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Atom (molecule) spin rotation and oscillation under refraction in constant electric field. Atomic-spin interferometry

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Abstract. It is shown that oscillation or precession phenomena of atom multipole moments occur when atoms with spin $S \geqslant 1$ are passing through the area occupied by a constant electric field. The presence of media located in the electric field leads to an additional phase shift of the atom wavefunction. This effect can be used for the creation of an atomic spin interferometer and the verification of the weak equivalence principle.

1. Introduction

It is well known that due to the magnetic moment interactions of neutrons with a magnetic field, the neutron wave, passing the border of the vacuum–magnetic field, experiences a refraction effect. This effect is characterized by two refractive indexes, $n_\pm$ for neutrons with spin parallel (anti-parallel) to the magnetic field,

$$n_\pm^2 = 1 \pm \frac{\mu_{\rm n} B}{W} \tag{1}$$

where $\mu_{\rm n}$ is the neutron magnetic moment, B is the magnetic induction, $W = \hbar^2 k^2/2m$ is the neutron kinetic energy before the area occupied by the magnetic field, k is the neutron wavenumber and m is its mass.

If the neutron spin before the field is oriented through some angle to the magnetic field, then its wavefunction ψ is the superposition of the states with spin parallel to the magnetic field $\psi_+ = C_1\binom{1}{0}$ and anti-parallel to the magnetic field $\psi_- = C_2\binom{0}{1}$ (quantization Z-axis is chosen parallel to the magnetic field $|C_1|^2 + |C_2|^2 = 1$). After passing in the field in the way l the neutron wavefunction is

$$\psi = C_1\,{\rm e}^{ikn_+l}\begin{pmatrix}1\\0\end{pmatrix} + C_2\,{\rm e}^{ikn_-l}\begin{pmatrix}0\\1\end{pmatrix}. \tag{2}$$

Hence we arrive at the well known result that the neutron polarization vector $\boldsymbol{P}$ $\left(\boldsymbol{P} = \langle\psi|\hat{S}|\psi\rangle/S\right)$ rotates in the field through an angle θ equal to

$$\theta = k(n_+ - n_-)l\,. \tag{3}$$

In this case

$$P_x = 2\,{\rm Re}\,\psi_+^*(l)\,\psi_-(l) \qquad P_y = 2\,{\rm Im}\,\psi_+^*(l)\,\psi_-(l) \qquad P_z = |\psi_+(l)|^2 - |\psi_-(l)|^2$$
$$\psi_+(l) = C_1\,{\rm e}^{ikn_+l} \qquad \psi_-(l) = C_2\,{\rm e}^{ikn_-l}\,.$$

So, the neutron spin-rotation effect in the magnetic field can be considered as a result of interference of neutron states with spin parallel and anti-parallel to the field. Let us note that the neutron spin-rotation effect in the magnetic field is kinematically similar to the Faraday effect, i.e. to the effect of light polarization plane rotation in the media, located in the magnetic field. The Faraday effect is also determined by the fact that, in this case, the photon refractive index with spin parallel to the magnetic field is not equal to the photon refractive index with spin anti-parallel to the magnetic field. Let us consider the case when the non-magnetic media are in the zone occupied by the magnetic field. In this case

$$n_{\pm}^2 = 1 - \frac{U}{W} \pm \frac{\mu_{\rm n} B}{W} \tag{4}$$

where $U = -(2\pi\hbar^2/m)\rho f(0)$ is the effective potential energy of neutron interaction with the media, m is the neutron mass, ρ is the number of scatters in cm^3, $f(0)$ is the amplitude of neutron forward elastic coherent scattering. So, the following important result follows from (4):

$$\begin{aligned} \theta &= k\left[\left(1 - \frac{U}{W} + \frac{\mu_{\rm n} B}{W}\right)^{1/2} - \left(1 - \frac{U}{W} - \frac{\mu_{\rm n} B}{W}\right)^{1/2}\right] l \\ &= k\frac{2\mu_{\rm n} B}{W(n_+ + n_-)} l \, . \end{aligned} \tag{5}$$

As may be seen, the spin rotation angle θ depends upon U, i.e. we can measure this media characteristic through the neutron-spin rotation angle. According to Baryshevsky *et al* (1991), in this case we deal with a peculiar interferometer in which the two waves necessary for wave interference move together unlike in the usual interferometer, in which, for example, a semi-transparent mirror splits the incident wave into two parts, which pass different spatial paths. Obviously, the arguments presented above refer to atoms (molecules) with spin $\frac{1}{2}$. The closest real situation for experimental investigation is when the target is gaseous, which increases the pathlength for the atoms without noticeable scattering.

2. Spin atom interferometry of atoms with spin $S = \frac{1}{2}$ in the electric field

Let us pay attention to the fact that the potential energy U arises not only due to collisions but also due to atomic interaction with a constant homogeneous electric field,

$$U = -\tfrac{1}{2}\alpha\mathcal{E}^2 \tag{6}$$

where α is the atom polarizability and $\mathcal{E}$ is the electric field intensity. As a result, the spin rotation angle when the particle is passing through the area occupied by the magnetic and electric fields can be given as

$$\theta = k\left[\left(1 + \frac{\alpha\mathcal{E}^2}{2W} + \frac{\mu B}{W}\right)^{1/2} - \left(1 + \frac{\alpha\mathcal{E}^2}{2W} - \frac{\mu B}{W}\right)^{1/2}\right] l \tag{7}$$

where μ is the atom magnetic moment. In the case where the particle kinetic energy is much larger than the energy of its interaction with the electric and magnetic fields we have the following approximate formula:

$$\theta \simeq k\frac{2\mu B}{W} l - k\frac{2\mu B}{W} l \frac{\alpha\mathcal{E}^2}{W} \, . \tag{8}$$

According to (7) and (8) the spin rotation angle measurement permits measurement of the polarizability α, i.e. the value which is not connected with spin.

3. Spin interferometry and birefrigence of atoms with spin $S \geqslant 1$ in the electric field

Let us consider the case where the atom with spin $S \geqslant 1$ passes the area occupied by the electric and magnetic fields. In this case the interaction energy (Landau *et al* 1991) of the atom with the field can be written as

$$\hat{V} = -g\boldsymbol{S}\cdot\boldsymbol{B} - \tfrac{1}{2}\hat{\alpha_{ik}}\mathcal{E}_i\mathcal{E}_k \qquad (9)$$

where g is the gyromagnetic relation,

$$\hat{\alpha}_{ik} = \alpha\delta_{ik} + \beta\left(\hat{S}_i\hat{S}_k + \hat{S}_k\hat{S}_i - \tfrac{2}{3}S\left(S+1\right)\delta_{ik}\right) \qquad (10)$$

is the tensor of atom polarizability and α is the scalar atom polarizability and β is the tensor atom polarizability. In the result,

$$\hat{n}^2 = 1 - \frac{\hat{V}}{W}. \qquad (11)$$

To analyse new aspects of the interaction of the atom with spin $S \geqslant 1$ and the electric field let us investigate the refraction in the absence of magnetic field ($B = 0$). Let the atom be travelling along the x-axis. Before the electric field area the atom wavefunction is

$$\psi = \psi_0 . \qquad (12)$$

Then after the atom passes in the x-direction in the electric field, its wavefunction will be

$$\psi = \mathrm{e}^{\mathrm{i}k\hat{n}x}\psi_0 . \qquad (13)$$

Let us consider the case when $V/E \ll 1$; then

$$\hat{n} \simeq 1 - \frac{\hat{V}}{2W}. \qquad (14)$$

Let the constant electric field be directed along the Z-axis. The spin wavefunction ψ_0 can be represented as a superposition of the basis spin functions χ_M which are eigenfunctions of the operators $\hat{S}$ and $\hat{S}_z^{\,2}$:

$$\psi_0 = \sum_M a^M\chi_M . \qquad (15)$$

In the result the solution (13) can be written in the following way:

$$\psi_0 = \sum_M \mathrm{e}^{\mathrm{i}kn(M)x}a^M\chi_M \qquad (16)$$

where $n(M)$ is the atom refraction index located in the quantum state χ_M:

$$n\left(M\right) = 1 + \frac{m\mathcal{E}^2}{2k^2\hbar^2}\left(p + p_1M^2\right) \qquad (17)$$

$$p_1 = 2\beta \qquad (18)$$

$$p = \alpha - \tfrac{2}{3}\beta S\left(S+1\right) . \qquad (19)$$

Then after the atom passes a way x in the area occupied by the field, the wavefunction ψ can be presented as

$$\psi = \mathrm{e}^{\mathrm{i}k\left(1+(m\mathcal{E}^2 2k^2\hbar^2)p\right)x}\sum_M a^M\mathrm{e}^{\mathrm{i}f_1M^2x}\chi_M \qquad (20)$$

where

$$f_1 = \frac{\mathcal{E}^2\beta}{\hbar}\frac{1}{v} = \omega_\beta\frac{1}{v} \qquad (21)$$

$\omega_\beta = \mathcal{E}^2\beta/\hbar$; v is the atom velocity. Let us introduce the following:

$$n_s = 1 + \frac{m\mathcal{E}^2}{2k^2\hbar^2}p \tag{22}$$

$$b^M = a^M \, \mathrm{e}^{\mathrm{i}f_1 M^2 x} . \tag{23}$$

Then the wavefunction (20) can be presented as

$$\psi = \mathrm{e}^{ikn_s x} \sum_M b^M \chi_M . \tag{24}$$

4. Refraction of atoms with spin $S = 1$ in the electric field

Let us consider the case where the atomic spin is equal to one. Using the wavefunction (24) let us calculate the values of polarization vector components. It is easy to show that

$$\begin{aligned} \langle\psi|\hat{S}_x|\psi\rangle &= \sqrt{2}\{\operatorname{Re} b^{1^*} b^0 + \operatorname{Re} b^{-1} b^{0^*}\} \\ \langle\psi|\hat{S}_y|\psi\rangle &= \sqrt{2}\{\operatorname{Im} b^{1^*} b^0 + \operatorname{Im} b^{0^*} b^{-1}\} \\ \langle\psi|\hat{S}_z|\psi\rangle &= |b^1|^2 - |b^{-1}|^2 . \end{aligned} \tag{25}$$

Let the atom polarization vector comprise the angle ϕ with the X-axis before the electric field. Then for the coefficients a^M we have the following expression:

$$a^M = C_M \, \mathrm{e}^{-\mathrm{i}M\phi} . \tag{26}$$

As a result, we have

$$\begin{aligned} \langle S_x\rangle &= \sqrt{2}C_0[C_1 \cos(\phi - f_1 x) + C_{-1}\cos(\phi + f_1 x)] \\ \langle S_y\rangle &= \sqrt{2}C_0[C_1 \sin(\phi - f_1 x) + C_{-1}\sin(\phi + f_1 x)] \\ \langle S_z\rangle &= C_1^2 - C_{-1}^2 . \end{aligned} \tag{27}$$

Let us consider some particular cases.

(a) Let the atom be polarized primarily in the plane XOY orthogonal to the Z-axis. According to (27) it is necessary that

$$C_1 = C_{-1} .$$

In the result we have

$$\begin{aligned} \langle S_x\rangle &= 2\sqrt{2}C_1 C_0 \cos\phi \cos f_1 x \\ \langle S_y\rangle &= 2\sqrt{2}C_1 C_0 \sin\phi \cos f_1 x \\ \langle S_z\rangle &= 0 . \end{aligned} \tag{28}$$

From (28) it follows that the atom polarization vector, passing the area occupied by the field, oscillates in the plane orthogonal to the field.

(b) Now, let us consider the case where

$$C_{-1} \ll C_1, C_0$$

i.e. the initial polarization vector is oriented within some small angle relative to the direction of the electric field. In the result, from (28) we find

$$\begin{aligned} \langle S_x\rangle &= \sqrt{2}C_0 C_1 \cos(\phi - f_1 x) \\ \langle S_y\rangle &= \sqrt{2}C_0 C_1 \sin(\phi - f_1 x) \\ \langle S_z\rangle &= C_1^2 . \end{aligned} \tag{29}$$

The values of the initial vector components of the polarization vector are determined by the following expression:

$$\begin{aligned}
\langle S_x\rangle_0 &= \sqrt{2}C_0C_1\cos\phi \\
\langle S_y\rangle_0 &= \sqrt{2}C_0C_1\sin\phi \\
\langle S_z\rangle_0 &= C_1^2\,.
\end{aligned}\tag{30}$$

Comparing (29) and (30) we find

$$\begin{aligned}
\langle S_x\rangle &= \langle S_x\rangle_0\cos f_1x + \langle S_y\rangle_0\sin f_1x \\
\langle S_y\rangle &= -\langle S_x\rangle_0\sin f_1x + \langle S_y\rangle_0\cos f_1x \\
\langle S_z\rangle &= \langle S_z\rangle_0\,.
\end{aligned}\tag{31}$$

According to (30) a very interesting effect of atom spin rotation around the direction of electric field arises. In this case the precession frequency around the field is equal to $(-\omega_\beta)$. While passing through the area occupied by the electric field the polarization vector turns through the angle

$$\theta = -f_1x = -\omega_\beta\frac{x}{v}\,.\tag{32}$$

The minus sign means that the rotation goes in the clockwise direction.

(c) Let us consider the case where the initial polarization vector is directed primarily against the electric field, i.e. the condition

$$C_1 \ll C_1, C_0$$

is fulfilled. Then

$$\begin{aligned}
\langle S_x\rangle &= \sqrt{2}C_0C_1\cos(\phi + f_1x) \\
\langle S_y\rangle &= \sqrt{2}C_0C_1\sin(\phi + f_1x) \\
\langle S_z\rangle &= -C_{-1}^2\,.
\end{aligned}\tag{33}$$

As we see the polarization vector is turned through an angle equal to f_1x (i.e. anticlockwise) around the field direction.

Now let us notice the fact that the polarization state of particles with spin $S \geqslant 1$ is characterized not only by the polarization vector but also by polarization moments of higher order. The maximum number of these moments is $2S$ (Ramsey 1956). So, in the case of particles with spin $S = 1$, there is a polarization moment of first order (polarization vector) and a moment of second order (quadrupolarization tensor) $\hat{Q}_{ik}$. According to the definition

$$\hat{Q}_{ik} = \frac{1}{2S(2S-1)}\left(\hat{S}_i\hat{S}_k + \hat{S}_k\hat{S}_i - \frac{2}{3}\hat{S}^2\delta_{ik}\right).$$

Now, let us consider the components of quadrupolarization tensor $\hat{Q}_{ik}$. It is easy to show that the components Q_{xx}, Q_{yy}, Q_{zz}, Q_{xy}, Q_{yx} are not changed while passing the area occupied by the field. The components Q_{xz}, Q_{yz} can be written as

$$\begin{aligned}
Q_{xz} &= \frac{1}{\sqrt{2}}C_0[C_1\cos(\phi - f_1x) - C_{-1}\cos(\phi + f_1x)] \\
Q_{yz} &= \frac{1}{\sqrt{2}}C_0[C_1\sin(\phi - f_1x) - C_{-1}\sin(\phi + f_1x)]\,.
\end{aligned}\tag{34}$$

Let us consider the following special cases:

(a) $C_1 = C_{-1}$. In this case

$$
\begin{aligned}
Q_{xz} &= \sqrt{2} C_1 C_0 \sin\phi \sin f_1 x \\
Q_{yz} &= -\sqrt{2} C_1 C_0 \cos\phi \sin f_1 x \,.
\end{aligned}
\tag{35}
$$

Comparing the above expressions with the components of the polarization vector (28) we can draw the conclusion that

$$
Q_{xz} = 2\left\langle S_x\left(\phi - \frac{\pi}{2}, f_1 x - \frac{\pi}{2}\right)\right\rangle \qquad Q_{yz} = 2\left\langle S_y\left(\phi - \frac{\pi}{2}, f_1 x - \frac{\pi}{2}\right)\right\rangle . \tag{36}
$$

The right-hand side of (36) indicates the dependency of polarization vector components on the azimuth angle ϕ and phase $f_1 x$. Introducing the vector

$$
\boldsymbol{n}_{1z} = Q_{xz}\boldsymbol{e}_1 + Q_{yz}\boldsymbol{e}_2 + Q_{zz}\boldsymbol{e}_3 \tag{37}
$$

we find that the components of the given vector, orthogonal to the field and the vector $\langle \boldsymbol{S}\rangle$ are orthogonal to each other and their oscillations are phase-shifted by $\pi/2$. According to (Baryshevsky 1992) the mentioned oscillation effect describes the exhibition of the birefringence effect for particles analogous to the birefringence effect in the media located in the electric field (Kerr effect).

(b) $C_{-1} \ll C_1, C_0$; then

$$
\begin{aligned}
Q_{xz} &= \frac{1}{\sqrt{2}} C_0 C_1 \cos(\phi - f_1 x) \\
Q_{yz} &= \frac{1}{\sqrt{2}} C_0 C_1 \sin(\phi - f_1 x) \\
Q_{zz} &= \tfrac{1}{3}\left(C_1^2 - 2C_0^2\right) .
\end{aligned}
\tag{38}
$$

From the derived expressions it follows that vector $\boldsymbol{n}_{1z}$ behaves similarly in the electric field to the polarization vector, namely, it turns through an angle $-f_1 x$ while the atom is passing through the area occupied by the field.

(c) When $C_1 \ll C_0, C_{-1}$, then

$$
\begin{aligned}
Q_{xz} &= -\frac{1}{\sqrt{2}} C_0 C_1 \cos(\phi + f_1 x) \\
Q_{yz} &= -\frac{1}{\sqrt{2}} C_0 C_1 \sin(\phi + f_1 x) \\
Q_{zz} &= \tfrac{1}{3}\left(C_{-1}^2 - 2C_0^2\right) .
\end{aligned}
\tag{39}
$$

From (38) it follows that the vector $\boldsymbol{n}_{1z}$ behaves similarly to the polarization vector, i.e. turns through the angle $f_1 x$ while the atom is passing through the area occupied by the field.

5. Refraction of atom with spin $S = \frac{3}{2}$ in the electric field

Now let us consider the atom with spin $S = \frac{3}{2}$. For the components of the polarization vector we find the following expressions:

$$
\begin{aligned}
\langle S_x\rangle &= \sqrt{3}[C_{3/2}C_{1/2}\cos(\phi - 2f_1 x) + C_{-3/2}C_{-1/2}\cos(\phi + 2f_1 x)] + 2C_{1/2}C_{-1/2}\cos\phi \\
\langle S_y\rangle &= \sqrt{3}[C_{3/2}C_{1/2}\sin(\phi - 2f_1 x) + C_{-3/2}C_{-1/2}\sin(\phi + 2f_1 x)] + 2C_{1/2}C_{-1/2}\sin\phi \\
\langle S_z\rangle &= 3/2\left(C_{3/2}^2 - C_{-3/2}^2\right) + 1/2\left(C_{1/2}^2 - C_{1/2}^2\right) .
\end{aligned}
\tag{40}
$$

As previously, let us study some special cases.

(a) The initial polarization vector is orthogonal to the electric field. In this case the condition

$$C_{3/2} = C_{-3/2} \qquad C_{1/2} = C_{-1/2}$$

is fulfilled. Then

$$\begin{aligned}\langle S_x\rangle &= 2\sqrt{3}C_{3/2}C_{1/2}\cos\phi\cos 2f_1x + 2C_{1/2}^2\cos\phi\\ \langle S_y\rangle &= 2\sqrt{3}C_{3/2}C_{1/2}\sin\phi\cos 2f_1x + 2C_{1/2}^2\sin\phi\\ \langle S_z\rangle &= 0\,.\end{aligned} \tag{41}$$

According to (41) the polarization vector consists of two components: the first oscillates at the frequency $2\omega_\beta$, the second is constant.

(b) Under the condition

$$C_{-3/2} \ll C_{-1/2}, C_{1/2}, C_{3/2}$$

we have

$$\begin{aligned}\langle S_x\rangle &= \sqrt{3}C_{3/2}C_{1/2}\cos(\phi - 2f_1x) + 2C_{1/2}C_{-1/2}\cos\phi\\ \langle S_y\rangle &= \sqrt{3}C_{3/2}C_{1/2}\sin(\phi - 2f_1x) + 2C_{1/2}C_{-1/2}\sin\phi\\ \langle S_z\rangle &= \tfrac{3}{2}C_{3/2}^2 + \tfrac{1}{2}\left(C_{1/2}^2 - C_{1/2}^2\right)\,.\end{aligned} \tag{42}$$

From (42) it follows that polarization vector is the sum of two vectors. The first turns through the angle $-2f_1x$ while the atom is passing through the area occupied by the field, the second does not change its direction.

(c) In the case where

$$C_{3/2} \ll C_{1/2}, C_{-1/2}, C_{-3/2}$$

the polarization vector is also the sum of two vectors. However, the first of them turns through the angle $2f_1x$, i.e. in the direction opposite to that described above in case (b).

Now let us consider the quadrupolarization tensor $\hat{Q}_{ik}$:

$$\hat{Q}_{ik} = 1/6\left(\hat{S}_i\hat{S}_k + \hat{S}_k\hat{S}_i - 5/2\delta_{ik}\right)\,. \tag{43}$$

The appropriate calculations leads to the following expressions:

$$Q_{zz} = 13/12\left(C_{3/2}^2 + C_{-3/2}^2\right) - 1/3\left(C_{1/2}^2 + C_{-1/2}^2\right) \tag{44}$$

$$\begin{aligned}Q_{xx} = {}&-1/6\left(C_{3/2}^2 + C_{-3/2}^2\right) + 1/6\left(C_{1/2}^2 + C_{-1/2}^2\right)\\ &+1/\sqrt{3}[C_{3/2}C_{-1/2}\cos 2(\phi - f_1x) + C_{-3/2}C_{1/2}\cos 2(\phi + f_1x)]\end{aligned} \tag{45}$$

$$\begin{aligned}Q_{yy} = {}&-1/6\left(C_{3/2}^2 + C_{-3/2}^2\right) + 1/6\left(C_{1/2}^2 + C_{-1/2}^2\right)\\ &-1/\sqrt{3}[C_{3/2}C_{-1/2}\cos 2(\phi - f_1x) + C_{-3/2}C_{1/2}\cos 2(\phi + f_1x)]\end{aligned} \tag{46}$$

$$Q_{xy} = 1/\sqrt{3}[C_{3/2}C_{-1/2}\sin 2(\phi - f_1x) + C_{-3/2}C_{1/2}\sin 2(\phi + f_1x)] \tag{47}$$

$$Q_{yz} = 1/\sqrt{3}[C_{3/2}C_{1/2}\sin(\phi - 2f_1x) - C_{-3/2}C_{-1/2}\sin(\phi + 2f_1x)] \tag{48}$$

$$Q_{xz} = 1/\sqrt{3}[C_{3/2}C_{1/2}\cos(\phi - 2f_1x) - C_{-3/2}C_{-1/2}\cos(\phi + 2f_1x)]\,. \tag{49}$$

From the expressions derived it follows that for particles with $S = \frac{3}{2}$ only one component is retained, namely Q_{zz}. When the particle polarization vector is orthogonal to the electric field, all the components of the quadrupolarization tensor, except Q_{zz}, oscillate with the frequency $2\omega_\beta$. In this case

$$Q_{xx} = -1/3\left(C_{1/2}^2 - C_{3/2}^2\right) + 2/\sqrt{3}C_{3/2}C_{1/2}\cos 2\phi\cos 2f_1x \tag{50}$$

$$Q_{yy} = -1/3\left(C_{1/2}^2 - C_{3/2}^2\right) - 2/\sqrt{3}C_{3/2}C_{1/2}\cos 2\phi\cos 2f_1x \tag{51}$$

$$Q_{xy} = 2/\sqrt{3}C_{3/2}C_{1/2}\sin 2\phi\cos 2f_1x \tag{52}$$

$$Q_{yz} = -2/\sqrt{3}C_{3/2}C_{1/2}\cos\phi\sin 2f_1x \tag{53}$$

$$Q_{xz} = 2/\sqrt{3}C_{3/2}C_{1/2}\sin\phi\sin 2f_1x\,. \tag{54}$$

The components Q_{xz}, Q_{yz} become proportional to the polarization vector components S_x, S_y, respectively, in which ϕ is changed to $\phi - \pi/2$ and $2f_x$ to $2f_1x - \pi/2$. In the case where the polarization vector forms some angle with the positive or negative field direction, the vector $\boldsymbol{n}_{1z}$ turns, respectively, through the angle $-2f_1x$ or $2f_1x$ around the field direction. Let us introduce two vectors $\boldsymbol{n}_{1y}$ and $\boldsymbol{n}_{1x}$, determining them in the following way:

$$\boldsymbol{n}_{1y} = Q_{xy}\boldsymbol{e}_1 + Q_{yy}\boldsymbol{e}_2 + Q_{zy}\boldsymbol{e}_3 \tag{55}$$

$$\boldsymbol{n}_{1x} = Q_{xx}\boldsymbol{e}_1 + Q_{yx}\boldsymbol{e}_2 + Q_{zx}\boldsymbol{e}_3\,. \tag{56}$$

Let us consider the behaviour of the vector $\boldsymbol{n}_{1y}$ in the case when we can neglect the value $C_{-3/2}$. Let us also suggest that

$$C_{1/2}C_{-3/2} \ll C_{-1/2}C_{3/2}\,.$$

Then

$$\begin{aligned} Q_{xy} &= 1/\sqrt{3}C_{3/2}C_{-1/2}\sin 2\left(\phi - f_1x\right) \\ Q_{yy} &= -1/\sqrt{3}C_{3/2}C_{-1/2}\cos 2\left(\phi - f_1x\right) - 1/6C_{3/2}^2 + 1/6\left(C_{1/2}^2 + C_{-1/2}^2\right) \\ Q_{zy} &= 1/\sqrt{3}C_{3/2}C_{1/2}\sin\left(\phi - 2f_1x\right)\,. \end{aligned} \tag{57}$$

Let us denote

$$\acute{Q}_{yy} = Q_{yy} + 1/6\left(C_{3/2}^2 - C_{1/2}^2 - C_{-1/2}\right)\,. \tag{58}$$

In the result we have

$$\begin{aligned} Q_{xy} &= Q_{xy}^0\cos 2f_1x + \acute{Q}_{yy}^0\sin 2f_1x \\ \acute{Q}_{yy} &= \acute{Q}_{xy}^0\cos 2f_1x - Q_{xy}^0\sin 2f_1x \\ Q_{zy} &= 1/\sqrt{3}C_{3/2}C_{1/2}\sin\left(\phi - 2f_1x\right) \end{aligned} \tag{59}$$

where $Q_{xy}^0, \acute{Q}_{yy}^0$ are the components of Q_{xy}, $\acute{Q}_{yy}$ under $x = 0$. From the derived expressions it follows that the orthogonal vector component $\boldsymbol{n}_{1y}$, in which the component Q_{yy} is changed to $\acute{Q}_{yy}$, turns through an angle $-2f_1x$ while the atom is passing the area occupied by the electric field. The Z-component of the given vector oscillates with frequency $2\omega_\beta$. So, the vector $\boldsymbol{n}_{1z}$ end will describe some ellipse-type trajectory.

6. Atom with spin $S = 2$

Let us consider one more special case of an atom with spin $S = 2$. Performing all necessary calculations we find for the polarization vector component the following expression:

$$\begin{aligned}\langle S_x\rangle &= 2[C_2C_1\cos(\phi-3f_1x)+C_{-2}C_{-1}\cos(\phi+3f_1x)]+\sqrt{6}C_0[C_1\cos(\phi-f_1x)\\&\quad +C_{-1}\cos(\phi+f_1x)]\\ \langle S_y\rangle &= 2[C_2C_1\sin(\phi-3f_1x)+C_{-2}C_{-1}\sin(\phi+3f_1x)]+\sqrt{6}C_0[C_1\sin(\phi-f_1x)\\&\quad +C_{-1}\sin(\phi+f_1x)]\\ \langle S_z\rangle &= 2\left(C_2^2-C_{-2}^2\right)+C_1^2-C_{-1}^2\,.\end{aligned} \tag{60}$$

From (60) it follows that the components x, y of the polarization vector contain the terms which oscillate at frequencies $3\omega_\beta$ and ω_β in the electric field. It is easy to show that in the case when the atom polarization vector is orthogonal to the electric field it can be presented as a sum of two vectors. One of them oscillates with frequency $3\omega_\beta$, the other with frequency ω_β. If the initial polarization vector is oriented primarily along or (opposite to) the field direction, then its further behaviour is determined by the behaviour of the two vector components. One of them is turned through the angle $\mp 3f_1x$ while the atom is passing the area, occupied by the field, the other through the angle $\mp f_1x$. Let us write the average meanings of the quadrupolarization tensor components:

$$Q_{ik} = 1/4\,(S_iS_k+S_kS_i-4/3\delta_{ik}) \tag{61}$$

$$Q_{zz} = C_2^2+C_{-2}^2-1/2\left(C_1^2+C_{-1}^2\right) \tag{62}$$

$$\begin{aligned}Q_{xx} &= -1/2\left(C_2^2+C_{-2}^2\right)+1/2C_0^2+1/4\left(C_1^2+c_{-1}^2\right)+3/2C_1C_{-1}\cos 2\phi\\&\quad +\sqrt{6}/2C_0[C_2\cos(2\phi-4f_1x)+C_{-2}\cos(2\phi+4f_1x)]\end{aligned} \tag{63}$$

$$\begin{aligned}Q_{yy} &= -1/2\left(C_2^2+C_{-2}^2\right)+1/2C_0^2+1/4\left(C_1^2+C_{-1}^2\right)-3/2C_1C_{-1}\cos 2\phi\\&\quad -\sqrt{6}/2C_0[C_2\sin(2\phi-4f_1x)+C_{-2}\sin(2\phi+4f_1x)]\end{aligned} \tag{64}$$

$$Q_{xy} = \sqrt{6}/2C_0[C_2\sin(2\phi-4f_1x)+C_{-2}\sin(2\phi+4f_1x)]+3/2C_1C_{-1}\sin 2\phi \tag{65}$$

$$\begin{aligned}Q_{yz} &= 3/2[C_2C_1\sin(\phi-3f_1x)-C_{-2}C_{-1}\sin(\phi+3f_1x)]\\&\quad +\sqrt{6}/4C_0[C_1\sin(\phi-f_1x)-C_{-1}\sin(\phi+f_1x)]\end{aligned} \tag{66}$$

$$\begin{aligned}Q_{xz} &= 3/2[C_2C_1\cos(\phi-3f_1x)-C_{-2}C_{-1}\cos(\phi+3f_1x)]\\&\quad +\sqrt{6}/4C_0[C_1\cos(\phi-f_1x)-C_{-1}\cos(\phi+f_1x)]\,.\end{aligned} \tag{67}$$

The components $\left(Q_{xx}, Q_{yy}, Q_{xy}\right)$, oscillating with frequency $4\omega_\beta$ appear in the derived expressions. This frequency exceeds the maximum oscillation frequency of polarization vector which equals $3\omega_\beta$.

As in cases previously considered it is easy to show that the behaviour of vector n_{1z} is similar to that of the polarization vector.

7. Atom with arbitrary spin S

Atom polarization vector with the arbitrary half-integer spin S located in the area occupied by the constant electric field can be given as a series

$$\langle S\rangle = \sum_{n=0}^{S-1/2} S_{2n}\,. \tag{68}$$

The vector S_{2n} has the following components:

$$
\begin{aligned}
S_{2nx} &= \sqrt{\left(S+\tfrac{1}{2}\right)^2 - n^2}\,[C_{n+1/2}C_{n-1/2}\cos(\phi - 2nf_1x) \\
&\qquad + C_{-(n+1/2)}C_{-(n-1/2)}\cos(\phi + 2nf_1x)] \\
S_{2ny} &= \sqrt{\left(S+\tfrac{1}{2}\right)^2 - n^2}\,[C_{n+1/2}C_{n-1/2}\sin(\phi - 2nf_1x) \\
&\qquad + C_{-(n+1/2)}C_{-(n-1/2)}\sin(\phi + 2nf_1x)] \\
S_{2nz} &= \left(n+\tfrac{1}{2}\right)\left(C^2_{n+1/2} - C^2_{-(n+1/2)}\right).
\end{aligned}
\tag{69}
$$

As follows from (69) the oscillation frequency of the x and y components of the vector under consideration equals $2n\omega_\beta$. In the case of integer spin S,

$$
\langle S\rangle = \sum_{n=0}^{S-1} S_{2n+1} \tag{70}
$$

$$
\begin{aligned}
S_{2n+1x} &= \sqrt{S(S+1)^2 - n(n-1)}\,[C_{n+1}C_n\cos(\phi - (2n+1)f_1x) \\
&\qquad + C_{-(n+1)}C_{-n}\cos(\phi + (2n+1)f_1x)] \\
S_{2n+1y} &= \sqrt{S(S+1)^2 - n(n-1)}\,[C_{n+1}C_n\sin(\phi - (2n+1)f_1x) \\
&\qquad + C_{-(n+1)}C_{-n}\sin(\phi + (2n+1)f_1x)] \\
S_{2n+1z} &= (n+1)\left(C^2_{n+1} - C^2_{-(n+1)}\right).
\end{aligned}
\tag{71}
$$

The components x, y of S_{2n+1} vector oscillate with the frequency $(2n+1)\,\omega_\beta$.

Considering the evolution of quadrupolarization tensor components we should note the following: the vector n_{1z} will behave similarly to the polarization vector. The components Q_{xx}, Q_{yy} will oscillate on the frequencies proportional to $\left[S^2 - (S-2)^2\right]\omega_\beta$; $4(S-2)\,\omega_\beta \ldots$.

8. Atom spin interferometry when the atom with spin $S \geqslant 1$ is passing through the electric field

As was mentioned above, the presence of an additional phase shift of the neutron wave in the media can be used as a basis of the experimental method called neutron spin interferometry. The potential energy of the atom in the electric field in quantum state χ_M equals

$$
U(M) = -1/2\mathcal{E}_z^2\left(p + p_1M^2\right). \tag{72}
$$

In the area occupied by the electric field we put some media with thickness l and optical potential V (for example a gas target); then the refraction index has the following form:

$$
n(M,V) = \left(1 - \frac{U(M)}{W} - \frac{V}{W}\right)^{1/2}. \tag{73}
$$

After passing through the media the spin components of the wavefunction, for which the value of the quantum number M differs per unit, obtain an additional phase difference

$$
\phi = k[n(M,V) - n(M-1,V)]l = \phi_0 + \Delta\phi \tag{74}
$$

where

$$
\phi_0 = (2M-1)\frac{\mathcal{E}^2\beta}{\hbar}\frac{l}{v} = (2M-1)\,\omega_\beta\frac{l}{v} \tag{75}
$$

the phase difference is connected with the rotation in constant electric field in the absence of potential V by

$$\Delta\phi = -\frac{1}{2}(2M-1)\,\omega_\beta \frac{V}{2W}\frac{l}{v}\,. \tag{76}$$

The additional difference of phases is caused by the presence the of media.

Thus, the existence of additional phase differences of the atom wavefunction in the media can be used as a basis for the spin interferometer. It is very interesting that an atom spin interferometer permits us to immediately detect the influence of the gravitational field on separate atoms. In fact, in the quasi-classical approximation the phase difference, which occurs when the particles are moving along a certain trajectory l (for example, up or down) is calculated according to the well known expression

$$\phi = k\int_{r_0}^{r}[n(M, V(r)) - n(M-1, V(r))]\,\mathrm{d}l\,. \tag{77}$$

Integration over l in (77) follows the trajectory l; r and r_0 are correspondingly the coordinates of the beginning and end of the trajectory. Using the same approximation as in (73) the gravitation effect value is determined by the expression

$$\Delta\phi_{gr} = -\frac{1}{2}(2M-1)\,\omega_\beta \frac{mgl}{2W}\frac{l}{v} = -\frac{1}{2}(2M-1)\,\omega_\beta \frac{gl^2}{v^3} \tag{78}$$

where g is the free-fall acceleration.

Let us note the fact that the given shift has a simple quasiclassical explanation. Namely, the given effect can be explained by a velocity change and as a consequence a time for which a particle, being in different quantum states, passes the given trajectory l.

As follows from (78) the gravitation shift is proportional to the frequency ω_β, to the square of the distance travelled, and is inversely related to the atom velocity cubed.

Let us estimate the given value for the atom ^{20}Ne (nuclei spin is equal to zero), located in the metastable $^3P_2$ state (lifetime of the given state is 20 s). According to Miller *et al* (1972) the neon atom tensor polarizability in the given state $\beta = -1.1\times10^{-24}$ cm^3 then under an electric field strength $\mathcal{E} = 30$ SGS $= 9$ kV cm^{-1}, frequency $\omega_\beta = (-)\,0.94$ MHz, where the minus sign in brackets should be taken into consideration when determining the direction of atom spin rotation. If the length of the way passed in the gravitation field equals 100 cm, then under the atom velocity $v = 10^5$ cm s^{-1} with $M = 2$, the gravitation shift $\Delta\phi_{gr} = -1.38\times10^{-2}$ rad, i.e. the effect has a rather considerable value.

9. Description of the proposed experiment

Let us briefly describe the experimental set-up for observation of the effects described in this paper. The atom beam exiting the atomizer passes through a system of diaphragms forming the necessary (rectangular) beam geometry. To reduce the velocity spread of atoms in the beam we are planning to use a velocity selector. After passing the system of diaphragms and velocity selector, the atom beam enters a polarizer (Stern–Gerlach magnet), where the atoms with the needed polarization are selected. In this case, if we use a Stern–Gerlach magnet as a polarizer, then for atoms with nuclear spin $I = 0$, the beams will possess the chosen polarization close to 100 % at the magnet exit.

One of the selected beams enters the reference hot-wire detector. The beam of interest for our polarization passes, without deposition on the wire (or strip), through the reference detector and enters the interaction chamber with electric field $\mathcal{E} = 50$ SGS. After passing the interaction chamber, the beam enters the analyser (in our case another Stern–Gerlach magnet). In the last unit of the set-up the positionally sensitive hot-wire detector is situated.

The analyser geometry is chosen so that its axis is parallel to the polarization vector of the atom beam before entering the interaction chamber. When there is a constant electric field in the interaction chamber with strength $\mathcal{E}$ the atom wavefunction will suffer the phase shift described in this paper, i.e. the rotation (oscillation) of the polarization vector. In the result the polarization vector of the atom beam is not parallel to the analyser vector. So at the detector entrance $2S + 1$ atom beams will arrive. The atom density in these beams changes proportionally to the rotation angle of polarization vector in electric field. Our hot-wire detector is positionally sensitive, i.e. it permits us to tune to each of the $2S + 1$ beam components.

The numerical modelling carried out leads us to the conclusion that the chosen geometry of the experiment permits to avoid swamping of the effect due to velocity spread of the particles. To measure the gravitational atom shift in the electric field we shall locate our set-up vertically.

In our experiment we are planning to work with atoms of the rare-earth elements Ce_{58}, Sm_{62}, Eu_{63}. The magnitude for Sm_{62} and Eu_{63} was determined experimentally (Angel and Sandars 1968). So for Sm_{62} $\beta\,(J = 6) = -3.64 \pm 0.17a_0^3$, for Eu_{63} $\beta\left(J = \frac{7}{2}\right) = 0.0141 \pm 0.0007a_0^3$. The value of total electron spin is given in brackets, a_0 is the Bohr radius. In the interaction chamber the intensity of the electric field $\mathcal{E}$ is 50 SGS, then for Sm_{62} $\omega_\beta = -1.35$ MHz, for Eu_{63} $\omega_\beta = 5.24$ kHz. Heat velocities are calculated according to the expression $v = \sqrt{3kT/m}$, where T is the temperature of the beam source $v_{\mathrm{Sm}} = 4.72 \times 10^4$ cm s^{-1}, $v_{\mathrm{Eu}} = 4.24 \times 10^4$ cm s^{-1}. To evaluate the rotation angle let us use the expression $\theta = \omega_\beta l/v$. Then at $l = 10$ cm, $\theta_{\mathrm{Sm}} = 287$ rad, $\theta_{\mathrm{Eu}} = 1.23$ rad, i.e. the observed effect has the correct magnitude. In the first stage of the experiment we are planning to work with Ce_{58} atoms. The choice of this element is based on the fact that Ce_{58} atom spin $S = 4$ and nucleus spin is equal to zero. Unfortunately, we have no data concerning the magnitude of Ce_{58} atom tensor polarizability (we are planning to measure it in our experiment). Nevertheless, the analysis of the value of the effect for the above-mentioned elements Sm_{62}, Eu_{63} gives us a basis to suggest that for Ce_{58} the given effects have essentially the same value.

Besides, Ce_{58} atoms possess a low ionization potential $V = 5.4$ eV. It essentially increases the effectiveness of hot-wire detection on the basis of surface ionization (for example, we are planning to work with Ir strips $\phi = 5.75$ eV, ϕ-work function).

The given experiment is of interest for the following reasons. First of all, we shall observe directly the effect connected with the atom (molecule) spin rotation in the electric field; secondly, the given experiment will permit us to obtain information about scalar and vector atom polarizabilities; thirdly, the effect considered in the last section of this paper is connected with the additional shift of atom wavefunction phase. This shift is caused by the medium located in the electric field and may be used for creation of an atomic spin interferometer. Let us also note that this effect may be used for the verification of the weak equivalence principle in quantum mechanics. We know that 'the weak equivalence principle, in the formation of general relativity, reads something like this: all small bodies free of any force (in practice the electromagnetic force) other than gravity, placed at the same inertial point in spacetime, and with the same inertial velocity, follow the same trajectories in spacetime geodesics of the spacetime metric), irrespective of their internal constitutions' (Schmiedmayer 1989). Up to now the experiments to verify the principle of equivalence for isolated atoms and elementary particles were not convincing, except for the experiments with neutrons (Clifford 1981). The interest in the verification of weak equivalence principle in a quantum field is caused by the search for the so-called fifth force introduced by Fishbach *et al* (1986).

10. Conclusions

The atom (molecule) interaction with spin $S \geqslant 1$ with constant electric field leads to dependence of effective potential interaction energy from atom spin projection. As a result, in the case where atom spin comprises an angle smaller than $\pi/2$ with the directed field, polarization vector spin rotation occurs clockwise. If the angle is larger than $\pi/2$ the rotation is anticlockwise. If the angle is equal to $\pi/2$ then the oscillation of particle spin occurs. It also appears that atom (molecule) quadrupole moment in the electric field depends on the time and the components Q_{xx}, Q_{yy} of atom quadrupolarization tensor oscillate with frequencies proportional to $[S^2 - (S-2)^2]\omega_\beta$; $4(S-2)\omega_\beta \ldots$. This effect may be used as the basis of a new method permitting experimentally the determination of the atom (molecule) polarizability. The effect considered in the last section connected with additional phase shift of atom wavefunction caused by the presence of the media located in the electric field can be used for the creation of an atomic spin interferometer.

The atomic spin interferometer may be used for verification of the weak equivalence principle in quantum mechanics.

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