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On thermal diffusivity

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Abstract

The two main properties related to heat conduction are thermal conductivity (K) and thermal diffusivity (D). However, while the meaning of K is very well known, the role played by D is usually undervalued and misunderstood. In this paper we will try to clarify the meaning of thermal diffusivity, first in homogeneous materials and then in composite materials. In this latter case, we will find that sometimes the association of two good thermal diffusers gives a bad diffuser. Moreover, by properly choosing the constituents of the composites we can manufacture materials with thermal properties that are not found in nature.

1. Introduction

Energy flow due to temperature difference is called heat. There are three different mechanisms of heat transfer: conduction, convection and radiation. Although in most real situations the three mechanisms appear at the same time, experimental conditions can be designed in such a way that only one of them takes place. In this paper we deal with pure heat conduction. The two main properties related to heat conduction are thermal conductivity (K) and thermal diffusivity (D). However, while K is very well known, the role played by D is usually undervalued and misunderstood. For instance, everybody knows that the thermal conductivity of a metal such as Ni is higher than the thermal conductivity of a polymer and that this is higher than that of air. In contrast, few physicists know that both Ni and air have the same thermal diffusivity, and even fewer are aware of the meaning of this. The situation is even worse for heterogeneous materials such as composites.

In this paper we will try to clarify the role played by thermal diffusivity; firstly in homogeneous materials and then in composites. In this latter case, we will find that unlike what happens with K, which only depends on the thermal conductivity of the constituents, D depends on both the thermal diffusivity and the thermal conductivity of the components. The main consequence is that sometimes the association of two good thermal diffusers gives a bad diffuser. Moreover, by choosing properly the constituents of the composites we can manufacture materials with thermal properties that are never found in nature.

2. Thermal properties of homogeneous materials

In this section we deal with the thermal properties of homogeneous and isotropic materials. The specific heat (c) is the only thermal property required in static problems, i.e. when the temperature is independent of position and time. Similarly, thermal conductivity is the only property required in steady problems, i.e. when temperature does not vary over time. However, the complete characterization of time-varying thermal phenomena requires the knowledge of two properties: thermal diffusivity and thermal effusivity (e). We specially focus on the meaning and role of thermal diffusivity and thermal effusivity.

2.1. Specific heat

For a thermally isolated material of mass m the relationship between heat deposition and temperature increase is given by the equation

$$\mathrm{d}Q = mc\,\mathrm{d}T. \tag{1}$$

Accordingly, the specific heat measures the amount of energy that should be added to the unit mass to produce a unit increase in temperature. Thus, it quantifies the material's ability to store internal energy (thermal energy).

2.2. Thermal conductivity

Whenever there is a temperature gradient ($\vec{\nabla}T$) into a material a heat flow ($\vec{q}$) is established that is given by Fourier's law, which in the case of homogeneous and isotropic materials is [1]

$$\vec{q} = -K\vec{\nabla}T. \tag{2}$$

Accordingly, thermal conductivity measures the heat that flows in unit time through a unit area of a layer of the material of unit thickness with unit temperature difference between its faces. This means that a good thermal conductor such as Cu or diamond in contact with a thermal reservoir at high temperature extracts from it much more energy per unit time than a bad conductor such as a polymer or a gas.

2.3. Thermal diffusivity

For a homogenous and isotropic medium with temperature-independent thermal properties and no heat generation inside it, the differential equation of heat conduction is [1]

$$\nabla^2 T - \frac{1}{D}\frac{\partial T}{\partial t} = 0. \tag{3}$$

Thermal diffusivity is the quantity that measures the change in temperature produced in unit volume of the material by the amount of heat that flows in unit time through a unit area of a layer of unit thickness with unit temperature difference between its faces. The physical meaning behind thermal diffusivity is associated with the speed of propagation of heat during changes of temperature over time. Therefore, a good thermal diffuser such as Cu or diamond placed in contact with a hot thermal reservoir at time $t = 0$ reaches its thermal equilibrium faster than bad diffusers such as polymers or glass. However, it is usually said that a material that is placed in contact with a hot/cold reservoir gets warm/cold quickly because it is a good thermal conductor, but actually the reason is that it is a good diffuser. The source of this confusion is that in condensed matter (liquids and solids) good/bad conductors are always good/bad diffusers. In figure 1 the thermal conductivity is represented against the thermal diffusivity for a wide variety of homogeneous materials: metals (closed circles), ceramics (squares), glasses (triangles), polymers (open squares) and liquids (open circles). As can be

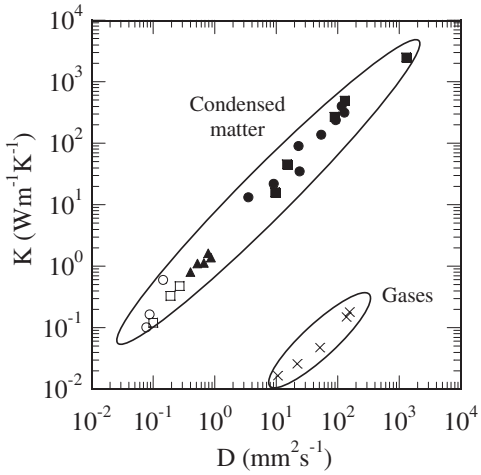

Figure 1. Thermal conductivity versus thermal diffusivity for a wide variety of homogeneous materials (closed circles, metals; squares, ceramics; triangles, glasses; open squares, polymers; open circles, liquids; and crosses, gases).

Table 1. Thermal properties of selected materials.

Material	Diamond	Cu	K	Co	Ni	Pb	AISI 304	Glass	PC	Water	Air	He
K (W m^{-1} K^{-1})	2300	400	102	100	91	35	14.5	1.11	0.20	0.60	0.026	0.15
D ($\times 10^6$ m^2 s^{-1})	1290	116	158	24.6	23	23	4.0	0.56	0.15	0.144	22	137
e (J m^{-2} K^{-1} s$^{-1/2}$)	64 040	37 140	8150	20 150	19 400	7300	7250	1480	515	1580	5.5	12.8
ρc ($\times 10^{-6}$ J m^{-3} K^{-1})	1.78	3.45	0.65	4.05	3.95	1.52	3.62	1.98	1.33	4.16	0.0012	0.0011

seen all these points are placed close to a 'straight line'. In fact K and D are not independent quantities but they are related through the following equation [1]:

$$D = \frac{K}{\rho c}, \tag{4}$$

where ρ is the density. The ratio K/D is the heat capacity per unit volume that is almost constant in condensed matter: its values fall in the range between 1×10^6 and 4×10^6 J m^{-3} K^{-1}.

However, this relationship between a better/worse conductor and a better/worse diffuser is not an exact equation. For instance let us consider two well known metals: Ni and Pb. As Ni is a better conductor than Pb we could say that Ni gets warm/cold faster than Pb, but the opposite happens according to their thermal diffusivities (see table 1). The confusion is more dramatic in the case of gases because at room conditions (1 atm and 20 °C) they are bad conductors but good diffusers. The relationship here between K and D is represented in figure 1 by crosses that fall onto a 'straight line' parallel to the one corresponding to condensed matter. Actually in the case of gases the heat capacity per unit volume is three orders of magnitude smaller than in condensed matter (between 0.7×10^3 and 2×10^3 J m^{-3} K^{-1}). Let us take air. Although its thermal conductivity is very small its thermal diffusivity is as high as Ni or Pb (see table 1). Therefore air gets warm/cold as fast as these elements[1]. We can conclude that, due to their low K, gases extract small amounts of energy from a thermal reservoir but that the absorbed energy is quickly diffused through them.

[1] Only heat conduction is taken into account.

Another noteworthy behaviour of gases is the dependence of their thermal properties on pressure at constant temperature. At low pressure ($P < 2$ atm) thermal conductivity and specific heat are independent of pressure [2]. On the other hand, at these low pressures all gases behave as ideal gases as follows:

$$P = \frac{\rho RT}{M} \tag{5}$$

where $R = 8.314$ J mol^{-1} K^{-1} is the universal gas constant and M is the molecular weight.

Consequently, according to equation (4), thermal diffusivity is inversely proportional to pressure

$$D = \frac{KRT}{PcM}. \tag{6}$$

This means that if pressure is reduced by a factor 10, then thermal diffusivity is multiplied by the same factor, while thermal conductivity remains unchanged. To explain this result we have to take into account that thermal diffusivity is proportional to the mean free path of the heat carriers [3]. As the pressure decreases the density decreases and therefore the collisions among the gas molecules decrease, increasing the mean free path and thus thermal diffusivity.

2.4. Thermal effusivity

The second thermal property needed in time-varying problems is thermal effusivity defined as [4]

$$e = \sqrt{\rho c K} = \frac{K}{\sqrt{D}} = \rho c \sqrt{D}, \tag{7}$$

which measures the material's ability to exchange heat with the environment. For instance, when a semi-infinite material initially at temperature T_0 is placed in perfect thermal contact at $t = 0$ with a thermal reservoir at temperature $T_1 > T_0$, the heat flow through the interface is given by [1]

$$q = \frac{e(T_1 - T_0)}{\sqrt{\pi t}}. \tag{8}$$

As can be seen, this heat flow is not proportional to the thermal conductivity of the material, as under steady conditions, but to its thermal effusivity. For instance, let us consider two metals, Co and K, that have the same thermal conductivity but different thermal effusivities (see table 1). Under steady conditions (e.g. a slab placed between two heat reservoirs at different temperatures) they extract the same amount of heat per unit time; but under transient conditions (e.g. the same slab placed suddenly in contact with a hot thermal reservoir) Co extracts 2.5 times more heat than K due to its higher effusivity.

On the other hand, when two semi-infinite solids, one at uniform temperature $T_{1\,0}$ and the other at $T_{2\,0)}$ ($T_{1\,0} > T_{2\,0}$), are placed together in perfect thermal contact at $t = 0$, their mutual interface temperature immediately reaches a steady value given by [5]

$$T_i = \frac{T_{10}e_1 + T_{20}e_2}{e_1 + e_2}. \tag{9}$$

Accordingly, if $e_1 = e_2$, T_i lies midway between $T_{1\,0}$ and $T_{2\,0}$, while if $e_1 > e_2$, T_i will be closer to $T_{1\,0}$ and if $e_1 < e_2$, T_i will be closer to $T_{2\,0}$. That is the reason why when touching with the hand ($T_{1\,0} \approx 37\,°$C) different objects that are at the same room temperature (e.g. 20 °C) the thermal feeling is different according to their thermal effusivities. When touching a piece of metal ($e_{metal} \gg e_{skin} \approx e_{water}$) the temperature of the skin immediately drops to 20 °C, and we sense the object as being cold. However, when touching a piece of plastic ($e_{plastic} < e_{skin}$) the temperature of the skin remains close to 37 °C, and we sense the object as being warm.

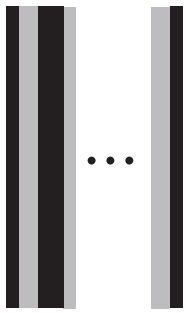

Figure 2. The geometry of a multilayer material made of thin slabs of two materials in alternate layers.

Finally, when the surface of a material is periodically heated plane thermal waves are propagated through it. If this material is made of two layers these thermal waves will be reflected and transmitted at the interface. Reflection and transmission coefficients for the thermal waves are defined in the same way as in mechanical waves by substituting the mechanical impedance by the thermal effusivity [4]. In this way the ratio of the thermal effusivities controls the thermal mismatch between two media.

3. Thermal properties of composites

For this type of heterogeneous material all the thermal properties are discontinuous functions of the location and consequently neither Fourier's law, nor the heat conduction equation can be applied. However, the thermal properties are such useful concepts in the study of materials that we want to retain them in some way. Therefore the concept of 'effective' properties, i.e. the properties of the equivalent homogeneous material that produces the same physical effects, appears to be useful.

The effective heat capacity always follows the mixture rule. Thus, for a composite made of two components, the effective heat capacity per unit volume is given by

$$(\rho c)_{eff} = v_1\rho_1 c_1 + v_2\rho_2 c_2 = v_1\frac{K_1}{D_1} + v_2\frac{K_2}{D_2} \tag{10}$$

where v_i is the volume fraction of components i. As can be seen, it only depends on the volume fraction of the components. By contrast, both the effective thermal conductivity and the diffusivity strongly depend on the internal distribution of the components inside the composite. In this work we focus on a multilayer material composed of thin slabs of two materials disposed alternately (see figure 2), for which simple analytical solutions exist.

The effective thermal conductivity parallel/perpendicular to the layers can be obtained using the in-parallel/in-series thermal resistor model [6]

$$K_{\parallel} = v_1 K_1 + v_2 K_2 \tag{11a}$$

$$\frac{1}{K_{\perp}} = \frac{v_1}{K_1} + \frac{v_2}{K_2}. \tag{11b}$$

According to these expressions the associations of two good/bad thermal conductors always give a good/bad thermal conductor while the association of a good conductor to a bad conductor always gives an intermediate thermal conductor.

On the other hand, using the generalized version of equation (4) for the effective properties we get the effective thermal diffusivity parallel/perpendicular to the layers as follows:

$$D_{\parallel} = \frac{K_{\parallel}}{(\rho c)_{eff}} = \frac{v_1 K_1 + v_2 K_2}{v_1\frac{K_1}{D_1} + v_2\frac{K_2}{D_2}} \tag{12a}$$

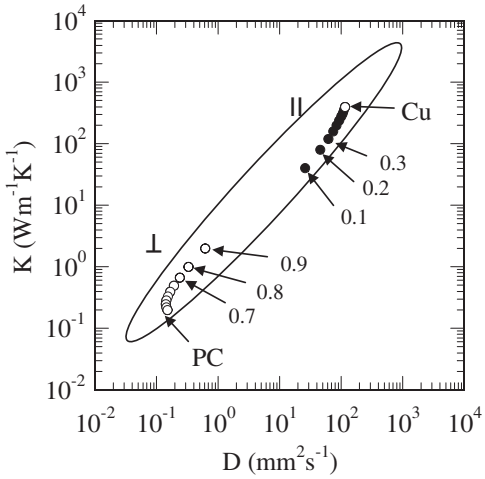

Figure 3. Effective thermal conductivity versus diffusivity for a multilayer composite made of Cu and PC for different proportions of each component. The numbers represent the volume fraction of Cu. The closed circles stand for $(D_{\parallel}, K_{\parallel})$ and the open circles for $(D_{\perp}, K_{\perp})$.

$$\frac{1}{D_{\perp}} = \frac{(\rho c)_{eff}}{K_{\perp}} = \frac{v_1^2}{D_1} + \frac{v_2^2}{D_2} + v_1 v_2 \left(\frac{K_2}{K_1 D_2} + \frac{K_1}{K_1 D_1} \right). \tag{12b}$$

Note that while the effective thermal conductivity only depends on the thermal conductivity of the components, the effective thermal diffusivity depends on both the thermal diffusivity and conductivity of the components. This is the source of some interesting results as follows.

(a) *The multilayer composite is made up of two solids and/or liquids.* According to equation (10) its effective heat capacity per unit volume will be intermediate between the values of the components, i.e. inside the typical range of condensed matter (1×10^6 to 4×10^6 J m^{-3} K^{-1}). Therefore the couples (D, K) of the resulting composite fall onto the 'straight line' graph of condensed matter (see figure 1). For instance, let us consider a multilayer composite made of a good conductor such as Cu and a bad conductor such as polycarbonate (PC). In figure 3 we show the values of the effective thermal conductivity versus diffusivity of the resulting composite for different proportions of each component. In this figure the closed circles stand for $(D_{\parallel}, K_{\parallel})$ and the open circles for $(D_{\perp}, K_{\perp})$. In both cases all points are inside the region corresponding to the condensed matter. Therefore, in the case of solids and/or liquids, when combining a good diffuser with a bad diffuser the resulting composite has intermediate thermal diffusivity. Anyway, it is worth noting that $D_{\perp}$ is close to the value of the polymer even for high concentrations of the metal, indicating the difficulty of improving $D_{\perp}$. Moreover, for volume fractions of Cu between 0.05 and 0.40 the thermal diffusivity of the composite is slightly smaller than the thermal diffusivity of the polymer, with the smallest value being equal to 0.14 mm^2 s^{-1}. By contrast, $D_{\parallel}$ is close to the Cu value even for small concentrations of the metal. As a result the composite shows a high degree of thermal anisotropy.

(b) *The multilayer composite is the combination of solid and gas.* According to equation (10) its effective heat capacity per unit volume will be intermediate between the typical values of condensed matter and gases. Therefore the couples (D, K) of the resulting composite fall outside the two regions that characterize condensed matter and gases (see figure 1). In this way we can manufacture composites with thermal properties other than those found in nature. For instance, let us consider a multilayer composite made of Cu and air. In figure 4 we show the values of the effective thermal conductivity versus the diffusivity of

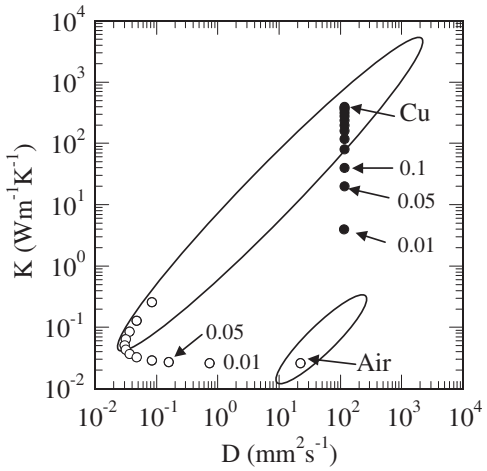

Figure 4. The same as in figure 3 for Cu and air.

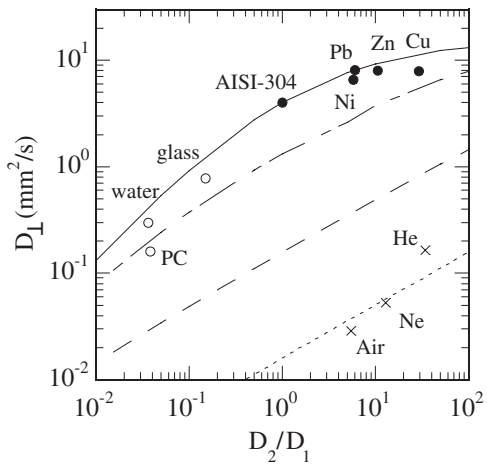

Figure 5. $D_\perp$ values for a layered composite made of AISI-304 reinforced by a second material as a function of the ratio D_2/D_1. The continuous line represents $e_2/e_1 = 1$, the dashed–dotted line represents $e_2/e_1 = 10^{-1}$ or 10, the dashed line represents $e_2/e_1 = 10^{-2}$ or 10^2 and the dotted line represents $e_2/e_1 = 10^{-3}$ or 10^3.

the resulting composite for different proportions of each component. As before the closed circles stand for $(D_\parallel, K_\parallel)$ and the open circles for $(D_\perp, K_\perp)$. As expected many points are outside the regions corresponding to condensed matter and gases. Focusing on $D_\perp$, it is surprising to find that the addition of two good diffusers (Cu and air) gives a bad diffuser.

To better understand this behaviour let us rewrite equation (12*b*) as a function of D and e

$$\frac{1}{D_\perp} = \frac{v_1^2}{D_1} + \frac{v_2^2}{D_2} + \frac{v_1 v_2}{\sqrt{D_1 D_2}}\left(\frac{e_2}{e_1} + \frac{e_1}{e_2}\right). \tag{13}$$

According to this equation, for given values of D_1 and D_2, $D_\perp$ shows a maximum when $e_1 = e_2$, i.e. for a perfect thermal coupling between the two materials. As the values of the thermal effusivities change, $D_\perp$ quickly decreases. This situation is shown in figure 5 for a composite made of AISI-304 stainless steel reinforced by a second material against the ratio

D_2/D_1, for different values of e_2/e_1. As can be seen, the highest value of $D_\perp$ is obtained when $e_1 = e_2$. In this way, it is worth noting that reinforcing the steel with Pb produces a better $D_\perp$ value than using Ni or Cu, although both are better thermal conductors than Pb. By contrast, when e_2 differs greatly from e_1, $D_\perp$ becomes extremely low because of the bad thermal mismatch between the two components. It is the case of gases ($e_{gases} \ll e_{solids}$), resulting in composites with very low $D_\perp$ values even though D_{gases} is high. In conclusion, if we want to manufacture a layered composite with high $D_\perp$ values then we have to look for materials with high thermal diffusivities and similar effusivity values.

4. Conclusions

In this work the role played by thermal diffusivity, as the quantity that governs the speed of heat propagation in transient problems, has been analysed. In this way, the difference between thermal conductivity and thermal diffusivity has been emphasized. It has been demonstrated that in homogeneous materials the relationship of better/worse thermal conductor–better/worse thermal diffuser is not always correct. In the case of layered composites it has been shown that while the associations of two good/bad thermal conductors always give a good/bad conductor, the association of two good diffusers gives a bad diffuser when their effusivities differ greatly.

All these results reinforce the role played by thermal diffusivity and effusivity in all time-varying thermal phenomena.

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