensional physics, electromagnetic vertices can be read out.

For a generalization to nucleons, it is instructive to recall how the vector dominance came about in the meson sector in the SS model. If we keep both the vector mesons and this external vector we have the following general structure of the lagrangian,

$$-\sum_n \frac{1}{4}|da^{(n)}|^2 - \frac{1}{2}m_n^2|a^{(n)}|^2 - \sum_k \frac{\zeta_k}{2}\langle da^{(2k+1)}, d(\mathcal{V}+i\beta)\rangle \tag{7.5}$$

with

$$\zeta_k = \int dw \frac{1}{e(w)^2}\,\psi_{(2k+1)}(w)\,, \tag{7.6}$$

which is related to $g_{v^{(k)}}$ of Sakai and Sugimoto as

$$g_{v^{(k)}} = m_{2k+1}^2\zeta_k\,. \tag{7.7}$$

The reason why only the vectors and not the axial vectors shifts by $\mathcal{V}$ is clear from the form λ's, for the eigenmodes for the axial vectors are odd functions.

[16] One can also introduce an axial vector $\mathcal{A}_\mu(x)$ added to $i\alpha_\mu(x)$. This would be relevant to the coupling of the hadrons to $\mathrm{SU}(2)_{\mathrm{weak}}$ but here we will not consider it.

JHEP09(2007)063

For canonical forms of the kinetic terms, then we must introduce shifted vector fields

$$v^{(k)} = a^{(2k+1)} + \zeta_k(\mathcal{V} + i\beta)\,, \tag{7.8}$$

where we now have

$$\sum_k \left[-\frac{1}{4}|da^{(2k)}|^2 - \frac{1}{2}m_{2k}^2|a^{(2k)}|^2 - \frac{1}{4}|dv^{(k)}|^2 - \frac{1}{2}m_{2k+1}^2|v^{(k)} - \zeta_k(\mathcal{V}+i\beta)|^2 \right] \tag{7.9}$$

up to a term that additively renormalizes the kinetic term of $\mathcal{V}$. This induces a quadratic vertex between vector mesons and the external gauge field $\mathcal{V}$, which induces in turn indirect couplings between $\mathcal{V}$ and pions. After some computation, one can show for the SS model that the cubic couplings between pions and these vectors are organized into the form

$$g_{v^{(k)}\pi\pi}\,\mathrm{tr}\left(v_\mu^{(k)}[\pi, \partial^\mu \pi]\right). \tag{7.10}$$

These cubic couplings between pions and $v^{(k)}$ plus the quadratic mixing between $v^{(k)}$ and $\mathcal{V}$ generates an effective cubic interaction of pions with $\mathcal{V}$, and summing over all intermediate vector mesons generates the form factor F_1^π. In particular, the zero momentum limit of this form factor is the electromagnetic charge of the pion, and the sum rules ensure consistency such as charge quantization.

Now let us see how this mixing of vector fields enters the coupling of baryons with electromagnetic vector field $\mathcal{V}$. Seemingly this case is very different from that of pions. For one thing, we have a minimal interaction term between nucleons and the 5D gauge field, A, and this is inherited by $\mathcal{V}$ without modification, since $\mathcal{V}$ is simply the non-normalizable part of A. Thus it may seem that we have a point-like interaction between baryons and $\mathcal{V}$, precluding the notion of vector dominance in the form factor. However, nucleons also couple minimally to the 4D massive vectors, $a^{(2k+1)}$, which mix with $\mathcal{V}$ in the propagator. They show up in the baryon effective Lagrangian as

$$\int dw\; \bar{\mathcal{B}}\gamma^m A_m \mathcal{B} = \bar{B}\gamma^\mu \mathcal{V}_\mu B + \sum_k g_{V,min}^{(k)} \bar{B}\gamma^\mu a_\mu^{(2k+1)} B + \cdots\,, \tag{7.11}$$

where the ellipsis denotes axial couplings to axial vectors as well as coupling to pions via α_μ and β_μ. There should be additional contribution from the 5D magnetic coupling, shifting $g_{V,min}$, to which we will come back shortly.

Alternatively, we may use the canonically normalized vectors $v^{(k)}$ instead, where we have the vector-current couplings of type

$$\bar{B}\gamma^\mu \mathcal{V}_\mu B + \sum_k g_{V,min}^{(k)} \bar{B}\gamma^\mu (v_\mu^{(k)} - \zeta_k \mathcal{V}_\mu) B + \cdots\,. \tag{7.12}$$

On the other hand,

$$\begin{aligned}
\sum_k g_{V,min}^{(k)}\zeta_k &= \sum_k \int dw'\; |f_L(w')|^2 \psi_{(2k+1)}(w') \times \int dw\; \frac{1}{e(w)^2}\,\psi_{(2k+1)}(w) \\
&= \sum_n \int dw'\; |f_L(w')|^2 \psi_{(n)}(w') \times \int dw\; \frac{1}{e(w)^2}\,\psi_{(n)}(w) \\
&= \int dw'\; |f_L(w')|^2 \times \int dw\; \frac{1}{e(w)^2}\,\sum_n \psi_{(n)}(w)\psi_{(n)}(w')\,,
\end{aligned} \tag{7.13}$$

where the second step makes use of the fact that $1/e(w)^2$ is an even function. Using the completeness of the normalizable eigenmodes ψ_n, we find that

$$\sum_k g^{(k)}_{V,min}\zeta_k = \int dw'\, |f_L(w')|^2 \times \int dw\, \delta(w-w') = \int dw'\, |f_L(w')|^2 = 1\,, \qquad (7.14)$$

which implies the crucial sum rule,

$$\sum_k g^{(k)}_{V,min}\zeta_k = 1\,. \qquad (7.15)$$

Therefore, in this shifted basis, we have the charge form factor

$$\bar{B}\gamma^\mu \mathcal{V}_\mu B + \sum_k g_V^{(k)} \bar{B}\gamma^\mu (v_\mu^{(k)} - \zeta_k \mathcal{V}_\mu) B + \cdots = \sum_k g_V^{(k)} \bar{B}\gamma^\mu v_\mu^{(k)} B + \cdots \qquad (7.16)$$

As in the case of the pion, we can see that the cubic electromagnetic interaction is mediated entirely by intermediate massive vector mesons, rendering the nucleon form factors entirely vector-dominated. This aspect will be highlighted in section 9.

Of course, this choice of basis is only for the sake of clarity. The $\{\mathcal{V}; v^{(k)}\}$ basis is such that the mixing between $\mathcal{V}$ and massive vector meson is maximal in the zero momentum limit, and thereby exhibits clearly how the minimal coupling to photon field is replaced by the mediation via massive vector mesons. However, the physics should be independent of such choices. In the following, we will compute the charge form factor and the Pauli form factor explicitly in the original $\{\mathcal{V}; a^{(2k+1)}\}$ basis and see how the physical quantities bear out the notion of the vector dominance.

7.2 Charge form factor F_1

To be more precise, let us compute the effective 3-point vertex of type $\bar{B}\gamma^\mu\mathcal{V}_\mu B$. Let us first put a cut-off along the fifth direction integrals, which effectively make $\mathcal{V}$ dynamical with a large kinetic term

$$\frac{L}{4}|d\mathcal{V}|^2 \qquad (7.17)$$

for some large number L, whose precise value will not matter. The propagator for $\{\mathcal{V}; a^{(2k+1)}\}$ is such that

$$\begin{aligned} \langle\mathcal{V}(q)\mathcal{V}(-q)\rangle &\sim \frac{i}{Lq^2 - \sum_k \zeta_k^2 q^4/(q^2+m_{2k+1}^2)}\,, \\ \langle a^{(2k+1)}(q)\mathcal{V}(-q)\rangle &\sim -\langle\mathcal{V}(q)\mathcal{V}(-q)\rangle \times \frac{\zeta_k q^2}{q^2+m_{2k+1}^2}\,. \end{aligned} \qquad (7.18)$$

The electromagnetic form factor F_1 can be found from this by computing tree-level correlator, $\langle\bar{B}\gamma^\mu\mathcal{V}_\mu B\rangle$, and amputating the external lines. The resulting charge form factor $F_{1,min}$ which arises from the minimal interaction term is

$$F_{1,min}(q^2) = 1 - \sum_k \frac{g^{(k)}_{V,min}\zeta_k q^2}{q^2+m_{2k+1}^2} = \sum_k \frac{g^{(k)}_{V,min}\zeta_k m_{2k+1}^2}{q^2+m_{2k+1}^2} = \sum_k \frac{g_{v^{(k)}} g^{(k)}_{V,min}}{q^2+m_{2k+1}^2} \qquad (7.19)$$

up to the electromagnetic charge operator. We used the sum rule $\sum_k g_{V,min}^{(k)}\zeta_k = 1$ and the definition $g_{v^{(k)}} = \zeta_k m_{2k+1}^2$. The first expression is natural in the $\{\mathcal{V}; a^{(2k+1)}\}$ basis while the second expression is natural in the $\{\mathcal{V}; v^{(k)}\}$ basis. The result is, of course, independent of the basis choice.

Note that there is no contact charge in the baryon, which would have resulted in $F_1(\infty) \neq 0$. However, since the holographic model used is defined by the mass scale $M_{\rm KK} \sim 1$ GeV, our form factor does not have the correct asymptotic behavior of perturbative QCD, $F_1(q^2) \sim 1/q^4$ [43]. This must be implemented by hand if one wanted to fit the experimental data at large momentum transfers.

The actual charge form factor picks up an additional contribution from the magnetic coupling, since the latter contributes couplings $g_{V,mag}^{(k)}$ between nucleon current and massive vector mesons as well. This does not induce an additional electric charge (as it should not) given the charge quantization, and this happens as a consequence of another sum rule:

$$
\begin{aligned}
&\sum_k g_{V,mag}^{(k)}\zeta_k \\
&\quad = \sum_k \int dw' \left(\frac{g_5(w')U(w')}{g_5(0)U_{\rm KK}M_{\rm KK}}\right)|f_L(w')|^2 \partial_{w'}\psi_{(2k+1)}(w') \times \int dw\, \frac{1}{e(w)^2}\,\psi_{(2k+1)}(w) \\
&\quad = \int dw' \left(\frac{g_5(w')U(w')}{g_5(0)U_{\rm KK}M_{\rm KK}}\right)|f_L(w')|^2 \times \int dw\, \partial_{w'}\delta(w-w') \\
&\quad = -\int dw'\, \partial_{w'}\left[\left(\frac{g_5(w')U(w')}{g_5(0)U_{\rm KK}M_{\rm KK}}\right)|f_L(w')|^2\right] = 0\,. \qquad (7.20)
\end{aligned}
$$

The contribution to the charge form factor from the magnetic coupling is then,

$$
F_{1,mag}(q^2) = -\sum_k \frac{g_{V,mag}^{(k)}\zeta_k q^2}{q^2+m_{2k+1}^2} = \sum_k \frac{g_{V,mag}^{(k)}\zeta_k m_{2k+1}^2}{q^2+m_{2k+1}^2} = \sum_k \frac{g_{v^{(k)}}g_{V,mag}^{(k)}}{q^2+m_{2k+1}^2}\,. \qquad (7.21)
$$

Since the minimal term couples nucleons to U(2) gauge field and the magnetic term couples nucleons to SU(2) gauge field, the two form factors, $F_{1,min}$ and $F_{1,mag}$ contribute differently to the proton and neutron charge form factors.

For this, note that both iso-scalars and iso-vectors part of $v^{(k)}$ enter this cubic coupling, unlike the case of pions where only the iso-vector vectors enter the story. The relative strength between the two are determined universally by the 5D $\mathrm{U}(N_F)$ charge of the baryon. For $N_c = 3$, v in the baryon vertex is in the representation

$$
v_\mu^{(k)} \simeq a^{(2k+1)} = \begin{pmatrix} 3/2 & 0 \\ 0 & 3/2 \end{pmatrix} \omega_\mu^{(k)} + \rho_\mu^{(k)}\,. \qquad (7.22)
$$

The mixing between v and $\mathcal{V}$ is computed in the representation which is appropriate for mesons, where

$$
v_\mu^{(k)} \simeq \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \omega_\mu^{(k)} + \rho_\mu^{(k)} \qquad (7.23)
$$

and

$$
\mathcal{V} = \left(\begin{pmatrix} 1/6 & 0 \\ 0 & 1/6 \end{pmatrix} + \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix}\right)\mathcal{V}_{\rm em}\,. \qquad (7.24)
$$

The representation of v for nucleons dictates the cubic coupling of v to the nucleon while the latter dictates the quadratic mixing of v and $\mathcal{V}$.

The electromagnetic interaction mediated by iso-scalars is thus proportional to $3/2 \times (1/2 \times 1/6)$ while its triplet counterpart is proportional to $\pm 1/2 \times (1/2 \times 1/2)$ with the sign choice corresponding to choosing proton or neutron. Since the two final products are equal in size, iso-scalar vectors and iso-vector vectors contribute to the nucleon form factor $F_{1,min}$ with the equal strength, adding up for the proton and cancelling each other for the neutron. On the other hand, only the iso-vector contribute to $F_{1,mag}$ with an opposite sign for proton and neutron, respectively. After taking into account the charge assignment for protons and neutrons carefully, the electromagnetic charge form factors are found as

$$\begin{aligned} F_1^{\text{proton}} &= F_{1,min} + \frac{1}{2}F_{1,mag} \, , \\ F_1^{\text{neutron}} &= -\frac{1}{2}F_{1,mag} \, . \end{aligned} \tag{7.25}$$

7.3 Pauli form factor F_2

The phenomenon of complete vector dominance, that is, the absence of direct coupling of photon to nucleons, is also seen in the Pauli form factor $F_2(q^2)$ defined as

$$\langle B|J^\mu(q)|B\rangle \sim \frac{F_2^a(q^2)}{2m_B}\bar{B}\gamma^{\mu\nu}p_\nu t_a B \, . \tag{7.26}$$

After inserting the mode expansion of the 5D gauge field in terms of vector mesons into our bulk 5D magnetic coupling with purely 4D polarizations, we obtain the interactions that are relevant for magnetic dipole coupling,

$$\frac{g_2}{4m_B}\bar{B}\gamma^{\mu\nu}\mathcal{F}_{\mu\nu}B + \sum_k \frac{g_2^{(k)}}{4m_B}\bar{B}\gamma^{\mu\nu}F_{\mu\nu}^{(2k+1)}B \, , \tag{7.27}$$

where $F_{\mu\nu}^{(2k+1)} = \partial_\mu a_\nu^{(2k+1)} - \partial_\nu a_\mu^{(2k+1)}$ is the "field strength" of the vector meson $a_\mu^{(2k+1)}$ and $\mathcal{F}_{\mu\nu}$ is the field strength of external source $\mathcal{V}_\mu$ for the current. The coupling constants are easily read off from overlap integrals,

$$\begin{aligned} g_2 &= 0.18N_c \times \frac{4m_B}{M_{\text{KK}}} \times \int_{-w_{\max}}^{w_{\max}} dw f_L^*(w) f_R(w) \, , \\ g_2^{(k)} &= 0.18N_c \times \frac{4m_B}{M_{\text{KK}}} \times \int_{-w_{\max}}^{w_{\max}} dw f_L^*(w) f_R(w)\psi_{(2k+1)}(w) \, . \end{aligned} \tag{7.28}$$

Note that contributions involving the axial vectors $a_\mu^{(2k)}$ are absent due to their odd profile in the 5-th coordinate $\psi_{(2k)}(-w) = -\psi_{(2k)}(w)$ and the property $f_L(-w) = f_R(w)$. In fact, the would-be terms like $\bar{B}\gamma^{\mu\nu}\gamma^5 F_{\mu\nu}^{(2k)}B = \epsilon^{\mu\nu\alpha\beta}\bar{B}\gamma_{\mu\nu}F_{\alpha\beta}^{(2k)}B$ is CP-violating.

Using the completeness relation for $\psi_{(2k+1)}$ as before, it is straightforward to check the sum rule

$$\sum_k g_2^{(k)}\zeta_k = g_2 \, , \tag{7.29}$$

which is saturated up to 99% by the lowest four vector mesons as can be seen in the table 2. Because of this sum rule, as we go to the shifted basis $v^{(k)} = a^{(2k+1)} + \zeta_k \mathcal{V}$, the direct photon coupling $\frac{g_2}{4m_B}\bar{B}\gamma^{\mu\nu}\mathcal{F}_{\mu\nu}B$ is exactly cancelled by the shift, and we are left with

$$\sum_k \frac{g_2^{(k)}}{4m_B}\bar{B}\gamma^{\mu\nu}(\partial_\mu v_\nu^{(k)} - \partial_\nu v_\mu^{(k)})B \,, \tag{7.30}$$

and the Pauli form factor is given as a sum over intermediate vector meson contributions,

$$F_2^3(q^2) = \sum_k \frac{g_2^{(k)}\zeta_k m_{2k+1}^2}{q^2 + m_{2k+1}^2} \,, \tag{7.31}$$

with the property due to the sum rule $F_2^3(0) = g_2$. It seems by now clear that the complete vector dominance is a generic phenomenon in the holographic QCD, as was first noticed in [44].

For each nucleon, we have

$$\begin{aligned} F_2^{\text{proton}} &= \frac{1}{2}F_2^3 \,, \\ F_2^{\text{neutron}} &= -\frac{1}{2}F_2^3 \,, \end{aligned} \tag{7.32}$$

since only the magnetic term contributes to F_2.

7.4 Numerics and a consistent truncation

It is tantalizing that there is a complete parallel between the vector dominance in the pion and that in the nucleon. Because the zero momentum limit of F_1 is the electromagnetic charge, which should be quantized, $F_1(0)$ of pions and nucleons must be the same, which would imply $g_{v^{(0)}\pi\pi} = g_V^{(0)}$ if the sums were saturated by the lowest vector meson $v^{(0)}$. This feature has been discussed much in old literatures and goes by the name of "universality."

To see whether the form factor is actually dominated by the first vector meson or not, we computed numbers for the first few lowest vector mesons and check the sum rules numerically. The result is shown in table 2. We have two independent sum rules for $g_{V,min}^{(k)}$ and for sum $g_V^{(k)}$. Since the sum rules for $g_{V,min}^{(k)}$ and $g_V^{(k)}$ are both tied to the net electromagnetic charge, we need to satisfy them both well, before discussing any comparison with data. As table 2 shows clearly, the sum rules that lead to the vector dominance cannot be satisfied with the lowest vector meson alone, indicating the truncation down to the first vector would be a bad approximation for the form factor. Instead, if we sum up to the fourth vector meson, both sum rules are obeyed with 0.2% accuracy, giving us a hope that F_1 may be well-approximated in the low momentum region by summing over the first four terms. A similar result can be seen for F_2, since, as also shown in the table 2, the sum rule for g_2 is saturated well within 1% accuracy.

Given this numerical data, the old "universality" seems to have found a new reincarnation. The table 2 shows that the sum rules for the nucleon (as well as for pions) are saturated within less than 5% by the four lowest vector mesons. We observe that

$$\zeta_k = (-1)^k/h \tag{7.33}$$

JHEP09(2007)063

k	m^2_{2k+1}	ζ_k	$g^{(k)}_{V,min}$	$g^{(k)}_{V,mag}$	$g^{(k)}_{V,min}\zeta_k$	$g^{(k)}_{V,mag}\zeta_k$	$g^{(k)}_2\zeta_k$
0	0.67	0.385	4.195	-0.577	1.615	-0.222	3.323
1	2.87	-0.387	2.280	-1.406	-0.882	0.544	-1.918
2	6.59	0.385	0.892	-1.366	0.343	-0.526	0.828
3	11.8	-0.383	0.220	-0.685	-0.084	0.262	-0.243
sum	-	-	-	-	0.992	0.058	$1.989(g_2 = 2.028)$

Table 2: Numerical results for vector meson couplings for the lowest four excitations in the case $\lambda N_c = 50$. Sum rules hold to a high precision. Our convention for the vector meson fields differ by sign from that of Sakai and Sugimoto for odd k. The vector meson mass squared is in the unit of M^2_{KK}.

where h is a constant independent — within less than 1% — of the species k. Assuming that the sum rule (7.2) is completely saturated by the four vector mesons, we arrive at the conclusion that[17]

$$\sum_{k=0}^{3}(-1)^k g_{v^{(k)}\pi\pi} = h. \tag{7.35}$$

Since the same relation holds with a nucleon replacing the π in (7.35), h could be identified with the HLS_∞ gauge coupling constant and that

$$\sum_{k=0}^{3}(-1)^k g_{v^{(k)}\pi\pi} \simeq \sum_{k=0}^{3}(-1)^k g_{v^{(k)}NN}. \tag{7.36}$$

We could consider this as a "generalized universality" relation, although we have no rigorous argument for such a relation.

If the sum rules (7.2) and (7.15) are saturated by the first four vector mesons then an interesting question is how the relation $\sum_{k=4}^{\infty}\zeta_k g_{v^{(k)}\pi\pi} = \sum_{k=4}^{\infty}\zeta_k g_{v^{(k)}NN} = 0$ is satisfied and what it means vis-a-vis with the short-range structure of the nucleon. We leave these issues for later publication.

7.5 The "old" vector dominance in light of the "new" vector dominance

Now that we have the form factors of both pions and nucleons completely vector-dominated with the infinite tower, it is interesting to review the old vector dominance involving the lowest vector mesons only, ρ, ω and ϕ — and in some works including the next-lying vector mesons [45] — in light of the new picture. This could bring light to the success and failure of the old vector dominance. We shall do this using the Harada-Yamawaki (HY) approach [23].

As has been suggested [46], HY's hidden local symmetry model can be considered as resulting from integrating out all excitations other than the pions and the lowest vector

[17] This is reminiscent of the nonet relation in three flavor HLS_1

$$g_\rho/m^2_\rho = 3g_\omega/m^2_\omega = -(3/\sqrt{2})g_\phi/m^2_\phi = 1/g \tag{7.34}$$

where g is the hidden gauge coupling constant.

mesons and matching the truncated action to the SS action at a matching scale Λ_M. It is however more natural to consider it as an emergent symmetry as mentioned in section 2.2. It is in this way via what is called "moose construction" [24] that the tower of vector mesons emerge in a dimensionally deconstructed QCD [4] with a five-dimensional YM action analogous to that reduced from string theory that we have been discussing.

What we would like to do here is to describe how the vector dominance and the putative violation thereof arise in this HLS approach (that will be referred to as HLS/VM below). In order to do so, we recall how massive vector meson degrees of freedom arise when one approaches hadron chiral dynamics from bottom up. At very low energy, $E \ll \Lambda_\chi$, i.e., the chiral scale, the chiral dynamics is given by the current algebra term with the lowest-derivative Lagrangian

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) \tag{7.37}$$

with the chiral field

$$U(x) = e^{2i\pi/f_\pi}. \tag{7.38}$$

By writing the U field as a product field

$$U = \xi_L^\dagger \xi_R \tag{7.39}$$

which can be done by introducing a redundant field σ as

$$\xi_{L/R} = e^{\mp i\pi/f_\pi} e^{i\sigma/f_\sigma} \tag{7.40}$$

with f_σ defined as the σ decay constant, one unearths a trivial local invariance

$$\xi_{L/R} \to h(x)\xi_{L/R} \tag{7.41}$$

with $h(x) \in \mathrm{U}(N_F)$. This local symmetry can be exploited by introducing a vector field v_μ to bring the energy scale from low, here that of the pion mass — which is zero in the chiral limit with the Lagrangian (7.37) — to high, say, the scale set by the mass of a meson v. This is essentially how the vector mesons (ρ, ω) were incorporated into the HLS theory of [14]. For convenience, we shall call it HLS_1.

As is well-known [47], the gauge theory so constructed does not lead to a unique higher-energy theory. In order to direct the hidden gauge theory of [14] toward a correct one, Harada and Yamawaki match à la Wilson the effective theory to QCD at a matching scale $\Lambda_M \sim 4\pi f_\pi$. Specifically the vector correlator Π_V and the axial-vector correlator Π_A calculated with the HLS Lagrangian are matched to those calculated in QCD, e.g., operator-product expansion (OPE). This allows the *bare* parameters of the HLS Lagrangian, g, f_π and f_σ, to be expressed in terms of the QCD variables, α_s, $\langle\bar{q}q\rangle$, $\langle G^2\rangle$ etc. Given the bare Lagrangian, the next step is to do renormalization group analysis to see how the theory flows as the scale is changed from the matching scale. Harada and Yamawaki find a variety of fixed points as well as a fixed line, to which the HLS_1 can flow [23]. In order to pick out the fixed point that maps to QCD, one has to impose the condition that when the chiral order parameter $\langle\bar{q}q\rangle$ is set equal to zero, the correlators are equal, i.e., $\Pi_V = \Pi_A$. This

JHEP09(2007)063

condition picks out the fixed point that corresponds to the fixed point to which the system flows when the condensate $\langle\bar{q}q\rangle$ goes to zero. This fixed point called "vector manifestation (VM) fixed point" (and the HLS theory with the VM fixed point called HLS/VM) [23] is characterized by

$$g^* = 0, \quad a^* = 1 \tag{7.42}$$

where a is the ratio of the decay constants $a \equiv (f_\sigma/f_\pi)^2$. This fixed point is reached when a hadronic system in Nambu-Goldstone phase makes the transition to the symmetry restored phase at high temperature T_c (as in the Early Universe) or at high density n_c (as in compact stars).

What this implies in the EM form factors of the pion and the nucleon in HLS_1 is as follows. In HLS theory with the lowest vector mesons (ρ, ω), the iso-vector photon coupling is given by

$$\delta\mathcal{L} = e\mathcal{A}^\mu_{\mathrm{EM}} \left(-2af_\pi^2 \mathrm{Tr}[g\rho_\mu Q] + 2i(1-a/2)\mathrm{Tr}[J_\mu Q]\right)\ ,) \tag{7.43}$$

where Q is the quark charge matrix, ρ_μ is the lowest-lying iso-vector vector meson and J_μ is the iso-vector vector current made up of the chiral field ξ (7.41). The first term of (7.43) represents the photon coupling through a ρ and the second term the direct coupling. The "old" vector dominance is obtained when $a = 2$ for which the well-known KSRF relation for the ρ meson holds, i.e., $m_\rho^2 = af_\pi^2 g^2 = 2f_\pi^2 g^2$. Now it has been established empirically that the way the vector dominance manifests itself is different between the pion and the nucleon. Let us look at them separately.

- Pion form factor: On-shell in matter-free space, the pion form factor is very well described by the vector dominance, hence $a = 2$, with no direct coupling. However in HLS/VM, in the framework of HLS_1, $a = 2$ is totally accidental, not even lying on a stable trajectory of the RGE [23, 48]. A small perturbation would take a away from the vector dominance point $a = 2$. Thus for instance, temperature [49] or density would push a toward 1, inducing what is referred to as "vector dominance violation." It is an interesting possibility that there is a connection, albeit indirect, between the departure from $a = 2$ toward $a = 1$ in HLS_1 and the role in medium of higher-lying vector mesons in HLS_∞. This is an important issue in CERN experiments on dilepton production in relativistic heavy ion collisions [50].

- Nucleon form factor: If one considers nucleon as a Skyrmion in HLS_1, then the second term in (7.43) corresponds to a direct photon coupling to the Skyrmion. As we will elaborate in section 9, experimental data clearly show that there is an important direct coupling with $a \sim 1$. This observation has been taken as an indication that vector dominance does not apply to nucleons, the reason put forward for this violation being that nucleons are extended objects. We will see in section 9 that this picture is drastically modified when the infinite tower of vector mesons enter in the structure of nucleons.

8. The anomalous magnetic dipole moment

While we can simply read out the magnetic moment from the form factors of previous section, here we would like to show a more direct computation, which does not depend on the structure of the vector dominance of the previous section. Since the magnetic moment comes from form factors at zero momentum limit, it is best to work in $\{\mathcal{V}; a^{(2k+1)}\}$ basis and ignore the vector mesons entirely. This is because $a^{(2k+1)}$'s mixes with $\mathcal{V}$ at the level of kinetic term and thus with two momentum factors while $v^{(k)}$ mixes with $\mathcal{V}$ at the mass term level. Thus, we may ask how the nonnormalizable mode $\mathcal{V}$ in

$$A_\mu(x;w) = \mathcal{V}_\mu(x) + i\alpha_\mu(x)\psi_0(w) + \cdots \tag{8.1}$$

couples to the nucleons. Let us insert the non-normalizable zero mode into the effective action for the five dimensional baryon, whereby we find the terms relevant

$$\int d^4x \int dw \left[-\bar{\mathcal{B}}\gamma^\mu \mathcal{V}'_\mu \mathcal{B} + g_5(w)\frac{\rho^2_{\rm baryon}}{e^2(w)} \bar{\mathcal{B}}\gamma^{\mu\nu}\mathcal{F}_{\mu\nu}\mathcal{B} \right] \tag{8.2}$$

with $g_5(0) = 2\pi^2/3$. Here we denoted the gauge field from the minimal coupling by $\mathcal{V}'$ because its generator is different from the one in the magnetic term.

Again recall that the 5D magnetic coupling that we obtained from comparing with long-range instanton tail must contain only the SU(2) isospin without an overall U(1), since the instanton tail involves only non-Abelian SU(2). On the contrary, the minimal coupling term contains $\mathrm{U}(1)_Y$ as well as SU(2) according to the charge of nucleons made out of N_c-quarks. As for the case $N_c = 3$ and $N_F = 2$, the quark doublet (u, d) has EM charge $(2/3, -1/3)$, which can be decomposed to $(1/6, 1/6)$ corresponding to $\mathrm{U}(1)_Y$ and $(1/2, -1/2)$ for the diagonal part of SU(2). As nucleons are made of 3 quarks in totally anti-symmetric fashion, the resulting $\mathrm{U}(1)_Y$ charge becomes $(1/2, 1/2)$ whereas the SU(2) charge remains fundamental representation $(1/2, -1/2)$. This tells us that we have to use EM charge $(1, 0)$ for (p, n) in the minimal coupling as expected, while we should instead have $(1/2, -1/2)$ in the term from 5D magnetic coupling.

The above descends down to similar 4D expression as

$$\int d^4x \left[-\bar{B}\gamma^\mu \mathcal{V}'_\mu B + g_5(0)\frac{\rho^2_{\rm baryon}}{e^2(0)} \bar{B}\gamma^{\mu\nu}\mathcal{F}_{\mu\nu}B \right] , \tag{8.3}$$

assuming that the eigenmode $f_{L,R}$ of the nucleon is sufficiently concentrated at origin $w = 0$, so that the w-dependence of the coupling does not enter the physics. The previous estimate gives us

$$g_5(0)\frac{\rho^2_{\rm baryon}}{e^2(0)} \simeq 0.18 N_c \times \frac{1}{M_{\rm KK}} . \tag{8.4}$$

This approximation becomes precise in the large N_c-limit. For later numerical calculations extrapolating to $N_c = 3$, the precise overlap integral replaces the above coefficient with

$$0.18 N_c \times \frac{1}{M_{\rm KK}} \times \int_{-w_{\rm max}}^{w_{\rm max}} dw\, \frac{g_5(w)U(w)}{g_5(0)U_{\rm KK}} f_L^*(w) f_R(w) , \tag{8.5}$$

where we used $f_L(w) = f_R(-w)$ for the lowest nucleon eigenmode.

Taking a non-relativistic limit, we will look for terms of type

$$\int d^4x \, \frac{\mu}{e_{\rm EM}} \, \mathbf{S} \cdot \mathbf{B} \tag{8.6}$$

with the magnetic field strength $\mathcal{B}$ and the spin $\mathbf{S}$. Here we have a factor of $e_{\rm EM}$ to correct the fact that our choice of gauge field is not canonically normalized. As in section (4), we introduce the two-component notation of B as

$$B = \begin{pmatrix} u \\ v \end{pmatrix} e^{-iEt+ip\cdot x} \,, \tag{8.7}$$

where the on-shell condition relates

$$v = \frac{E - \sigma \cdot p}{-im_B} \, u \,. \tag{8.8}$$

Isolating the magnetic dipole coupling, we find

$$\frac{1}{m_B} \int d^4x \, \left[u^\dagger \mathbf{B}' \cdot \sigma u \right] + \left(\frac{4g_5(0)\rho^2_{\rm baryon}}{e(0)^2} \right) \int d^4x \, \left[u^\dagger \mathbf{B} \cdot \sigma u \right] \,. \tag{8.9}$$

where, again, the prime on the magnetic field reminds us that the charge generator for the minimal coupling term is different from the one for the magnetic term. Given the normalization, ${\rm tr} u^\dagger u = 1/2$ (see section. 4), one can identify ${\rm tr} u^\dagger \sigma u$ as the spin operator $\mathbf{S}$ of the nucleon. This leads to

$$\frac{\mu_{\rm proton}}{e_{\rm EM}} = \frac{1}{2m_B} + \left[\frac{g_5(0)\rho^2_{\rm baryon}}{e(0)^2} \right] , \qquad \frac{\mu_{\rm neutron}}{e_{\rm EM}} = - \left[\frac{g_5(0)\rho^2_{\rm baryon}}{e(0)^2} \right] \,. \tag{8.10}$$

However, we do not have a reliable estimate of the nucleon mass m_B. One way to bypass this difficulty is to look at the anomalous part of the magnetic dipole moment. In fact, the anomalous part is the dominant part in the large N_c limit, and thus is likely to be more reliable. We have

$$\frac{\mu^{\rm an}_{\rm proton}}{e_{\rm EM}} = \frac{0.18 N_c}{M_{\rm KK}} \,, \qquad \frac{\mu^{\rm an}_{\rm neutron}}{e_{\rm EM}} = - \frac{0.18 N_c}{M_{\rm KK}} \,. \tag{8.11}$$

For comparison with experiments, let us first consider the difference of the anomalous magnetic moment $\Delta\mu^{\rm an} = \mu^{\rm an}_{\rm proton} - \mu^{\rm an}_{\rm neutron}$,

$$\frac{\Delta\mu^{\rm an}}{e_{\rm EM}} \simeq \frac{0.36 N_c}{M_{\rm KK}} \,. \tag{8.12}$$

Experimentally, $(\Delta\mu^{\rm an})_{\rm exp} = (2.79 - (-1.91) - 1) \times \mu_N = 3.7\mu_N$ where $\mu_N = e_{\rm EM}/2m_N$ is the nuclear magneton. Once we take $M_{\rm KK} = 0.94{\rm GeV}$ as determined by the meson sector fit, it happens to be approximately the physical nucleon mass, denoted as m_N. Thus our prediction is $\Delta\mu^{\rm an} \simeq 0.72 N_c \times \mu_N = 2.16\mu_N$ for $N_c = 3$.

JHEP09(2007)063

However, if we replace N_c by $(N_c + 2)$ again guided by CQM, then it becomes

$$\Delta\mu^{\rm an} \simeq 3.6\mu_N \,, \tag{8.13}$$

which agrees with experiment value, $3.7\mu_N$, very well. With the same shift, the individual anomalous magnetic moment are

$$\mu^{\rm an}_{\rm proton} \simeq 1.8\mu_N \,, \qquad \mu^{\rm an}_{\rm neutron} \simeq -1.8\mu_N \,:, \tag{8.14}$$

which again compare quite favorably to the experimental values, $1.79\mu_N$ and $-1.91\mu_N$, respectively. Such a shift $N_c \to N_c + 2$ was discussed in section 5.4.2 for the leading chiral coupling between the pion and the nucleon. As mentioned there, the spin-isospin structure is the same for the axial coupling and the iso-vector magnetic moment, so the collective quantization leads to the same shifting for both.

A thorny issue here, and also for much of next section where we consider electromagnetic form factors, is the matter of the nucleon mass m_B. For instance, the non-anomalous part of the proton magnetic moment would be computed to be $e_{\rm EM}/2m_B$ and the question of whether the model predicts $m_B \simeq m_N$ becomes an important issue.

In this article we did not attempt to compute m_B within our model. In fact, it is unclear if there should exist an unambiguous prediction for the ground state mass in this approach, since the quantity is additively renormalized, since an infinitely many oscillators around the classical soliton contribute zero-point energy. Hata et.al. [29] computed the mass spectra of various excited baryons but, for this reason, chose to the treat the ground state mass (m_B in our notation) as a free parameter instead. For a bona fide comparison of quantities that depends on the nucleon mass sensitively, this issue should be resolved first.

9. Electromagnetic form factors

9.1 Two-component description

The full electromagnetic form factors are encoded in three functions, $F_{1,2,3}$. Before we compute the form factors to compare with the experimental data, we review briefly the past theoretical status on the subject.

For qualitative illustration of what the problem is, we take the iso-vector Dirac form factor F_1 of the nucleon. This form factor will receive contributions from the vector mesons in the tower in the ρ channel, ρ, ρ', etc. Other channels can be discussed in a similar way.

In the literature, analysis have been made by including one [51] or two [45] lowest vector mesons in the ρ channel, i.e., $\rho(770)$ and $\rho'(1450)$. Let us just take the lowest only for the discussion, relegating the role of ρ' to a short comment later.

It has been known since a long time that the nucleon form factors at low momentum transfers cannot be fitted by a monopole form factor of the type $\sim 1/(1 + cq^2)$ with c a constant where q is Euclidean four momentum transfer. In fact, one obtains a much better fit by a dipole form factor of the form $\sim 1/(1 + dq^2)^2$ with $d \approx 1/(0.71 \text{ GeV})^2$. This meant that the single-vector-meson mediated mechanism along the line of reasoning used for the

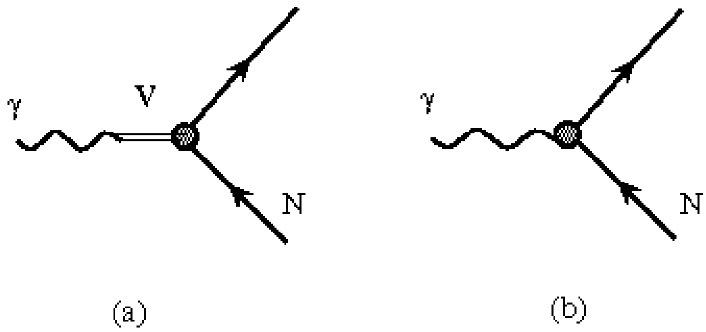

Figure 2: (a) Photon coupling to the nucleon via vector meson V and (b) direct photon coupling to the nucleon. The blob represents the intrinsic form factor accounting for short-distance effects (referred to as "intrinsic core" in some circles) unaccounted for in the effective theory, e.g., asymptotically free QCD property.

pion form factors could not explain the process. This led to a two-component description figure 2 which can be put in the form [52]

$$F_1(q^2) = \frac{1}{2}\left[A(q^2) + B(q^2)\frac{m_\rho^2}{q^2 + m_\rho^2}\right] \tag{9.1}$$

with the normalization

$$A(0) + B(0) = 1. \tag{9.2}$$

In (9.1), the momentum dependence in A and B represents a form-factor effect corresponding to an intrinsic structure of the nucleon which is expected from both the confinement and the asymptotic behavior of perturbative QCD [43]. The first term corresponds to a direct coupling to the intrinsic component of the nucleon ("nucleon core" in short), figure 2b and the second term via one or more vector mesons in the ρ channel, figure 2a. Let us for simplicity consider only one vector meson exchange. The same reasoning applies to the case where more than one vector mesons are considered. Making the reasonable assumption that the photon and the ρ meson couple to the nucleon core with a same form factor, one can rewrite (9.1) as

$$F_1(q^2) = \frac{1}{2}h(q^2)\left[(1-\beta_\rho) + \beta_\rho\frac{m_\rho^2}{q^2 + m_\rho^2}\right] \tag{9.3}$$

with the core form factor normalized as $h(0) = 1$. Perturbative QCD indicates that asymptotically $h(q^2) \approx (1 + \gamma q^2)^{-2}$. The coefficient γ is not given by the model but can be fixed by experimental data. One can make a very good fit to the data with (9.3) with the coefficients $\gamma \approx 0.52$ GeV^{-2} and $\beta_\rho \approx 0.51$ [51].

Let us consider what this result means with regard to our prediction of section (7.1). The VD prediction (7.19), as mentioned above, lacks the intrinsic short-distance form factor but this can be implemented, albeit phenomenologically as in the two-component model, since it involves physics intervening at a scale above the KK scale. What is significant is the role of the first term of (9.3). In the two-component model, this part, characterized by a size of $\sim 0.4\,\mathrm{fm}$, is to represent the short-distance physics of the microscopic degrees of freedom of QCD that are extraneous to long-wavelength excitations — π, ρ etc. — in the baryon. Adding the ρ' meson and higher in the second term of (9.3) is expected to further reduce the size of the core. One interpretation of the core component was given in terms of a "chiral bag" in which quarks and gluons are confined with the broken chiral symmetry of QCD suitably implemented outside of the bag [53]. The baryon charge was assumed to be divided roughly half and half between the quark-gluon sector and the hadron sector. This hybrid model met with a fair success in reproducing the data available up to late 1980's [53]. Interestingly, it has been claimed that there is an (albeit indirect) evidence for a core of $\sim 0.2\,\mathrm{fm}$ from the Nachtmann moment of the unpolarized proton structure function measured at JLab [54].

Within the framework of the two-component picture, an alternative description using the Skyrmion as an extended object to which the photon couples both directly and via the exchange of the lowest member of the vector-meson tower has been constructed [55]. With one parameter that represents the amount of direct coupling, the model is found to agree quite well with the dipole form factors up to $q^2 \sim 1\,\mathrm{GeV}^2$ and can explain satisfactorily the deviation from the dipole form for $q^2 \gtrsim 1\,\mathrm{GeV}^2$. What this implies is that the nucleon form factors at low momentum transfers, say, $q^2 \lesssim 1\,\mathrm{GeV}^2$, can be well understood given the three basic ingredients: (a) an extended object, (b)partial coupling to vector mesons and (c) relativistic recoil corrections.

What we have found in the holographic dual model in section 7.1 is that by a suitable field re-definition and using a sum rule involving the spread in the fifth dimension, one can transform away the "contact" coupling figure 2b — here to the soliton — at the expense of saturating figure 2a with the infinite tower of the vector mesons. The novel structure of this model is that the "intrinsic core" is largely replaced by the higher-lying vector mesons in the infinite tower encapsulated in the instanton baryon — modulo the asymptotically free property relevant at very high momentum transfer not captured in the model, say, physics of $\lesssim 0.2\,\mathrm{fm}$. We will see indeed that this small core size is needed for phenomenology.

9.2 Instanton baryon prediction for the form factors

The nucleon form factors are defined from the matrix elements of the external current operator J^μ as

$$\langle p'|\, J^\mu(x)\,|p\rangle = e^{iqx}\,\bar{u}(p')\,\mathcal{O}^\mu(p,p')\,u(p)\,, \tag{9.4}$$

where $q = p' - p$. By the Lorentz invariance and the current conservation we may expand the operator $\mathcal{O}^\mu$ as

$$\mathcal{O}^\mu(p,p') = \gamma^\mu\left[\frac{1}{2}F_1(q^2) + F_1^a(q^2)\tau^a\right] + \frac{\gamma^{\mu\nu}}{2m_B}q_\nu\left[F_2(q^2) + F_2^a(q^2)\tau^a\right]\,, \tag{9.5}$$

where F_1 and F_2 are the Dirac and Pauli form factors for iso-scalar current respectively, and F_1^a, F_2^a are for iso-vector currents. Our convention is $\tau^a = \sigma^a/2$.

JHEP09(2007)063

In the AdS/CFT correspondence the matrix element is given by the overlap integral of the normalizable modes, corresponding to the nucleon states, and a non-normalizable mode of gauge fields $A_\mu(x,z)$, which becomes an external source for the current at the UV boundary. By matching the correct operators from the 5D effective action in eq. (4.6), one can read off the corresponding form factors.

We first Fourier-transform the gauge fields of the external source of currents as

$$A_\mu(x,z) = \int_q A_\mu(q) A(q,z)\, e^{iqx}\,. \tag{9.6}$$

From the equation of motion for the gauge field we get

$$\left(1+z^2\right)^{4/3} \partial_z^2\, A(q,z) + 2z\left(1+z^2\right)^{1/3} \partial_z\, A(q,z) - q^2\, A(q,z) = 0 \tag{9.7}$$

with boundary conditions for all q

$$\lim_{z\to\pm\infty} A(z,q) = 1, \qquad \lim_{z\to\pm\infty} \partial_z A(q,z) = 0\,. \tag{9.8}$$

After solving this and inserting it into our 5D action, we can read off suitable form factors at momentum q^2. We note that the Dirac form factor is a sum of a term, $F_{1\mathrm{min}}$, coming from the minimal coupling and a term, $F_{1\mathrm{mag}}$, coming from the magnetic coupling, which are

$$\begin{aligned} F_{1\mathrm{min}}(q^2) &= \int_{-w_{\max}}^{w_{\max}} dw\, \left|f_L(w)\right|^2\, A(q,z(w))\,, \\ F_{1\mathrm{mag}}(q^2) &= 2\times 0.18 N_c \int_{-w_{\max}}^{w_{\max}} dw \left(\frac{g_5(w)U(w)}{g_5(0)U_{\mathrm{KK}}M_{\mathrm{KK}}}\right) \left|f_L(w)\right|^2 \partial_w A(q,z(w))\,. \end{aligned} \tag{9.9}$$

where $f_{L,R}(z)$ are the left(right)-handed normalizable modes, corresponding to the nucleon state. The Pauli form factor is given as

$$F_2^3(q^2) = 0.18 N_c \times \frac{4m_B}{M_{\mathrm{KK}}} \int_{-w_{\max}}^{w_{\max}} dw\, \frac{g_5(w)U(w)}{g_5(0)U_{\mathrm{KK}}} f_L^*(w) f_R(w)\, A(q,z(w))\,. \tag{9.10}$$

One salient prediction of instanton baryons on the form factor is that the $\mathrm{U}(1)$ part of the Pauli form factor $F_2(q^2)=0$, because the instanton does not have a $\mathrm{U}(1)$ tail, while $F_{1\mathrm{min}}^3(q^2) = F_{1\mathrm{min}}(q^2)$. We also note that our expressions for the form factors are from the AdS/CFT correspondence, for which we have to use full 5D effective action rather than using the leading two terms in the derivative expansion. Therefore, our results cannot be trusted for $q^2 \gtrsim M_{\mathrm{KK}}^2$.

The experimentally measured nucleon form factors (Sachs form factors) are defined for the space like momentum transfer, $q^2>0$, as

$$G_M^p(q^2) = F_{1\mathrm{min}}(q^2) + \frac{1}{2}F_{1\mathrm{mag}}(q^2) + \frac{1}{2}F_2^3(q^2)\ :, \tag{9.11}$$

$$G_E^p(q^2) = F_{1\mathrm{min}}(q^2) + \frac{1}{2}F_{1\mathrm{mag}}(q^2) - \frac{q^2}{4m_B^2}\frac{1}{2}F_2^3(q^2)\,, \tag{9.12}$$

$$G_M^n(q^2) = -\frac{1}{2}F_{1\mathrm{mag}}(q^2) - \frac{1}{2}F_2^3(q^2)\,, \tag{9.13}$$

$$G_E^n(q^2) = -\frac{1}{2}F_{1\mathrm{mag}}(q^2) + \frac{q^2}{4m_B^2}\frac{1}{2}F_2^3(q^2)\,. \tag{9.14}$$

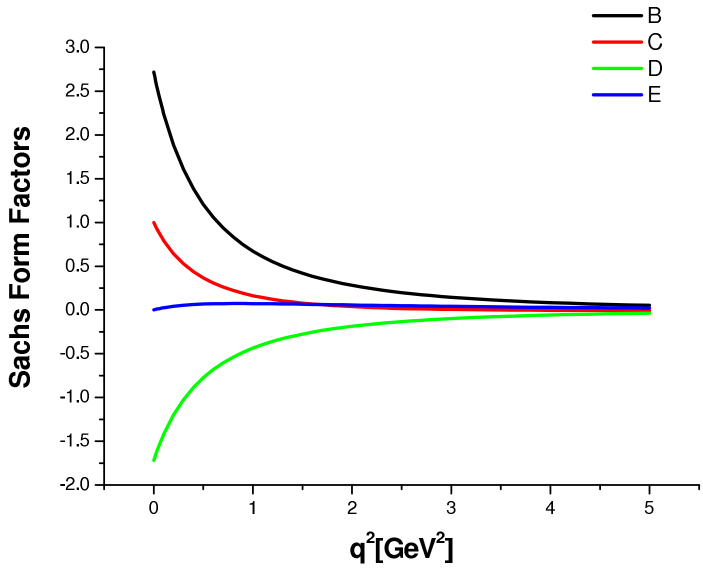

Figure 3: The Sachs form factors vs. q^2 in GeV^2: B=G_M^p, C=G_E^p, D=G_M^n, and E=G_E^n, where we take $m_B = M_{KK}$ and have shifted $N_C \to N_C + 2$.

For the numerical analysis we need to know the coordinate dependence of the magnetic coupling $g_5(w)/e^2(w)$, which is for simplicity approximated as $g_5(w)/e^2(w) \simeq g_5(0)/e^2(0)$.

Our results are plotted in figure 3. To meaningfully compare our results with experiments, there are several corrections to be taken into account that are left out in our theory. One of the most important of them that influences the iso-vector from factors at low momentum transfer is that the lowest iso-vector vector meson ρ has a large width, $\sim 150\,\mathrm{MeV}$, which in our treatment corresponds to higher order in $1/N_c$ expansion and hence is absent. As mentioned above, the short-distance physics involving a scale higher than the KK mass M_{KK} given in QCD as an asymptotic scaling [12] is also missing. It is therefore with these caveats in mind that our results for the Sachs form factors given in figure 3 should be viewed. To have an idea as to how they fare with Nature, let us look at the first nontrivial moment of the proton form factors, namely, $dG^p(q^2)/dq^2|_{q^2=0}$ corresponding to charge (magnetic) square radius. For very low momentum transfers, $q^2 \ll 1\,\mathrm{GeV}^2$, the form factors can be written as

$$G^p(q^2) \approx 1 - \frac{1}{6}\langle r^2\rangle q^2 + \cdots , \tag{9.15}$$

Our results of figure 3 give

$$\sqrt{\langle r^2\rangle_E^p} \simeq 0.80 \ \text{ fm}, \quad \sqrt{\langle r^2\rangle_M^p} \simeq 0.74 \ \text{ fm}. \tag{9.16}$$

The empirical values [56] determined from experiments via dispersion relation analysis are

$$\sqrt{\langle r^2\rangle_E^p} = 0.886 \ \text{ fm}, \quad \sqrt{\langle r^2\rangle_M^p} = 0.855 \ \text{ fm}. \tag{9.17}$$

By comparing the predictions with the empirical results, we can note that the predicted sizes — both electric and magnetic — are smaller than the experimental sizes by $\sim 0.15-0.17\,\mathrm{fm}$, roughly the size of the "intrinsic core" seen in inelastic electron scattering experiments [54]. Since the radii are smaller, the form factors are expected to fall more slowly than observed at low momentum transfers. However what is significant is that the deviations are of the same magnitude, i.e., $\sim 0.15-0.17\,\mathrm{fm}$ for both charge and magnetic radii. That they come out to be the same can be understood by that the "core" reflects short-distance physics more or less "blind" to flavor and spin. This suggests that the "core" effect should cancel out in the ratio $R_p \equiv \frac{\mu_p G_E^p}{G_M^p}$. It indeed does. The predicted value at $q^2 = 0.1\,\mathrm{GeV}^2$ is

$$R_p(q^2 = 0.1\mathrm{GeV}^2) \approx 0.966 \tag{9.18}$$

to be compared with the empirical value

$$R_p(q^2 = 0.1\mathrm{GeV}^2) \approx 0.97. \tag{9.19}$$

Another way of calculating the form factors is to expand the non-normalizable mode in terms of the normalizable modes, $\psi_{(2k+1)}(z)$, of vector mesons in the overlap integrations (9.10) and (9.10),

$$A(q,z) = \sum_k \frac{g_{v^{(k)}}\psi_{(2k+1)}(z)}{q^2 + m_{2k+1}^2}\,, \tag{9.20}$$

where m_{2k+1} and $g_{v^{(k)}}$ are the mass and the decay constant of the k-th vector mesons [44]. (Note that the axial vector mesons should enter to form a complete set when we expand the non-normalizable mode. However, since the overlap integration for the Dirac and Pauli form factors is parity even under the parity flip of the 5th coordinate, the axial vectors do not contribute.) Then we will get the previously defined form factors Eq's (7.19) and (7.31), where the vector meson decay constant is given by

$$g_{v^{(k)}} = \zeta_k\, m_{2k+1}^2\,. \tag{9.21}$$

This shows that as noted in [44], the vector dominance in the form factors for both the pion and the nucleon is a direct consequence of AdS/CFT.

To illustrate that the vector-dominance description captures the same physics as the instanton picture, we calculate the iso-vector charge radius (ICR) of the proton by saturating the charge form factor by the four lowest vector mesons in the ρ channel. Numerically, ζ_k are a constant. We take $\zeta = 0.27$ and find from table 2

$$\sqrt{\langle r^2\rangle_C^p} \simeq \left(6\zeta \sum_{k=0}^{3} \frac{g_V^{(k)}}{m_{v^k}^2}\mathrm{sign}\zeta_k\right)^{1/2} \simeq 0.63\ \ \mathrm{fm}. \tag{9.22}$$

The "empirical value" represented by the dipole parametrization $1/(1+Q^2/m_V^2)^2$ with $m_V = 0.84\,\mathrm{MeV}$ is $\sqrt{\langle r^2\rangle_C^p} = 0.81\,\mathrm{fm}$, so we find the predicted charge radius is smaller than the empirical one by $\sim 0.18\,\mathrm{fm}$, about the same as what we found with the Sachs form factors.

10. Summary and comments

In this article, we pursued an holographic realization of baryons in the SS model of QCD. In this model, the entire meson sector of quenched QCD is collectively realized as a five-dimensional $\mathrm{U}(N_F)$ gauge field and the KK tower produced upon a dimensional reduction gives the towers of vector mesons and axial vector mesons, while an open Wilson line corresponds to the chiral field of pions, U. The string theory, in which the model is embedded, tells us unambiguously that the baryon arises by quantizing an instanton soliton of $\mathrm{SU}(N_F)$ gauge field in five dimensions. We studied its static property in large λ and large N_c limit, as demanded by the classical approximation on the bulk side to the AdS/CFT correspondence, and found the soliton size scales as $\sim 1/(M_{\rm KK}\sqrt{\lambda})$. The small size motivates us to set up an effective action approach treating the soliton as a point-like object, and we explored its consequence in detail. The picture that arises is consistent with heavy-baryon chiral effective field theory where baryons are taken as local fields, with higher order corrections in derivative and/or $1/N_c$ expansion, accounting for the finite size of the baryons.

One might wonder how this small instanton soliton is related to the usual Skyrmion in the four dimensional chiral lagrangian approach[18] in which the size of the baryon is mostly given by the soliton size. Both objects are classified by the topological charge $\pi_3(\mathrm{SU}(N_F = 2))$, and the topological relation can be made precise by the following mapping [26]: Let A be an instanton in $R^3 \times I$ with the unit Pontryagin number. Then the open Wilson line

$$U = Pe^{i\int_I A}\,, \tag{10.1}$$

as a function $R^3 \to \mathrm{SU}(2)$, carries a unit winding number in $\pi_3(\mathrm{SU}(2))$. The latter is of course the definition of the Skyrmion winding number. Topologically this shows why the instanton soliton is the underlying five-dimensional object which produces the Skyrmion upon dimensional reduction to four dimensions.

However, the question of size must be addressed. The Skyrmion solution that would have come out of the chiral lagrangian is of size $\sim 1/M_{\rm KK}$. Yet, the instanton soliton we found has the size $\sim 1/(M_{\rm KK}\sqrt{\lambda})$, which is much smaller when compared to the expected Skyrmion size. Upon the above map from the instanton soliton to Skyrmion, we can see also that the size of the latter essentially is the size of the former. So what went wrong? The answer is that the usual Skyrmion is a bad approximation to the baryon once we begin to include massive vector and axial-vector mesons. Likewise, the truncation down to the usual chiral Lagrangian involving only the pion field is also a bad approximation once we begin to consider the baryonic sector of QCD as previously suspected [18, 19].

From the four dimensional viewpoint, one can understand this disparity of sizes by incorporating more and more of massive vector and axial mesons into the chiral Lagrangian. The Skyrme solution will source these vector mesons through various cubic couplings such as $g_{v^{(k)}\pi\pi}$, and in turn will backreact to these classical excitations of vector fields. It just so happens that the net result tends to shrink the size of the Skyrmion while preserving the

[18] Here and in what follows, by "usual Skyrmion," we mean the Skyrmion arising from the Skyrme Lagrangian consisting of the pion hedgehog. This should be distinguished from the Skyrmion involving the infinite tower of vector mesons that emerges from the 5D instanton.

winding number. This tendency was also demonstrated some time ago by incorporating ρ meson in the chiral Lagrangian with HLS_1 in the conventional field theory setting [13]. What we found here is that in the strong coupling limit with the entire tower of vector mesons included this backreaction of the Skyrmion is rather extreme.

While we identified the instanton soliton as the carrier of baryonic quantum numbers, we actually set up effective action for a subclass of baryons. We restricted our attention to $N_F = 2$ case, and considered dynamics of the iso-doublet under $\mathrm{SU}(N_F = 2)$. These are of course the proton-neutron pair. For these nucleons, we found a simple five-dimensional effective action where all cubic and quartic interaction with mesons are encoded in two interaction terms: a minimal coupling of the baryon current to $\mathrm{U}(N_F = 2)$ gauge field of the form, $\bar{\mathcal{B}} A_m \gamma^m \mathcal{B}$, and a magnetic coupling to $\mathrm{SU}(N_F = 2)$ gauge field strength of the form $\bar{\mathcal{B}} F_{mn} \gamma^{mn} \mathcal{B}$. Considering that the gauge field includes the entire tower of vector, axial-vector mesons, and pions this universal form of the interaction is simply staggering.

Electromagnetic vertices are also extracted, more indirectly using the AdS/CFT prescription relating source terms to the boundary operators and bulk fields. The most prominent feature found here is the vector dominance in a *generalized sense*. The *conventional* vector dominance refers to the assertion that photon couples to hadrons only indirectly via mixing with the lowest lying vector mesons, namely ρ and ω. Here, instead, we showed explicitly that the photon field couples to nucleons indirectly by mixing with the infinite tower of vector mesons in the manner similar to the case of pions.

In contrast to the *conventional* vector dominance which holds poorly for the nucleons, photon has no direct contact coupling with the nucleons (and pions). For small momentum transfer, where our model is valid, in turns out that the first four vector mesons, respectively in the iso-scalar and the iso-vector sectors, dominate the form factors. While the vector dominance for pions is relatively well-established even with the lowest vector mesons, the vector dominance for nucleons has been more controversial [51 – 53]. A remarkable result of our findings is that while in the usual Skyrme model, the direct photon coupling to the soliton is mandatory [55], in terms of the instanton, the direct coupling can be transformed away and the full vector dominance, albeit with the infinite tower, is recovered. One can think of this as a "derivation" of vector dominance model for the nucleon.

We devoted much of this article to exploring consequences of this effective action, and made effort to match with experimental data. Qualitative predictions, such as large N_c behaviors of chiral couplings and ratios between various vector meson couplings, seem to match with data fairly well, and general tendencies of subleading corrections also concur with experiments. Upon some extra assumptions on subleading corrections, motivated by CQM, quantities like the axial coupling to pions and anomalous magnetic moment seem to match rather well. We should however admit that so far our effort to reach out to the experimental data is at best rudimentary. In particular, the extraction of coupling constants are usually quite model-dependent and we must fill the gap between the model and the data by computing actual amplitudes, which would require going beyond the large 't Hooft and large N_c approximations. Thus a lot more work is needed before our theory can confront the real data, e.g., the precise JLab data on nucleon form factors etc.

Also, as a theoretical model, we have various improvements that are still desired. In practice, the biggest hurdle in using our effective action to its full potential ability lies with the magnetic coupling $g_5(w)$. As we emphasized earlier, we have an accurate number for its central value $g_5(0)$ only, owing to the fact that the instanton solution exists only when centered at $w = 0$. The simple procedure we adopted in section 4 cannot be used to extract $g_5(w \neq 0)$. The uncertainty due to this ignorance can be minimized in the large λN_c limit whereby the baryon wavefunction gets squeezed near the center $w = 0$ along the fifth direction, and the large N_c limit of quantities like g_A and the anomalous magnetic moment are insensitive to this problem. However, the extrapolation to small λN_c will be hampered by this ignorance to various degrees, especially for quantities whose dominant contribution arises from the magnetic term. We took a simplifying assumption, $g_5(w)/e(w)^2 = g_5(0)/e(0)^2$, for our numerical estimates but this must be improved further.

Another immediate problem to address is the question of excited baryons. The general approach we took should be certainly applicable to more general baryons, such as Δ, but the precise form and the coupling constant of the magnetic term in section 4 will be modified since the details of the latter depended on the spin and iso-spin structure of the baryon field in question. How the magnetic term will be modified for higher iso-spin baryons remains unaddressed at the moment.

Finally, one would like to generalize the instanton picture to hyperons. It is known that the conventional approach with the Skyrme Lagrangian becomes inefficient when going from U(2) to U(3). We should expect no better result with our model if we tried to consider U(3), especially since we do not know of natural way to incorporate the strange quark mass.[19] A possible approach to hyperons is the Callan-Klebanov bound-state model [58] where the kaons are introduced as extra massive pseudo-scalars in doublets under $\mathrm{U}(N_F = 2)$ and bound to the SU(2) soliton. This approach was far more successful than U(3)-based models, particularly if vector mesons were included in the Lagrangian [19, 59]. It would be interesting to work out the effective action for hyperons as well as exotic baryon (e.g., pentaquark) structure with kaons bound to the instanton.

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[19]ref. [57] studied related issues in a more general D-branes/anti-D-branes setting which allows bare masses of matter fermions from open string tachyon field.

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References

[1] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113] [hep-th/9711200];
S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109];
E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[2] T. Sakai and S. Sugimoto, *Low energy hadron physics in holographic QCD*, *Prog. Theor. Phys.* **113** (2005) 843 [hep-th/0412141]; *More on a holographic dual of QCD*, *Prog. Theor. Phys.* **114** (2006) 1083 [hep-th/0507073].

[3] A. Karch and E. Katz, *Adding flavor to AdS/CFT*, *JHEP* **06** (2002) 043 [hep-th/0205236];
M. Kruczenski, D. Mateos, R.C. Myers and D.J. Winters, *Meson spectroscopy in AdS/CFT with flavour*, *JHEP* **07** (2003) 049 [hep-th/0304032];
J. Babington, J. Erdmenger, N.J. Evans, Z. Guralnik and I. Kirsch, *Chiral symmetry breaking and pions in non-supersymmetric gauge/gravity duals*, *Phys. Rev.* **D 69** (2004) 066007 [hep-th/0306018].

[4] D.T. Son and M.A. Stephanov, *QCD and dimensional deconstruction*, *Phys. Rev.* **D 69** (2004) 065020 [hep-ph/0304182].

[5] J. Erlich, E. Katz, D.T. Son and M.A. Stephanov, *QCD and a holographic model of hadrons*, *Phys. Rev. Lett.* **95** (2005) 261602 [hep-ph/0501128];
L. Da Rold and A. Pomarol, *Chiral symmetry breaking from five dimensional spaces*, *Nucl. Phys.* **B 721** (2005) 79 [hep-ph/0501218];
N. Evans and A. Tedder, *Perfecting the ultra-violet of holographic descriptions of QCD*, *Phys. Lett.* **B 642** (2006) 546 [hep-ph/0609112];
C. Csáki and M. Reece, *Toward a systematic holographic QCD: a braneless approach*, *JHEP* **05** (2007) 062 [hep-ph/0608266].

[6] D.K. Hong, T. Inami and H.-U. Yee, *Baryons in AdS/QCD*, *Phys. Lett.* **B 646** (2007) 165 [hep-ph/0609270].

[7] G. Policastro, D.T. Son and A.O. Starinets, *The shear viscosity of strongly coupled $N = 4$ supersymmetric Yang-Mills plasma*, *Phys. Rev. Lett.* **87** (2001) 081601 [hep-th/0104066];
P. Kovtun, D.T. Son and A.O. Starinets, *Viscosity in strongly interacting quantum field theories from black hole physics*, *Phys. Rev. Lett.* **94** (2005) 111601 [hep-th/0405231];
D. Mateos, R.C. Myers and R.M. Thomson, *Holographic viscosity of fundamental matter*, *Phys. Rev. Lett.* **98** (2007) 101601 [hep-th/0610184].

[8] R.A. Janik and R. Peschanski, *Asymptotic perfect fluid dynamics as a consequence of AdS/CFT*, *Phys. Rev.* **D 73** (2006) 045013 [hep-th/0512162];
S. Nakamura and S.-J. Sin, *A holographic dual of hydrodynamics*, *JHEP* **09** (2006) 020 [hep-th/0607123];
S.-J. Sin, S. Nakamura and S.P. Kim, *Elliptic flow, Kasner universe and holographic dual of RHIC fireball*, *JHEP* **12** (2006) 075 [hep-th/0610113];
D. Bak and R.A. Janik, *From static to evolving geometries: R-charged hydrodynamics from supergravity*, *Phys. Lett.* **B 645** (2007) 303 [hep-th/0611304].

JHEP09(2007)063

[9] S.-J. Sin and I. Zahed, *Holography of radiation and jet quenching*, *Phys. Lett.* **B 608** (2005) 265 [hep-th/0407215];
S.S. Gubser, *Drag force in AdS/CFT*, *Phys. Rev.* **D 74** (2006) 126005 [hep-th/0605182];
H. Liu, K. Rajagopal and U.A. Wiedemann, *Calculating the jet quenching parameter from AdS/CFT*, *Phys. Rev. Lett.* **97** (2006) 182301 [hep-ph/0605178];
A. Buchel, *On jet quenching parameters in strongly coupled non-conformal gauge theories*, *Phys. Rev.* **D 74** (2006) 046006 [hep-th/0605178];
N. Armesto, J.D. Edelstein and J. Mas, *Jet quenching at finite 'T Hooft coupling and chemical potential from AdS/CFT*, *JHEP* **09** (2006) 039 [hep-ph/0606245];
C.P. Herzog, A. Karch, P. Kovtun, C. Kozcaz and L.G. Yaffe, *Energy loss of a heavy quark moving through $N = 4$ supersymmetric Yang-Mills plasma*, *JHEP* **07** (2006) 013 [hep-th/0605158];
Y.-h. Gao, W.-s. Xu and D.-f. Zeng, *Jet quenching parameters of Sakai-Sugimoto model*, hep-th/0611217.

[10] G.E. Brown, B.A. Gelman and M. Rho, *Matter formed at Brookhaven Heavy Ion Collider*, *Phys. Rev. Lett.* **96** (2006) 132301.

[11] A. Buchel and J.T. Liu, *Universality of the shear viscosity in supergravity*, *Phys. Rev. Lett.* **93** (2004) 090602 [hep-th/0311175];
A. Buchel, *On universality of stress-energy tensor correlation functions in supergravity*, *Phys. Lett.* **B 609** (2005) 392 [hep-th/0408095].

[12] S.J. Brodsky and G.F. de Teramond, *Hadronic spectra and light-front wavefunctions in holographic QCD*, *Phys. Rev. Lett.* **96** (2006) 201601 [hep-ph/0602252];
G.F. de Teramond, *Mapping string states into partons: form factors and the hadron spectrum in AdS/QCD*, hep-ph/0606143;
H.R. Grigoryan and A.V. Radyushkin, *Form factors and wave functions of vector mesons in holographic QCD*, hep-ph/0703069;
T. Hambye, B. Hassanain, J. March-Russell and M. Schvellinger, *On the $\Delta(I) = 1/2$ rule in holographic QCD*, *Phys. Rev.* **D 74** (2006) 026003 [hep-ph/0512089].

[13] T. Fujiwara et al., *An effective lagrangian for pions, ρ mesons and skyrmions*, *Prog. Theor. Phys.* **74** (1985) 128;
U.G. Meissner, N. Kaiser, A. Wirzba and W. Weise, *Skyrmions with ρ and ω mesons as dynamical gauge bosons*, *Phys. Rev. Lett.* **57** (1986) 1676;
U.G. Meissner and I. Zahed, *Skyrmions in the presence of vector mesons*, *Phys. Rev. Lett.* **56** (1986) 1035.

[14] M. Bando, T. Kugo and K. Yamawaki, *Nonlinear realization and hidden local symmetries*, *Phys. Rept.* **164** (1988) 217.

[15] T.H.R. Skyrme, *A unified field theory of mesons and baryons*, *Nucl. Phys.* **31** (1962) 556.

[16] G.S. Adkins, C.R. Nappi and E. Witten, *Static properties of nucleons in the Skyrme model*, *Nucl. Phys.* **B 228** (1983) 552.

[17] E. Witten, *Anti-de Sitter space, thermal phase transition and confinement in gauge theories*, *Adv. Theor. Math. Phys.* **2** (1998) 505 [hep-th/9803131].

[18] Y. Brihaye, C.T. Hill and C.K. Zachos, *Bounding gauged skyrmion masses*, *Phys. Rev.* **D 70** (2004) 111502 [hep-th/0409222].

JHEP09(2007)063

[19] M. Rho, *Pentaquarks, skyrmions and the vector manifestation of chiral symmetry*, hep-ph/0502049.

[20] K. Nawa, H. Suganuma and T. Kojo, *Brane-induced skyrmions: baryons in holographic QCD*, hep-th/0701007.

[21] D.K. Hong, M. Rho, H.-U. Yee and P. Yi, *Chiral dynamics of baryons from string theory*, hep-th/0701276.

[22] S. Sugimoto, private communication.

[23] For review M. Harada and K. Yamawaki, *Hidden local symmetry at loop: a new perspective of composite gauge boson and chiral phase transition*, *Phys. Rept.* **381** (2003) 1 [hep-ph/0302103].

[24] N. Arkani-Hamed, H. Georgi and M.D. Schwartz, *Effective field theory for massive gravitons and gravity in theory space*, *Ann. Phys. (NY)* **305** (2003) 96 [hep-th/0210184].

[25] G.T. Horowitz and J. Polchinski, *Gauge/gravity duality*, gr-qc/0602037.

[26] M.F. Atiyah and N.S. Manton, *Skyrmions from instantons*, *Phys. Lett.* **B 222** (1989) 438.

[27] E. Witten, *Baryons and branes in Anti de Sitter space*, *JHEP* **07** (1998) 006 [hep-th/9805112].

[28] S.K. Domokos and J.A. Harvey, *Baryon number-induced Chern-Simons couplings of vector and axial-vector mesons in holographic QCD*, arXiv:0704.1604.

[29] H. Hata, T. Sakai, S. Sugimoto and S. Yamato, *Baryons from instantons in holographic QCD*, hep-th/0701280.

[30] G. 't Hooft, *Computation of the quantum effects due to a four-dimensional pseudoparticle*, *Phys. Rev.* **D 14** (1976) 3432.

[31] R. Jackiw, C. Nohl and C. Rebbi, *Conformal properties of pseudoparticle configurations*, *Phys. Rev.* **D 15** (1977) 1642.

[32] A.V. Manohar, *Large-N QCD*, hep-ph/9802419.

[33] R.F. Dashen and A.V. Manohar, $1/N_c$ *corrections to the baryon axial currents in QCD*, *Phys. Lett.* **B 315** (1993) 438 [hep-ph/9307242].

[34] R.D. Amado, R. Bijker and M. Oka, *Algebraic description of the skyrmion and its quantization for finite n*, *Phys. Rev. Lett.* **58** (1987) 654.

[35] S. Peris, g_A *in the constituent quark model*, *Phys. Lett.* **B 268** (1991) 415.

[36] S.J. Dong, J.F. Lagae and K.F. Liu, *Flavor singlet g_A from lattice QCD*, *Phys. Rev. Lett.* **75** (1995) 2096 [hep-ph/9502334];
M. Gurtler et al., *A lattice determination of g_A and from overlap fermions*, *Nucl. Phys.* **140** *(Proc. Suppl.)* (2005) 707 [hep-lat/0409164].

[37] H.-W. Lin and S. Ohta, *Nucleon structure in lattice QCD with dynamical domain-wall fermions quarks*, PoS(LAT2006)118 [hep-lat/0610028].

[38] D.O. Riska, *Dynamical interpretation of the nucleon nucleon interaction and exchange currents in the large-N_c limit*, *Nucl. Phys.* **A 710** (2002) 55 [nucl-th/0204016].

[39] G. Höhler and E. Pietarinen, *The ρNN vertex in vector dominance models*, *Nucl. Phys.* **B 95** (1975) 210.

JHEP09(2007)063

[40] V.G.J. Stoks, R.A.M. Klomp, C.P.F. Terheggen and J.J. de Swart, *Construction of high quality N N potential models*, *Phys. Rev.* **D 49** (1994) 2950 [nucl-th/9406039].

[41] R. Machleidt, *The high-precision, charge-dependent bonn nucleon-nucleon potential (CD-Bonn)*, *Phys. Rev.* **D 63** (2001) 024001 [nucl-th/0006014].

[42] F. Gross and A. Stadler, *High-precision covariant one-boson-exchange potentials for NP scattering below* 350 MeV, arXiv:0704.1229.

[43] G.P. Lepage and S.J. Brotsky, *Exclusive processes in quantum chromodynamics: the form-factors of baryons at large momentum transfer*, *Phys. Rev. Lett.* **43** (1979) 545; *Exclusive processes in perturbative quantum chromodynamics*, *Phys. Rev.* **D 22** (1980) 2157.

[44] S. Hong, S. Yoon and M.J. Strassler, *On the couplings of vector mesons in AdS/QCD*, *JHEP* **04** (2006) 003 [hep-th/0409118].

[45] E.L. Lomon, *Effect of recent $R(p)$ and $R(n)$ measurements on extended Gari-Kruempelmann model fits to nucleon electromagnetic form factors*, *Phys. Rev.* **D 66** (2002) 045501 [nucl-th/0203081].

[46] M. Harada, S. Matsuzaki and K. Yamawaki, *Implications of holographic QCD in CHPT with hidden local symmetry*, *Phys. Rev.* **D 74** (2006) 076004 [hep-ph/0603248].

[47] S. Weinberg, *Effective field theories in the large-N limit*, *Phys. Rev.* **D 56** (1997) 2303 [hep-th/9706042].

[48] M. Harada and K. Yamawaki, *Fate of vector dominance in the effective field theory*, *Phys. Rev. Lett.* **87** (2001) 152001 [hep-ph/0105335].

[49] M. Harada and C. Sasaki, *Vector manifestation and violation of vector dominance in hot matter*, *Nucl. Phys.* **A 736** (2004) 300 [hep-ph/0304282].

[50] M. Harada and C. Sasaki, *Thermal dilepton production from dropping ρ based on the vector manifestation*, *Phys. Rev.* **D 74** (2006) 114006 [hep-ph/0608237].

[51] R. Bijker and F. Iachello, *Re-analysis of the nucleon space- and time-like electromagnetic form factors in a two-component model*, *Phys. Rev.* **D 69** (2004) 068201 [nucl-th/0405028].

[52] F. Iachello, A.D. Jackson and A. Lande, *Semiphenomenological fits to nucleon electromagnetic form- factors*, *Phys. Lett.* **B 43** (1973) 191.

[53] G.E. Brown, M. Rho and W. Weise, *Phenomenological delineation of the quark-gluon structure from nucleon electromagnetic form-factors*, *Nucl. Phys.* **A 454** (1986) 669.

[54] R. Petronzio, S. Simula and G. Ricco, *Possible evidence of extended objects inside the proton*, *Phys. Rev.* **D 67** (2003) 094004 [hep-ph/0301206].

[55] G. Holzwarth, *Electro-magnetic nucleon form factors and their spectral functions in soliton models*, *Z. Phys.* **A356** (1996) 339 [hep-ph/9606336].

[56] M.A. Belushkin, H.W. Hammer and U.G. Meissner, *Dispersion analysis of the nucleon form factors including meson continua*, *Phys. Rev.* **D 75** (2007) 035202 [hep-ph/0608337].

[57] R. Casero, E. Kiritsis and A. Paredes, *Chiral symmetry breaking as open string tachyon condensation*, hep-th/0702155.

[58] C.G. Callan Jr. and I.R. Klebanov, *Bound state approach to strangeness in the Skyrme model*, *Nucl. Phys.* **B 262** (1985) 365.

[59] N.N. Scoccola, D.P. Min, H. Nadeau and M. Rho, *The strangeness problem: an* SU(3) *skyrmion with vector mesons*, *Nucl. Phys.* **A 505** (1989) 497.

JHEP09(2007)063

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Addendum

The normalization convention for spin one mesons (vector mesons and axial-vector mesons) used in this article needs a further clarification. In terms of the actual convention that was used for computation, equation (2.26) should read

$$\mathcal{L}_{massive} = -\sum_n \mathrm{tr}\left\{ \frac{1}{4}\, F^{(n)}_{\mu\nu} F^{(n)\mu\nu} + \cdots \right\} .$$

with the trace explicitly put in. Equations (7.5), (7.9), and (7.17) should be similarly clarified. This coefficient 1/4 is canonical for a $U(N_F)$ gauge theory, especially in its $U(1)^{N_F}$ Coulomb phase. Upon $U(N_F) \to SU(N_F) \times U(1)$ decomposition, however, the natural $SU(N_F)$ gauge generators and the overall $U(1)$ generator obey tr $T^a T^b = \delta^{ab}/2$. This results in a non-canonical normalization of conventional vector mesons and axial-vector mesons. For instance, the kinetic term for the ρ-meson becomes

$$-\frac{1}{8}\, (\partial_\mu \rho^a_\nu - \partial_\nu \rho^a_\mu)(\partial^\mu \rho^\nu_a - \partial^\nu \rho^\mu_a)$$

with the iso-vector index a when $N_F = 2$.

Although the convention was used self-consistently, it would have been more helpful for readers if we had employed the canonical normalization. The conversion to the latter can be performed by rescaling spin one mesons so that, for example, the coefficient of ρ-meson kinetic term above is 1/4 instead of 1/8. This rescaling would result in an overall multiplicative factor of $\sqrt{2}$ for all cubic couplings to the baryons (nucleons): all numerical entries for $g^{(k)}_{V,min}$'s and $g^{(k)}_{V,mag}$'s in table 1 and in table 2 would be multiplied by $\sqrt{2}$, in particular.

This factor of $\sqrt{2}$ can be alternatively traced to the modification of the normalization condition and the completeness condition for the meson wavefunctions $\psi_{(n)}$'s with $n \geq 1$, since the original five-dimensional $U(N_F)$ gauge field is not modified. The normalization of ψ_0 is not affected. Therefore, in addition to the quadratic action (7.5) and (7.9) which should be more explicitly put in the canonical form, equations (7.6), (7.13), and (7.20) would also be modified by replacing $1/e(w)^2$ by $1/2e(w)^2$. This forces all numerical entries for ζ_k in table 2 to be multiplied by $1/\sqrt{2}$ as well. Finally, equation (7.17) would be written as $(L/2)\,\mathrm{tr}\,|d\mathcal{V}|^2$.

Physical observables in general and all quantities we explicitly compared to experimental data in this article are unaffected by this convention issue. Also, the pseudo-scalar mesons such as pions are already in the canonical form and completely unaffected. A revised version of this article with the canonical normalization convention for spin one mesons is available as an e-print arXiv:0705.2632v3.

JHEP09(2007)063