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Stable phase-locking of an external-cavity diode laser subjected to external optical injection

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Abstract
We analyse the locking phenomena arising when an external-cavity diode laser is subjected to optical injection from another uncontrolled diode laser. The system stability is investigated as a function of coupled cavity time delay and the optical injection strength. Different regimes, spanning from 'in-phase locking' to 'out-of-phase locking' with ultimate amplitude death of low-frequency fluctuations/pulsations, are described experimentally as well as numerically for weak to moderate injection. Qualitative agreements between numerically and experimentally observed results for amplitude quenching are shown. Numerical studies describe the shifting of phase-flip bifurcation as the optical injection strength is varied for a particular time delay. Stable phase-locking behaviours, which are desired from the point of view of practical applications, are observed numerically in a wide range of control parameter space.

1. Introduction

Semiconductor diode lasers are widely used in modern technologies because of their compactness, low cost and durability. In order to improve the diode laser characteristics in terms of narrowing of linewidth [1] and wavelength tunability [2], optoelectronic feedback [3] or optical feedback [4] or injection [5] in an external cavity [6] is generally used. A semiconductor laser subjected to external optical feedback exhibits a large variety of dynamic behaviours, such as periodic and quasi-periodic oscillations, chaos [7], coherence collapse [8] and low-frequency fluctuations [9] or regular pulsations that degrade the laser characteristics. The low-frequency fluctuation/pulsation (LFF/LFP) regime is typically observed when laser diodes are pumped near threshold and subjected to moderate optical re-injection from a distant reflector. This complex dynamical regime is characterized by a succession of sudden drop-outs and slow recoveries of the laser's mean intensity in between the drop-outs [10]. The recovery process of LFF/LFPs has been described as a sequence of transitions among compound cavity modes [11, 12]. A complete understanding of LFF regimes has been under debate since the very first observation. There is no analytical or experimental picture showing how the dynamics of the injected laser depends on the parameters on a global scale [13]. The only map showing various dynamical properties of the system for a relatively large range of parameter space comes from experimentalists [14]. But from the experimental point of view, a complete characterization of low-frequency dynamics is also quite difficult because of very diverse time scales involved in the dynamics. The LFF poses a serious problem in applications where a constant average output power from the laser is desired [15]. It is thus important to investigate minutely the nonlinear dynamical behaviour induced by external optical feedback in an external-cavity diode laser (ECDL), and explore methods of suppressing or controlling chaos and LFFs. The external optical feedback can induce additional noise, which is considered as feedback-induced deterministic. On the other hand, it has been shown that noise characteristics of a free-running laser can be improved by injection locking techniques [16]. We wish to consider whether the optical injection can be used to suppress the feedback-induced noise, and study the effect of optical injection on the feedback-induced noise performance and the phase locking [17] behaviour of the ECDL system. Here 'phase' refers to intensity oscillation cycles and it is not related to the optical phase of the electric field.

A few experimental and theoretical studies have been performed on the quenching of the amplitude of the LFF via a second optical feedback [18] or optical injection from another laser [19]. But detailed information about the phase locking and route to ultimate amplitude death is lacking, which is the crucial point we wish to pay attention to in the present paper in the context of a mutually-coupled laser system [20]. The coupling between the lasers could be in either unidirectional [21, 22] or bidirectional fashion [12] leading to different kinds of outputs [23]. The mutually-coupled diode laser system is very interesting because of its potential application in a wide range of fields, specially in communications [24]. Apart from this, the coupled diode laser system also provides a simple and powerful tool to unveil the collective behaviour within a wide range of control parameter space. Among the collective behaviour, amplitude death can occur when two identical or non-identical coupled chaotic systems drive each other to a fixed point and stop the oscillations [25]. The amplitude death of low-frequency dynamics may arise in different scenarios. First, if the coupling between two diode lasers is sufficiently strong that it can create a saddle node pair of fixed points on the limit circle when the natural frequencies of the diode lasers are sufficiently separated [26], amplitude death can occur. Secondly, if there exists a time delay in coupling when the frequency mismatch between the coupled lasers is zero, a variation in the time delay eventually leads to amplitude death [27, 28] of the laser output. In the second case, the time delay initiates a Hopf bifurcation [29] in which the two lasers pull each other off their limit cycles and collapse to a steady state. The most accessible control parameters, from an experimental implementation point of view, are probably the injection current, the coupling strength and the time delay. The number of adjustable external and internal parameters and their combinations is so high that new dynamical regimes are still being observed by careful adjustment of the laser parameters. An important focus of this paper is on investigating some intriguing aspects of the collective behaviour and route to ultimate amplitude death of the low-frequency dynamics.

In our system of a single-mode ECDL (slave) [11, 30, 31], we present a novel scheme of stabilization by phase locking and suppression of LFPs via bidirectional coupling with a multimode free-running (master) diode laser having similar characteristics. Optical injection locking [32] usually arises in one of the two ways: either both the lasers emit in the same phase (in-phase solution), or the phase difference between the laser outputs is π (out-of-phase solution). The in-phase locking produces constructive interference in the output whereas the out-of-phase locking produces destructive interference. We also observe, in a certain parameter range, that the phase difference between the laser outputs is either in between 0 and π (signifying phase synchronization) or unbounded (signifying desynchronization). In our set-up, the ECDL is subjected to both delayed optical feedback from a reflector (grating) and bidirectional coupling of the light from a driving laser into the external cavity. This system is interesting from several viewpoints, as it can combine desirable features of both external-cavity lasers and injection-locked lasers. For example, feedback and injection can both contribute to a narrowing of the laser linewidth [33], while strong injection inhibits coherence collapse. For optical communications using a chaotic carrier, it has been predicted numerically that optical injection may enhance the bandwidth in an external-cavity semiconductor laser operating on a high-dimensional chaotic state [34]. This allows the possibility of higher data transmission rates. This interest in optically injected external-cavity semiconductor lasers motivates the characterization of their simplest modes of operation. The dynamical behaviour has been investigated theoretically, tracing a route from stability to coherence collapse at increasing injection levels [35]. In this paper we concentrate on the regimes of continuous-wave emission of the injected laser (slave ECDL), phase-locked on the master laser, and study of phase-flip bifurcation [36] of the coupled laser system. We have found a new route to complete amplitude death in this purely optically-coupled diode laser system and obtained results for the locking of different dynamical regimes with low to high coupling strengths.

The organization of the paper is as follows. The experimental setup is introduced in section 2. This is followed by discussion of numerical results in section 3. The conclusions are presented in section 4.

2. Experimental set-up

The schematic of the experimental set-up is shown in figure 1 where a single-mode tunable ECDL (Advanced Laser Systems LA-5C-830 semiconductor laser, with a grating in the Littrow configuration) is operated at a wavelength of 825 nm at a threshold current of 63.0 mA from a well-stabilized home-made current controller within an accuracy of ± 0.01 mA. The temperature of the ECDL is set by a home-made temperature controller at 23.25 °C within an accuracy of ± 0.01 °C. A second uncontrolled injection diode laser (Advanced Laser Systems LA-5C-830) operating at 830 nm with a solitary threshold of 73.0 mA is coupled to the ECDL via a polarizing beam-splitter. Special attention has been paid to achieve a well-defined coupling condition and time delay. The slave laser is subjected to self-feedback from its external cavity and a coupling from the master laser, and the master laser to only coupling from the slave ECDL. The self-feedback strength is adjusted by a piezo-disc in order to get the perfect Littrow configuration, and the coupling strength is adjusted using a polarizer. The feedback and coupling strengths are tuned so that the proportionate injection field to each laser is the same ($=\eta$). The path length between the lasers is 52.6 cm (coupled-cavity round-trip time = 3.5 ns). The length of the external cavity is set to 4.6 cm (round-trip time = 0.306 ns). A fast avalanche photodiode (Hamamatsu C5331-02) is used to monitor the ECDL intensity, which is simultaneously recorded with a digital oscilloscope (Tektronix TDS350, resolution = 2.5 ns).

The experimental results of the LFPs from the ECDL with no coupling: $\eta = 0.0$, and the amplitude suppression of these LFPs at a particular delay time (=coupled-cavity round-trip time) and varying coupling strengths, $\eta = 0.0283$, 0.0286 and 0.0311, are shown in figure 2. The figure clearly

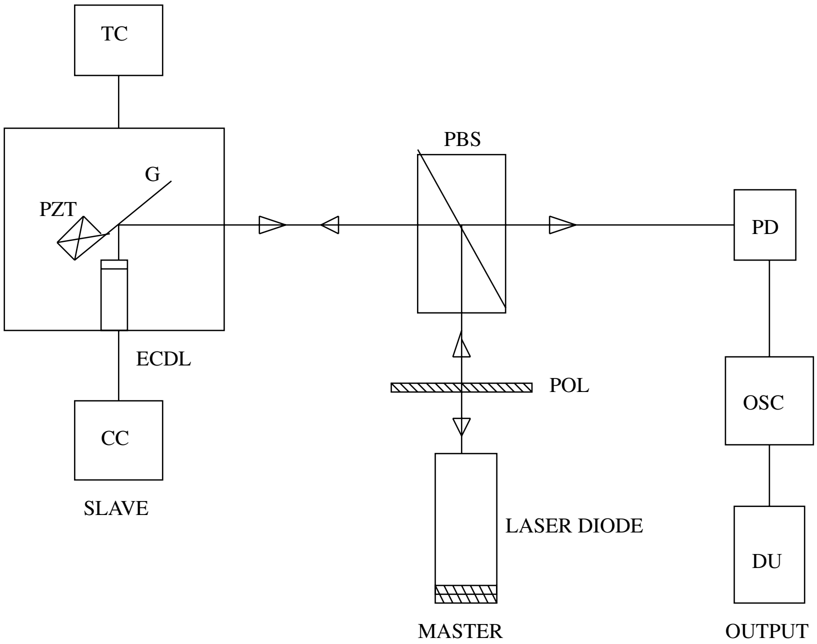

Figure 1. The schematic experimental set-up; ECDL: external-cavity diode laser, TC: temperature controller, CC: current controller, G: grating, PZT: piezo-electric transducer, PBS: polarizing beam-splitter, POL: polarizer, PD: photodidode, DU: detection unit and OSC: oscilloscope.

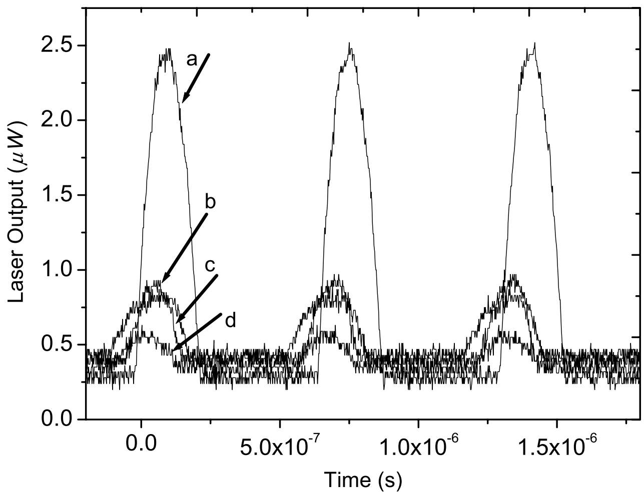

Figure 2. Experimentally observed slave laser output powers (in microwatts) versus time (in seconds), showing amplitude suppression of low-frequency pulsations. The curves are for injection strengths (a) $\eta = 0.0$, (b) $\eta = 0.0283$, (c) $\eta = 0.0286$ and (d) $\eta = 0.0311$, at a fixed time delay =1.75 ns.

indicates that as we increase the coupling strength, amplitudes get suppressed.

3. Results of numerical simulation

The fundamental equations modelling a single ECDL is a set of delay-differential equations, known as Lang–Kobayashi (LK) equations [37], which describe the time evolutions of the complex electric field $E(t)$ of a single longitudinal mode and the carrier density $N(t)$ (with the threshold value subtracted out) averaged spatially over the laser medium. In order to analyse the behaviour of the proposed scheme of the two coupled lasers, the LK equations can be written in a standard normalized form:

$$\begin{aligned}
\frac{\mathrm{d}E_1}{\mathrm{d}t} &= (1+\mathrm{i}\alpha)N_1(t)E_1(t) + \eta\,\mathrm{e}^{-\mathrm{i}\omega_2\tau}E_2(t-\tau),\\
T\frac{\mathrm{d}N_1}{\mathrm{d}t} &= J_1 - N_1(t) - [2N_1(t)+1]\,|E_1(t)|^2,\\
\frac{\mathrm{d}E_2}{\mathrm{d}t} &= (1+\mathrm{i}\alpha)N_2(t)E_2(t) + \eta\,\mathrm{e}^{\mathrm{i}\omega_1\tau}E_1(t-\tau),\\
T\frac{\mathrm{d}N_2}{\mathrm{d}t} &= J_2 - N_2(t) - [2N_2(t)+1]\,|E_2(t)|^2,
\end{aligned} \tag{1}$$

where η is the coupling parameter, i.e., the fraction of light of ECDL injected into the other laser and vice versa, J are the injected constant current densities (with the threshold value subtracted out), T is the ratio of the carrier lifetime to the photon lifetime, the delay time $\tau = 2L/c$ is the round-trip time taken by the light to cover the distance L between the lasers, and α is the linewidth enhancement factor of the diode lasers. The measured quantities are output powers of the lasers, $P_{1,2} = |E_{1,2}(t)|^2$.

The LK equations represent a simple and generally accepted model in the literature to describe a semiconductor laser. This model is mono-mode and quasi-monochromatic. We also ignore the gain compression term, spontaneous emission, and the Langevin noise term in our model. Since the aim of our work is to gain a physical insight by fast numerical analysis, we have not tried to develop a more accurate model including noise sources or describing the laser technology.

We study the dynamical behaviour of a diode laser subjected to an external optical injection from a coupled diode

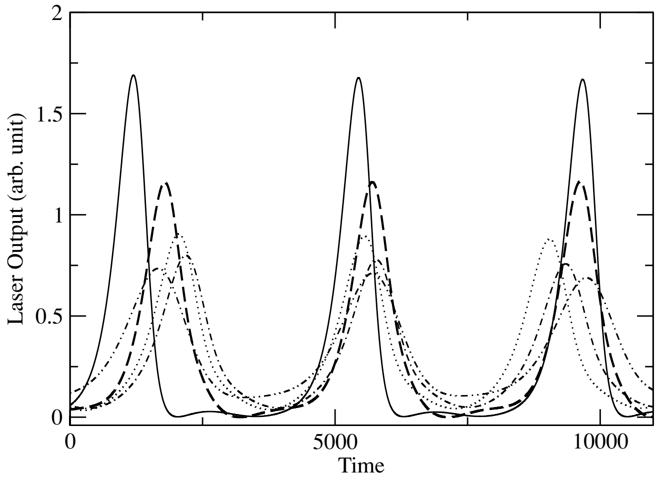

Figure 3. Plot of laser output power P_1 versus time (in units of cavity photon lifetime) for a fixed time delay $\tau = 14$. The curves are at different coupling strengths, $\eta = 0.0321$ (continuous line), 0.0322 (dashed line), 0.0323 (dotted line), 0.0324 (dash-dotted line) and 0.0325 (dash-double dotted line).

laser, as a function of the injection time delay τ (in units of cavity photon lifetime) and of the relative amplitude η of the injected field. Numerical integration is done using a Runge–Kutta fourth-order scheme with a step size, $\Delta t = \tau/n$, where $n = 1000$ is chosen based on the accuracy criteria. We have checked the stability of this method and found that for a fixed time delay, it gives reliable results if n is sufficiently large. The dimensionless parameters in equations (1) are taken as $J_1 = 0.165$, $J_2 = 0.175$, $T = 1000$ and $\alpha = 5.6$.

We start by analyzing the modulation properties of the coupled laser system from the rate equations. The effect of modulation is strongly dependent on the injection locking parameters and injection locking range. Three control parameters (J, η, τ) defined above are relatively straightforward to determine experimentally, even though the internal parameters of diode lasers are typically not very accurately known. An example of suppression of low-frequency pulsations at a particular time delay τ and varying coupling strengths η is shown in figure 3.

Numerically, we see that there are two types of amplitude modulation of low-frequency dynamics in our coupled diode laser system. The first one is the almost non-transient amplitude suppression with constant time delay as shown in figure 3, which is in good qualitative agreement with the experimental result shown in figure 2. In the other interesting dynamics, the amplitude death occurs with time in low-frequency dynamics as shown in figure 4. At a constant time delay, the injection strength does not change the drop-out behaviour (pattern or shape) except changing its in-phase amplitude. A similar dynamics of amplitude death with out-of-phase motion for a different η is shown in figure 5. The experimental observation of amplitude suppression of LFPs at a particular injection strength and time delay is in good agreement with the simulated result; however, the detailed comparison between the model and experiments for the other different observed in-phase and out-of-phase dynamics is left for future work. Here the in-phase and out-of-phase amplitude death are dependent on the type of the coupling (resonant or non-resonant) parameters. For the resonant case, both the lasers are in phase with amplitude death occurring as shown in figure 4. For the non-resonant case, when the injected light does not meet the laser round-trip time phase condition, then, to a good approximation, we can assume that the amplified light only makes a single pass through the device before being lost from the laser cavity. This non-resonant amplification of light does not couple into the slave laser modes and decreases the carrier number density, which results in increased refractive index and subsequently decreased cavity resonance frequency

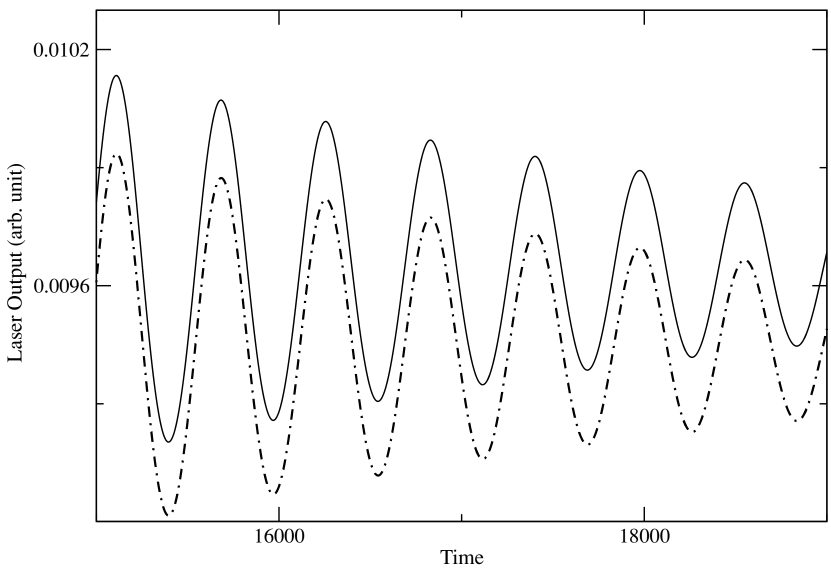

Figure 4. The laser output powers, P_1 and P_2, versus time (in units of cavity photon life-time) for a fixed time delay $\tau = 14$ and coupling strength $\eta = 0.25$, showing in-phase motion in the amplitude-death region B in figure 6. The initial transients are not shown.

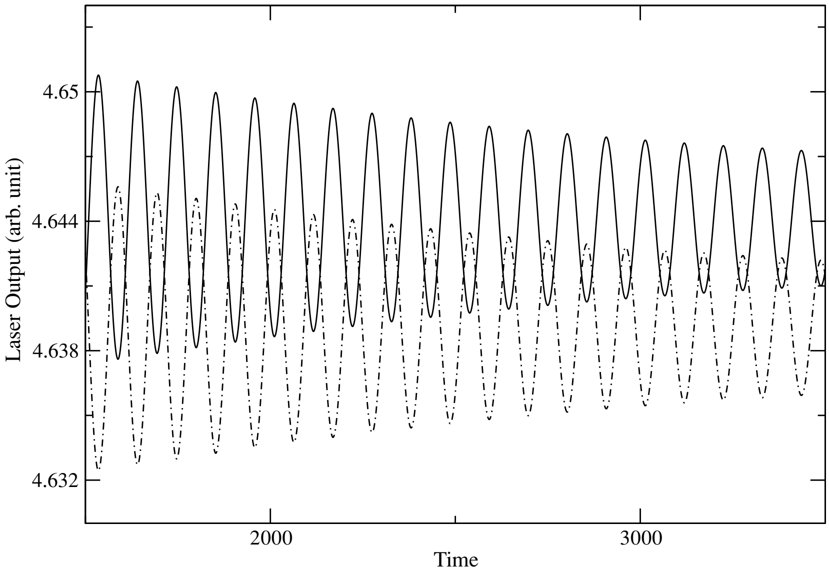

Figure 5. The laser output powers, P_1 and P_2, versus time (in units of cavity photon life-time) for a fixed time delay $\tau = 14$ and coupling strength $\eta = 0.30$, showing out-of-phase motion in the amplitude-death region A in figure 6. The initial transients are not shown.

of the slave laser. This means that fewer carriers take part in the gain process for the slave laser mode reducing the optical power. This leads to the out-of-phase phenomenon between the master and slave lasers as shown in figure 5.

The different dynamical regimes are shown in figure 6 in a schematic phase diagram in control parameters η and τ. Due to the presence of the delay in the optical injection, our system has an infinite-dimensional phase space making any rigorous analysis of equations (1) very difficult. In our simulation of equations (1), we first fix the delay time τ, and vary the coupling strength η. At each set of parameters, in order to remove the initial transients, it is sufficient to discard the first 10^5 data points, and then we look for sustained dynamical behaviour within the next 10^5 data points. We plot the output powers P_1 and P_2 of both the lasers versus time (as shown in figure 7) and the phase relation between P_1 and P_2 (as shown in figure 8). Based on these two figures, we identify the dynamics for each set of parameters, whether it is in-phase or out-of-phase or mixed phase or amplitude death state. After a rigorous scanning of the (η, τ)-parameter space, we mark the transition line across which the dynamical states are of different behaviour. In figure 6, symbols A to H mark the different types of dynamical regimes. The phase-flip bifurcation [28, 36] within an (η, τ)-parameter space takes place along the thick line marked by '•', across which the dynamics are given by in-phase and out-of-phase motions. Such typical in-phase and out-of-phase motions are shown in figures 4 and 5 for regions B and A, respectively. We have checked our results for different values of the current densities J_1 and J_2. We do not find any new dynamics, although the locus of the transition curves, defining the boundaries of the different types of motion, gets shifted or modified. We have also checked the dynamical states with many different uniformly-distributed random initial conditions, and found that there is no multistability.

We now attempt to provide an understanding of the route to ultimate amplitude death. We find distinct stages during the transition to complete amplitude death where the system gets locked as the coupling strength is increased from zero. In the first stage, the ECDL decreases the amplitude to death, but before reaching complete death, a laser transition takes place to another dynamical state, as can be seen from the results in figures 2 and 3. The first stage describes the transition to partial amplitude death. In the second stage, the injected ECDL shows different dynamical output states as shown in figure 7. In the final stage, the amplitude decreases until ultimate amplitude death, in-phase or out-of-phase, is established, as shown in figures 4 and 5. The variation of the phase difference with coupling strength η is shown in figure 8(a) while phase relations in between P_1 and P_2 are shown in figures 8(b), (c) and (d) for in-phase, out-of-phase and mixed phase, respectively, in different regimes of the parameter space (figure 6). It is worth mentioning here again that our description of the route to ultimate amplitude death is based on a set of numerical observations, rather than any rigorous analysis of equations (1), as given for example in [38] for bifurcation scenarios in an ECDL, which is difficult because of the delayed coupling. We have checked the robustness of these dynamics by considering small noise. Experimental results, where noises are unavoidable, also confirm the robustness.

4. Conclusions

We have presented an extended analysis of the dynamics of a semiconductor laser subjected to external optical injection. Due to the presence of the delay in the optical injection, which makes the system infinite-dimensional, an analytical treatment of the problem is quite difficult. We have investigated the transition of the coupled diode laser system to ultimate amplitude death, in-phase or out-of-phase, by using numerical simulation. We have found a new route to complete amplitude death in this purely optically coupled diode laser system.

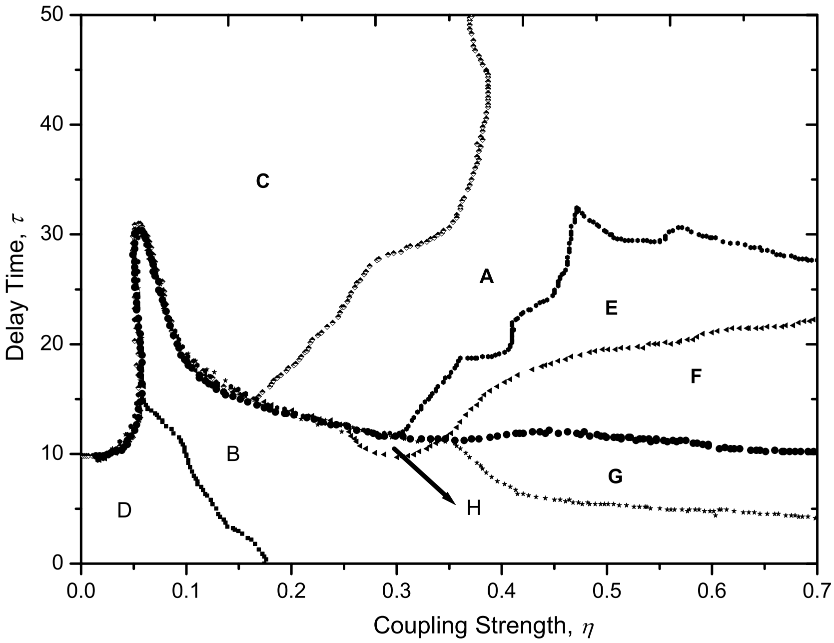

Figure 6. Schematic phase diagram in the parameter space of delay time τ and coupling strength η. The different regions are marked as A: out-of-phase amplitude death, B: in-phase amplitude death, C: out-of-phase desynchronized, D: in-phase desynchronized, E: out-of-phase quasiperiodic; F: out-of-phase periodic, G: in-phase limit circle, and H: in-phase quasiperiodic motions.

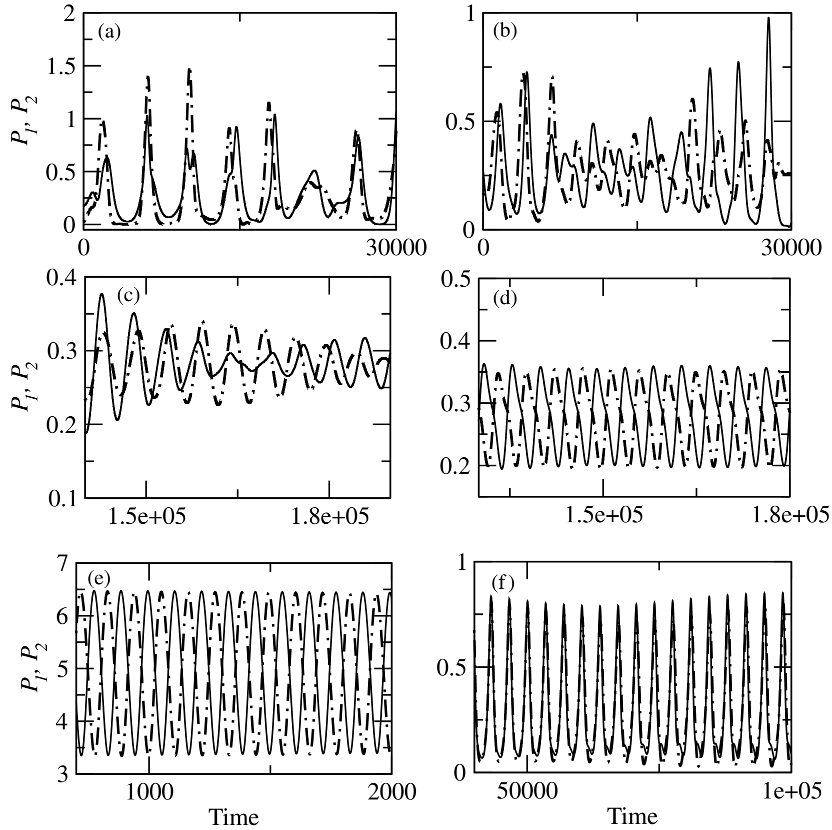

Figure 7. Plots of laser output powers, P_1 and P_2 versus time (in units of cavity photon life-time) for a fixed time delay $\tau = 14$, corresponding to different regimes of figure 6, with (a) $\eta = 0.05$ in the in-phase unsynchronized state in regime D, (b) $\eta = 0.15$ in the out-of-phase unsynchronized state in regime C, (c) $\eta = 0.27$ in the in-phase quasiperiodic state in regime H, (d) $\eta = 0.32$ in the out-of-phase quasiperiodic state in regime E, (e) $\eta = 0.36$ in the out-of-phase periodic state in regime F, and (f) $\eta = 0.38$ in the in-phase periodic state in regime G. The initial transients are not shown.

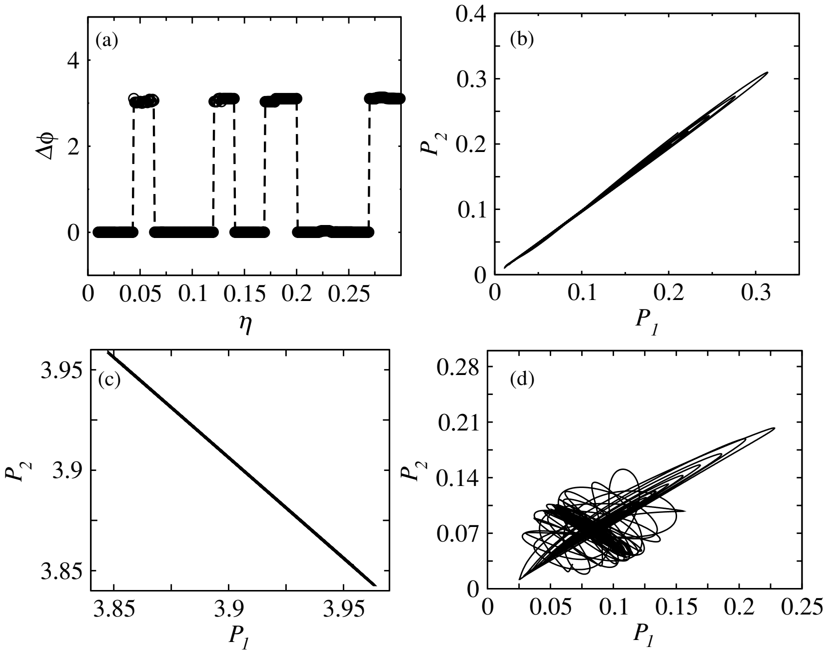

Figure 8. Plots of (a) phase difference $\Delta\phi$ versus injection strength η showing phase flip, (b) P_2 versus P_1 for $\eta = 0.25$ in regime B of figure 6, (c) P_2 versus P_1 for $\eta = 0.30$ in regime A of figure 6, and (d) P_2 versus P_1 for $\eta = 0.20$ near the bifurcation line of figure 6, each for a time delay $\tau = 14$.

The different transitions are characterized in this way: low-frequency periodic pulsations $\rightarrow$ irregular pulsations $\rightarrow$ constant periodic motion $\rightarrow$ amplitude death state.

The transition to complete amplitude death is manifested by two parameters: the coupling strength η and coupled-cavity delay time τ. The results presented in figures 2 and 3 show that for a small η the amplitude does not reach zero in physically accessible time scales and complete amplitude death is not seen. However, for sufficiently large η, ultimate amplitude death could exist in a range of coupling strengths as shown in figure 6. Therefore, there exists a first stage which describes the process of transition to partial amplitude death and after that the laser jumps to another LFF state. At the onset of partial amplitude death, the master laser provides a sufficient delayed optical injection to the slave ECDL to suppress the LFF/LFP by destroying or pushing the antimodes far away from the external cavity mode, which are responsible for the power drop-out crisis.

We have obtained results for the locking of different dynamical regimes with low to high coupling strengths. The locking diagrams have been drawn in a window in the (η, τ)-plane, where separate first and final stable locking regions have been found. The injected light decreases the carrier density which results in an increase of the refractive index and subsequently a lowering of the cavity resonance frequency of the slave ECDL. The master laser provides a physical means to the slave ECDL for generating almost zero feedback through bidirectional coupling between the lasers. For very small injection strengths, both the lasers are essentially independent (out-of-phase) and for large values of the injection strength, both the lasers are tightly coupled (in-phase) acting as one laser. At a particular set of values of η and τ the laser transitions to amplitude death take place. An important advantage of this technique is that one can apply it, unlike most existing locking or stabilization techniques, without changing any parameter of the ECDL.

Acknowledgments

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