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Spin waves in superlattices: III. Magnetic polaritons in the Voigt configuration

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Abstract. Dispersion relations for bulk and surface magnetic polaritons in semi-infinite layered structures composed of two different magnetic materials are analysed theoretically. The considerations are restricted to the case of in-plane magnetisation and to wave propagation in the Voigt geometry. Some exemplary wave spectra are presented for structures composed of magnetic and non-magnetic materials as well as for structures where both constituents are magnetic.

1. Introduction

Magnetic layered structures are a subject of growing interest. Recently, for example, interesting magnetic phase transitions (Camley and Tilley 1988) and electronic transport properties (Baibich *et al* 1988) have been found in artificially layered systems with an effective antiferromagnetic coupling between ferromagnetic layers. Excitation spectra of magnetic superlattices also provide an attractive subject for theoretical and experimental investigations. Such spectra have been analysed in many special cases including exchange (Albuquerque *et al* 1986, Dobrzynski *et al* 1986, Hinchey and Mills 1986, Vayhinger and Kronmüller 1986, McKnight and Vittoria 1987, Barnaś 1988a, Hillebrands 1988, Morkowski and Szajek 1988, Mathon 1989), magnetostatic (Camley *et al* 1983, Kueny *et al* 1984, Hillebrands *et al* 1986, Camley and Cottam 1987, Rupp *et al* 1987, Barnaś 1988b, Villeret *et al* 1989) and retarded (Barnaś 1987, Raj and Tilley 1987, 1989, Zhu and Cao 1987, Zhou and Gong 1989, How and Vittoria 1989) limits.

In a previous paper (Barnaś 1988a) some general dispersion equations for bulk and surface excitations have been derived for semi-infinite superlattices with arbitrary number of different magnetic layers in an elementary unit. These equations have subsequently been applied to magnetostatic modes (Barnaś 1988b). Here, we apply them to retarded waves propagating in the Voigt geometry.

As is well known, the retardation corrections to the spectrum of magnetic excitations are significant in the long-wavelength limit, where the 'pure' magnetic excitations of the system (magnons) interact with 'pure' electromagnetic waves (photons). The resulting magnon–photon mixed states are called magnetic polaritons or simply magnetic retarded waves. The retarded waves supported by a single layer magnetised in the film plane have been studied theoretically by Karsono and Tilley (1978), Marchand and Caille (1980),

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Lima and Oliveira (1983) and others. If the magnetic films form an infinite or semi-infinite periodic structure, the wave spectrum has new interesting features which are characteristic for artificially layered systems.

Bulk magnetic polaritons in magnetic/non-magnetic (M/N) layered structures were discussed by the present author (Barnaś 1987). Retarded waves in magnetic/magnetic (M/M) infinite superlattices were analysed by Raj and Tilley (1987), who developed also the effective-medium theory for such systems. Here we extend the considerations to surface modes propagating in semi-infinite M/M superlattices.

General dispersion equations for bulk and surface retarded modes in semi-infinite superlattices are given in section 2. The explicit form of these equations for M/M superlattices are derived in section 3. Numerical results for wave spectra in M/N and M/M structures are presented and discussed in section 4 and section 5, respectively.

2. General dispersion equations

Let us first discuss some general dispersion equations for bulk and surface retarded modes propagating in the Voigt geometry. As derived in the previous paper (Barnaś 1988a) the bulk waves are described by the equation

$$\cos(ql) = \tfrac{1}{2}(T_{11} + T_{22}) \tag{1}$$

where q is the wavevector component along the superlattice direction ($-\pi/l \leqslant q < \pi/l$) and l is the superlattice parameter (thickness of the elementary unit). T_{11} and T_{22} are the diagonal elements of the appropriate 2×2 'transfer matrix' $\mathbf{T}$ which depends on the frequency ω and the in-plane wavevector $\boldsymbol{k}_{\parallel}$. This dependence is not written explicitly in equation (1) and, for simplicity, will also be dropped in the following.

Frequencies of surface waves propagating in a semi-infinite structure are determined by the equation

$$D_1 D_2 (T_{11} - T_{22}) - D_1^2 T_{21} + D_2^2 T_{12} = 0 \tag{2}$$

where D_1 and D_2 result from the appropriate boundary conditions and depend on ω and $\boldsymbol{k}_{\parallel}$, in general, whereas T_{12} and T_{21} are the off-diagonal elements of the 'transfer matrix'. The corresponding decay parameter $\alpha_<$ in the free half-space is given by

$$\alpha_< = (k_{\parallel}^2 - \omega^2/c^2)^{1/2}. \tag{3}$$

In the occupied half-space the decay parameter β can be determined from the equation

$$\exp(-\beta l) = T_{11} + T_{12} D_2/D_1 \tag{4}$$

for $T_{12} \neq 0$. If $T_{12} = 0$ then $\exp(-\beta l) = T_{11}$ for $D_2 = 0$ and $\exp(-\beta l) = T_{22}$ for $D_1 = 0$. Only those solutions of equation (2) for which

$$\mathrm{Re}\,\beta > 0 \tag{5a}$$

and

$$\alpha_< > 0 \tag{5b}$$

correspond to surface waves. For real ω the parameter $\alpha_<$ can be either real (and positive by definition) or pure imaginary. The conditions (5b) correspond then to real and positive $\alpha_<$.

In a general case there are three possible solutions for β:

(i) β real;
(ii) β imaginary;
(iii) β complex.

The third possibility was omitted previously (Barnaś 1988a). However, it may occur for complex 'transfer matrix' **T**. If the matrix **T** is real, as in the case of magnetostatic modes in the Voigt configuration (Barnaś 1988b), only the first two possibilities can occur.

Equations (1)–(5) determine bulk and surface modes in semi-infinite structures. Dynamical properties of a system are contained in the appropriate 'transfer matrix' **T** which is characteristic of a particular case.

3. Explicit dispersion equations for bulk and surface polaritons in magnetic/magnetic superlattices

Consider a semi-infinite layered structure composed of two different magnetic materials (ferromagnetic or antiferromagnetic). Suppose that the films are magnetised in the film plane and parallel to the z axis of the coordinate system, along which a static magnetic field H_0 is applied. The axis x of this system is assumed to be normal to the films.

We shall consider only transverse electric (TE) modes with an electric field parallel to the axis z. The second polarisation (transverse magnetic modes with a magnetic field along the z axis) is a trivial case since there is no linear coupling of the corresponding electromagnetic field with the magnetic structure of the system and the appropriate modes are 'pure' photons. For simplicity we neglect also dielectric properties of both materials.

Employing now the recurrence formulae derived previously (Barnaś 1988a) one finds the following expressions for the matrix **T** in the Voigt geometry ($\boldsymbol{k}_\parallel = k\boldsymbol{e}_y$ with $\boldsymbol{e}_y$ being the unit vector along the y axis):

$$T_{11(22)} = \exp(\pm\alpha^{(1)}d_1)\,[\cosh(\alpha^{(2)}d_2) \pm \sinh(\alpha^{(2)}d_2) \times (2\mu_X^{(1)}\mu_X^{(2)}k^2 + \mu_\perp^{(1)}\mu_V^{(1)}\gamma^{(2)} + \mu_\perp^{(2)}\mu_V^{(2)}\gamma^{(1)})/2\mu_\perp^{(1)}\mu_\perp^{(2)}\alpha^{(1)}\alpha^{(2)}] \tag{6a}$$

$$T_{12(21)} = \pm\{[\exp(\mp\alpha^{(1)}d_1)\sinh(\alpha^{(2)}d_2)]/2\mu_\perp^{(1)}\mu_\perp^{(2)}\alpha^{(1)}\alpha^{(2)}\}\,[2\mu_X^{(1)}\mu_X^{(2)}k^2 \pm 2\mu_\perp^{(1)}\mu_X^{(2)}\alpha^{(1)}k + \mu_\perp^{(2)}\mu_V^{(2)}\gamma^{(1)} - (\gamma^{(2)}/\gamma^{(1)})(\pm\mu_\perp^{(1)}\alpha^{(1)} + \mu_X^{(1)}k)^2] \tag{6b}$$

where the lower sign refers to the indices in parentheses, and $\alpha^{(i)}$ and $\gamma^{(i)}$ (the index i distinguishes the two different materials; $i = 1, 2$) are defined as follows:

$$\alpha^{(i)} = (k^2 - (\omega^2/c^2)\mu_V^{(i)})^{1/2} \tag{7}$$

$$\gamma^{(i)} = (k^2 - (\omega^2/c^2)\mu_\perp^{(i)}). \tag{8}$$

In the above equations, $\mu_\perp^{(i)}$ and $\mu_X^{(i)}$ ($i = 1, 2$) are the diagonal and off-diagonal components of the linear dynamic magnetic permeability tensor:

$$\mu_{xx}^{(i)} = \mu_{yy}^{(i)} = \mu_\perp^{(i)} \tag{9a}$$

$$\mu_{xy}^{(i)} = -\mu_{yx}^{(i)} = \mathrm{i}\mu_X^{(i)} \tag{9b}$$

and $\mu_V^{(i)}$ is the magnetic permeability in the Voigt configuration:

$$\mu_V^{(i)} = \mu_\perp^{(i)} - \mu_X^{(i)2}/\mu_\perp^{(i)}. \tag{10}$$

Equations (6a) and (6b) do not apply at frequencies at which $\gamma^{(1)} = 0$, $\alpha^{(1)} = 0$ or

$\alpha^{(2)} = 0$. These special cases have to be treated separately. One can show, however, that the boundary conditions cannot be fulfilled at these frequencies.

Dispersion equation (1) for the bulk polaritons can be written explicitly as

$$\cos(ql) = \cosh(\alpha^{(1)}d_1)\cosh(\alpha^{(2)}d_2) + \sinh(\alpha^{(1)}d_1)\sinh(\alpha^{(2)}d_2) \times (2\mu_X^{(1)}\mu_X^{(2)}k^2 + \mu_\perp^{(1)}\mu_V^{(1)}\gamma^{(2)} + \mu_\perp^{(2)}\mu_V^{(2)}\gamma^{(1)})/2\mu_\perp^{(1)}\mu_\perp^{(2)}\alpha^{(1)}\alpha^{(2)} \tag{11}$$

where $l = d_1 + d_2$ with d_i ($i = 1, 2$) being the thicknesses of the elementary layers.

If the semi-infinite superlattice occupies the half-space $x > 0$ and the first layer corresponds to the material index $i = 1$, then the surface magnetic polaritons are described by equation (2) with

$$D_{1(2)} = 1 \pm (\gamma^{(1)} + \mu_X^{(1)}\alpha_< k)/\mu_\perp^{(1)}\alpha^{(1)}\alpha_< \tag{12}$$

and with the 'transfer matrix' given by equations (6*a*) and 6*b*). The explicit form of this dispersion equation can be written as

$$f_1 \sinh(\alpha^{(1)}d_1)\cosh(\alpha^{(2)}d_2) + f_2\cosh(\alpha^{(1)}d_1)\sinh(\alpha^{(2)}d_2) + f_3\sinh(\alpha^{(1)}d_1)\sinh(\alpha^{(2)}d_2) = 0 \tag{13}$$

where

$$f_1 = \mu_\perp^{(2)}\alpha^{(2)}(\alpha_<^2\mu_\perp^{(1)}\mu_V^{(1)} - 2\alpha_<\mu_X^{(1)}k - \gamma^{(1)}) \tag{14a}$$

$$f_2 = \mu_\perp^{(1)}\alpha^{(1)}(\alpha_<^2\mu_\perp^{(2)}\mu_V^{(2)} - 2\alpha_<\mu_X^{(2)}k - \gamma^{(2)}) \tag{14b}$$

$$f_3 = (\mu_X^{(1)}\gamma^{(2)} - \mu_X^{(2)}\gamma^{(1)})k + \alpha_<\mu_\perp^{(2)}\mu_V^{(2)}(\alpha_<\mu_X^{(1)}k + \gamma^{(1)}) - \alpha_<\mu_\perp^{(1)}\mu_V^{(1)}(\alpha_<\mu_X^{(2)}k + \gamma^{(2)}). \tag{14c}$$

Equation (13) determines surface solutions for a superlattice with two different magnetic layers in the elementary unit. The corresponding decay parameters in the free and occupied half-spaces can be then found from equation (3) and equation (4), respectively. However, only those solutions of equation (13), for which equations (5*a*) and (5*b*) are fulfilled describe true surface waves.

The dispersion equations (11) and (13) are applicable to superlattices composed of ferromagnetic as well as uniaxial (with the in-plane easy axis) antiferromagnetic materials. Equation (11) was first derived by Raj and Tilley (1987). It includes also the known results for M/N layered structures (Barnaś 1987). In the magnetostatic limit $c \to \infty$, the above expressions reduce to those given previously (Barnaś 1988b). In the following sections we apply them to some special cases.

4. Application to magnetic/non-magnetic structures

Consider now the case when one of the two materials, say that with $i = 2$, is non-magnetic: $\mu_\perp^{(2)} = 1$ and $\mu_X^{(2)} = 0$. The dispersion equation for surface modes reduces then considerably. In this particular case one finds that

$$f_1 = -f_3 = \alpha_<^2\mu_\perp^{(1)}\mu_V^{(1)} - 2\alpha_<\mu_X^{(1)}k - \gamma^{(1)} \tag{15a}$$

$$f_3 = 0. \tag{15b}$$

Equation (13) can then be written as two independent equations

$$\alpha_<^2\mu_\perp^{(1)}\mu_V^{(1)} - 2\alpha_<\mu_X^{(1)}k - \gamma^{(1)} = 0 \tag{16a}$$

$$\sinh(\alpha^{(1)}d_1) = 0. \tag{16b}$$

Equation (16*b*) can be rewritten equivalently as

$$\alpha^{(1)} = \mathrm{i}n\pi/d_1 \qquad n = 1, 2, 3, \ldots \tag{17}$$

from which one can conclude that the corresponding solutions consist of bulk states in

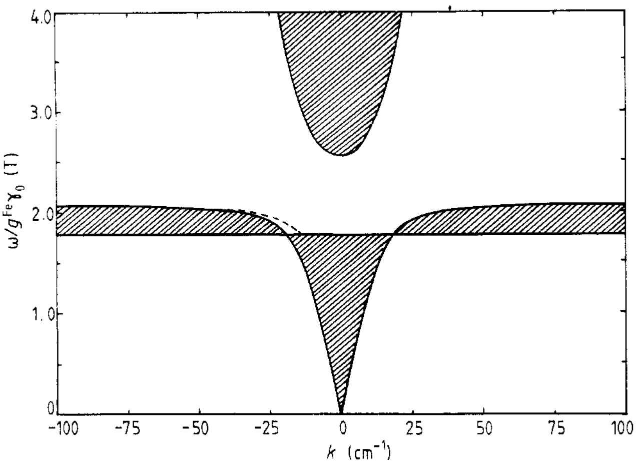

Figure 1. Spectrum of bulk polaritons (hatched regions) and surface polaritons (broken curve) in ferromagnetic (Fe)/non-magnetic superlattices for $d_1 = d_2 = 5 \times 10^{-4}$ cm and $\mu_0 H_0 = 1$ T. The other parameters are given in the text.

each magnetic layer. To describe a surface wave each solution of equation (16*a*) and equation (16*b*) has to fulfil the stability conditions as discussed in section 2. These conditions will be analysed numerically in each particular case.

Consider first the polariton spectrum of ferromagnetic/non-magnetic (F/N) superlattices. For the magnetic permeabilities $\mu_\perp$ and μ_X we assume that

$$\mu_\perp = 1 + \Omega_0 \Omega_m / (\Omega_0^2 - \omega^2) \qquad \mu_X = -\Omega_m \omega / (\Omega_0^2 - \omega^2)$$

where $\Omega_0 = g\mu_0\gamma_0 H_0$ (with the uniaxial anisotropy neglected) and $\Omega_m = g\mu_0\gamma_0 M_0$. The parameters g, μ_0 and M_0 denote the Landé factor, magnetic permeability of the vacuum and spontaneous magnetisation, respectively, and $\gamma_0 = e/2m$ with e (>0) and m being the electron charge and electron mass. The appropriate polariton spectrum in the limit of thick films, $|k|d_{1(2)} \gg 1$ has already been analysed (Barnaś 1987, How and Vittoria 1989). In figure 1 the mode spectrum is shown for equal film thicknesses, $d_1 = d_2$, which are small in comparison with the appropriate wavelength. The parameters typical for Fe, i.e. $\mu_0 M_0^{Fe} = 2.15$ T and $g^{Fe} = 2.15$, have been assumed here. The hatched regions represent the bands of bulk polaritons whereas the broken curve describes the surface modes. Two interesting features of the wave spectrum from figure 1 are worth noting. The first is the occurrence of the surface mode. In the corresponding magnetostatic limit the surface Damon–Eshbach modes occur only for $d_1 > d_2$ (and, of course, only for negative k). When the retardation corrections are included, the surface modes exist also for $d_1 \leqslant d_2$, as is visible in figure 1. However, the surface modes exist then only in a restricted range of the wavevectors. The appropriate dispersion curves start at some cut-off wavevector and then merge into the band of bulk modes at a sufficiently large negative k. For $d_1 > d_2$ they exist for $k \to -\infty$. The second feature is the band structure of bulk waves, which differs considerably from that in the case of relatively thick films where a characteristic multiband structure appears. In the limit of small film thicknesses, $|k|d_{1(2)} \ll 1$, the appropriate wave spectrum, like that in figure 1, can also be obtained from the effective-medium theory developed by Raj and Tilley (1987).

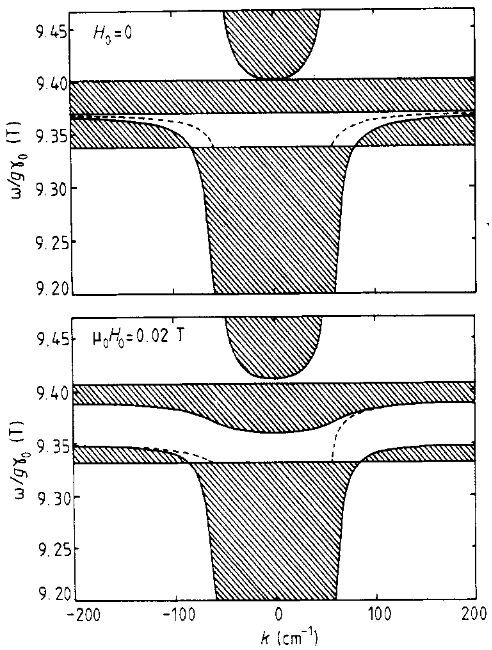

Figure 2. Spectrum of bulk polaritons (hatched regions) and surface polaritons (broken curves) in antiferromagnetic (MnF_2)/non-magnetic superlattices for $d_1 = d_2 = 5 \times 10^{-4}$ cm. The other parameters are given in the text.

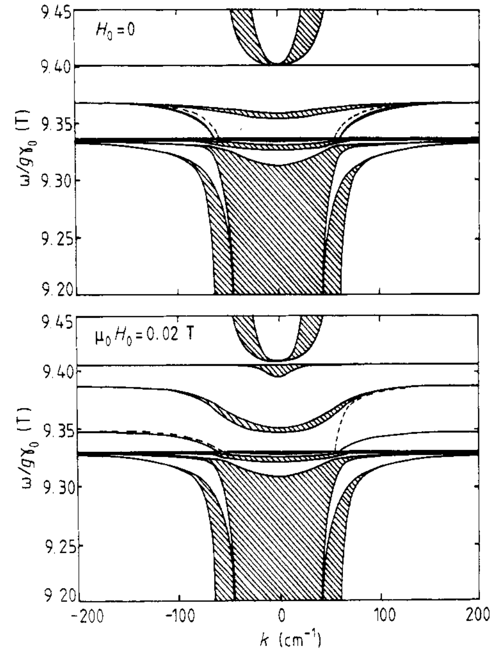

Figure 3. Same as in figure 2 but for $d_1 = d_2 = 0.035$ cm.

Equations (16*a*) and (16*b*) are also applicable to layered structures in which the magnetic constituent is a uniaxial antiferromagnet (AF/N structures) with an easy axis in the film plane and with an external field H_0 parallel to the easy direction. The appropriate permeability components can be written in the form

$$\mu_\perp = 1 + \Omega_a\Omega_m/(\Omega_1^2 - \omega_+^2) + \Omega_a\Omega_m/(\Omega_1^2 - \omega_-^2)$$

$$\mu_X = \Omega_a\Omega_m/(\Omega_1^2 - \omega_+^2) - \Omega_a\Omega_m/(\Omega_1^2 - \omega_-^2)$$

where Ω_m is defined as in the case of a ferromagnetic material but with M_0 being the sublattice magnetisation, $\omega_\pm$ are given as $\omega_\pm = \omega \pm g\mu_0\gamma_0H_0$, Ω_1 is determined by the anisotropy H_a and exchange H_{ex} fields:

$$\Omega_1 = g\mu_0\gamma_0[H_a(H_a + 2H_{ex})]^{1/2}$$

and Ω_a is defined as $\Omega_a = g\mu_0\gamma_0H_a$.

In figure 2 the spectrum of bulk and surface polaritons in the limit of small film thicknesses is presented for $H_0 = 0$ and $H_0 > 0$. Only the frequency range around the antiferromagnetic resonance is shown. Equal film thicknesses, $d_1 = d_2 = d$, and parameters typical for MnF_2, $\mu_0H_{ex} = 55$ T, $\mu_0H_a = 0.787$ T, $\mu_0M_0 = 0.754$ T and $g = 2$, have been assumed. The hatched regions represent the bands of bulk waves and the broken curves correspond to surface modes. The surface waves occur in a restricted range of wavevectors, similarly to the case of F/N structures. (It is worth recalling now

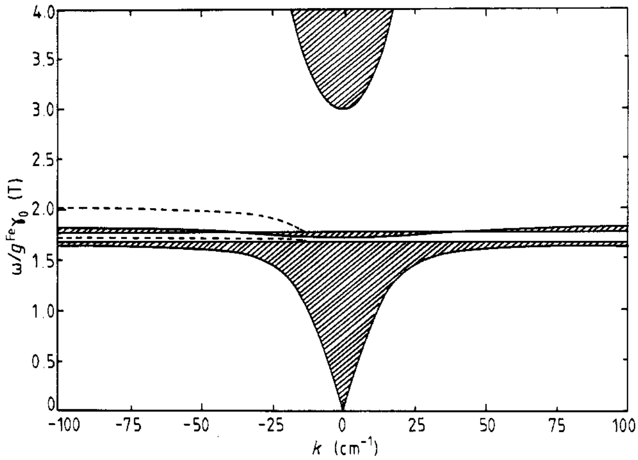

Figure 4. Spectrum of bulk polaritons (hatched regions) and surface polaritons (broken curves) in ferromagnetic (Fe)/ferromagnetic (Co) superlattices. The film thicknesses assumed here are $d_1 = d_2 = 5 \times 10^{-4}$ cm. The other parameters are given in the text.

that no surface waves can propagate at $d_1 = d_2$ in the corresponding magnetostatic limit (Barnaś 1988b).)

The surface waves in figure 2 exist for $k < 0$ as well as for $k > 0$ and $\omega(-k) = \omega(k)$ at $H_0 = 0$. For $H_0 > 0$, however, the propagation of surface waves is non-reciprocal: $\omega(-k) \neq \omega(k)$.

If the film thicknesses increase, the multiband structure of bulk modes appears, similarly to the case of F/N superlattices. This is shown in figure 3, where the same parameters as in figure 2 have been assumed, except the film thicknesses which are now larger.

5. Magnetic/magnetic structures

Consider now the case when both constituents are ferromagnetic. For the basic parameters we assume values typical for Fe ($i = 1$), as given in section 4, and Co ($i = 2$); $\mu_0 M_0^{\mathrm{Co}} = 1.76\,\mathrm{T}$ and $g^{\mathrm{Co}} = 2.17$. The appropriate spectrum of magnetic polaritons in the semi-infinite structure is shown in figure 4 for equal film thicknesses, $d_1 = d_2 = d$, and for $|k|d \ll 1$. As in the corresponding magnetostatic limit (Barnaś 1988b) there are two surface modes at $k < 0$. They start now at some cut-off wavevector at the photon line and for sufficiently large negative k they coincide with the magnetostatic waves. In comparison with the F/N structures (figure 1) there is an additional narrow band of bulk modes.

The wave spectrum for larger film thicknesses is shown in figure 5. It is worth noting the existence of some additional surface modes which can propagate in some range of wavevectors.

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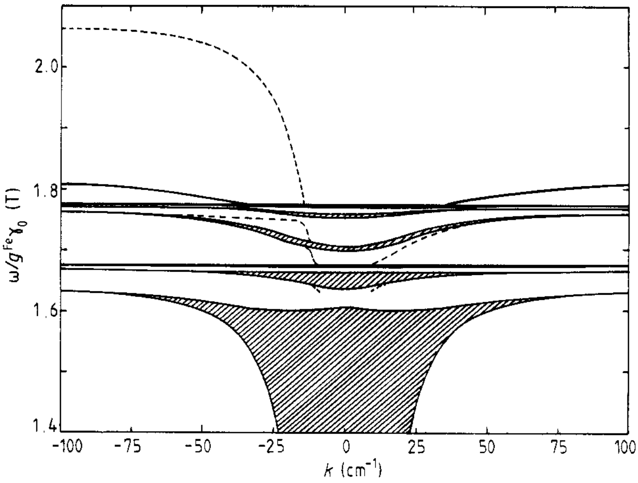

Figure 5. Same as in figure 4 but for $d_1 = d_2 = 0.035$ cm.

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