Journal of Nuclear Energy. Part C, Plasma Physics, Accelerators, Thermonuclear Research

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To cite this article: V Kopecký and J Václavík 1966 *J. Nucl. Energy, Part C Plasma Phys.* **8** 645

Plasma Physics (Journal of Nuclear Energy Part C) 1966, Vol. 8, pp. 645 to 656. Pergamon Press Ltd. Printed in N. Ireland

ON STRONG TURBULENCE OF AN INHOMOGENEOUS WEAKLY IONIZED PLASMA IN CROSSED ELECTRIC AND MAGNETIC FIELDS

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(*Received* 30 *March* 1966)

Abstract—Instability of the drift-dissipative type, of a weakly ionized non-isothermal plasma in crossed electric and magnetic fields (Simon; Hoh) has been studied in the quasi-classical approximation for two cases: (a) $\Omega_e\tau_e \gg 1$, $\Omega_i\tau_i \gg 1$, (b) $\Omega_e\tau_e \gg 1$, $\Omega_i\tau_i \ll 1$. The increment and frequency of the instability of such a plasma against potential disturbances have been deduced in the linear approximation. Equations of strong turbulence evolved with the aid of Mikhailovskii's diagram method have been used in estimating the maximum value of the spectrum function of the electric field of oscillations and of the coefficient of turbulent diffusion. The concluding part of the paper presents a set of equations of strong turbulence in the dimensionless form suitable for computer analyses.

1. INTRODUCTION

AS THE papers by both Simon and Hoh suggest, an inhomogeneous weakly ionized plasma in crossed electric and magnetic fields is unstable against potential oscillations with a characteristic frequency much smaller than the cyclotron frequency of ions ($\omega \ll \Omega_i$), An instability of this sort belonging by virtue of its character among instabilities of the drift-dissipative type, is caused by the difference between the drift velocities of ions and electrons in a transversal electric field. It follows from an analysis made by SIMON (1963) and HOH (1963) that the oscillations are unstable for a magnetic field H larger than a certain critical value H_c. It is indicated that the order of the increment of unstable oscillations can reach up to that of the frequency. This instability occurs in discharges of the Penning type where anomalous diffusion is observed at $H > H_c$ (BONNAL *et al.*, 1962; REICHRUDEL *et al.*, 1958; CHEN *et al.*, 1962).

The present paper concentrates on a study of the non-linear effects of the above instability, in particular on the spectrum of the electric field of oscillations and determination of the coefficient of anomalous diffusion.

2. FORMULATION OF THE PROBLEM AND THE INITIAL EQUATIONS

We shall consider a weakly ionized plasma with density n_0 inhomogeneous along the y-axis, inserted in a homogeneous magnetic field $H = H_z$. The electron temperature T is assumed to be constant, the ion temperature equal to zero. There is applied along the y-axis an external homogeneous electric field E_0. The following two distinct cases of the magnetic field magnitude will be considered: (a) $\Omega_e\tau_e \gg 1$, $\Omega_i\tau_i \gg 1$, (b) $\Omega_e\tau_e \gg 1$, $\Omega_i\tau_i \ll 1$. The two cases will be studied at the same time and formulae which might differ from one to another, marked with the respective letters (a) or (b). In the discussion that follows, Ω_i, Ω_e are the cyclotron frequencies of ions and electrons, respectively, and τ_i^{-1}, τ_e^{-1} the collision frequencies of ions and electrons with a neutral gas considered an immobile background.

Since we are dealing with potential oscillations ($\mathbf{E} = -\nabla\varphi$) with a characteristic frequency $\omega \ll \Omega_i$, we may neglect the terms corresponding to the inertia across the magnetic field in the equations of motion of both electrons and ions. Moreover, we may also neglect the inertia along the magnetic field, which corresponds to a diffusion regime. The problem is solved in the quasi-classical approximation, i.e. the space–time dependence of the oscillation is sought in the form $\exp\{-i\omega t + i\mathbf{kr}\}$; this is justified on the assumption that $k_\perp \gg \kappa$ where $\kappa = \mathrm{d}\ln n_0/\mathrm{d}y$.

Equations describing the plasma oscillations under the above-stated assumptions are in the following form:

$$\frac{\partial n_{e,i}}{\partial t} + \operatorname{div}(n_{e,i}\mathbf{v}_{e,i}) = 0 \tag{1}$$

$$0 = -T\nabla n_e - en_e\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}_e, \mathbf{H}]\right) - \frac{n_e m\mathbf{v}_e}{\tau_e}, \tag{2}$$

$$0 = en_i\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}_i\mathbf{H}]\right) - \frac{n_i M\mathbf{v}_i}{\tau_i}, \tag{3a}$$

$$0 = en_i\mathbf{E} - \frac{n_i M\mathbf{v}_i}{\tau_i}, \tag{3b}$$

$$\operatorname{div}\mathbf{E} = 4\pi e(n_i - n_e). \tag{4}$$

We express quantities $n_e\mathbf{v}_e$ and $n_i\mathbf{v}_i$ from (2) and (3) and introduce to (1). Following simple rearrangements we obtain with accuracy terms $(\Omega_i\tau_i)^{-1}$ and $(\Omega_e\tau_e)^{-2}$ in equations

$$\begin{aligned}\frac{\partial n_e}{\partial t} = \frac{e\mathbf{E}_\perp}{m\Omega_e}[\nabla_\perp n_e, \mathbf{h}] + \frac{1}{m\Omega_e^2\tau_e}(T\Delta_\perp n_e + en_e \operatorname{div}_\perp \mathbf{E}_\perp \\ + e\mathbf{E}_\perp \,.\, \nabla_\perp n_e) + \frac{\tau_e}{m}\left(T\frac{\partial^2 n_e}{\partial z^2} + en_e\frac{\partial E_z}{\partial z} + eE_z\frac{\partial n_e}{\partial z}\right),\end{aligned} \tag{5}$$

$$\begin{aligned}\frac{\partial n_i}{\partial t} = \frac{e\mathbf{E}_\perp}{M\Omega_i}[\nabla_\perp n_i, \mathbf{h}] - \frac{e}{M\Omega_i^2\tau_i}(n_i \operatorname{div}_\perp \mathbf{E}_\perp + \mathbf{E}_\perp \,.\, \nabla_\perp n_i) \\ - \frac{e\tau_i}{M}\left(n_i\frac{\partial E_z}{\partial z} + E_z\frac{\partial n_i}{\partial z}\right),\end{aligned} \tag{6a}$$

$$\frac{\partial n_i}{\partial t} = -\frac{e\tau_i}{M}(n_i \operatorname{div}\mathbf{E} + \mathbf{E}\,.\,\nabla n_i), \tag{6b}$$

where $\mathbf{h} = \mathbf{H}/H$. These equations together with equation (4) represent the initial set of equations for the given problem.

3. LINEAR THEORY

In the linear approximation we obtain from equations (5) and (6) the following expressions for the disturbance in the densities of electrons and ions

$$n_e = n_0 e\varphi\left(\frac{\omega^*}{T} + \frac{ik_\perp^2}{m\Omega_e^2\tau_e} + \frac{ik_z^2\tau_e}{m}\right)\left(\omega - \omega_E + \frac{\bar{\omega}_E}{\Omega_e\tau_e} + \frac{iTk_\perp^2}{m\Omega_e^2\tau_e} + \frac{iTk_z^2\tau_e}{m}\right)^{-1}, \tag{7}$$

$$n_i = -n_0 e\varphi\left(-\frac{\omega^*}{T} + \frac{ik_\perp^2}{M\Omega_i^2\tau_i} + \frac{ik_z^2\tau_i}{M}\right)\left(\omega - \omega_E - \frac{\bar{\omega}_E}{\Omega_i\tau_i}\right)^{-1}, \tag{8a}$$

$$n_i = -n_0 e\varphi\frac{ik^2\tau_i}{M}(\omega - \bar{\omega}_E\Omega_i\tau_i)^{-1} \tag{8b}$$

using the notation

$$\omega^* = \frac{k_x T\kappa}{m\Omega_e}, \qquad \omega_E = \frac{k_x e E_0}{m\Omega_e}, \qquad \bar{\omega}_E = \frac{k_y e E_0}{m\Omega_e}.$$

We shall furthermore assume that the plasma is sufficiently dense ($C_A \ll c$, C_A—Alfvén velocity); consequently, equation (4) changes into the condition of plasma quasi-neutrality, i.e. $n_i = n_e$. Introducing in this condition the above found disturbances of the densities of electrons and ions, (7), (8) we get the dispersion equation as

$$(\omega - \omega_E)\left[\frac{k_\perp^2}{m\Omega_e\Omega_i\tau_i} + \frac{k_z^2\tau_e}{m}\right](1+\alpha) = \frac{\bar{\omega}_E k_z^2\tau_e}{\Omega_i\tau_i m}(1-\alpha^2) + \omega^*\left(\frac{k_\perp^2}{m\Omega_e^2\tau_e} + \frac{k_z^2\tau_e}{m}\right)$$
$$- i\left[\frac{\bar{\omega}_E\omega^*}{T\Omega_i\tau_i}(1+\alpha) + \frac{k_\perp^4 T}{(m\Omega_e)^2\Omega_e\tau_e\Omega_i\tau_i} + \frac{k_\perp^2 k_z^2\tau_e}{m^2\Omega_e\Omega_i\tau_i}(1+\alpha^2)\right], \quad \text{(9a)}$$

$$\omega\left[\frac{\omega^*}{T} + i\left(\frac{k_\perp^2}{m\Omega_e^2\tau_e} + \frac{k_z^2\tau_e}{m} + \frac{k^2\tau_i}{M}\right)\right] = \frac{\bar{\omega}_E\omega^*}{T}\Omega_i\tau_i$$
$$+ \frac{Tk^2\tau_i\tau_e}{mM}\left(\frac{k_\perp^2}{(\Omega_e\tau_e)^2} + k_z^2\right) + i\left(\frac{\bar{\omega}_E k_z^2\tau_e\Omega_i\tau_i}{m} + \frac{\omega_E k^2\tau_i}{M}\right) \quad \text{(9b)}$$

where $\alpha = m\tau_i/M\tau_e$. Thence we immediately obtain the frequency ω and the increments γ of the oscillations as

$$\omega = \omega_E + \left[\frac{\bar{\omega}_E k_z^2\tau_e}{m\Omega_i\tau_i}(1-\alpha^2) + \omega^*\left(\frac{k_\perp^2}{m\Omega_e^2\tau_e} + \frac{k_z^2\tau_e}{m}\right)\right]$$
$$\times\left(\frac{k_\perp^2}{m\Omega_e\Omega_i\tau_i} + \frac{k_z^2\tau_e}{m}\right)^{-1}(1+\alpha)^{-1}, \quad \text{(10a)}$$

$$\gamma = -\left[\frac{\bar{\omega}_E\omega^*}{T\Omega_i\tau_i}(1+\alpha) + \frac{k_\perp^4 T}{(m\Omega_e)^2\Omega_e\tau_e\Omega_i\tau_i} + \frac{k_\perp^2 k_z^2\tau_e T}{m^2\Omega_e\Omega_i\tau_i}(1+\alpha^2)\right]$$
$$\times\left(\frac{k_\perp^2}{m\Omega_e\Omega_i\tau_i} + \frac{k_z^2\tau_e}{m}\right)^{-1}(1+\alpha)^{-1}, \quad \text{(11a)}$$

$$\omega = \left\{\frac{\bar{\omega}_E\omega^{*2}}{T^2}\Omega_i\tau_i + \frac{\omega^* k^2\Omega_e\tau_e\Omega_i\tau_i}{(m\Omega_e)^2}\left(\frac{k_\perp^2}{(\Omega_e\tau_e)^2} + k_z^2\right) + \left(\frac{\bar{\omega}_E k_z^2\Omega_e\tau_e\Omega_i\tau_i}{m\Omega_e} + \frac{\omega_E k^2\Omega_i\tau_i}{m\Omega_e}\right)\right.$$
$$\left.\times\left(\frac{k_\perp^2}{m\Omega_e^2\tau_e} + \frac{k_z^2\tau_e}{m} + \frac{k^2\tau_i}{M}\right)\right\}\cdot\left[\left(\frac{\omega^*}{T}\right)^2 + \left(\frac{k_\perp^2}{m\Omega_e^2\tau_e} + \frac{k_z^2\tau_e}{m} + \frac{k^2\tau_i}{M}\right)^2\right]^{-1}, \quad \text{(10b)}$$

$$\gamma = -\left\{-\frac{\omega^*\omega_E k^2\Omega_i\tau_i}{Tm\Omega_e} + \frac{Tk^2\Omega_e\tau_e\Omega_i\tau_i}{(m\Omega_e)^2}\left(\frac{k_\perp^2}{(\Omega_e\tau_e)^2} + k_z^2\right)\left(\frac{k_\perp^2}{m\Omega_e^2\tau_e} + \frac{k_z^2\tau_e}{m} + \frac{k^2\tau_i}{M}\right)\right\}$$
$$\times\left[\left(\frac{\omega^*}{T}\right)^2 + \left(\frac{k_\perp^2}{m\Omega_e^2\tau_e} + \frac{k_z^2\tau_e}{m} + \frac{k^2\tau_i}{M}\right)^2\right]^{-1}, \quad \text{(11b)}$$

where term $Tk_z^4\tau_i\tau_e/mM$ was neglected in formula (11a). This is fully justified in agreement with (12a) if inequality $(k_{z\,\min})^2/\kappa^2 \gg \kappa^2\alpha^2/k_\perp^2$ holds.

It is clear that the denominator and the last terms of the numerator of formulae (11) are always positive; thus instability is possible only in the case where the first term of the numerator is negative and sufficiently large. In case (b) the latter can be negative only when quantity κE_0 is positive. From the condition that $\gamma = 0$ on the boundary of instability, we obtain from (11) the following expressions for the critical magnetic fields:

$$H_{c1,2} = \frac{1}{2}\frac{E_0\kappa mc}{Tk_z^2\tau_e}\frac{1+\alpha}{1+\alpha^2}\left(1 \pm \sqrt{1 - \frac{4k_\perp^2k_z^2T^2(1+\alpha^2)}{(eE_0)^2\kappa^2(1+\alpha)^2}}\right), \tag{12a}$$

$$H_{c1,2}^2 = \frac{m^2c^2k_\perp^2}{2e^2\tau_e^2k_z^2(k_z^2+k^2\alpha)}\left[\frac{\kappa eE_0}{T} - 2k_z^2 - k^2\alpha \pm \sqrt{\left(\frac{\kappa eE_0}{T}\right)^2 + k^4\alpha^2 - \frac{2\kappa eE_0}{T}(2k_z^2+k^2\alpha)}\right]. \tag{12b}$$

Hence the plasma considered in our study is unstable for magnetic fields fulfilling the inequality $H_{c1} < H < H_{c2}$ with respect to disturbances whose transverse wave vector satisfies the condition of

(a) $$k_\perp^2 < \frac{(eE_0)^2\kappa^2(1+\alpha)^2}{4k_z^2T^2(1+\alpha^2)},$$

(b) $$k^2 < \frac{\kappa eE_0}{T\alpha}\left(1 - 2\sqrt{\frac{Tk_z^2}{\kappa eE_0}}\right).$$

The increment of instability has a maximum at minimum $k_z = k_{z\,\min} = \pi/L$ (L is the length of the device), and at $k_\perp = k_0$, where

$$k_0^2 = k_{z\,\min}^2\Omega_e\tau_e\Omega_i\tau_i\left[\sqrt{1 + \frac{eE_0\kappa(1+\alpha)}{Tk_{z\,\min}^2\Omega_i\tau_i}} - \left(\alpha + \frac{1}{\alpha}\right) - 1\right], \tag{13a}$$

$$k_0^2 \cong \frac{\kappa^2}{\left(\dfrac{1}{\Omega_e\tau_e} + \Omega_i\tau_i\right)^2}\left[\sqrt{1 + \frac{eE_0}{T\kappa}(1 + \Omega_e\tau_e\Omega_i\tau_i)} - 1\right], \tag{13b}$$

and can attain values up to the order of the frequency. As formulae (13) imply, $k_z^2/k_\perp^2 \ll 1/\Omega_e\tau_e \ll 1$; hence the unstable disturbances are intensely elongated along the magnetic field.

4. NON-LINEAR THEORY

We shall study the strongly turbulent state of the plasma that arises in consequence of the above-described instability, using the diagram method suggested by MIKHAILOVSKII (1964).

To find equations which describe the strongly turbulent plasma state considered herein, we shall start from equations (5), (6). We shall put $n_{i,e} = n_{0i,e} + \tilde{n}_{i,e}$, $\mathbf{E} = \mathbf{E}_0 - \nabla\varphi$, where $n_{0i,e} = \langle n_{i,e}\rangle$, $\mathbf{E}_0 = \langle \mathbf{E}\rangle$ are quantities satisfying the quasi-equilibrium equation (see below), $\tilde{n}_{i,e}$ and φ are the oscillating functions for which it holds that $\langle \tilde{n}_{i,e}\rangle = \langle\varphi\rangle = 0$; symbol $\langle\cdots\rangle$ denotes the averaging over the set of oscillations. In the zero approximations we shall consider $n_{0i,e}$ and $\mathbf{E}_0$ the equilibrium ones, i.e. quantities independent of time.

On averaging equations (5) and (6) over the set of oscillations we obtain the quasi-equilibrium equations as follows

$$\frac{\partial n_{0e}}{\partial t} = \frac{1}{m\Omega_e^2\tau_e}\Big\{T\Delta_\perp n_{0e} + e\mathbf{E}_{0\perp}\,.\,\nabla_\perp n_{0e} - e\langle\tilde{n}_e\Delta_\perp\varphi\rangle$$
$$- e\langle\nabla_\perp\varphi\,.\,\nabla_\perp\tilde{n}_e\rangle - \Omega_e\tau_e e\langle\nabla_\perp\tilde{n}_e\,.\,[\mathbf{h},\nabla_\perp\varphi]\rangle - e(\Omega_e\tau_e)^2\Big\langle\frac{\partial}{\partial z}\Big(\tilde{n}_e\frac{\partial\varphi}{\partial z}\Big)\Big\rangle\Big\}, \quad (14)$$

$$\frac{\partial n_{0i}}{\partial t} = \frac{e}{M\Omega_i^2\tau_i}\Big\{-\mathbf{E}_{0\perp}\,.\,\nabla_\perp n_{0i} + \langle\tilde{n}_i\Delta_\perp\varphi\rangle + \langle\nabla_\perp\varphi\,.\,\nabla_\perp\tilde{n}_i\rangle$$
$$- \Omega_i\tau_i\langle\nabla_\perp\tilde{n}_i\,.\,[\mathbf{h},\nabla_\perp\varphi]\rangle + (\Omega_i\tau_i)^2\Big\langle\frac{\partial}{\partial z}\Big(\tilde{n}_i\frac{\partial\varphi}{\partial z}\Big)\Big\rangle\Big\}, \quad (15a)$$

$$\frac{\partial n_{0i}}{\partial t} = -\frac{e\tau_i}{M}(\mathbf{E}_{0\perp}\,.\,\nabla_\perp n_{0i} - \langle\tilde{n}_i\Delta\varphi\rangle - \langle\nabla\varphi\,.\,\nabla\tilde{n}_i\rangle) \quad (15b)$$

Substracting these equations from the corresponding un-averaged equations (5) and (6) we obtain equations for the oscillating quantities

$$\frac{\partial\tilde{n}_e}{\partial t} = \frac{1}{m\Omega_e^2\tau_e}\Big\{T\Delta_\perp\tilde{n}_e - en_{0e}\Delta_\perp\varphi + e\mathbf{E}_{0\perp}\,.\,\nabla_\perp\tilde{n}_e - e\nabla_\perp\varphi\,.\,\nabla_\perp n_{0e}$$
$$+ e\Omega_e\tau_e\nabla_\perp\tilde{n}_e\,.\,[\mathbf{h},\mathbf{E}_0] - e\Omega_e\tau_e\nabla_\perp n_{0e}\,.\,[\mathbf{h},\nabla_\perp\varphi] + (\Omega_e\tau_e)^2\Big(T\frac{\partial^2\tilde{n}_e}{\partial z^2}$$
$$- en_{0e}\frac{\partial^2\varphi}{\partial z^2}\Big)\Big\} - \frac{e}{m\Omega_e^2\tau_e}\Big\{\tilde{n}_e\Delta_\perp\varphi + \nabla_\perp\varphi\,.\,\nabla_\perp\tilde{n}_e + \Omega_e\tau_e\nabla_\perp\tilde{n}_e\,.\,[\mathbf{h},\nabla_\perp\varphi]$$
$$+ (\Omega_e\tau_e)^2\frac{\partial}{\partial z}\Big(\tilde{n}_e\frac{\partial\varphi}{\partial z}\Big) - \langle\cdot\cdot\cdot\cdot\cdot\rangle\Big\}, \quad (16)$$

$$\frac{\partial\tilde{n}_i}{\partial t} = \frac{e}{M\Omega_i^2\tau_i}\Big\{n_{0i}\Delta_\perp\varphi - \mathbf{E}_{0\perp}\,.\,\nabla_\perp\tilde{n}_i + \nabla_\perp\varphi\,.\,\nabla_\perp n_{0i} + \Omega_i\tau_i\nabla_\perp\tilde{n}_i\,.\,[\mathbf{h},\mathbf{E}_0]$$
$$- \Omega_i\tau_i\nabla_\perp n_{0i}\,.\,[\mathbf{h},\nabla_\perp\varphi] - (\Omega_i\tau_i)^2 n_{0i}\frac{\partial^2\varphi}{\partial z^2}\Big\}$$
$$+ \frac{e}{M\Omega_i^2\tau_i}\Big\{\tilde{n}_i\Delta_\perp\varphi + \nabla_\perp\varphi\,.\,\nabla_\perp\tilde{n}_i - \Omega_i\tau_i\nabla_\perp\tilde{n}_i\,.\,[\mathbf{h},\nabla_\perp\varphi]$$
$$+ (\Omega_i\tau_i)^2\frac{\partial}{\partial z}\Big(\tilde{n}_i\frac{\partial\varphi}{\partial z}\Big) - \langle\cdot\cdot\cdot\cdot\rangle\Big\}, \quad (17a)$$

$$\frac{\partial\tilde{n}_i}{\partial t} = -\frac{e\tau_i}{M}\{-n_{i0}\Delta\varphi + \mathbf{E}_{0\perp}\,.\,\nabla_\perp\tilde{n}_i - \nabla\varphi\,.\,\nabla n_{0i} - \tilde{n}_i\Delta\varphi - \nabla\varphi\,.\,\nabla\tilde{n}_i$$
$$+ \langle\tilde{n}_i\Delta\varphi\rangle + \langle\nabla\varphi\,.\,\nabla\tilde{n}_i\rangle\}. \quad (17b)$$

In agreement with the assumption of the quasi-classical approximation, we can transform the above equations to Fourier representation, i.e. put

$$\tilde{n}_{i,e} = \int_{-\infty}^{+\infty} n_{i,e}(k)\exp\{-i\omega t + i\mathbf{k}\cdot\mathbf{r}\}\,\mathrm{d}k,$$

where $k \equiv \{\mathbf{k}, \omega\}$ and analogously for φ. Equations (16) and (17) then change to the form of

$$n_e(k)\left(\omega - \omega_E + \frac{\bar{\omega}_E}{\Omega_e\tau_e} + \frac{ik_\perp{}^2T}{m\Omega_e{}^2\tau_e} + \frac{ik_z{}^2\tau_eT}{m}\right) = n_{0e}e\varphi(k)\left(\frac{\omega^*}{T} + \frac{ik_\perp{}^2}{m\Omega_e{}^2\tau_e} + \frac{ik_z{}^2\tau_e}{m}\right)$$
$$+ \frac{ie}{m\Omega_e{}^2\tau_e}\int dk'\{\mathbf{k}_\perp\cdot\mathbf{k}_\perp{}' - \Omega_e\tau_e[\mathbf{k},\mathbf{k}']_z + (\Omega_e\tau_e)^2k_zk_z{}'\}\{n_e(k-k')\varphi(k')$$
$$- \langle n_e(k-k')\varphi(k')\rangle\}, \quad (18)$$

$$n_i(k)\left(\omega - \omega_E - \frac{\bar{\omega}_E}{\Omega_i\tau_i}\right) = n_{0i}e\varphi(k)\left(\frac{\omega^*}{T} - \frac{ik_\perp{}^2}{M\Omega_i{}^2\tau_i} - \frac{ik_z{}^2\tau_i}{M}\right)$$
$$- \frac{ie}{M\Omega_i{}^2\tau_i}\int dk'\{\mathbf{k}_\perp\cdot\mathbf{k}_\perp{}' + \Omega_i\tau_i[\mathbf{k},\mathbf{k}']_z + (\Omega_i\tau_i)^2k_zk_z{}'\}$$
$$\times\{n_i(k-k')\varphi(k') - \langle n_i(k-k')\varphi(k')\rangle\}, \quad (19a)$$

$$n_i(k)(\omega - \bar{\omega}_E\Omega_i\tau_i) = -in_{0i}e\varphi(k)\frac{k^2\tau_i}{M} - i\frac{e\tau_i}{M}\int dk'\,\mathbf{k}\cdot\mathbf{k}'\{n_i(k-k')\varphi(k')$$
$$- \langle n_i(k-k')\varphi(k')\rangle\}. \quad (19b)$$

A solution of these equations is sought in the form of a power series with respect to amplitudes φ:

$$n_{i,e}(k) = \sum_{l=1}^{\infty} n_{i,e}^{(l)}(k). \quad (20)$$

In the first (linear) approximation we obtain for $n_{i,e}^{(1)}(k)$ expressions (7), (8). For $n_{i,e}^{(l)}(k)$ we can easily find a recurrent expression for $l \geqslant 2$:

$$n_e^{(l)}(k) = \frac{ie\Theta_e{}^{-1}(k)}{m\Omega_e{}^2\tau_e}\int dk'\{\mathbf{k}_\perp\cdot\mathbf{k}_\perp{}' - \Omega_e\tau_e[\mathbf{k},\mathbf{k}']_z + (\Omega_e\tau_e)^2k_zk_z{}'\}$$
$$\times\{n_e^{(l-1)}(k-k')\varphi(k') - \langle\cdots\rangle\}, \quad (21)$$

$$\text{(a)}\quad n_i^{(l)}(k) = -\frac{ie\Theta_i{}^{-1}(k)}{M\Omega_i{}^2\tau_i}\int dk'\{\mathbf{k}_\perp\cdot\mathbf{k}_\perp{}' + \Omega_i\tau_i[\mathbf{k},\mathbf{k}']_z + (\Omega_i\tau_i)^2k_zk_z{}'\}$$
$$\times\{n_i^{(l-1)}(k-k')\varphi(k') - \langle\cdots\rangle\},$$

$$\text{(b)}\quad n_i^{(l)}(k) = -\frac{ie\tau_i\Theta_i{}^{-1}(k)}{M}\int dk'\,\mathbf{k}\cdot\mathbf{k}'\{n_i^{(l-1)}(k-k')\varphi(k') - \langle\cdots\rangle\},$$

where

$$\Theta_e(k) = \omega - \omega_E + \frac{\bar{\omega}_E}{\Omega_e\tau_e} + \frac{ik_\perp{}^2T}{m\Omega_e{}^2\tau_e} + \frac{ik_z{}^2\tau_eT}{m}, \quad (22)$$

$$\text{(a)}\qquad \Theta_i(k) = \omega - \omega_E - \frac{\bar{\omega}_E}{\Omega_i\tau_i},$$

$$\text{(b)}\qquad \Theta_i(k) = \omega - \bar{\omega}_E\Omega_i\tau_i.$$

Using formulae (7), (8) and (21) we obtain expression for the general term $n_j^{(l)}(k)$

($l \geqslant 1$) of series (2) with an accuracy to the terms of the order of $(\Omega_j\tau_j)^{-1}$ and $\Omega_j\tau_j k_z^2/k_\perp^2$

$$
\begin{aligned}
n_j^{(l)}(k) = \left(\frac{-ie}{m\Omega_e}\right)^{l-1} \frac{en_0}{\Theta_j(k)} \int \mathrm{d}k_1 \ldots \mathrm{d}k_l\, A_j(k_l) \\
\times \frac{[\mathbf{k}, \mathbf{k}_1]_z[\mathbf{k}-\mathbf{k}_1, \mathbf{k}_2]_z \ldots [\mathbf{k}-\mathbf{k}_1-\ldots-\mathbf{k}_{l-2}, \mathbf{k}_{l-1}]_z}{\Theta_j(k-k_1)\Theta_j(k-k_1-k_2)\ldots\Theta_j(k-k_1-\ldots-k_{l-1})} \\
\times R\{\varphi(k_1)\ldots\varphi(k_l)\}\delta(k-k_1-\ldots-k_l),
\end{aligned}
\tag{23a}
$$

$$
\begin{aligned}
n_i^{(l)}(k) = \left(-\frac{ie\tau_i}{M}\right)^{l-1} \frac{en_0}{\Theta_i(k)} \int \mathrm{d}k_1 \ldots \mathrm{d}k_l\, A_i(k_l)\delta(k-k_1-\ldots-k_l) \\
\times \frac{(\mathbf{k}_1\cdot\mathbf{k})(\mathbf{k}_2\cdot(\mathbf{k}-\mathbf{k}_1))\ldots(\mathbf{k}_{l-1}\cdot(\mathbf{k}-\ldots-\mathbf{k}_{l-2}))}{\Theta_i(k-k_1)\Theta_i(k-k_1-k_2)\ldots\Theta_i(k-k_1-\ldots-k_{l-1})} R\{\varphi(k_1)\ldots\varphi(k_l)\}
\end{aligned}
\tag{23b}
$$

after having introduced the following notation therein:

$$A_e(k) = \frac{\omega^*}{T} + \frac{ik_\perp^2}{m\Omega_e^2\tau_e} + \frac{ik_z^2\tau_e}{m}, \tag{24}$$

(a) $$A_i(k) = \frac{\omega^*}{T} - \frac{ik_\perp^2}{M\Omega_i^2\tau_i} - \frac{ik_z^2\tau_i}{M},$$

(b) $$A_i(k) = -\frac{i\tau_i}{M}k^2,$$

$$
\begin{aligned}
&R\{\varphi(k_1)\ldots\varphi(k_l)\} \\
&\quad \equiv \varphi(k_1)\ldots\varphi(k_l) - \varphi(k_1)\ldots\langle\varphi(k_{l-1})\varphi(k_l)\rangle - \ldots - \langle\varphi(k_1)\ldots\varphi(k_l)\rangle.
\end{aligned}
$$

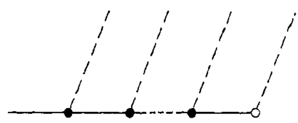
$n_j^{(l)}(k) =$ 

FIG. 1

Quantities $n_j^{(l)}(k)$ can be represented graphically (refer to Fig. 1). The elements of Fig. 1 denote the respective quantities as follows:

solid line $= 1/\Theta_j(k)$

dashed line $= \varphi(k_p)$

$$\text{solid circle} = -\frac{ie}{m\Omega_e}[\mathbf{k}-\mathbf{k}_1-\ldots-\mathbf{k}_{p-1}, \mathbf{k}_p]_z \tag{a}$$

$$= -\frac{ie\tau_i}{M}\mathbf{k}_p\cdot(\mathbf{k}-\mathbf{k}_1-\ldots-\mathbf{k}_{p-1}) \tag{b}$$

open circle $= en_0\,\delta(k-k_1-\ldots-k_l)A_j(k_l)$.

Integration over k_p is carried out around each circle.

Since the structure of expressions (23) is analogous to that of the expression giving the general term of the distribution function series in MIKHAILOVSKII's paper, we can hereafter use the diagram technique evolved there. Some modifications must be made, however,

By introducing Q—products of potentials (MIKHAILOVSKII), series (20) transform to series

$$n_j(k) = \sum_{l=1}^{\infty} n_j^{*(l)}(k), \tag{25}$$

the general term of which is given by expression

$$n_j^{*(l)}(k) = g_j(k) \int \gamma_j^{(l+1)}(k; k_1, \ldots, k_l) Q\{\varphi(k_1) \ldots \varphi(k_l)\} \, \mathrm{d}k_1 \ldots \mathrm{d}k_l, \tag{26}$$

$\gamma_j^{(l+1)}$ is the sum of topologically non-equivalent renormalized vortex parts of diagrams of the same order, and $g_j(k)$ is the modified propagator linked to the elementary propagator $g_{0j}(k) = \Theta_j^{-1}(k)$ and the self-energy part $\sigma_j(k)$ of the modified propagator through Dayson's equation

$$g_j(k) = g_{0j}(k) + g_{0j}(k)\sigma_j(k)g_j(k).$$

Equation (4) can then be rewritten with the aid of (25) and (26) in the form

$$\varphi(k) = G_0(k) \sum_{l=2}^{\infty} \int \mathrm{d}k_1 \ldots \mathrm{d}k_l \Gamma_{l+1}^0(k; k_1, \ldots, k_l) Q\{\varphi(k_1) \ldots \varphi(k_l)\}, \tag{27}$$

where

$$G_0^{-1}(k) = 1 - \frac{4\pi e}{k^2} \left[g_i(k) \int \mathrm{d}k_1 \gamma_i^{(2)}(k; k_1) - g_e(k) \int \mathrm{d}k_1 \gamma_e^{(2)}(k; k_1) \right] \tag{28}$$

$$\Gamma_{l+1}^0 = \frac{4\pi e}{k^2} [g_i(k) \gamma_i^{(l+1)}(k; k_1, \ldots, k_l) - g_e(k) \gamma_e^{(l+1)}(k; k_1, \ldots, k_l)]. \tag{29}$$

The subsequent procedure of deriving the equations of strong turbulence, i.e. the equations for the spectrum function of potential $I(k) = \langle \varphi(k) \varphi^*(k) \rangle$ is identical with that given in MIKHAILOVSKII's paper.

Next to the above-stated quantities characterizing of the turbulent state of plasma, there figures in the quasi-equilibrium equations (14), (15) transformed to Fourier representation

$$\frac{\partial n_{0e}}{\partial t} = \frac{1}{m\Omega_e^2 \tau_e} \left\{ T \frac{\partial^2 n_{0e}}{\partial y^2} + eE_0 \frac{\partial n_{0e}}{\partial y} - \mathrm{Im}\, e \frac{\partial}{\partial y} \int \mathrm{d}k (k_y + \Omega_e \tau_e k_x) \langle n_e(k) \varphi(-k) \rangle \right\}, \tag{30}$$

$$\frac{\partial n_{0i}}{\partial t} = \frac{-e}{M\Omega_i^2 \tau_i} \left\{ E_0 \frac{\partial n_{0i}}{\partial y} - \mathrm{Im} \frac{\partial}{\partial y} \int \mathrm{d}k (k_y - \Omega_i \tau_i k_x) \langle n_i(k) \varphi(-k) \rangle \right\}, \tag{31a}$$

$$\frac{\partial n_{0i}}{\partial t} = -\frac{e\tau_i}{M} \left\{ E_0 \frac{\partial n_{0i}}{\partial y} - \mathrm{Im} \frac{\partial}{\partial y} \int \mathrm{d}k \, k_y \langle n_i(k) \varphi(-k) \rangle \right\}, \tag{31b}$$

also quantity $\langle n_j(k) \varphi(-k) \rangle$ for the expression of which we can obtain, with the aid of (25), a relation analogous to that of mixed correlation, proceeding similarly as in MIKHAILOVSKII's paper.

In the sections that follow we shall consider only those diagrams that give an approximate set of equations for strong turbulence corresponding to the set of equations for weak coupling (KADOMTSEV). A graphic representation of this set is indicated in Fig. 2. The meaning of the various symbols is the same as in MIKHAILOVSKII's paper.

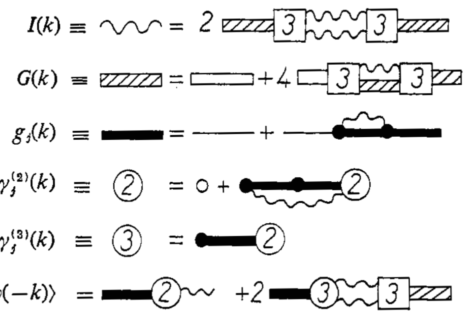


FIG. 2

The analytic expression is given by the following set of equations:

$$g_e(k) = \frac{1}{\omega - \omega_E + \dfrac{\bar{\omega}_E}{\Omega_e\tau_e} + \dfrac{iT\tau_e}{m}\left(\dfrac{k_\perp{}^2}{(\Omega_e\tau_e)^2} + k_z{}^2\right) - \sigma_e(k)},$$

(a) $$g_i(k) = \frac{1}{\omega - \omega_E - \dfrac{\bar{\omega}_E}{\Omega_i\tau_i} - \sigma_i(k)},$$

(b) $$g_i(k) = \frac{1}{\omega - \bar{\omega}_E\Omega_i\tau_i - \sigma_i(k)}, \tag{32}$$

(a) $$\sigma_j(k) = \left(\frac{e}{m\Omega_e}\right)^2 \int \mathrm{d}\mathrm{k}_1 g_j(k - k_1)[\mathbf{k}, \mathbf{k}_1]_z{}^2 \mathrm{I}(k_1),$$

(b) $$\sigma_i(k) = \left(\frac{e\tau_i}{M}\right)^2 \int \mathrm{d}k_1 g_i(k - k_1)(\mathbf{k}_1 \cdot \mathbf{k})(\mathbf{k}_1 \cdot (\mathbf{k} - \mathbf{k}_1))\mathrm{I}(k_1),$$

$$\gamma_e^{(2)}(k) = en_0\left[\frac{\omega^*}{T} + \frac{i\tau_e}{m}\left(\frac{k_\perp{}^2}{\Omega_e{}^2\tau_e{}^2} + k_z{}^2\right)\right]$$
$$- \left(\frac{e}{m\Omega_e}\right)^2 \int \mathrm{d}k_1[\mathbf{k}, \mathbf{k}_1]_z{}^2\, g_e(k - k_1) g_e(-k_1)\gamma_e^{(2)}(-k_1)\mathrm{I}(k_1),$$

(a) $$\gamma_i^{(2)}(k) = en_0\left[\frac{\omega^*}{T} - \frac{i\tau_i}{M}\left(\frac{k_\perp{}^2}{\Omega_i{}^2\tau_i{}^2} + k_z{}^2\right)\right]$$
$$- \left(\frac{e}{m\Omega_e}\right)^2 \int \mathrm{d}k_1[\mathbf{k}, \mathbf{k}_1]_z{}^2 g_i(k - k_1) g_i(-k_1)\gamma_i^{(2)}(-k)\mathrm{I}(k_1),$$

(b) $$\gamma_i^{(2)}(k) = -\frac{ien_0\tau_i}{M}k^2 - \left(\frac{e\tau_i}{M}\right)^2 \int \mathrm{d}k_1(\mathbf{k}_1 \cdot \mathbf{k})(\mathbf{k} \cdot (\mathbf{k} - \mathbf{k}_1)) g_i(k - k_1).$$
$$g_i(-k_1)\gamma_i^{(2)}(-k_1)\mathrm{I}(k_1),$$

$$G_0^{-1}(k) = 1 - \frac{4\pi e}{k^2}[g_i(k)\gamma_i^{(2)}(k) - g_i(k)\gamma_e^{(2)}(k)],$$

$$G(k) = \frac{1}{G_0^{-1}(k) - \sum(k)},$$

$$\sum(k) = 4\int dk_1\, \Gamma^0_{3\,\mathrm{sym}}(k; k_1)G(k-k_1)\Gamma^0_{3\,\mathrm{sym}}(k-k_1, k)\mathrm{I}(k_1),$$

$$\Gamma^0_{3\,\mathrm{sym}}(k; k_1) = \tfrac{1}{2}[\Gamma_3{}^0(k; k_1) + \Gamma_3{}^0(k; k-k_1)],$$

$$\Gamma_3{}^0(k; k_1) = \frac{4\pi e}{k^2}[g_i(k)\gamma_i^{(3)}(k; k_1) - g_e(k)\gamma_e^{(3)}(k; k_1)],$$

(a) $$\gamma_j^{(3)}(k, k_1) = -\frac{ie}{m\Omega_e}[\mathbf{k}, \mathbf{k}_1]_z g_j(k-k_1)\gamma_j^{(2)}(k-k_1),$$

(b) $$\gamma_i^{(3)}(k, k_1) = -\frac{ie\tau_i}{M}\mathbf{k}\cdot\mathbf{k}_1 g_i(k-k_1)\gamma_i^{(2)}(k-k_1),$$

$$\mathrm{I}(k) = 2\,|G(k)|^2\int dk_1\,|\Gamma^0_{3\,\mathrm{sym}}(k; k_1)|^2\,\mathrm{I}(k_1)\mathrm{I}(k-k_1),$$

$$\langle n_j(k)\varphi(-k)\rangle = g_j(k)\gamma_j^{(2)}(k)\mathrm{I}(k) + 2g_j(k)\int dk_1\gamma_j^{(3)}(k; k_1)\mathrm{I}(k_1)\mathrm{I}(k-k_1)\Gamma^0_{3\,\mathrm{sym}}(-k; -k_1)G(-k).$$

It is clear that this set of integral equations is too intricate to yield to an analytic solution. Hence the only solution that might come into consideration, is that effected on an automatic computer. It would, therefore, serve a useful purpose to rewrite the set of equations in a dimensionless form which is moreover convenient for qualitative estimates.

5. DIMENSIONLESS FORM OF EQUATIONS AND QUALITATIVE ESTIMATES

We shall introduce the following dimensionless variables

$$\tilde{k}_\perp = \frac{k_\perp}{k_0}, \qquad \tilde{k}_z = \frac{k_z L}{\pi}, \qquad \tilde{\omega} = \frac{\omega}{\omega_0}, \tag{33}$$

where k_0 is given by formula (13) and $\omega_0 = eE_0k_0/m\Omega_e$ and the dimensionless functions

(a) $$\tilde{g}_j = g_j\omega_0, \qquad \tilde{\sigma}_j = \sigma_j/\omega_0, \qquad \tilde{\gamma}_j^{(2)} = \gamma_j^{(2)}\frac{m\Omega_e}{en_0k_0\kappa},$$

(b) $$\tilde{g}_i = g_i\omega_0\Omega_i\tau_i, \qquad \tilde{\sigma}_i = \sigma_i/\omega_0\Omega_i\tau_i, \qquad \tilde{\gamma}_i^{(2)} = \gamma_i^{(2)}\frac{m\Omega_e}{en_0k_0\kappa\Omega_i\tau_i},$$

$$\tilde{I} = I\frac{e}{m\Omega_e}\frac{k_0{}^5\pi,}{E_0L}, \qquad \tilde{G}_0 = G_0\frac{4\pi en_0\kappa}{E_0k_0{}^2}, \tag{34}$$

(a) $$\tilde{\gamma}_j^{(3)} = \gamma_j^{(3)}\frac{E_0m\Omega_e}{en_0\kappa k_0{}^2},$$ (b) $$\tilde{\gamma}_i^{(3)} = \gamma_i^{(3)}\frac{E_0m\Omega_e}{en_0\kappa k_0{}^2\Omega_i\tau_i},$$

$$\tilde{\Gamma}_3{}^0 = \frac{E_0{}^2k_0}{4\pi en_0\kappa}\Gamma_3{}^0, \qquad \tilde{G} = G\frac{4\pi en_0\kappa}{E_0k_0{}^2},$$

$$\tilde{\sum} = \sum\frac{E_0k_0{}^2}{4\pi en_0\kappa}, \qquad \widetilde{\langle n_j\varphi\rangle} = \langle n_j\varphi\rangle\frac{\pi k_0{}^5e}{m\Omega_eLn_0\kappa}.$$

The system of equations (32) then assumes the following form (with the symbol denoting the dimensionless quantities left out):

(a) $$g_i(k) = \frac{1}{\omega - k_x - \dfrac{k_y}{\Omega_i\tau_i} - \sigma_i(k)},$$

(b) $$g_i(k) = \frac{1}{\dfrac{\omega}{\Omega_i\tau_i} - k_y - \sigma_i(k)},$$

$$g_e(k) = \frac{1}{\omega - k_x + \dfrac{k_y}{\Omega_e\tau_e} + i\,\dfrac{Tk_0k_\perp{}^2}{\Omega_e\tau_e eE_0} + i\,\dfrac{k_z{}^2T\Omega_e\tau_e}{eE_0k_0}\left(\dfrac{\pi}{L}\right)^2 - \sigma_e(k)},$$

(a) $$\sigma_j(k) = \int \mathrm{d}k_1 g_j(k-k_1)[\mathbf{k}, \mathbf{k}_1]_z{}^2 \mathrm{I}(k_1),$$

(b) $$\sigma_i(k) = \int \mathrm{d}k_1 g_i(k-k_1)(\mathbf{k}_{\perp 1}\cdot\mathbf{k}_\perp)(\mathbf{k}_{\perp 1}\cdot(\mathbf{k}_\perp - \mathbf{k}_{\perp 1}))\mathrm{I}(k_1),$$

$$\gamma_e^{(2)}(k) = k_x + ik_\perp{}^2\frac{k_0}{\Omega_e\tau_e\kappa} + ik_z{}^2\frac{\Omega_e\tau_e}{k_0\kappa}\left(\frac{\pi}{L}\right)^2 - \int \mathrm{d}k_1[\mathbf{k}, \mathbf{k}_1]_z{}^2 g_e(k-k_1)g_e(-k_1)\gamma_e^{(2)}(-k_1)\mathrm{I}(k_1),$$

(a) $$\gamma_i^{(2)}(k) = k_x - ik_\perp{}^2\frac{k_0}{\Omega_i\tau_i\kappa} - ik_z{}^2\frac{\Omega_i\tau_i}{k_0\kappa}\left(\frac{\pi}{L}\right)^2 - \int \mathrm{d}k_1[\mathbf{k}, \mathbf{k}_1]_z{}^2 g_i(k-k_1)g_i(-k_1)\gamma_i^{(2)}(-k_1)\mathrm{I}(k_1),$$

(b) $$\gamma_i^{(2)}(k) = -ik_\perp{}^2\frac{k_0}{\kappa} - \int \mathrm{d}k_1(\mathbf{k}_\perp\cdot\mathbf{k}_{\perp 1})(\mathbf{k}_\perp\cdot(\mathbf{k}_\perp - \mathbf{k}_{\perp 1}))g_i(k-k_1) \times g_i(-k_1)\gamma_i^{(2)}(-k_1)\mathrm{I}(k_1),$$

$$G_0^{-1}(k) = \frac{1}{k_\perp{}^2}[g_e(k)\gamma_e^{(2)}(k) - g_i(k)\gamma_i^{(2)}(k)],$$

$$G(k) = \frac{1}{G_0^{-1}(k) - \sum(k)},$$

$$\sum(k) = 4\int \mathrm{d}k_1 \Gamma^0_{3\,\mathrm{sym}}(k; k_1)G(k-k_1)\Gamma^0_{3\,\mathrm{sym}}(k-k_1; k)\mathrm{I}(k_1),$$

$$\Gamma_3{}^0(k_i\, k_1) = \frac{1}{k_\perp{}^2}[g_i(k)\gamma_i^{(3)}(k; k_1) - g_e(k)\gamma_e^{(3)}(k; k_1)],$$

(a) $$\gamma_j^{(3)}(k; k_1) = -i[\mathbf{k}, \mathbf{k}_1]_z g_j(k-k_1)\gamma_j^{(2)}(k-k_1),$$

(b) $$\gamma_i^{(3)}(k; k_1) = -i\mathbf{k}_\perp\cdot\mathbf{k}_{\perp 1} g_i(k-k_1)\gamma_i^{(2)}(k-k_1),$$

$$\mathrm{I}(k) = 2|G(k)|^2\int \mathrm{d}k_1|\Gamma^0_{3\,\mathrm{sym}}(k; k_1)|^2\mathrm{I}(k_1)\mathrm{I}(k-k_1),$$

$$\langle n_j(k)\varphi(-k)\rangle = g_j(k)\gamma_j^{(2)}(k)\mathrm{I}(k) + 2g_j(k)\int \mathrm{d}k_1\gamma_j^{(3)}(k; k_1)\mathrm{I}(k_1) \times \mathrm{I}(k-k_1)\Gamma^0_{3\,\mathrm{sym}}(-k; -k_1)G(-k),$$

where we have neglected 1 in the expression for G_0 because of condition $C_A \ll c$, and in expressions G_0, $\Gamma_3{}^0$ and σ_i, $\gamma_i^{(2)}$, $\gamma_i^{(3)}$ in case (b) changed k^2 to $k_\perp{}^2$ because of condition $k_z{}^2/k_\perp{}^2 \ll 1$.

Since it may be expected that in case (a) all the dimensionless quantities are of the order of unity, and in case (b) $g_i \sim \Omega_i\tau_i$, $\gamma_i^{(2)} \sim (\Omega_i\tau_i)^{-1}$ and the remaining terms are also of the order of unity, we obtain from (34) the following estimate of the maximum value of the spectrum function

$$I_{\max} \approx \left(\frac{ek_0{}^5}{m\Omega_e E_0}\frac{\pi}{L}\right)^{-1} \tag{36}$$

and from equation (30) the estimate of the coefficient of turbulent diffusion

$$D_\perp \approx \frac{eE_0}{m\Omega_e k_0}. \tag{37}$$

On introducing k_0 from (13) in expression (37) and bearing in mind that in experiments it is usually $eE_0/\kappa T \sim 1$. we arrive at the following estimates of the diffusion coefficient

case (a) $$D_\perp \sim D_B/(\Omega_i\tau_i)^{1/2},$$

case (b) $$D_\perp \sim \frac{D_B\Omega_e\tau_e}{1 + \Omega_e\tau_e\Omega_i\tau_i},$$

where $D_B = T/m\Omega_e$. It is evident that in case (a) the diffusion coefficient is smaller and in case (b) much larger than Bohm's coefficient.

If a more accurate determination of the diffusion coefficient and of the form of the spectrum function is desired, the system of dimensionless equations (35) must be solved numerically for each concrete case because equations (35) contain four dimensionless independent parameters $\Omega_e\tau_e$, $\Omega_i\tau_i$, $eE_0/\kappa T$, $(\pi/L)^2/k_0\kappa$. This problem will be dealt with in one of our forthcoming papers.

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