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Corrigendum

Gravitational deflection of massive particles in classical and semiclassical general relativity

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In this paper, hereafter referred to as I, the angle of particle deflection by the sun was calculated to order G^3 in the gravitational constant. The result obtained is, however, incorrect for two reasons. One comes from the procedure of adopting Newtonian values for two, u_1 and u_2, of the three roots of equation (I1) (the differential equation for $(\mathrm{d}u/\mathrm{d}\phi)^2$ with $u = 1/r$, as usual) instead of using the values which are defined by the differential equation itself. This led to the loss of the relativistic post-Newtonian terms that one is looking for. The other is a missing term in the expansion of the integrand of the equation defining the deflection angle Φ a few lines after equation (I1), there called Φ_C.

As for the roots, writing $(\mathrm{d}u/\mathrm{d}\phi)^2 = 2GM(u - u_0)(u - u_1)(u - u_2)$ as done in I, M being the mass of the sun with radius R, the three relations that come from equation (I1) are

$$2GM(u_0 + u_1 + u_2) = 1 \tag{1}$$

(this was used in I to express u_0 in terms of the other two roots in the differential equation written just above),

$$2GMu_0u_1u_2 = -\frac{1}{b^2} \tag{2}$$

and

$$(u_1 + u_2)u_0 + u_1u_2 = \frac{1}{L^2}. \tag{3}$$

In these last two equations, b is the impact parameter of the particle approaching the sun with initial velocity v_0 and, in unities $c = 1$, $L = b\ v_0\sqrt{1 - v_0^2}$. This comes from the conserved $L = r^2\mathrm{d}\phi/\mathrm{d}\tau$ calculated at infinity, $L(\infty) = r^2(\mathrm{d}\phi/\mathrm{d}t)(\mathrm{d}t/\mathrm{d}\tau) = l(1 - v_0^2)^{-1/2}$ where $l = r^2\mathrm{d}\phi/\mathrm{d}t = bv_0$ is the conserved angular momentum per unity mass. Also, the right-hand side of equation (2) comes from the relation $\lambda = (E^2 - 1)/L^2 = 1/b^2$, where $E = E(\infty) = \mathrm{d}t/\mathrm{d}\tau = (1 - v_0^2)^{-1/2}$ is the second conserved quantity. If we use equation (1) for u_0 in equations (2) and (3), we obtain a pair of equations for the determination of the other two roots. To the desired order the result is

$$u_1u_2 = -\frac{1}{b^2} - \frac{4G^2M^2}{b^4v_0^2} + O(G^4), \tag{4}$$

and

$$u_1 + u_2 = \frac{2GM}{b^2v_0^2} + \frac{8G^3M^3}{b^4v_0^2}\left(1 + \frac{1}{v_0^2}\right) + O(G^5), \tag{5}$$

which can be easily checked. Only the first, Newtonian, terms were taken in I. Before we proceed we re-emphasize that one is considering the deflection for relativistic particles, $v_0 \lesssim 1$.

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In this case the last two terms in both of these equations can be taken as post-Newtonian corrections of order $G^2M^2/b^2 \leqslant G^2M^2/R^2 \sim 10^{-12}$ to the first ones. This, *a fortiori*, will entail the validation of the the expansion for the integrand of the equation defining the deflection angle Φ, as done in I. Calling, as before, $X = 2GM(u_1 + u_{2.} + u_1 \sin^2 x + u_2 \cos^2 x)$ the expansion of the integrand of that equation is, to the desired order, $1+X/2+3X^2/8+5X^3/16$. The last term was, however, forgotten in I. From equations (4) and (5) the value of $\bar{x}$ written just below equation (I2) becomes

$$\bar{x} = \frac{\pi}{4} - \frac{GM}{2bv_0^2} + \left(\frac{GM}{bv_0^2}\right)^3 \left(\frac{1}{6} - v_0^2 - 2v_0^4\right) \tag{6}$$

to the desired order G^3. We shall comment on 'orders' in a moment. With these corrections the final result for the deflection angle turns out to be, in natural unities,

$$\Phi = \frac{2GM}{bv_0^2}\Bigg[1 + \frac{v_0^2}{c^2} + \frac{3\pi}{2}\frac{GM}{bc^2} + \frac{3\pi}{8}\frac{GM}{bc^2}\frac{v_0^2}{c^2} + 15\frac{G^2M^2}{b^2c^4} + \frac{G^2M^2}{b^2c^4}\left(\frac{5c^2}{v_0^2} - \frac{c^4}{3v_0^4} + \frac{5v_0^2}{3c^2}\right)\Bigg]. \tag{7}$$

This result is valid to $O(G^3)$ leaving aside the value of v_0 with the sole condition of being relativistic, $v_0 \lesssim c$. It should be noted that to say that v_0^2 is of the order of GM/b does not apply here first because we are dealing with relativistic particles and second because we are not dealing with a bound state problem where the virial theorem holds. Therefore, one cannot for instance argue that the second and third terms in the square brackets are of the same order or that the next two are of the same order and of one order higher than the previous ones, as implied in I. Also, for the same reason, we cannot argue that the last term gives a contribution which is of one order higher than G^3 so the calculation should be pushed explicitly to order G^4. The final outcome of the present calculation is that it corrects the result derived in I in two respects. First, we have the factor 15 in the fifth term in the square brackets instead of the factor 9 there at present. Second, we have the extra G^3 last term depending on v_0.

As a final word we mention that in a forthcoming article we shall calculate the light deflection to order G^3 following the same steps as for the particle. We shall show that if one *formally* puts v_0 equal to c in the particle result in equation (7), the light result is reproduced. The reason for that will also be discussed and explained.