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Bearing fault diagnosis based on variational mode decomposition and total variation denoising

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Abstract

Feature extraction plays an essential role in bearing fault detection. However, the measured vibration signals are complex and non-stationary in nature, and meanwhile impulsive signatures of rolling bearing are usually immersed in stochastic noise. Hence, a novel hybrid fault diagnosis approach is developed for the denoising and non-stationary feature extraction in this work, which combines well with the variational mode decomposition (VMD) and majoriation–minization based total variation denoising (TV-MM). The TV-MM approach is utilized to remove stochastic noise in the raw signal and to enhance the corresponding characteristics. Since the parameter λ is very important in TV-MM, the weighted kurtosis index is also proposed in this work to determine an appropriate λ used in TV-MM. The performance of the proposed hybrid approach is conducted through the analysis of the simulated and practical bearing vibration signals. Results demonstrate that the proposed approach has superior capability to detect roller bearing faults from vibration signals.

Keywords: variational mode decomposition, weighted kurtosis index, rolling element bearing, denoising, fault diagnosis

(Some figures may appear in colour only in the online journal)

1. Introduction

Rolling element bearings are widely applied in rotating machinery and play an important role in modern manufacturing industries. The failure of rolling element bearings results in the deterioration of machine performance; it is thus necessary to accurately detect faults in the bearings [1, 2]. Vibration signal analysis is the most commonly used approach due to its easy measurement and high correlation with structural dynamics [3–6]. However, there are two challenging issues in vibration signal analysis: the non-stationary collected signals and signatures usually mingled with heavy noise caused by coupled machine components and the working environment [7, 8].

For the past few years, a range of methods have been developed to analyze the measured non-stationary vibration signals in the fields of fault diagnosis. For example, wavelet transform (WT) becomes a powerful tool for feature extraction because of its advantage of multi-resolution analysis [9, 10]. Yan [10] presented a review of the recent developments and applications of WT in the fields of fault diagnosis of rotary machines. Actually, similarity between the wavelet basis function and the defect-related signal characteristics plays a decisive role for successful detection. Thus, wavelet basis function should be appropriately selected in the WT. Nevertheless, it is sometimes not practical for a specific application. Some adaptive signal decomposition techniques emerge as the time requires, for example, empirical mode decomposition (EMD) pioneered by Huang in [11]. Subsequently, EMD technique finds wide applications in the fault diagnosis of rotating machines [12–14], due to its good intrinsic locally adaptive property in processing non-stationary signals. However, EMD still has some drawbacks, such as mode mixing and end effects. Rilling *et al* [15, 16] first completely probed into these issues of EMD.

Chen and Wang [17] presented a new analytical mode decomposition theorem based on the Hilbert transform which can address a number of issues associated with EMD. Ensemble empirical mode decomposition (EEMD), an extended version of EMD, has been developed to overcome the mode mixing of EMD in [18]. However, in order to reduce the errors caused by added white noise in EEMD, the algorithm of EMD should be conducted many times, which inevitably results in a large amount of computation burden [19]. The empirical wavelets transform (EWT) is proposed as a new adaptive data analysis method in [20], which could also partially address the limitations (sensitivity to noise and sampling) of EMD [21]. But EWT relies on robust preprocessing for peak detection, and the constructed frequency bands are still slightly strict.

Variational mode decomposition (VMD) has been lately proposed by Dragomiretskiy and Zosso [21]. Unlike the EMD, VMD is a non-recursive signal processing method with a pretty firm theoretical foundation. Meanwhile, its performances of no stationary signal analysis have been thoroughly studied and compared with other adaptive signal processing techniques. For example, the equivalent filter bank property of the VMD has been investigated in [22], and the results show that VMD can be considered as a wavelet packet decomposition or a generalized short-time Fourier transform with a varying window width. It is also demonstrated that VMD is more powerful in tone-separation and noise robustness in comparison with EMD [23]. In addition, feature vectors extracted by VMD are better than those achieved by EWT, which is much more suitable for the support vector machine based classifications [24]. Due to the advantages of VMD, it has been successfully utilized for identifying rubbing faults [25], instantaneous detection of voiced/non-voiced regions in speech signals [23] as well as trends analysis of financial markets [26], etc. However, VMD is still not appropriate for analysis of a vibration signal with strong background noise [21]. The presence of strong noise in the measured vibration signal is unavoidable. It is thus necessary to carry out denoising preprocessing of the raw signals prior to the VMD analysis.

In practice, in order to extract a true signal in noise, the most frequently used methods are based on filters. Unlike traditional low-pass filter denoising, total variation denoising (TVD) is actually defined based on an optimization problem. TVD was initially proposed by Rudin and was applied to remove noise in images [27]. Since then, TVD has been widely researched and extensively adopted in one-dimensional (1D) signal processing [28–30] due to its advantages in preserving the sharp edges of the given signals. Since the output of the TVD is obtained by minimizing a particular cost function, TVD has been further developed with the majorization–minimization algorithm (TV-MM) [31]. It has been demonstrated that it can take little time for TV–MM to achieve a good de-noising result [32]. Nevertheless, it is also worth noting that the regularization parameter λ used in TV-MM could seriously affect its denoising performance.

Consequently, a hybrid approach based on TV-MM and VMD techniques for bearing fault diagnosis is proposed in this work. The weighted kurtosis index is also first proposed to determine an appropriate λ used in TV-MM. The rest of this paper is organized as follows: TV-MM and VMD are briefly introduced in section 2. The weighted kurtosis index is proposed to select parameter λ in section 3, where its performances in denoising are also compared with the morphological filter technique through simulated bearing vibration signals. In section 4, the effectiveness of the proposed approach is further verified using the practical bearing vibration signals. Conclusions are drawn in section 5.

2. Background

2.1. A brief introduction of VMD

VMD can non-recursively decompose a multi-component input signal into a discrete set of quasi-orthogonal band-limited intrinsic mode functions (IMFs) (BLIMFs) [21], which are in accordance with the new definition of IMF described in [33]. Each mode u_k is almost compact around a matching center frequency ω_k, and its bandwidth is assessed by means of H^1 Gaussian smoothness.

The process of VMD can be considered as a constrained variational problem, while the formulation of the constrained variational problem is written [21]:

$$\min_{\{u_k\},\{\omega_k\}}\left\{\sum_{k=1}^{K}\left\|\partial_t\left[\left(\delta(t)+\frac{\mathrm{j}}{\pi t}\right)*u_k(t)\,\mathrm{e}^{-\mathrm{j}\omega_k t}\right]\right\|_2^2\right\}$$

$$\text{subject to }\sum_{k=1}^{K}u_k(t)=f(t) \tag{1}$$

The solution to equation (1) can be easily achieved via an unstrained optimization problem using the augmented Lagrangian method [21]

$$\mathcal{L}(\{u_k\},\{\omega_k\},\lambda)=\alpha\sum_{k=1}^{K}\left\|\partial_t\left[\left(\delta(t)+\frac{\mathrm{j}}{\pi t}\right)\mathrm{e}^{-\mathrm{j}\omega_k t}\right]\right\|_2^2$$
$$+\left\|f(t)-\sum_{k=1}^{K}u_k(t)\right\|_2^2+\left\langle\lambda(t),f(t)-\sum_{k=1}^{K}u_k(t)\right\rangle \tag{2}$$

An alternating direction method of multipliers is adopted to solve equation (2). The estimated modes u_k and the corresponding updated center frequency in the frequency domain can be written as follows:

$$\hat{u}_k^{n+1}(\omega)=\frac{\hat{f}(\omega)-\sum_{i<k}\hat{u}_i^{n+1}(\omega)-\sum_{i>k}\hat{u}_i^{n}(\omega)+\frac{\hat{\lambda}^n(\omega)}{2}}{1+2\alpha(\omega-\omega_k^n)^2} \tag{3}$$

$$\omega_k^{n+1}=\frac{\int_0^\infty\omega\left|\hat{u}_k^{n+1}(\omega)\right|^2\mathrm{d}\omega}{\int_0^\infty\left|\hat{u}_k^{n+1}(\omega)\right|^2\mathrm{d}\omega} \tag{4}$$

where α is the balancing parameter of the data-fidelity constraint. More specifically, the process of VMD can be summarized as follows:

(1) Initialize mode $\{u_k^1\}$, central frequency $\{\omega_k^1\}$, Lagrangian multiplier λ^1 and iterations n.
(2) Update u_k according to equations (2) and (3)
(3) For all $\omega \geqslant 0$, update λ_k using

$$\widehat{\lambda}^{n+1}(\omega) \leftarrow \widehat{\lambda}^{n}(\omega) + \tau\left(\widehat{f}(\omega) - \sum_k \widehat{u}^{n+1}(\omega)\right)$$

(4) ε is denoted as the accuracy for convergence. The iteration of the algorithm does not stop until convergence, i.e.

$$\sum_k \|\widehat{u}_k^{n+1} - \widehat{u}_k^n\|_2^2 / \|\widehat{u}_k^n\|_2^2 < \varepsilon$$

More details of the VMD algorithm can be found in [21]. In addition, initialization and input parameters (balancing parameter α and number of modes k) are both key parameters of VMD. Compared with uniformly spaced distribution, VMD can get much more reliable and meaningful results with zero initial for the detecting transient signatures, according to the equivalent filter structure of VMD given in [22, 25]. Meanwhile, a small value of α parameter will be used for the purpose of detecting impacts [22]. Thus, α is set to 2500, and the number of mode k is set to 4 in this paper. Lagrangian multipliers can strictly enforce constraints and it will be shut-off when its update parameter τ is 0 [21]. Therefore, an reasonable update parameter should be considered in a Lagrangian multiplier. However, it is very difficult to explicitly determine the update parameter τ in theory. It has been demonstrated in the following simulations and experimental investigations that the update parameter $\tau = 0.3$ is reasonable. Thus, the update parameter used in the Lagrangian multiplier τ is set to 0.3, which can ensure the fidelity of the signal decomposition, especially in the presence of Gaussian noise. Although VMD has some robustness to noise, Lagrangian multipliers are not useful for recovering modes any more when the given signal contains stronger random noises [21]. Therefore, a proper denoising method prior to VMD is necessary for bearing fault diagnosis.

2.2. Majoriation–minization algorithm based TV denoising

Considering the presence of strong noise in the raw vibration signal, total variation de-noising based on the MM algorithm is introduced. The total variation de-noising method can be considered as a numerical optimization algorithm involving a quadratic data fidelity term and a convex regularization term [31, 32]. Given the 1D signal $x(n)$ $(0 \leqslant n \leqslant N-1)$, the total variation of signal $x(n)$ is defined as:

$$\mathrm{TV}(x) := \|Dx\|_1 = \sum_{n-1}^{N-1} |x(n) - x(n-1)| \qquad (5)$$

in which Dx represents the first order differential, $\|\cdot\|_p$ $(p \geqslant 1)$ is the ℓ_p norm. Assuming that signal $x(n)$ is contaminated by additive white Gaussian noise $w(n)$, i.e.

$$y(n) = x(n) + w(n)$$

the estimates of the signal $x(n)$ using TV de-noising can be written below,

$$F(x) = \|y - x\|_2^2 + \lambda\|Dx\|_1 \qquad (6)$$

where $\|y - x\|_2^2$ is referred to as the data fidelity term and $\lambda \geqslant 0$ is a regularization parameter. That is to say, the aim is to find the signal x that minimizes the objective function with respect to

$$x \in R^N,\ x = \arg\min_x F(x) \qquad (7)$$

where the matrix D is defined as follows

$$D = \begin{bmatrix} -1 & 1 & & & & \\ & -1 & 1 & & & \\ & & \ddots & \ddots & & \\ & & & -1 & 1 & \\ & & & & -1 & 1 \end{bmatrix}$$

Then, DD^T is a three diagonal matrix:

$$DD^T = \begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & & \ddots & & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{bmatrix}$$

As is well known, the MM algorithm is an effective method in solving optimization problems that are difficult to solve directly. Although the MM algorithm needs to address a linear system at each iteration, its rate of convergence is faster than that of the 'iterative clipping' algorithm [32]. For more discussions of this algorithm, interested readers can refer to [31]. As a result, the optimized objective function given in equation (7) can be formulated as [32],

$$X_{k+1} = y - D^T\left(\frac{1}{\lambda}\mathrm{diag}(|DX_k|) + DD^T\right)^{-1} Dy \qquad (8)$$

The TV de-noising algorithm can be achieved based on equation (8). However, parameter λ tends to have great influences on the performance of the TV-MM denoising in practical applications. Thus, a feasible and effective selection of the parameter method for TV-MM should be explored. A novel parameter selection method is developed for the TV-MM denoising in the following section.

3. The proposed hybrid fault diagnosis approach

3.1. Parameter selection of TV-MM

The parameter λ is important in TV-MM, which may greatly affect its denoising performance. As can be found in equation (6), λ controls the weight of the second term that indicates the fluctuation. Obviously, as λ is infinitely close to zero, the term of the total variation ceases to be a penalty function, namely, the signal obtained by this denoising algorithm is the same as the original signal. On the contrary, as λ approaches to infinity, the total variation is dominant. Meanwhile, the objective function will be very small. Nevertheless, the fidelity will be very poor, which leads to the deviation of the de-noised signal from the original signal x. As a result, the denoising

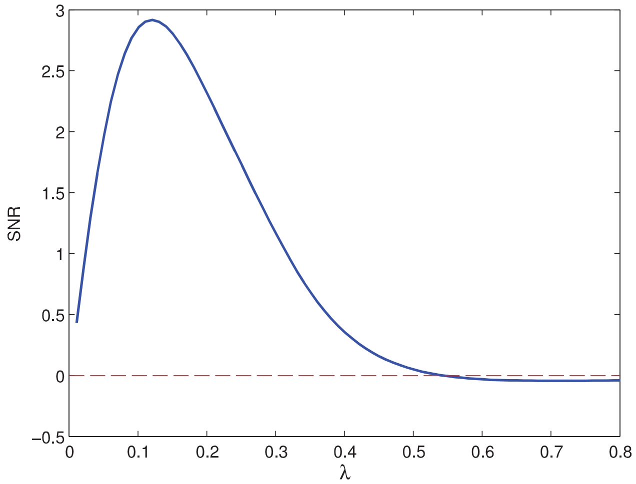

Figure 1. The influence of parameter λ on the signal to noise ratio.

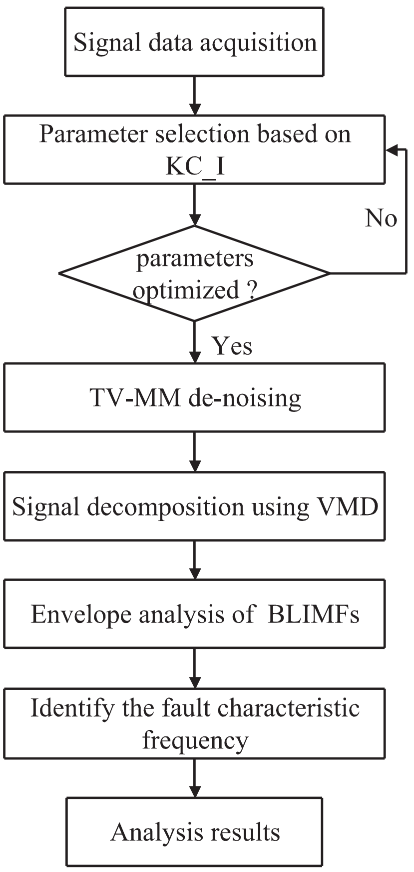

Figure 2. The flow chart of the proposed method.

performance will be very poor in this case. The influences of the parameter λ on the signal to noise ratio (SNR) are illustrated in figure 1. As can be seen in figure 1, a large value of λ will excessively remove noise as well as fine signatures, which results in a large kurtosis of the output signal. On the contrary, a small value of λ used in the algorithm will not effectively remove noise.

With regards to the selection of parameter λ, an approach has been described in [34] based on the Stein unbiased risk estimator (SURE). However, the noise variance σ^2 is required for the SURE, while errors may be caused by the estimation of

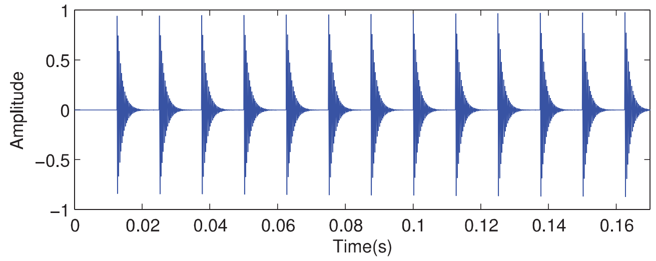

Figure 3. Simulated vibration signal $s(t)$ without noise.

the noise variance. In addition, $\lambda = \sqrt{3}\,\sigma$ is a usually used and verified experimentally in [31, 35]. In this work, a weighted kurtosis index approach is proposed to select a suitable λ. The weighted kurtosis index (KC_I) is achieved by the kurtosis index (K_I) and correlation coefficient index (C_I), as represented in equation (11). As is well known, the kurtosis of the bearing vibration signal is a very important indicator, and its maximization is often adopted in fault diagnosis. Whereas it is not always appropriate to simply use kurtosis maximization as a signal denoising indicator. The reason is that features embedded in a given signal may be sometimes removed to some extent, if we only consider the kurtosis maximization criterion. Correlation coefficients, as a supplement, can ensure a certain similarity between the original signal and the denoised signal. Therefore, the proposed method can still ensure fidelity of the denoised signal. The regularization parameter λ is set to $\lambda \geqslant 0$ which controls the smoothness of the signal. It can be seen in figure 1 that SNR is close to a constant value when $\lambda \geqslant 0.6$ is satisfied. As a result, [0, 1] is selected as a region in order to get a more reasonable parameter λ.The process of the parameter λ selection is described as follows: First, the range of parameter λ is set to [0, 1]. Then, the maximum KC_I of the denoised signal will be searched in this region. Thus, the parameter λ value and the corresponding maximum KC_I are achieved. Finally, the value of parameter λ can be considered as an optimal parameter.

$$K\_I = \frac{\frac{1}{n}\sum_{i=1}^{N}(x_i - \bar{x})}{\left(\frac{1}{n}\sum_{i=1}^{N}(x_i - \bar{x})^2\right)^2} \tag{9}$$

$$C\_I = \frac{\mathrm{cov}(x, y)}{\sigma_x \sigma_y} = \frac{E((x - \bar{x})(y - \bar{y}))}{\sigma_x \sigma_y} \tag{10}$$

$$KC\_I = K\_I \times |C\_I|^r \tag{11}$$

where r is an adjustable positive real number and is applied to add fidelity of the denoised signal. In this paper, r is set to 2 based on the results of the experiment.

3.2. TV-MM denoising combined with VMD

When the parameter λ used in the TV-MM algorithm is determined, a hybrid approach is developed to detect bearing fault based on the TV-MM and VMD. The proposed method for bearing fault diagnosis is schematically shown in figure 2.

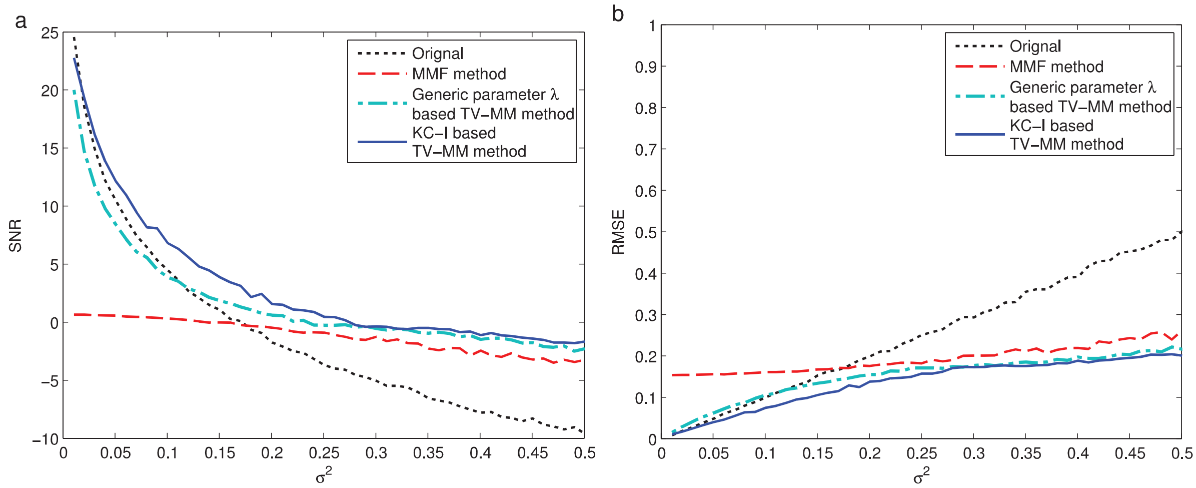

Figure 4. SNR and RMSE of the de-noised signal with TV-MM and MMF.

3.3. Validations and comparisons

Compared with total variation de-noising, the morphological morphology filter (MMF) has been initially used in image processing and has attracted lots of attention due to its little computation burden [36, 37]. To further evaluate the performance of the TV-MM method, it is compared with MMF through simulated signals in this section. A simulated signal of rolling bearing failure is written as follows:

$$s(t)=\sum_{k=0}^{\infty}A_k h(t-kT-\tau_k)+n(t)=x(t)+n(t) \tag{12}$$

$$A_k=1+A_0\sin(2\pi f_r t) \tag{13}$$

$$h(t)=\mathrm{e}^{-Ct}\sin(2\pi f_n t) \tag{14}$$

where $x(t)$ is the original periodical impulsive signal, and $n(t)$ is Gaussian white noise. The fault characteristic frequency $f_0=1/T$, random fluctuation of T due to slippage τ_k, initial amplitude of the impulse A_0, rotational frequency f_r, decay factor C and resonant frequency f_n have been set to 80 Hz, 0, 0.3, 20 Hz, 700 and 3 kHz, respectively. In this paper, sampling frequency f_s is set to12 kHz, and the data length is 2048.

The simulated signal $s(t)$ without Gaussian white noise is shown in figure 3. The performance of the two denoising methods are evaluated based on SNR and the root mean square error (RMSE), which are defined below:

$$\mathrm{SNR}=10\log\left\{\frac{\sum_{i=1}^{N}s^2(i)^2}{\sum_{i=1}^{N}(s(i)-\hat{s}(i))^2}\right\} \tag{15}$$

$$\mathrm{RMSE}=\sqrt{\frac{1}{N}\sum_{i=1}^{N}(s(i)-\hat{s}(i))^2} \tag{16}$$

in which $s(i)\,(i=1,2,\dots N)$ is the original periodical signal, while $s(i)\,(i=1,2,\dots N)$ is the purified signal.

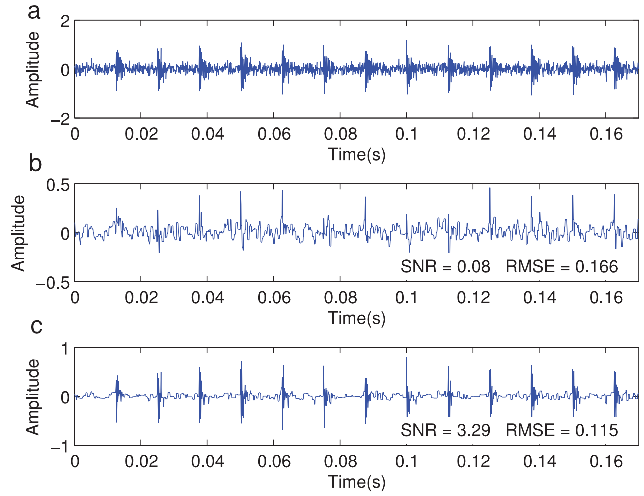

Figure 5. The time domain waveform of de-noising results obtained using two de-noising algorithms. (a) Original vibration signal, (b) the filtered signal with MMF, (c) the filtered signal with TV-MM.

It can be seen in figure 4 that the SNR and RMSE are better when parameter λ is selected based on the proposed weighted kurtosis index, rather than using the universal value ($\lambda=\sqrt{3}\,\sigma$). Figure 4 also indicates that the TV-MM algorithm is much more effective than the MMF algorithm. In what follows, the SNR of a noisy signal with 1.5 dB is used to show its denoised waveforms. The results of the two methods are shown in figure 5. It can be seen that the TV-MM algorithm can eliminate noise and preserve impulsive characteristics as much as possible, but several features of the original signal are removed when MMF is adopted. Results indicate that the TV-MM algorithm outperforms MMF through the comparisons. Thus, TV-MM will be used as a preprocessing technique prior to VMD.

VMD is applied to decompose the denoised signal with TV-MM and MMF techniques, respectively. The decomposed four BLIMFs are shown in figure 6. VMD has a good ability to detect impacts hidden in a signal [22]. As is illustrated in figure 6, impulsive components have been well extracted. Meanwhile, the results indicate that figure 6(b) is better than

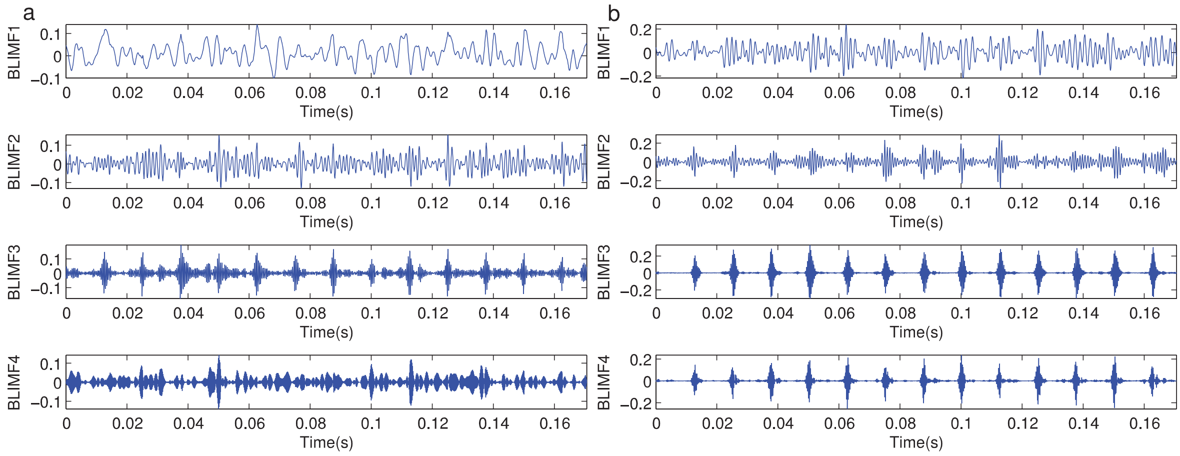

Figure 6. BLIMFs of the de-noised signal using VMD (a) denoising with MMF (b) denoising with TV-MM.

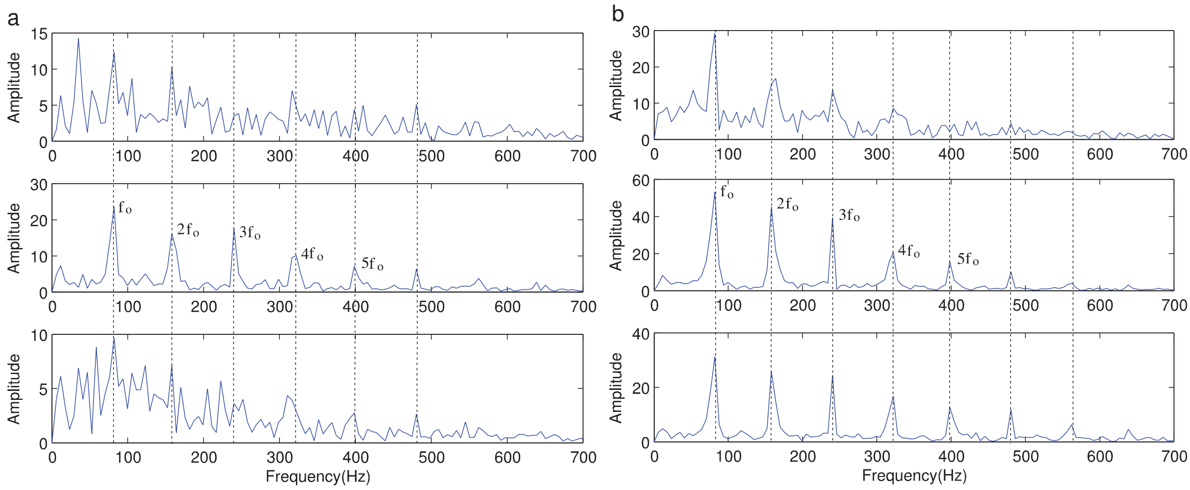

Figure 7. Envelopes of the BLIMF2, BLIMF3 and BLIMF4.

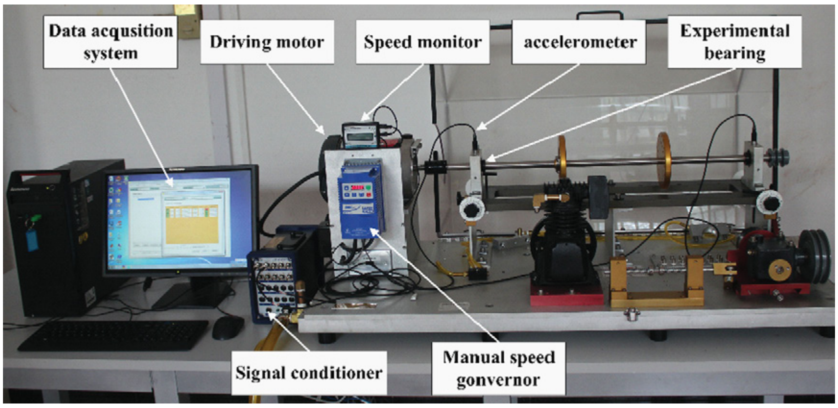

Figure 8. Machinery fault simulator (MFS)-magnum experimental platform.

figure 6(a), which demonstrates that the proposed hybrid technique is effective in detecting periodical impulsive signatures. Except for the first low-frequency BLIMF1 components, the Hilbert envelope spectra of the BLMF2–BLMF4 components are all shown in figure 7. Although both methods can extract the fault characteristic frequency, it is obvious that TV-MM combined with VMD can achieve a better performance than MMF. Its effectiveness will be further investigated using practical bearing vibration signals.

4. Experimental evaluations

In this section, two kinds of bearing (outer race defect and inner race defect) signals acquired on an MFS-Magnum test-rig are adopted to verify the effectiveness of the proposed method. The test-rig is illustrated in figure 8. Type ER-12K bearings are used in the experiments, whose specifications are provided in table 1. Parameters relating to the fault signature are listed in table 2, in which the theoretical values

Table 1. Specifications of the bearing.

Inside diameter (mm)	Outside diameter (mm)	Pitch diameter (mm)	Number of rolling elements (mm)	Ball diameter (mm)
25.4	52	33.4772	8	7.9375

Table 2. Parameters related to fault.

Defect location	Rotational speed (r min^{-1})	Sampling frequency f_s (kHz)	Rotation frequency f_r (Hz)	Fault characteristic frequency (Hz)
Outer race	1790	12.8	29.83	90.96
Inner race	1792	25.6	29.87	147.84

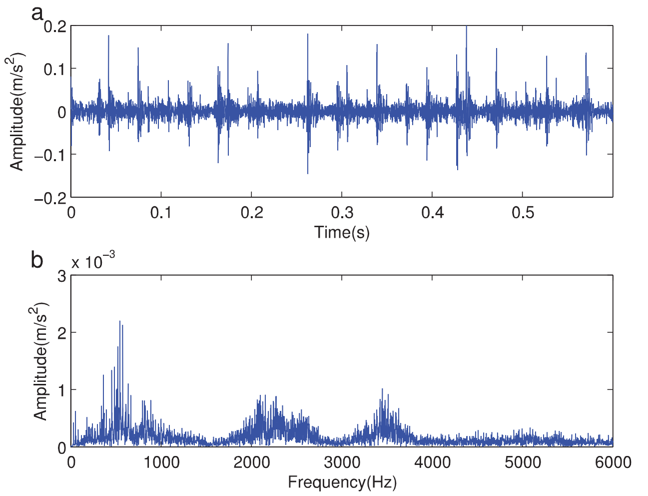

Figure 9. Vibration signal of bearing with an outer race fault (a) time domain waveform, (b) fast Fourier transform (FFT) spectrum.

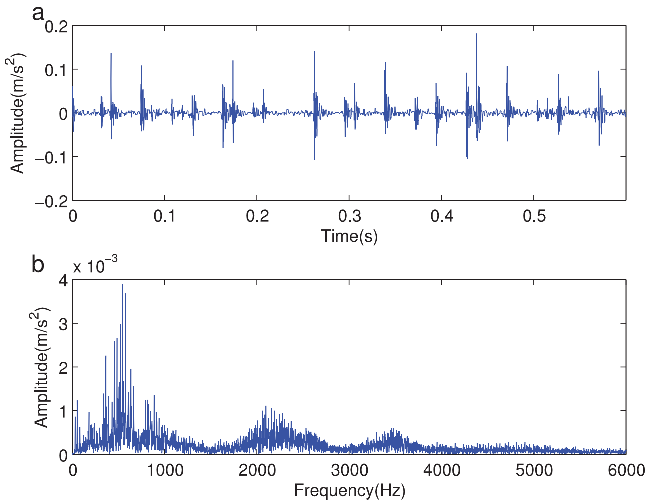

Figure 10. Result of TV-MM denoising (a) time-domain waveform, (b) its FFT spectrum.

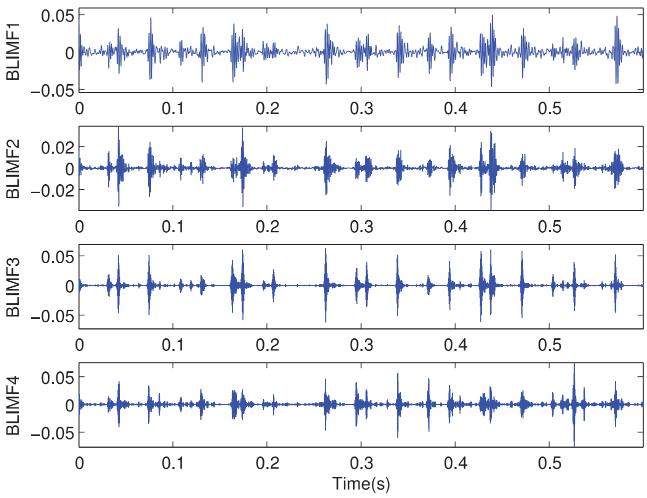

Figure 11. The decomposed BLIMFs of the purified signal using VMD.

of fault characteristic frequency are calculated via equations (17) and (18).

$$\mathrm{BPFO} = \frac{N_b}{2} * f_r * \left(1 - \frac{B_d}{P_d} * \cos\theta\right) \tag{17}$$

$$\mathrm{BPFI} = \frac{N_b}{2} * f_r * \left(1 + \frac{B_d}{P_d} * \cos\theta\right) \tag{18}$$

where BPFO denotes the ball passing frequency of the outer race, BPFI is the ball passing frequency of the inner race, N_b is the number of balls, f_r is the rotational frequency, B_d is the ball diameter, P_d is the pitch diameter, and θ is the contact angle. Vibration data were collected using a VibraQuest data acquisition system with an accelerometer (sensitivity 98 mV g^{-1}) fixed near the bearing bases.

4.1. Bearing outer race fault detection

The raw vibration signal of the bearing with an outer race defect is shown in figure 9. As is well known, the vibration signal should present the impulsive features due to the presence of a defect in the rolling bearing. However, weak signatures in the time domain are often contaminated because of the existing strong ambient noises.

The purified signal using TV-MM is shown in figure 10. Compared with the original signal, noises are successfully removed in figure 10, and impulsive features are well retained. In addition, characteristic frequencies are concentrated in the low frequency band.

The VMD method is then applied to the purified signal, and the decomposed results are shown in figure 11. The envelope spectra of the BLIMF2–BLIMF4 are shown in figure 12. As is illustrated in figure 12, the rotating frequency f_r and the outer race fault characteristic frequency BPFO is clearly revealed.

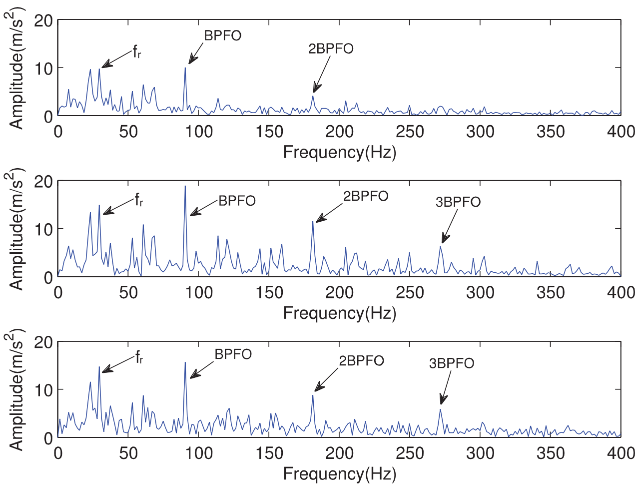

Figure 12. The envelope spectra of the BLIMF2–BLIMF4 (the purified signal with TV-MM).

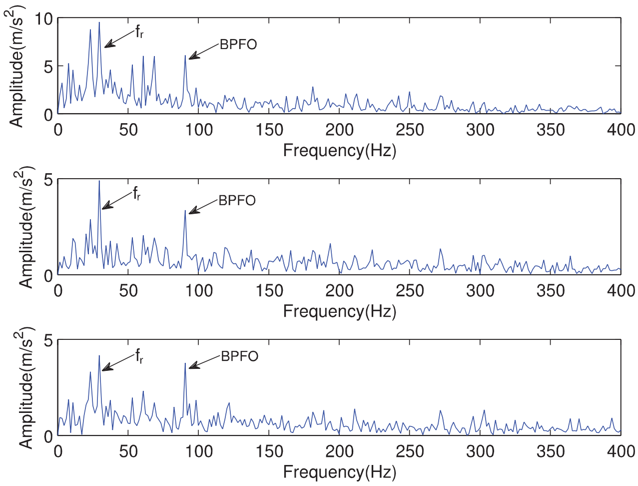

Figure 13. The envelope spectra of the latter three BLIMFs (the purified signal with MMF).

Moreover, the purified signal using MMF is also decomposed with VMD. The achieved Hilbert envelope spectra of the BLIMF2–BLIMF4 are shown in figure 13. It can be seen that the detected characteristic frequencies are not as good as those given in figure 12.

4.2. Detection of bearing inner race fault

In the case of the bearing inner race defect, the vibration signal is an amplitude modulated waveform due to the rotating of the inner race. Therefore, the side-band is expected at two frequencies, i.e. BPFI $\pm f_r$. The original vibration signal and the purified signal using TV-MM are presented in figures 14(a) and (b), respectively. The envelope spectrum of BLIMF4 is shown in figure 15(a), which contains the most abundant fault information among the four BLIMFs. In figure 15(a), it can be seen that BPFI, 2BPFI and 3BPFI along with the side bands (147.84 $\pm$ 29.87 Hz, 295.68 $\pm$ 29.87 Hz, 443.52 $\pm$ 29.87 Hz, 591.36 Hz + 29.87 Hz) are all prominent. It reveals that there exists an inner race fault

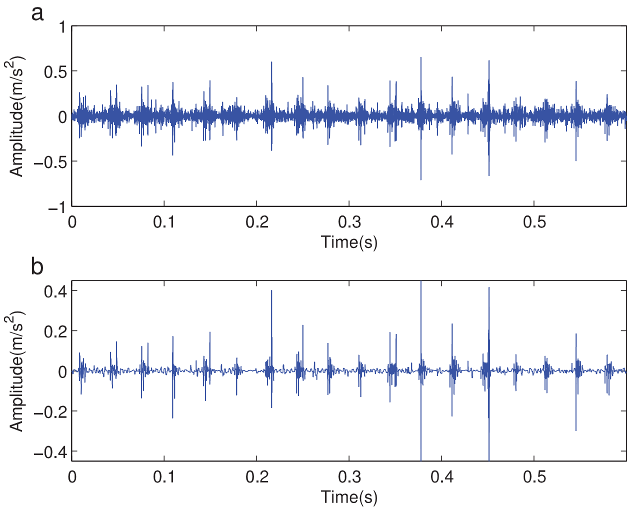

Figure 14. Vibration signal of bearing with an inner race fault (a) the raw signal, (b) the purified signal with TV-MM.

in the bearing. On the contrary, as is shown in figure 15(b), the envelope spectrum of BLIMF4 achieved by VMD and MMF can only indicate part of the signatures. In addition, it is also demonstrated that parameter λ selection technique based on the proposed weighted kurtosis index is appropriate for TV-MM denoising.

5. Conclusions

In this paper, the weighted kurtosis index is introduced into TV-MM de-noising, which can adaptively select an appropriate regularization parameter during the iterations. The de-noising results are better when the adaptive selection of λ is used in TV-MM, compared with the other general choice of λ. Moreover, a hybrid method which combines TV-MM and VMD is proposed to the bearing fault detection. VMD is applied to extract impulsive signatures of the purified signals via TV-MM denoising. The proposed hybrid approach is evaluated by simulated signals and real bearing vibration signals. A series of experimental tests corresponding to different bearing health conditions have demonstrated the superior capability of the proposed technique to the related classical bearing fault detection methods, especially in the aspect of denoising and non-stationary feature extraction.

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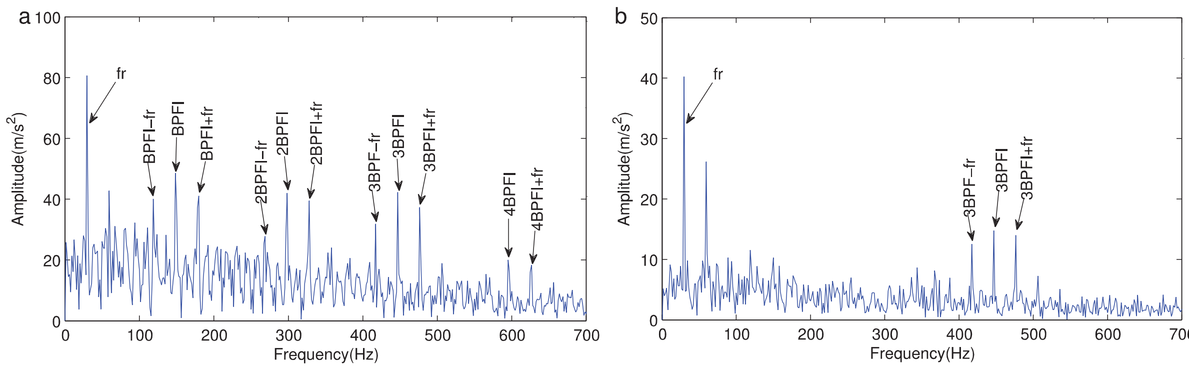

Figure 15. (a) The envelope spectrum of the fourth BLIMF (the purified signal with TV-MM), (b) the envelope spectrum of the fourth BLIMF (the purified signal with MMF).

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