

## The elastic constants of piezoelectric crystals

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# The elastic constants of piezoelectric crystals

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The values available in the literature for the elastic constants of the piezoelectric crystals Rochelle salt, potassium dihydrogen phosphate, ammonium dihydrogen phosphate, quartz, sodium chlorate and sodium bromate are assembled. The averages and standard deviations of the estimates are calculated, and the variability of the results discussed. The direct extension and lateral contraction stiffnesses of Rochelle salt show great variability which, however, is much reduced when the elastic constants are expressed as compliances. Little, if any, evidence is found of similar behaviour in the other materials. For comparison with the results on piezoelectric crystals, the available determinations on the non-piezoelectric crystals sodium chloride, lithium fluoride and pyrites are also summarized. It is concluded that elastic constants of crystals are seldom known to better than 1%, and in many cases the uncertainty is considerably larger. This variability is probably due to inherent variability in the materials rather than to experimental error or to differences caused by the use of different experimental methods.

The elastic constants of piezoelectric crystals are of importance, practically in the calculation of the frequencies of vibration of rotated cuts, and theoretically in connexion with the atomic theory of piezoelectric and elastic effects in crystals. The constants are measured either as compliances  $S_{ik}$  or stiffnesses  $C_{ik}$ <sup>(1)</sup>; the former are appropriate when the strains are expressed in terms of elastic constants and stresses, and the latter when the stresses are expressed in terms of elastic constants and strains. A number of determinations of the elastic constants of the piezoelectric crystals Rochelle salt, ammonium dihydrogen phosphate (ADP), potassium dihydrogen phosphate (KDP), quartz, sodium chlorate (NaClO<sub>3</sub>) and sodium bromate (NaBrO<sub>3</sub>) are now available; the object of this paper is to collect these results, and to discuss their accuracy. The methods which have been used in the determinations are summarized in Table 1; for brevity the methods will be referred to later by the letter given in the second column of Table 1.

Brief descriptions of methods *R*, *T*, and *D* are given in Hearmon's review,<sup>(2)</sup> together with a more detailed discussion of method *S*. Methods *T*, *P* and *D* are dealt with at some length by Bergmann,<sup>(10)</sup> but for completeness, short accounts of the methods are given here.

Table 1. Methods of measuring elastic constants

Method	Abbreviation	References
Static	<i>S</i>	2, 3, 4, 5
Resonant frequencies	<i>R</i>	6, 7, 8, 9
Ultrasonic transmission	<i>T</i>	10, 11, 12, 13, 14, 15
Pulse transmission	<i>P</i>	10, 16, 17
Light diffraction	<i>D</i>	10, 18, 19, 20

In method *S*, the strain under static loading is measured, and the appropriate compliance is calculated from the ratio of stress to strain.<sup>(5)</sup> In method *R*, the resonant frequencies of appropriately cut and excited specimens are measured and the elastic constants are derived from the resonant frequencies and the dimensions of the specimens.<sup>(7, 9)</sup>

Methods *T*, *P* and *D* are of more recent development and are therefore less familiar. In method *T*, a quartz wedge is attached to one face of a plate of the crystal, and the other face is immersed in a suitable liquid. The wedge is set into vibration by application of an alternating voltage and the frequency of excitation is varied. At the resonant frequencies of the specimen, the transmission of ultrasonic energy through the specimen and into the liquid is a maximum; this state of affairs is detected by passing a beam of light through the liquid and observing the light diffraction effects produced by

Table 2. Rochelle salt (orthorhombic)

Author	Method	Reference	$S_{11}$	$S_{22}$	$S_{33}$	$S_{44}$	$S_{55}$	$S_{66}$	$S_{12}$	$S_{13}$	$S_{23}$
Mandell	<i>S</i>	4	46.9	32.0	28.2	60.9	306	80.2	-8.0	-21.8	+16.8
Hinz	<i>S</i>	3	52.3	34.3	32.4	96.3	337	118	-21.8	-16.9	-13.4
Mason	<i>R</i>	9	51.8	34.9	33.4	79.8	328	101	-15.3	-21.1	-10.3
Sundara Rao	<i>T</i>	15	50.2	30.4	31.7	82.0	333	106	-11.6	-21.4	-8.9
Huntington	<i>P</i>	16	52.4	35.0	33.7	74.6	311	102	-15.4	-9.8	-9.1
All results	$\bar{S}$		50.7	33.3	31.9	78.7	323	101	-14.4	-18.2	-5.0
	$\sigma_s$		2.37	1.98	2.37	15.2	13.3	16.3	5.9	5.2	13.0
	$V_s$		4.7	5.9	7.4	19.3	4.1	16.1	41	29	260
Omitting static results	$\bar{S}$		51.5	33.4	32.9	78.8	324	103	-14.1	-17.4	-9.4
	$\sigma_s$		1.3	2.7	1.18	4.4	13.0	2.95	2.2	6.8	0.83
	$V_s$		2.5	8.1	3.6	5.6	4.0	2.9	16	39	8.8
Author	Method	Reference	$C_{11}$	$C_{22}$	$C_{33}$	$C_{44}$	$C_{55}$	$C_{66}$	$C_{12}$	$C_{13}$	$C_{23}$
Mandell	<i>S</i>	4	3.46	4.73	8.01	1.64	0.327	1.25	-0.78	3.14	-3.42
Hinz	<i>S</i>	3	6.82	10.31	9.66	1.04	0.297	0.848	6.83	6.38	7.83
Mason	<i>R</i>	9	4.25	5.15	6.29	1.25	0.304	0.996	2.96	3.57	3.42
Sundara Rao	<i>T</i>	15	4.06	5.20	6.40	1.22	0.300	0.95	2.56	3.46	3.20
Huntington	<i>P</i>	16	2.55	3.81	3.71	1.34	0.321	0.979	1.41	1.16	1.46
Jona	<i>D</i>	18	2.58	3.80	3.75	—	0.314	0.997	1.40	1.12	—
Omitting Jona's results	$\bar{C}$		4.23	5.84	6.81	1.30	0.310	1.005	2.60	3.54	2.50
	$\sigma_c$		1.84	2.80	2.56	0.26	0.013	0.173	3.28	2.25	4.8
	$V_c$		43	48	38	20	4.2	17	126	63	192
Omitting Jona's and static results	$\bar{C}$		3.62	4.72	5.47	1.27	0.308	0.975	2.31	2.73	2.69
	$\sigma_c$		1.0	0.82	1.59	0.071	0.012	0.027	0.91	1.42	1.15
	$V_c$		28	17	29	5.6	3.9	2.8	39	52	43

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the ultrasonic "grating" in the liquid. The elastic constants are then found from the frequencies of maximum transmission and the thickness of the specimen. This method was devised and developed by Bhagavantam<sup>(11)</sup> and his co-workers in India, and its use has apparently been confined to that country.

Method *D* was devised and developed by continental workers and it has been used by them and by one Japanese.<sup>(19)</sup> A quartz transducer is attached to the specimen, which is set into vibration on excitation of the quartz by an alternating voltage. A beam of light is passed through the specimen itself, and the light is diffracted by the supersonic "grating" in the vibrating crystal. The diffraction pattern is photographed, and the elastic constants are derived from certain dimensions of the diffraction pattern, in conjunction with an apparatus constant.<sup>(10)</sup>

Table 3. *KDP (tetragonal)*

Author	Method	Reference	$S_{11}$	$S_{33}$	$S_{44}$	$S_{66}$	$S_{12}$	$S_{13}$
Mason	<i>R</i>	9	18	20	79	164	-4	-7
Zwicker	<i>D</i>	20	15.3	19.6	77.5	168	2.0	-3.8
Bantle and others	<i>R</i>	20	15.2	19.5	77.9	158	1.8	-4.5
Price and Huntington	<i>P</i>	17	14.7	19.5	79	159	1.7	-3.8
All results	$\bar{S}$		15.8	19.7	78.3	162	0.4	-4.8
	$\sigma_s$		1.62	0.25	0.73	4.9	2.9	1.57
	$V_s$		10	1.3	0.93	3.0	725	33
Omitting Mason's results	$\bar{S}$		15.1	19.5	78.1	162	1.8	-4.0
	$\sigma_s$		0.35	0.059	0.89	5.9	0.18	0.41
	$V_s$		2.3	0.30	1.1	3.6	10	10

Author	Method	Reference	$C_{11}$	$C_{33}$	$C_{44}$	$C_{66}$	$C_{12}$	$C_{13}$
Mason	<i>R</i>	9	7.8	7.7	1.27	0.61	3.23	3.84
Zwicker	<i>D</i>	20	6.91	5.56	1.29	0.600	-0.600	1.22
Bantle and others	<i>R</i>	20	7.08	5.84	1.28	0.633	-0.383	1.55
Price and Huntington	<i>P</i>	17	7.14	5.61	1.27	0.628	-0.49	1.29
All results	$\bar{C}$		7.23	6.18	1.28	0.618	0.439	1.98
	$\sigma_c$		0.44	1.05	0.0098	0.016	1.87	1.28
	$V_c$		6.1	17	0.77	2.6	426	65
Omitting Mason's results	$\bar{C}$		7.04	5.67	1.28	0.620	-0.491	1.35
	$\sigma_c$		0.14	0.17	0.012	0.019	0.128	0.195
	$V_c$		2.0	3.0	0.94	3.1	26	14

In method *P*, the elastic constants are obtained from the velocity of propagation of ultrasonic pulses in the crystal. A quartz transducer is attached to one face of the specimen, and pulses are applied to the quartz. These are transmitted through the material, reflected at the opposite face of the specimen, travel back and are picked up by the quartz. The pulses are produced, and the interval between launching and reception timed, by apparatus and techniques similar to those used in radar; this method has been devised, developed and applied mainly in the United States.<sup>(16, 17)</sup> From the transmission times, and the path lengths, the velocities of propagation are found, from which in turn the elastic constants are calculated.

Method *S* yields the compliances  $S_{ik}$  directly, method *R* yields the compliances or the stiffnesses  $C_{ik}$  according to the type of vibration, and the other three methods yield the stiffnesses directly. The original authors do not always give both the compliances and the stiffnesses, and in these cases the figures have been converted by the writer, using the appropriate conversion equations.<sup>(2, 5, 20, 21)</sup> In piezoelectric materials, the values of some of the elastic constants depend

on the electrical conditions obtaining during the measurements, and are different, for example, at constant field *E* and at constant displacement *D*.<sup>(1, 9, 21)</sup> The results quoted in the present paper refer as a rule to constant field conditions, at ordinary temperature (ca. 20-30° C); Mason's results for Rochelle salt<sup>(9)</sup> are an exception since they refer to constant displacement conditions.\* However, in practice the only elastic constants which are affected appreciably at ordinary temperatures by the electrical conditions are  $S_{44}$  and  $C_{44}$  of Rochelle salt and  $S_{66}$  and  $C_{66}$  of ADP.

Table 4. *ADP (tetragonal)*

Author	Method	Reference	$S_{11}$	$S_{33}$	$S_{44}$	$S_{66}$	$S_{12}$	$S_{13}$
Mason	<i>R</i>	9	17.5	43.5	114	163	7.5	-11
Zwicker	<i>D</i>	20	20.0	45.7	117	169	1.7	-12.9
Price and Huntington	<i>P</i>	17	17.4	42.2	117	168	2.0	-10.9
Bechmann	<i>R</i>	7	18.1	43.5	115	165	1.9	-11.8
All results	$\bar{S}$		18.3	43.7	116	166	3.3	-11.7
	$\sigma_s$		1.27	1.72	1.47	2.94	2.84	0.98
	$V_s$		6.9	3.9	1.3	1.8	86	8.4
Omitting Mason's results	$\bar{S}$		18.5	43.8	116	167	1.9	-11.9
	$\sigma_s$		1.53	2.07	1.18	2.36	0.18	1.18
	$V_s$		8.3	4.7	1.0	1.4	9.5	10

Author	Method	Reference	$C_{11}$	$C_{33}$	$C_{44}$	$C_{66}$	$C_{12}$	$C_{13}$
Mason	<i>R</i>	9	7.58	2.96	0.87	0.614	-2.43	1.30
Zwicker	<i>D</i>	20	6.17	3.28	0.85	0.592	0.72	1.94
Price and Huntington	<i>P</i>	17	6.89	3.35	0.856	0.595	0.40	1.89
Bechmann	<i>R</i>	7	6.77	3.38	0.868	0.608	0.59	1.99
All results	$\bar{C}$		6.85	3.24	0.861	0.602	-0.18	1.78
	$\sigma_c$		0.69	0.21	0.0098	0.0108	1.54	0.34
	$V_c$		10	6.5	1.1	1.8	855	19
Omitting Mason's results	$\bar{C}$		6.61	3.34	0.858	0.598	0.57	1.94
	$\sigma_c$		0.42	0.059	0.0106	0.0094	0.19	0.059
	$V_c$		6.4	1.8	1.2	1.6	33	3.0

Table 5. *Quartz (trigonal)*

Author	Method	Reference	$S_{11}$	$S_{33}$	$S_{44}$	$S_{12}$	$S_{13}$	$S_{14}$
Voigt	<i>S</i>	5	12.99	9.90	20.05	-1.66	-1.52	4.31
Atanasoff and Hart*	<i>R</i>	6	12.75	9.61	20.09	-1.79	-1.16	4.52
Mason	<i>R</i>	9	12.79	9.56	19.78	-1.53	-1.10	4.46
Bhagavantam	<i>T</i>	11	12.80	9.84	20.00	-1.59	-1.64	4.35
Bechmann	<i>R</i>	7	12.77	9.60	20.00	-1.79	-1.22	4.48
Nomoto	<i>D</i>	19	12.87	9.93	20.42	-1.85	-1.43	4.49
All results	$\bar{S}$		12.83	9.74	20.06	-1.70	-1.35	4.43
	$\sigma_s$		0.096	0.148	0.256	0.128	0.216	0.084
	$V_s$		0.75	1.52	1.28	7.5	16.0	1.9

Author	Method	Reference	$C_{11}$	$C_{33}$	$C_{44}$	$C_{12}$	$C_{13}$	$C_{14}$
Voigt	<i>S</i>	5	8.51	10.55	5.70	0.70	1.41	-1.69
Atanasoff and Hart*	<i>R</i>	6	8.67	10.68	5.79	0.69	1.13	-1.80
Mason	<i>R</i>	9	8.61	10.71	5.87	0.51	1.05	-1.83
Bhagavantam	<i>T</i>	11	8.69	10.68	5.76	0.69	1.56	-1.74
Bechmann	<i>R</i>	7	8.67	10.72	5.80	0.71	1.19	-1.78
Nomoto	<i>D</i>	19	8.64	10.46	5.66	0.79	1.36	-1.73
All results	$\bar{C}$		8.63	10.63	5.76	0.68	1.28	-1.76
	$\sigma_c$		0.072	0.104	0.084	0.112	0.204	0.056
	$V_c$		0.83	0.98	1.46	16.5	16.0	3.2

\* Results corrected by Lawson.<sup>(8)</sup>

The compliances and stiffnesses are given in Tables 2-8; the units throughout are 10<sup>-13</sup> cm<sup>2</sup>/dyn for  $S_{ik}$  and 10<sup>11</sup> dyn/cm<sup>2</sup> for  $C_{ik}$ . It should be noted that hitherto in the literature the sign of  $S_{14}$  for quartz has been negative and that of  $C_{14}$  has been positive, whereas in Table 5  $S_{14}$  is

\* Mason's stiffnesses are quoted incorrectly by Huntington<sup>(16)</sup> and Jona.<sup>(18)</sup>

positive and  $C_{14}$  is negative. This change in sign is made in accordance with the recent I.R.E. standard.<sup>(1)</sup> Voigt's values for  $\text{NaClO}_3$ <sup>(5)</sup> have not been included in Table 6 because all the recent determinations show these values to be in error. The values given by Ramachandran and Wooster<sup>(22)</sup> for  $\text{NaClO}_3$  have also been omitted. These workers used a new and ingenious method depending on the diffuse reflexion of X-rays from the crystal, and although the method is extremely promising it has not yet been developed to give accurate results.

Table 6.  $\text{NaClO}_3$  (cubic)

Author	Method	Reference	$S_{11}$	$S_{44}$	$S_{12}$	$C_{11}$	$C_{44}$	$C_{12}$
Mason	R	9	23.3	85.4	-5.15	4.90	1.17	1.39
Sundara Rao	T	14	23.5	84.1	-5.30	4.92	1.19	1.45
Bhagavantam	T	12	22.9	84.7	-5.35	5.09	1.18	1.55
Jona	D	18	23.4	85	-5.2	4.89	1.17	1.39
Bechmann	R	7	22.9	85.4	-5.05	4.99	1.17	1.41
All results	Mean		23.2	84.9	-5.21	4.96	1.18	1.44
	$\sigma$		0.26	0.56	0.13	0.086	0.0086	0.069
	$V$		1.1	0.66	2.5	1.7	0.73	4.8

Table 7.  $\text{NaBrO}_3$  (cubic)

Author	Method	Reference	$S_{11}$	$S_{44}$	$S_{12}$	$C_{11}$	$C_{44}$	$C_{12}$
Mason	R	9	20.6	65.0	-5.7	6.16	1.54	2.36
Sundara Rao	T	14	22.4	66.7	-5.8	5.45	1.50	1.91
Bechmann	R	7	20.4	65.7	-4.8	5.73	1.52	1.77
All results	Mean		21.1	65.8	-5.4	5.78	1.52	2.01
	$\sigma$		1.18	1.00	0.59	0.42	0.024	0.35
	$V$		5.6	1.5	11	7.2	1.6	17

Table 8.  $\text{NaCl}$  and  $\text{LiF}$  (cubic)

		$S_{11}$	$S_{44}$	$S_{12}$	$C_{11}$	$C_{44}$	$C_{12}$
NaCl (9 observations)	Mean	23.3	79.2	-4.9	4.83	1.26	1.25
	$\sigma$	0.61	2.1	0.27	0.11	0.031	0.041
	$V$	2.6	2.7	5.5	2.3	2.5	3.3
LiF (4 observations)	Mean	11.6	17.8	-3.4	11.3	5.64	4.5
	$\sigma$	1.46	1.36	0.55	0.99	0.46	0.66
	$V$	13	8	16	9	8	15

In addition to the individual values, Tables 2-8 also contain the average value of each elastic constant, and the standard deviation of the results. For a variable  $x$ , the former,  $(\bar{x})$ , is  $(\sum x)/n$ , where  $n$  is the number of observations, and the latter,  $(\sigma_x)$ , is calculated from

$$\sigma_x = K(x_{max} - x_{min}) \quad (1)$$

where  $K$  depends on  $n$  as follows<sup>(23)</sup>:

$n$	3	4	5	6
$K$	0.59	0.49	0.43	0.40

The standard deviation  $\sigma_x$  is usually estimated from the formula

$$\sigma_x^2 = \frac{\sum(x - \bar{x})^2}{n - 1} \quad (2)$$

but for low values of  $n$ , it has been shown<sup>(23)</sup> that the estimate given by equation (1) is almost as efficient as that given by equation (2), and is, of course, simpler to calculate. However, as a check, calculations were made on all the constants of Rochelle salt and ADP using equation (2). Comparison with the results obtained from equation (1) showed that the differences between the two estimates, though sometimes appreciable, were not large enough to affect the conclusions drawn from the results, and the simpler method involving the range (equation 1) was therefore used in the remaining calculations. For the sake of uniformity, the standard deviations given in Tables 2 and 4 for Rochelle salt and ADP are also those obtained using equation (1).

Finally, the coefficient of variation ( $V$ ) defined by

$$V = 100\sigma_x/\bar{x}$$

is included in the tables; this quantity gives a measure of the dispersion of the results which is independent of the units employed.

For comparison with the piezoelectric crystals, the results available in the literature for the non-piezoelectric crystals sodium chloride ( $\text{NaCl}$ ) and lithium fluoride ( $\text{LiF}$ ) have been summarized and the values are given in Table 8. The results for  $\text{NaCl}$  are based on nine sets of observations and those for  $\text{LiF}$  on four sets.

DISCUSSION

(1) *Rochelle salt.* The most significant result emerging from the observations on Rochelle salt is the large variation in the direct extension and lateral contraction stiffnesses  $C_{11}$ ,  $C_{22}$ ,  $C_{33}$ ,  $C_{12}$ ,  $C_{13}$  and  $C_{23}$ ; this effect has been previously commented on by Sundara Rao.<sup>(15)</sup> The stiffnesses measured by Jona<sup>(18)</sup> are included for comparison, although Jona was unable to measure  $C_{44}$  and  $C_{23}$ , and for this reason his results have been omitted from the calculation of  $\bar{C}$ ,  $\sigma_c$  and  $V_c$ . His results actually agree very closely with Huntington's<sup>(16)</sup> and Sundara Rao's agree with Mason's,<sup>(9)</sup> but the discrepancy between the two groups is very considerable. If the static results of Mandell<sup>(4)</sup> and Hinz<sup>(3)</sup> are included,  $V_c$  for the extensional constants  $C_{11}$ ,  $C_{22}$  and  $C_{33}$  is approximately 50%, and for the lateral extension constants  $C_{12}$ ,  $C_{13}$  and  $C_{23}$  is approximately 100%. By omitting the static results,  $V_c$  can be approximately halved, but is still very large. The picture, however, is quite different when the observations are expressed as compliances,  $S_{ik}$ . The value of  $V_s$  for the constants  $S_{11}$ ,  $S_{22}$ ,  $S_{33}$  is about 5%, and for  $S_{12}$  and  $S_{13}$  is about 30%. It is evident that Mandell's value<sup>(4)</sup> of  $S_{23}$  is open to suspicion and affects the  $V_s$  for this constant; the value of  $V_s$  when the static observations are omitted is 8.8%. In the orthorhombic system, to which Rochelle salt belongs, the shear stiffnesses  $C_{44}$ ,  $C_{55}$ ,  $C_{66}$  are simply the reciprocals of the corresponding shear compliances  $S_{44}$ ,  $S_{55}$ ,  $S_{66}$ , and there is accordingly no significant difference in the variability of these two sets of results. It is not possible, on the information available, to explain the large variation in the stiffnesses, but inherent variability of the material, reflected to a greater extent by the stiffnesses than by the compliances, cannot be ruled out, and further work would evidently be necessary to settle the question.

Recent work on the anomalous elastic, piezoelectric and dielectric behaviour of Rochelle salt has indicated that between the Curie points ( $-18^\circ\text{C}$  and  $24^\circ\text{C}$ ), Rochelle salt normally consists of an aggregate of single domains, possessing monoclinic, rather than orthorhombic, symmetry.<sup>(21)</sup> The gross elastic properties of an assemblage of monoclinic domains may depend on the size and disposition of the individual domains, and it is probable that the domain structure of the specimens of Rochelle salt used by the workers quoted in Table 2 varied considerably. It therefore seems that domain structure may have to be taken into account in interpreting the variability of the elastic and other constants of Rochelle salt, but information on this point is not at present available. It is, however, a matter which should be borne in mind in any future work.

(2) *KDP and ADP.* There is no evidence for these two substances of the effects noted for Rochelle salt. The values of  $V_s$  and  $V_c$  are all quite small, except on the constants with suffixes 12 and 23 when Mason's results<sup>(9)</sup> are included. The figures attributed to Mason in Tables 3 and 4 have

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actually been read as  $S_{ik}$  at 20° C from rather small-scale graphs, and converted to  $C_{ik}$ . Some of the discrepancies may be due to errors in reading the graphs, but the discrepancies in  $S_{12}$  on both materials, and to a lesser extent in  $S_{13}$  on KDP, make it probable either that Mason's material was genuinely different from that of the other workers, or that his results for the above constants are erroneous. For these reasons, the results have been summarized in two groups, the first including all the values and the second omitting Mason's values.

(3) Quartz, NaClO<sub>3</sub>, NaBrO<sub>3</sub>, NaCl and LiF. Quartz, NaClO<sub>3</sub>, NaBrO<sub>3</sub>, although piezoelectric, are not ferroelectric. There are no significant differences between the variability of the stiffnesses and the compliances, and on quartz and NaClO<sub>3</sub>, the direct extension and shear constants (those having suffixes 11, 33, 44) are concordant to about 1%. The dispersion associated with the other constants ranges up to about 16%, which is the value for  $C_{12}$ ,  $C_{13}$  and  $S_{13}$  of quartz. The values of  $V$  for NaCl are in the region of 2-3%, and for LiF and NaBrO<sub>3</sub> they are somewhat higher.

(4) General. Table 9 gives the highest and lowest values of  $V$  found when the constants are divided into three groups: (a) direct extension constants, i.e. those having suffixes 11, 22, 33; (b) direct shear constants, i.e. those having suffixes 44, 55, 66; (c) transverse constants, i.e. those having suffixes 12, 13, 23, 14.

Table 9 summarizes conveniently the points already made, namely that the direct extension and transverse stiffnesses  $C_{ik}$  of Rochelle salt show much greater variability than do the compliances  $S_{ik}$ , and that the evidence for such an effect with other materials is slight or non-existent. It also brings out the fact that for all substances the variability of the transverse constants is greater than that of the direct extension and shear constants. This is presumably a reflection of the difficulty of determining the transverse constants, which normally are measured by indirect procedures.

Table 9. Maximum and minimum values of  $V$

Material	Suffixes	Constants	11, 22, 33		44, 55, 66		12, 13, 23, 14	
			Max.	Min.	Max.	Min.	Max.	Min.
Rochelle salt, omitting Jona's results	S	C	7.4	4.7	19.3	4.1	260	29
			48	38	20	4.2	192	63
Rochelle salt, omitting Jona's and static results	S	C	8.1	2.5	5.6	2.9	39	8.8
			29	17	5.6	2.8	52	39
KDP, all results	S	C	10	1.3	3.0	0.93	725	33
			17	6.1	2.6	0.77	426	65
KDP, omitting Mason's results	S	C	2.3	0.30	3.6	1.1	10	10
			3.0	2.0	3.1	0.94	26	14
ADP, all results	S	C	6.9	3.9	1.8	1.3	86	8.4
			10	6.5	1.8	1.1	855	19
ADP, omitting Mason's results	S	C	8.3	4.7	1.4	1.0	10	9.5
			6.4	1.8	1.6	1.2	33	3.0
Quartz, all results	S	C	1.52	0.75	1.28	1.28	16	1.9
			0.98	0.83	1.46	1.46	16.5	3.2
NaClO <sub>3</sub> , all results	S	C	1.1	1.1	0.66	0.66	2.5	2.5
			1.7	1.7	0.73	0.73	4.8	4.8
NaBrO <sub>3</sub> , all results	S	C	5.6	5.6	1.5	1.5	11	11
			7.2	7.2	1.6	1.6	17	17
NaCl, all results	S	C	2.6	2.6	2.7	2.7	5.5	5.5
			2.3	2.3	2.5	2.5	3.3	3.3
LiF, all results	S	C	13	13	8	8	16	16
			9	9	8	8	15	15

The material showing the least variability in the direct constants is quartz, but even here  $V$  is about 1%. It may be recalled that if the observations are normally distributed, approximately 32% of them will have deviations from the mean numerically greater than the standard deviation. The

value of  $V$  for the direct constants of NaCl is about 2.5% and for some other crystals reaches even higher values.

The present survey includes nearly all the materials for which more than three independent sets of measurements are available. It leads to the conclusion that the elastic constants of crystals are not, in general, known with high precision. As with the discrepancies for Rochelle salt discussed earlier, the cause of this lack of precision is not known, but it is probably due mainly to inherent variability in the materials themselves. Some support is given to this suggestion by the results of Doraiswami<sup>(13)</sup> who measured the elastic constants of four samples of pyrites by method  $T$  (Table 1). Doraiswami's results are analysed in Table 10, which also contains the average elastic constants obtained by including the earlier results of Voigt<sup>(5)</sup> (method  $S$ ) and Bhagavantam<sup>(11)</sup> (method  $T$ ).

Table 10. Pyrites (cubic)

	$S_{11}$	$S_{44}$	$S_{12}$	$C_{11}$	$C_{44}$	$C_{12}$
Mean (all results)	2.87	9.49	0.43	36.27	10.54	-4.73
Doraiswami { Mean	2.86	9.49	0.43	36.33	10.55	-4.74
σ	0.083	0.31	0.054	0.85	0.35	0.44
$V$	2.9	3.3	12.5	2.3	3.3	9.3

The results are very concordant in the sense that differences between the two means are never greater than about 0.35%. Nevertheless, the values of  $V$  derived from Doraiswami's results for the direct constants are very similar to those for NaCl (Table 8). For the transverse constants,  $V$  on pyrites is 2-3 times the value for NaCl. Thus, even when the results of a single worker, using a single method, are examined, the elastic constants show appreciable variability, indicating that there is almost certainly inherent variability in the material. This behaviour in pyrites is not, of course, proof that other materials behave similarly; pyrites is, in fact, abnormal in that  $S_{12}$  is positive (i.e. Poisson's ratio is negative), but the probability is that other materials will also show inherent variability. A final assessment of the magnitude of inherent variability, and of the accuracy of the experimental methods, could only be reached by a co-operative investigation in which the constants of different samples of a series of materials were determined by different workers, using different methods.

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## A radio-frequency power supply system for high voltage accelerators

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A radio-frequency power supply system for operating an ion source on a high-voltage accelerator is described. A multi-channel link for radio-frequency power is provided between the ground and the high-voltage terminal, each separate power requirement of the ion source assembly having its own channel. Oscillators at ground potential provide the power, and adjustment of their outputs gives positive control of the beam intensity and focus.

A problem in the production of a beam of high-energy particles in a high-voltage accelerator is the provision of electrical power to operate and control the ion source. The normal practice is to use a motor-generator driven by insulating belts and to operate the necessary controls by means of insulating strings. The disadvantages of having heavy machinery mounted at the high-voltage terminal of an accelerator are obvious.

In the power supply system described, a multi-channel link for radio-frequency power is provided between the ground and the high-voltage terminal. Each separate power requirement of the ion source assembly has its own channel; where d.c. power is necessary it is provided by rectification at the high-voltage terminal. Thus the operation of the ion source can be controlled completely from the ground without the use of mechanical remote controls.

### METHOD

The radio-frequency ion source<sup>(1)</sup> used on the 1.2 MV accelerator in this laboratory is shown in Fig. 1. The power requirements for the complete assembly are:

- (a) 300 W at 20 Mc/s for excitation;
- (b) 20 W for heating the palladium leak controlling the flow of gas (hydrogen or deuterium) into the ion source;
- (c) 5 kV at 20 mA for extracting the ions;
- (d) 25 kV at 0.5 mA for the first focusing electrode of the accelerator.

These are the maximum powers required; (a) may be pre-set, but (b), (c), and (d) must be controllable independently during operation.

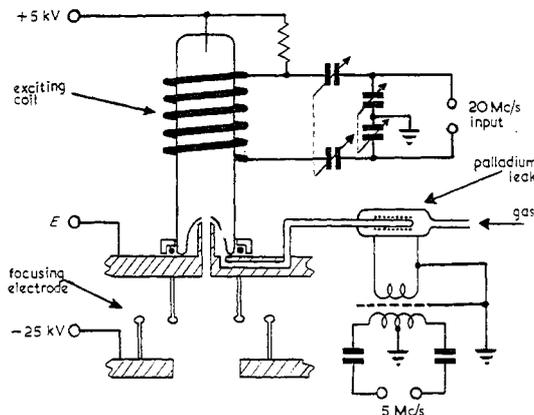


Fig. 1. Schematic diagram of ion source assembly

The four power requirements are provided by four oscillators at ground potential, operating at different frequencies. The oscillator outputs are mixed in a filter unit, and the power is taken up to the high-voltage terminal by a high-pass artificial transmission line designed to withstand the overall accelerator voltage between the input and output ends. The four frequencies are there separated by an identical filter unit. (a) is taken to the ion source via an impedance-matching unit; (b) to the palladium leak via a transformer. (c) and (d) are obtained by rectifying the power in the remaining two channels.

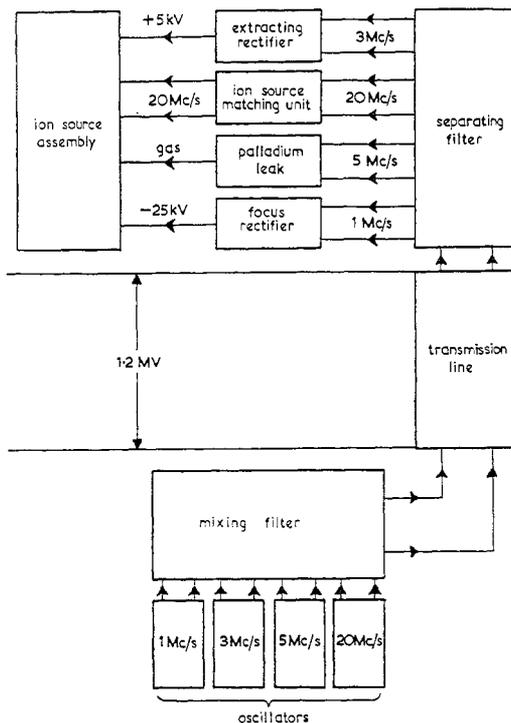


Fig. 2. Block diagram of power supply system

The degree of ion source excitation, the rate of gas flow, and the extracting and focusing voltages are all controlled by adjusting the power outputs of the appropriate oscillators. The complete system is shown schematically in Fig. 2.