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Massive string theories from M-theory and F-theory

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Abstract: The massive IIA string theory whose low energy limit is the massive supergravity theory constructed by Romans is obtained from M-theory compactified on a 2-torus bundle over a circle in a limit in which the volume of the bundle shrinks to zero. The massive string theories in 9-dimensions given by Scherk-Schwarz reduction of IIB string theory are interpreted as F-theory compactified on 2-torus bundles over a circle. The M-theory solution that gives rise to the D8-brane of the massive IIA theory is identified. Generalisations of Scherk-Schwarz reduction are discussed.

Keywords: M-Theory, String Duality, F-Theory, Supergravity Models.

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There is a massive version of the ten dimensional type IIA supergravity due to Romans [1] and it has long been a mystery as to whether it has an eleven dimensional origin, in which the mass might arise from an explicit mass in eleven dimensions, or from a parameter in the dimensional reduction ansatz, as in Scherk-Schwarz dimensional reduction in which the fields have non-trivial dependence on the coordinates of the internal dimensions [2]. In [3], it has been argued, subject to certain assumptions, that no covariant massive deformation of 11-dimensional supergravity is possible, which would mean that the massive IIA supergravity cannot come from a conventional reduction of such a massive theory. Dimensionally reducing the Romans theory on a circle gives a massive 9 dimensional theory which can also be obtained from the type IIB theory by a Scherk-Schwarz reduction [4], but the Romans theory cannot be obtained by a Scherk-Schwarz reduction of 11-dimensional supergravity; a different 10-dimensional massive supergravity theory, for which there is no action, was proposed in [5], and obtained via a Scherk-Schwarz reduction of 11-dimensional supergravity in [6]. In [7], the Romans supergravity was lifted to a massive deformation of 11-dimensional supergravity where the terms in the 11-dimensional action depending on the mass parameter m also depend explicitly on the Killing vector used in the dimensional reduction to 10-dimensions, and so this 11-dimensional theory is not fully covariant.

The IIA supergravity is the field theory limit of the IIA superstring, and the strong coupling limit of the IIA superstring is M-theory, which has 11-dimensional supergravity as its field theory limit. There is a massive version of type IIA string theory [8] whose field theory limit is the Romans supergravity theory (see for instance [4]), and the question arises as to how this massive IIA string theory arises from M-theory. Our purpose here is to argue that although the Romans supergravity theory may not be derivable from 11-dimensional supergravity, or any covariant massive deformation thereof, the massive IIA superstring, whose low energy limit is the Romans theory, can be obtained from M-theory.

The type IIB supergravity theory also cannot be obtained from 11-dimensional supergravity, but the type IIB string theory can be obtained from M-theory by compactifying on a 2-torus and taking a limit in which the area of the torus tends to zero while the modulus τ tends to a constant, the imaginary part of which is the string coupling constant of the IIB string theory [9]. The massive IIA string theory compactified on a circle of radius R is T-dual to a Scherk-Schwarz compactification of the IIB superstring on a circle of radius $1/R$, with mass-dependent modifications of the usual T-duality rules [4]. Thus the massive IIA string can be obtained from M-theory by first reducing on a 2-torus that shrinks to zero size to obtain the IIB string, and then using a 'twisted' T-duality to obtain the massive IIA string, by making a Scherk-Schwarz reduction on a circle and then shrinking the radius to zero size. Moreover, we shall argue that the Scherk-Schwarz compactification of the IIB superstring has a natural formulation in terms of F-theory. The Scherk-Schwarz reduction of the IIB string theory can then be obtained from a limit of a compactification of M-theory, using the relation between F-theory and M-theory, and it will be shown that the massive IIA

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string can be obtained by reducing M-theory on a torus bundle over a circle and taking a limit in which the bundle shrinks to zero size, with all three radii tending to zero. It will be seen that this relates the D8-brane, which only occurs in the IIA string with non-vanishing mass, to a brane-like solution of M-theory, which might be thought of as an M9-brane, and to a related 12-dimensional F-theory 'solution'.

The Scherk-Schwarz mechanism and its generalisations [2, 10, 11, 12, 13, 14, 15, 16] introduces mass parameters into toroidal compactifications of supergravities and string theories. If the original theory has a global symmetry G acting on fields ϕ by $\phi \to g(\phi)$, then in a generalised Scherk-Schwarz reduction or twisted reduction the fields are not independent of the internal coordinates, but are chosen to depend on the torus coordinates y through an ansatz

$$\phi(x^\mu, y) = g_y(\phi(x^\mu)) \tag{1}$$

for some y-dependent symmetry transformation $g_y = g(y)$ in G. In many cases this leads to a spontaneous breaking of supersymmetry [2], while in others it results in the gauging of certain symmetries of the conventionally reduced theory, and the introduction of a scalar potential and cosmological constant [13, 14, 16]. Here, we will restrict ourselves to compactifications on a circle, with periodic coordinate $y \sim y + 1$. For example, for reducing a theory with a linearly realised $U(1)$ symmetry on a circle, a massless field ϕ of charge q can be given a y dependence $\phi(x, y) = e^{2\pi i q m y}\phi(x)$, so that the field $\phi(x)$ is given a mass of qm.

The map $g(y)$ is not periodic, but has a *monodromy*

$$\mathcal{M}(g) = g(1)g(0)^{-1} \tag{2}$$

for some $\mathcal{M}$ in G. We will consider here maps of the form

$$g(y) = \exp(My) \tag{3}$$

for some Lie algebra element M, so that the monodromy is

$$\mathcal{M}(g) = \exp M\,. \tag{4}$$

Then

$$M = g^{-1}\partial_y g \tag{5}$$

is proportional to the mass matrix of the dimensionally reduced theory and is independent of y [16].

The next question is whether two different choices of $g(y)$ give inequivalent theories. The ansatz breaks the symmetry G down to the subgroup preserving $g(y)$, consisting of those h in G such that $h^{-1}g(y)h = g(y)$. Acting with a general constant element k in G will change the mass-dependent terms, but will give a $D-1$ dimensional theory related to the original one via the field refinition $\phi \to k(\phi)$. This same theory could have been obtained directly via a reduction using $k^{-1}g(y)k$ instead of $g(y)$, so two choices of $g(y)$

in the same conjugacy class give equivalent reductions (related by field-redefinitions). As a result, the reductions are classified by conjugacy classes of the mass-matrix M.

The map $g(y)$ is a local section of a principal fiber bundle over the circle with fibre G and monodromy $\mathcal{M}(g)$ in G. Such a bundle is constructed from $I \times G$, where $I = [0, 1]$ is the unit interval, by gluing the ends of the interval together with a twist of the fibres by the monodromy $\mathcal{M}$. Two such bundles with monodromy in the same G-conjugacy class are equivalent. Only those monodromies $\mathcal{M}$ that can be written as e^M for some M arise in this way, and for those monodromies that are in the image of the exponential map, there are in general an infinite number of possible choices of mass-matrix. Indeed, if M, M' are two such mass matricies for a given monodromy such that $e^M = e^{M'} = \mathcal{M}$, then $e^M e^{-M'} = 1$ and so there is a λ satisfying $e^\lambda = 1$ such that $M - M' - \frac{1}{2}[M, M'] + \ldots = \lambda$. The general solution of $e^M = \mathcal{M}$ is then of the form $M = M' + \lambda + \frac{1}{2}[M', \lambda] + ...$ where M' is a particular solution and λ is any solution of $e^\lambda = 1$. The algebra elements λ with $e^\lambda = 1$ fall into adjoint orbits, as, for any group element g, $\lambda' = g\lambda g^{-1}$ satisfies this condition if λ does. The set of all Lie algebra elements λ with $e^\lambda = 1$ is given by the adjoint orbits of all points in the dual of the weight lattice of the maximal compact subalgebra H of G, sometimes called the integer lattice.

Of particular interest are the D-dimensional supergravity theories with rigid duality symmetry G and scalars taking values in G/H [17, 18], which can be Scherk-Schwarz-reduced on a circle to $D - 1$ dimensions. The reduction requires the choice of a map $g(y)$ of the form (3) from S^1 to G, which then determines the y-dependence of the fields through the ansatz (1), and any choice of Lie algebra element M is allowed. In the quantum theory, the symmetry group G is broken to a discrete sub-group $G(\mathbb{Z})$ [19]. A consistent twisted reduction of a string or M-theory, whose low-energy effective theory is the supergravity theory considered above, then requires that the monodromy be in the U-duality group $G(\mathbb{Z})$. (In the classical supergravity theory, any element of G can be used as the monodromy.) Then the choice of M is restricted by the constraint that e^M should be in $G(\mathbb{Z})$. As before, if $M = kM'k^{-1}$ where k is in G, the theories are related by field redefinitions. However, only if k is in $G(\mathbb{Z})$ will the redefinition preserve the charge lattice [19]. Once the conventions for the definitions of charges are fixed, it is necessary to restrict to conjugation by elements of $G(\mathbb{Z})$, and so reductions are specified by $G(\mathbb{Z})$ conjugacy classes of maps (3) with monodromy (4) in $G(\mathbb{Z})$.

Here we will concentrate on the examples relevant to the massive IIA superstring. The type IIB supergravity theory has $G = SL(2, \mathbb{R})$ global symmetry and any element M of the $SL(2, \mathbb{R})$ Lie algebra can be used in the ansatz (1),(3) to give a Scherk-Schwarz reduction to 9-dimensions to obtain a class of massive 9-dimensional supergravity theories. Such reductions for particular elements of $SL(2, \mathbb{R})$ were given in [4, 15, 6], and the general class of $SL(2, \mathbb{R})$ reductions of IIB supergravity was obtained in [16]. Note that this ansatz does not allow the monodromy to be an arbitrary $SL(2, \mathbb{R})$ group element, but requires it to be in the image of the exponential map. Acting with an $SL(2, \mathbb{R})$ transformation leaves the mass-independent part of the theory unchanged but changes the mass matrix by $SL(2,\mathbb{R})$ conjugation, and so there are three distinct classes of inequivalent theories, corresponding to the hyperbolic, elliptic and parabolic $SL(2,\mathbb{R})$ conjugacy classes, represented by monodromy matrices of the form

$$\begin{pmatrix} a & 0 \\ 0 & a^{-1} \end{pmatrix}, \qquad \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \qquad \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}, \tag{6}$$

respectively. The details of the reduction of the bosonic sector of the supergravity theory for general M were given in [16].

In the quantum theory, only an $SL(2,\mathbb{Z})$ symmetry remains [19]. The quantum-consistent Scherk-Schwarz reductions of this theory to 9 dimensions are those for which the monodromy is in $SL(2,\mathbb{Z})$, and are defined up to $SL(2,\mathbb{Z})$ conjugacy. The fact that the monodromy must be in $SL(2,\mathbb{Z})$ implies a quantization of the masses.

The IIB supergravity scalars take values in $SL(2,\mathbb{R})/U(1)$ and can be represented by a complex scalar $\tau = C_0 + ie^{-\Phi}$ transforming under $SL(2)$ by fractional linear transformations, so that $g \in SL(2)$ acts as

$$g = \begin{pmatrix} a & b \\ c & d \end{pmatrix}: \quad \tau \to \tau_g = \frac{a\tau + b}{c\tau + d}, \tag{7}$$

The Scherk-Schwarz ansatz, $\tau(x,y) = \tau(x)_{g(y)}$, gives a complex scalar $\tau(x)_{g(y)}$ of the reduced theory and for fixed x, $\tau(x)_{g(y)}$ depends on y and is a section of the bundle over the circle with fibre $SL(2,\mathbb{R})/U(1)$ obtained as a quotient of the principle $SL(2,\mathbb{R})$ bundle by $U(1)$.

For the IIB string theory, the monodromy must be restricted to lie in $SL(2,\mathbb{Z})$. If $g(y)$ has $SL(2,\mathbb{Z})$ monodromy, the local section $\tau(y) = \tau_{g(y)}$ can be used to construct a torus bundle over the circle in which τ is the T^2 modulus, and depends on the position on the circle. The total space of the torus bundle is a 3-dimensional space B with metric

$$ds_B^2 = R^2 dy^2 + \frac{A}{Im(\tau)} |dz_1 + \tau(y) dz_2|^2, \tag{8}$$

where the fibre is a T^2 with real periodic coordinates z_1, z_2, $z_i \sim z_i + 1$, constant area modulus A and complex structure $\tau(y)$, which depends on the coordinate y of the circular base space, and this has circumference R. The Scherk-Schwarz reduction of the IIB superstring with an ansatz $\tau(y) = \tau_{g(y)}$ associated with a particular torus bundle B is precisely what is meant by F-theory compactified on the three dimensional total space B [20, 21, 22, 23].

This generalises; for theories in which the global symmetry is $G = SL(n,\mathbb{R})$ with quantum symmetry $SL(n,\mathbb{Z})$, a twisted reduction on an m-torus in which all monodromies are in $SL(n,\mathbb{Z})$ corresponds to a torus bundle with fibres T^n over a base T^m. For $m = 1$, this gives a T^n bundle over a circle. Certain torus bundles over a circle are also circle bundles over a torus, and the latter was the interpretation used in [6]. However, the torus bundle over a circle is both more general and more useful, as it has an F-theory interpretation. For example, the 7-dimensional maximal supergravity theory has $G = SL(5,\mathbb{R})$ symmetry, while the 7-dimensional type II string theory has $SL(5,\mathbb{Z})$ U-duality. The general twisted reduction from 7 to 6 dimensions would involve a map $g(y): S^1 \to SL(5,\mathbb{R})$ with $SL(5,\mathbb{Z})$ monodromy, which is also the data for a T^5 bundle over S^1. Then the general $SL(5)$ Scherk-Schwarz reduction can be re-interpreted as a reduction of the F'-theory of [23] on a T^5 bundle over S^1. (The F'-theory is an analogue of F-theory, also in 12 dimensions, which can be compactified on spaces admitting a T^5 fibration [23].)

Any twisted reduction of the IIB string to 9 dimensions can be recast as the reduction of F-theory on a bundle B which is a T^2 bundle over S^1. One can also consider compactifications of M-theory on B, and the two are related by fibre-wise duality as follows. For M-theory compactified on B in which the T^2 fibres have a constant area A, the limit $A \to 0$ keeping the modulus $\tau(x,y)$ fixed gives F-theory compactified on B with fixed torus area $A = 1$, say. For a trivial bundle, this follows from the fact that M-theory compactified on T^2 becomes, in the limit in which the torus shrinks to zero size, the IIB string theory, and the generalisation to non-trivial bundles follows from the adiabatic argument [24].

Consider the Scherk-Schwarz reduction using the map $S^1 \to SL(2,\mathbb{R})$

$$g(y) = \begin{pmatrix} 1 & my \\ 0 & 1 \end{pmatrix}, \qquad M = \begin{pmatrix} 0 & m \\ 0 & 0 \end{pmatrix}, \tag{9}$$

so that (1) leads to the linear ansatz

$$\tau(x,y) = \tau(x) + my\,. \tag{10}$$

The monodromy is

$$\mathcal{M} = \begin{pmatrix} 1 & m \\ 0 & 1 \end{pmatrix} \tag{11}$$

and in the quantum theory this must be in $SL(2,\mathbb{Z})$ so that m must be an integer, and the mass is quantized, as it is proportional to m. This is precisely the reduction studied in [4], and is T-dual to the massive IIA string theory, with mass parameter m, conventionally compactified on S^1. The bundle B has a metric given by (8),(10), which takes the simple form

$$ds^2 = dy^2 + (dz_1 + my dz_2)^2 + dz_2^2 \tag{12}$$

if $\tau_0 = i, A = R = 1$. This 3-space B is also a circle bundle over a 2-torus with fibre coordinate z_1, base-space coordinates y, z_2 and connection 1-form $\mathbf{A} = my dz_2$ [6].

The massive IIA string theory arises from M-theory as follows. Let $B(A,R)$ be the the torus bundle over a circle of radius R, where the torus has modulus τ depending on the S^1 coordinate y through

$$\tau = \tau_0 + my \tag{13}$$

for some constant τ_0, and y-independent area A. Compactifying M-theory on $B(A,R)$ and taking the limit $A \to 0$ gives F-theory compactified on $B(1,R)$, or equivalently the Scherk-Schwarz reduction of the IIB string on a circle of radius R using the ansatz (10). This is T-dual to the massive IIA string with mass parameter m compactified on a circle of radius $1/R$, and so the uncompactified massive IIA string is obtained by taking the limit $R \to 0$. Putting this together, we obtain the massive IIA string by compactifying M-theory on $B(A,R)$ and taking the zero-volume limit $A \to 0$, $R \to 0$. The bundle also depends on τ_0 and m, and is trivial if $m=0$, in which case B is a 3-torus, and M-theory on a 3-torus indeed gives, in the limit in which the torus shrinks to zero size, the massless IIA string theory or M-theory, depending on the value of the string coupling. The IIB string coupling constant g_B is given by the imaginary part of τ_0, $g_B = 1/Im(\tau_0)$, and the coupling constant g_A for the T-dual IIA theory is related to this by $g_A = g_B/R$, so that

$$g_A = \frac{1}{Im(\tau_0)R} \,. \tag{14}$$

Then if $Im(\tau_0) \to \infty$ as $R \to 0$ so that $Im(\tau_0)R$ remains fixed, the massive IIA theory at finite string coupling (14) is obtained. The massive IIA string theory can also be obtained from F-theory on $B(1,R)$ by taking the limit $R \to 0$, keeping $Im(\tau)R$ fixed.

The massive IIA supergravity theory doesn't have a Minkowski or (anti) de Sitter solution, and there is no maximally supersymmetric solution. There is a D8-brane solution which preserves half of the supersymmetries, however [4]. The string-frame metric is

$$ds^2 = H^{-1/2} d\sigma_{8,1}^2 + H^{1/2} dx^2 \,, \tag{15}$$

where $d\sigma_{p,1}^2$ is the $p+1$ dimensional Minkowski metric on $\mathbb{R}^{p,1}$. There is an 8+1 dimensional longitudinal space and a one-dimensional transverse space with coordinate x. The function $H(x)$ is harmonic, $H''=0$, and the solution

$$H = \begin{cases} c + m'|x| & \text{for } x<0 \\ c + m|x| & \text{for } x>0 \end{cases} \tag{16}$$

for some constant c represents a domain wall at $x=0$, separating regions with two different (integer) values of the mass parameter, m and m'. If one of the longitudinal coordinates, y say, is made periodic, a T-duality in the y-direction leads to the circularly symmetric IIB D7-brane solution of [4], with string-frame metric

$$ds^2 = H^{-1/2} d\sigma_{7,1}^2 + H^{1/2}(dx^2 + dy^2) \tag{17}$$

and

$$e^{-\phi} = H, \qquad C_0' = H' \,, \tag{18}$$

where ϕ is the dilaton and C_0 is the RR scalar. In Einstein frame, the metric is

$$ds^2 = d\sigma_{7,1}^2 + H(dx^2 + dy^2) \tag{19}$$

Dimensional reduction in the y direction of the D8-brane (15) or D7-brane (17) leads to the 7-brane solution [13] of the massive 9-dimensional theory (obtained by twisted reduction of the IIB theory using (10)) with metric

$$ds^2 = H^{-1/2} d\sigma_{7,1}^2 + H^{1/2} dx^2 \,. \tag{20}$$

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Conventional dimensional reduction of 11-dimensional supergravity on a 2-torus gives massless 9-dimensional type II theory with scalars in the coset space $\mathbb{R}^+ \times SL(2,\mathbb{R})/U(1)$, which is the moduli space of the torus [25]. A Scherk-Schwarz reduction of this to 8-dimensions using the ansatz (10) for the complex scalar in $SL(2,\mathbb{R})/U(1)$ gives a massive type II supergravity in 8-dimensions [13] and this theory has a 6-brane solution [13] with metric

$$ds^2 = H^{2/3}\left(H^{-1/2}d\sigma_{6,1}^2 + H^{1/2}dx^2\right). \tag{21}$$

However, this massive 8-dimensional theory arises directly from reduction from 11-dimensions on the torus bundle B, and we will now check that the 6-brane solution arises from an 11-dimensional solution reduced on the torus bundle B. The moduli τ, A, R of the bundle become scalar fields in the dimensionally reduced theory, and for the 11-dimensional oxidation of the solution (21), these moduli can be expected to be functions of transverse coordinate x. The 11-dimensional oxidation of (21) was given in [13, 6, 26], with metric

$$ds^2 = d\sigma_{6,1}^2 + Hdx^2 + H(dy^2 + Adz_2^2) + AH^{-1}(dz_1 + mydz_2)^2\,, \tag{22}$$

where A is a constant that can be absorbed into a rescaling of z_1, z_2. This can be rewritten in the form

$$ds^2 = H^{1/2}\left(H^{-1/2}d\sigma_{6,1}^2 + H^{1/2}dx^2\right) + ds_B^2\,, \tag{23}$$

where ds_B^2 is a B-metric of the form (8),(10), but where the moduli τ, R depend on x as well as y:

$$R = H^{1/2}, \qquad \tau = my + iH\,. \tag{24}$$

The metric is of the form $\mathbb{R}^{6,1} \times M_4$ where M_4 is of the form $\mathbb{R} \times B$ with coordinates x, y, z_1, z_2 and Ricci-flat metric

$$ds^2 = Hdx^2 + ds_B^2\,, \tag{25}$$

with the moduli of B given by (24). The 11+1 dimensional space $\mathbb{R}^{7,1} \times M_4$ is Ricci-flat and is the F-theory 'solution' that gives rise to the Einstein-frame 7-brane solution (19), which can be reduced further to the 9-dimensional 7-brane (20). Note that for domain walls separating regions of mass m, m', as in (16), then there are two different bundles B, B' arising on either side of the wall, one with monodromy (11) and one with monodromy given by (11) with m replaced by m'.

Now taking the limit in which the total spaces B, B' shrink to zero size, the solution (23) becomes the D8-brane solution of the massive IIA string, while taking the limit in which the T^2 fibres shrink to zero size ($A \to 0$) gives the circularly symmetric D7-brane (17). This can be seen in a number of ways. For example, first dimensionally reducing in the z_1 direction and Weyl rescaling to obtain the IIA string-frame metric, (22) becomes the D6-brane solution

$$ds^2 = H^{-1/2}d\sigma_{6,1}^2 + H^{1/2}(dx^2 + dy^2 + Adz_2^2)\,, \tag{26}$$

where the harmonic function depends only on x, so that this can be thought of as a D6-brane 'smeared' over the y and z_2 directions. Thus regarding B as a circle bundle over T^2 with fibre coordinate z_1, we can shrink the fibre to obtain the smeared D6-brane solution of the IIA theory with charge proportional to m. Now the limit $A \to 0$ is obtained by T-dualising in the z_2 direction, using the rules of [25], gives the circularly symmetric D7-brane (17) of the IIB theory. A further T-duality in the y direction gives the D8-brane solution (15). Then taking the limit of (22) in which the T^2 fibres are shrunk is given by first reducing on z_1 to obtain (26) and then T-dualising in the z_2 direction to obtain the D7-brane (17), while the limit in which the total space shrinks is given by making a further T-duality in the y direction to obtain the D8-brane (15).

In [27], it was argued that there should be an 'M9-brane' that gives rise to the D8-brane of the IIA theory, arising as a domain wall in M-theory, and in [28, 29], such branes were considered further. In particular, in [29] it was shown that such branes could not be $SO(9,1)$ invariant, but that one of the directions was special, in the same way that the KK monopole solution giving rise to the D6-brane is not $SO(7,1)$ invariant, and has a special compact direction corresponding to the Taub-NUT fibre. The solution (22) is a domain wall solution of M-theory that gives the D8-brane of the massive IIA theory in the limit in which the 3-space B shrinks to zero size, and so might be thought of as a type of M9-brane, with three special compact directions.

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