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Recent advances in the study of wind waves

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Abstract

The field of wind-generated ocean surface gravity waves is reviewed for the period covering the last fifteen years. Theories and observations relevant to understanding the physics of wind waves are discussed, as well as techniques for measuring and forecasting waves.

It is found that although a great deal of recent progress has been made on certain aspects of the wind wave problem, there are still important aspects which are poorly understood. In particular, the central problem of how the wind generates waves in the ocean has not yet been solved; the primary physical mechanism(s) by which the wind makes waves has not been found. When the wind blows how much energy and momentum goes into waves and how much goes into currents? At the present time it is not possible to give a very definite answer to this important question. However, wave generation theories are available which perhaps can be modified to give better agreement with observations than they do now. New wave measuring techniques developed recently and/or under development now may provide the badly needed field observations necessary for future advances in understanding wind waves.

Very little is known about the dissipation of wind waves, either in the open ocean or near coastal boundaries. Relevant theories and observations are not sufficient for understanding wave dissipation at present. Relatively little work has been done in the general area of the interaction of wind waves with currents. Some preliminary theoretical work suggests that this interaction could be quite important in the particular problem of the propagation of wind waves into major ocean currents, but an experimental test of the theory is lacking.

Present methods of wave forecasting incorporate more physics than the earlier empirical methods. However, these methods still require a set of 'engineering approximations' in order to produce decent forecasts. With proper tuning the numerical forecast models produce good estimates of the one-dimensional wave spectrum.

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Contents

1. Introduction

This article is an attempt to summarize recent developments in the field of wind-generated ocean waves. The entire field was last reviewed comprehensively in a stimulating article by Ursell (1956). Shorter but more recent reviews of the subject are included in the papers by Longuet-Higgins (1961), Miles (1967), Stewart (1967), and Hasselmann *et al* (1973). Longer discussions useful for review are contained in the works of Roll (1957) and Phillips (1966). The tone of the present article is set by Ursell's often quoted opening sentence: 'Wind blowing over a water surface generates waves in the water by physical processes which cannot be regarded as known.' Although the same statement can still be made today, it must be understood against a broader background which includes extensive recent theoretical and experimental efforts to understand how the wind makes waves.

Of all the various types of wave motion that are possible in the ocean, wind waves are one of the most energetic and easily observed and, therefore, one of the most studied. Wind waves are generally considered to be surface gravity waves which are caused by the wind and propagate under the restoring force of gravity. Their wave lengths range from about 10 cm to about 1 km, with maximum energy density typically centred at wave lengths of about 150 m. These water waves have maximum particle motion right at the air–sea interface, the particle motion decreasing rapidly with depth. The waves are dispersive over most of the ocean, with the long wave lengths travelling faster than the short wave lengths. Popular names for wind waves include 'sea' for the rough, irregular waves in and near a storm area, and 'swell' for the smooth, sinusoidal waves at some distance from the storm area.

An important distinction is made in this article between 'wind' and 'capillary' waves. Capillary waves or 'ripples' are surface waves with wave lengths less than about 1 cm which propagate under the restoring force of surface tension. Typically, the energy density of capillary waves in the ocean is orders of magnitude less than that of gravity waves. Capillary waves are also generated by the wind, and the generation of the longer gravity waves may depend intimately on the presence of the capillary waves, which may effect the coupling between the turbulent air boundary layer and the sea. Although the term 'wind waves' could logically include capillary waves, we choose to reserve it for the longer, more energetic gravity waves. Thus work specifically on capillary waves will not be reviewed here.

Other types of wave motion are outside the scope of this review but will be mentioned here briefly to give some perspective to wind waves. Tsunamis, also (improperly) called 'tidal' waves, are surface gravity waves with wave lengths much greater than the total depth of the ocean (4 km) and are caused by earthquakes. Internal waves are gravity waves which owe their existence to vertical density gradients within the ocean, and they have their maximum expression between the sea surface and sea floor (it is very difficult to detect them at the air–sea interface); their cause is largely unknown. Very large-scale wave motions include the tides which are forced by the gravitational attractions of the sun and moon, and Rossby waves which propagate under a restoring force associated with the variation with latitude of the local vertical component of the earth's rotation. In addition there exist several types of waves which are confined to the vicinity of continental boundaries and are trapped by bottom topography effects. The names of some of these are edge waves, continental shelf waves, and Kelvin waves.

A review, today, of wind-generated waves cannot be done in the detailed manner of Ursell because of the large amount of work which has taken place in the meantime. Therefore we have had to make our own judgments as to which works to include, and, unfortunately, we have had to exclude reference to many recent papers. Although less detailed, this article will be more comprehensive, for not only will the process of wave generation be covered, but also we will summarize results on wave propagation, dissipation of wave energy, and techniques for measuring and forecasting waves. However, we will not cover in depth the large amount of work which has been done on wave breaking (surf) in shallow water.

The reader wishing to obtain a more detailed description of the fundamental properties of wind waves is referred to Kinsman's (1965) very readable book. The monograph of Phillips (1966) will provide the more advanced reader with a concise summary of the major aspects of wave theory. The more classical aspects of wave theory are provided by Lamb's (1932) hydrodynamical treatise. A rather unique summary of an exciting conference on ocean wind waves exists in book form under the name *Ocean Wave Spectra*, published in 1963. Some of the earlier oceanographic work on wind waves is summarized by Defant (1961, vol 2), and a few articles also appear in *The Sea* (ed N M Hill 1962, vol I).

The outline of the remainder of the article is as follows. Section 2 will be background material intended to give the reader familiarity with the state of knowledge through the Ursell article. Following this will be sections on theoretical considerations, measurement techniques, observations and how they compare with theory, prediction of wind waves, and finally a summary section.

2. Background

At the time Ursell reviewed the field, the body of theoretical work exceeded that of the experimental work, but both were in an unsatisfactory state. Today the same ratio holds in that the theoretical ideas are still ahead of the experimental testing. In particular, field observations relevant to wave generation and dissipation in the ocean in 1955 were nearly non-existent. Today they are simply very scarce.

2.1. Theory

The first theoretical idea applied to the generation of water waves by wind was the classical Kelvin–Helmholtz instability mechanism (see Lamb 1932, p462). In this theory the air pressure is 180° out of phase with the surface elevation, and for a sufficiently large wind speed the pressure distribution over the surface due to the Bernoulli effect can cause an infinitesimal-amplitude sinusoidal wave to grow against the restoring forces of gravity and surface tension. The theory assumes a wind speed which is a constant (independent of height above the water surface) and it leads to the prediction of a minimum wind speed necessary to make the waves grow. This minimum is an absolute minimum for waves which travel in the same direction as the wind. However, common experience shows that waves exist at wind speeds much lower than the predicted (absolute) minimum speed of $6{\cdot}5\ \mathrm{m\ s^{-1}}$.

The next major idea was put forth by Jeffreys (1924, 1925) and is known as the 'sheltering hypothesis'. Jeffreys made the plausible suggestion that a turbulent wind blowing over a pre-existing wave crest would act like air blowing past a blunt body in that boundary layer 'separation' would occur on the downwind side of a wave crest (the boundary layer would presumably 're-attach' itself on the upwind side of the next wave crest). The resulting asymmetrical properties of the wind relative to the wave would cause a pressure distribution which could feed energy into the wave, provided the wave moved slower than the wind. The component of air pressure in phase with the wave slope (the vertical velocity of the water surface) does work on the wave and can make the wave grow if frictional dissipation can be overcome. This theory lost support when laboratory measurements of air flowing over stationary solid waves showed that the pressure forces produced were too small for Jeffreys' mechanism to be effective. However, Ursell pointed out that different sets of laboratory measurements were conflicting and the experiments were not relevant to evaluate the generation of moving water waves. Jeffreys' theory may yet emerge as being important since more recent theories (though not completely evaluated yet) based on perturbation techniques have not yielded the major growth mechanism for wind waves. It is still not known, though, whether or not air flow separation does in fact occur over wind waves.

Another early model of wave generation is due to Eckart (1953). Eckart represented the wind by a random distribution of normal pressures in the form of idealized circular gusts over a finite storm area. He calculated the resulting waves at distances large compared with the storm diameter. The normal pressures were assumed to be independent of the waves already produced. Although reliable measurements of the air pressure had not been made, Eckart concluded that what evidence there was suggested that his theory was not a very effective generation mechanism. It has been suggested since (Phillips 1957) that Eckart's representation of the wind field was a little too specific to be realistic.

Next, Lock (1954) (also Wuest 1949) developed a boundary layer instability model which in some ways is analogous to flow past a semi-infinite flat plate. These authors examined the case of air flowing over water wherein the boundary layer between the two media is laminar and viscous, and is assumed to start at a definite point on the interface and grow in thickness downstream. The problem then is to determine the stability of the motion when it is perturbed by a small sinusoidal oscillation with a wave length assumed to be much smaller than the thickness of the boundary layer, and also much smaller than the distance from the boundary layer origin. The equations describing the flow then reduce to a pair of ordinary differential equations of the Orr–Sommerfeld type. The solution of these equations shows zones of amplification and decay of the (water and air) waves in wave number and coordinate phase space. The solutions are, however, somewhat complicated and difficult to visualize and interpret.

By stressing the unsatisfactory state of knowledge about the nature of wind wave generation, Ursell's review sparked two major independent and complementary theories by Phillips (1957) and Miles (1957) (discussed in §3.2) which stimulated a new period of advancement in wind wave research. Phillips' theory was an improvement over Eckart's theory of wave generation by normal pressures, and Miles' theory was an advance over the previous shear flow instability theories of Kelvin, Helmholtz, Wuest, and Lock.

One additional conceptual advancement was developed around the 1950s which has flavoured research in the field of wind waves ever since. This was the realization that the best first-order description of the sea surface was in terms of average or statistical quantities. The statistical approach leads naturally to the concept of the energy spectrum as the most important statistic for describing the rough sea surface. The appreciation of the statistical nature of the ocean surface started with the application of the work of Rice (1944, 1945) and Wiener (1960) in the fields of communication and time series analysis, and developed slowly through the work of several investigators, including Barber and Ursell (1948), Longuet-Higgins (1950, 1952), and St Dennis and Pierson (1953); longer summaries are given by Longuet-Higgins (1962a), Cartwright (1962), and Kinsman (1965).

2.2. Measurement techniques

Prior to the mid-1950s most wave measurements were obtained visually. This procedure usually involved a 'calibrated eyeball' or some simple arrangement of fixed, graduated staff and optical recording device. It was perhaps for this reason that the concept of the significant wave (the mean of the highest third of all waves present) arose. The visual measurement techniques were clearly a hindrance to the progress of ocean wave research.

A major contradiction to the above statements may be found in the early work of Barber and Ursell (1948) and Snodgrass (1958). These authors were among the first to utilize pressure transducers mounted on the sea floor to obtain time histories of sea surface elevations. The low-pass filter effect of the overlying water column was both convenient for sampling purposes and easily correctable via linear theory (Kinsman 1965).

Another technique for measuring waves just coming into its own in the early 1950s was the use of an accelerometer. Double integration of the acceleration time history of the sea surface yields a record of the sea surface displacement. Alternatively, the acceleration spectrum can be directly related to the height spectrum through linear theory. Tucker (1956) was among the first to utilize this technique in the development of a ship-borne wave recorder for British weather ships.

For historical interest it should be mentioned that Barber (1949) described a photographic technique for measuring wave direction. As we shall see in §4, a variant of the method has recently been applied to estimating the directional wave spectrum via optical Fourier transform techniques. Barber's work is most remarkable because it *preceded* the invention of the laser, an instrument vital in the optical Fourier transform process.

2.3. Observations pertaining to wave generation

Stanton *et al* (1932) and Motzfeld (1937) investigated the distribution of normal pressure induced by air flow over a fixed solid wave profile. The observed pressure differences along the 'wave' profile, which would induce growth, were too small to support the sheltering coefficient called for by Jeffreys' (1924) theory. Thijsse (1951) obtained observations in contradiction to the above, but the methods and scaling problems associated with his experiment leave considerable doubt as to the applicability of the results to the wave generation problem. Prior to 1955 (for that matter, 1966) these were virtually the only measurements with which to test theories of wave generation by wind.

A number of other measurements designed to provide information on the effect of wind on the sea surface had been made prior to 1955. Many notable works can be cited in this regard, although none of the data was used to investigate the processes of wave generation. Some studies, based on many years of observations, are reported by Barber and Ursell (1948) and Darbyshire (1952, 1955). A set of quite remarkable observations was obtained by Roll (1951) in the tidal flats off the Frisian coast. The works of these authors have been summarized by Ursell (1956) and mentioned here only for historical interest.

2.4. *Methods of wave prediction*

A practical application of wind wave research is obviously wave forecasting. The earliest method of forecasting seems to have been that developed by Sverdrup and Munk (1947). Using basic hydrodynamic theory and a tiny amount of observational evidence, these authors were able to construct a reasonable wave prediction method. In particular, they combined theory and observations into a set of nomographs so that a relatively skilled meteorologist/oceanographer using weather maps could derive several simple parameters with which to predict wave heights. Surprisingly enough, the method worked with some accuracy, although it was not particularly reliable in complex geophysical situations. Bretschneider (1952) made some improvements on the basic method in later years which enhanced its accuracy and utility. At about the same time in Britain, Darbyshire (1952, 1955) was developing similar prediction techniques.

A major advance in methods of wave prediction was made by Pierson, Neumann and James (1955). Like Darbyshire, these authors introduced the concept of the wave spectrum (§3.1.2) into the forecast where the previous work had only considered significant wave height. The PNJ method, as it was called, developed a large set of complex rules that required rather sophisticated interpretation in order to produce a reasonable wave forecast. Tests of the forecast method were not as satisfactory as might have been desired. Nevertheless, the authors had made the giant stride of trying to predict the spectrum of the sea surface.

One of the prime difficulties with both of the methods mentioned above was that they were largely subjective. Therefore their success or failure was highly dependent upon the skill of the person using the method. Also, the methods tried to handle an extremely complex problem through rather simple manual techniques. It is not surprising that both techniques left much to be desired.

2.5. *Summary*

In the year 1955, knowledge concerning the mechanics and properties of the wind wave field was quite unsatisfactory. There were several theories that purported to describe wave generation by the wind, yet none of the theories could be substantiated. Observations of wave growth needed to verify the theories were almost non-existent. Methods of obtaining observational data were in even worse condition. Considering these drawbacks, it is not surprising that a reliable wave prediction method did not exist. It is against this background, then, that the advances to be described in the following sections have been made.

3. Theoretical considerations

3.1. *Basic formulation*

3.1.1. The equations of fluid dynamics. Theoretical studies of surface gravity waves begin with simplified forms of the basic equations of fluid dynamics, the Navier–Stokes equation (conservation of momentum) on a rotating earth

$$\rho \mathrm{D}\boldsymbol{u}/\mathrm{D}t = -\nabla p - \rho\boldsymbol{g} - 2\rho\boldsymbol{\Omega}\times\boldsymbol{u} + \mu\nabla^2\boldsymbol{u} + \tfrac{1}{3}\mu\nabla(\nabla\cdot\boldsymbol{u}) \qquad (3.1)$$

and the continuity equation (conservation of mass)

$$\mathrm{D}\rho/\mathrm{D}t+\rho\nabla.\boldsymbol{u}=0 \tag{3.2}$$

where $\boldsymbol{u}$ is the velocity vector relative to the earth's frame, p is the fluid pressure, ρ is the fluid density, $\boldsymbol{g}$ is the gravitational acceleration vector, μ is the coefficient of viscosity, $\boldsymbol{\Omega}$ is the angular velocity of the earth, and the substantial derivative operator $\mathrm{D}/\mathrm{D}t=\partial/\partial t+(\boldsymbol{u}.\nabla)$.

When these equations are applied to the water motion associated with wind waves, several simplifications can be made. First, because the frequency of wind waves is more than three orders of magnitude larger than the earth's angular velocity, the Coriolis term $(-2\rho\boldsymbol{\Omega}\times\boldsymbol{u})$ can usually be neglected compared with the acceleration term $(\rho\mathrm{D}\boldsymbol{u}/\mathrm{D}t)$ in equation (3.1). Backus (1962) showed that the effect of the earth's rotation on the trajectory of small-amplitude surface gravity waves, even over great distances on the earth, is one or two orders of magnitude smaller than could be observed by the measurements of Munk *et al* (1963). However, Ursell (1950) pointed out that the Coriolis force could have a significant effect on the motion of fluid particles in finite-amplitude waves, as discussed in §3.7.

Since the Reynolds number, based on wave length, wave period, and laminar viscosity coefficient, is very large for wind waves (of order 10^5 for a wave length of order 1 m and period of order 1 s), the friction terms in equation (3.1) are negligible compared with the acceleration term. Phillips (1959) showed that the effects of friction on wind waves should be small in the ocean even when the much larger turbulent 'eddy' viscosity coefficient is considered.

Next, both density gradients within the water and the compressibility of water have negligible effects on wind waves for typical ocean conditions, so that the density can be considered constant in equations (3.1) and (3.2). In other words, surface gravity waves are well separated from both internal gravity waves and sound waves in frequency and wave number space (Eckart 1960).

An idealization usually made is that the water motion is irrotational, which means that the curl of the velocity vector is zero everywhere. This allows the velocity to be written as the gradient of a scaler 'velocity' potential. The main justification for making this assumption comes from Kelvin's circulation theorem which says that the fluid will be irrotational if the motion is started from rest by conservative forces. Finally, the effects of surface tension can be neglected for wave lengths greater than about 10 cm (Lamb 1932), so that gravity is the only restoring force acting on the free surface.

With these assumptions the basic equations reduce to

$$\nabla^2\phi=0 \tag{3.3}$$

which satisfies (3.2), where $\phi(\boldsymbol{x},t)$ is the velocity potential $(\boldsymbol{u}=\nabla\phi)$. Therefore Laplace's equation for the velocity potential is to be satisfied in the interior of the fluid subject both to initial conditions and to boundary conditions at the air–sea interface and the sea–sediment interface. The two boundary conditions at the air–sea interface are

$$\partial\xi/\partial t+\nabla\xi.\nabla\phi=\partial\phi/\partial z \quad \text{at } z=\xi \tag{3.4}$$

$$\partial\phi/\partial t+g\xi+\tfrac{1}{2}(\nabla\phi)^2=0 \quad \text{at } z=\xi. \tag{3.5}$$

The first condition, equation (3.4), is the kinematical constraint that at the free surface, $z=\xi(x,y,t)$, the velocity of the fluid must equal the velocity of the air–sea interface.

The second condition, equation (3.5), is the dynamical constraint that the pressure be constant (zero in this case) at the free surface and results from a first integral (Bernoulli's equation) of equation (3.1). The coordinate system is chosen so that the horizontal coordinates x, y lie in the plane of the equilibrium free surface ($z=0$) and the z axis points vertically upward (antiparallel to gravity).

The bottom boundary is normally taken to be rigid, flat, and parallel to the undisturbed free surface, the total depth being h. Then at the bottom ($z=-h$) the vertical component of the fluid velocity must be zero:

$$\partial\phi/\partial z=0 \quad \text{at } z=-h. \tag{3.6}$$

The fluid is also usually taken to be horizontally infinite, so that no further boundary conditions are required.

Once the initial conditions are specified, equations (3.3)–(3.6) completely describe the fluid motion, which can then be studied in terms of wave solutions. Two major difficulties must be overcome immediately. The first difficulty is that although equation (3.3) is linear, the boundary conditions (3.4) and (3.5) are nonlinear. The second difficulty is that the boundary conditions (3.4) and (3.5) are to be applied at the free surface $z=\xi$, which is not known *a priori* but is to be found as part of the solution. The second difficulty is overcome by expanding equations (3.4) and (3.5) in a Taylor series about $z=0$. The terms of increasing order in the expansion become increasingly small provided the (nondimensional) wave slope is small. This procedure, however, creates more nonlinear terms. The nonlinearity is handled by again assuming small wave slopes and expanding equations (3.4) and (3.5) in perturbation series

$$\begin{aligned}\phi&=\epsilon\phi_1+\epsilon^2\phi_2+\epsilon^3\phi_3+\ \ldots\\ \xi&=\epsilon\xi_1+\epsilon^2\xi_2+\epsilon^3\xi_3+\ \ldots\end{aligned} \tag{3.7}$$

where ϵ is a small nondimensional parameter proportional to the wave slope. It is to be noted that the wave slope is not the only relevant nondimensional parameter; the ratio of the wave length to the water depth is another one. However, this ratio is very small for most ocean situations, except near coastal boundaries.

The resulting linearized problem is then

$$\begin{aligned}&\nabla^2\phi_1=0 && -h\leqslant z\leqslant 0\\ &\left.\begin{aligned}\partial\xi_1/\partial t&=\partial\phi_1/\partial z\\ \partial\phi_1/\partial t+g\xi_1&=0\end{aligned}\right\} && z=0\\ &\partial\phi_1/\partial z=0 && z=-h.\end{aligned} \tag{3.8}$$

Elimination of ξ_1 between the second and third equations of (3.8) gives a boundary value problem to be solved for ϕ_1. Once solved the velocity is given by $\boldsymbol{u}_1=\nabla\phi_1$ and the perturbation pressure by $p_1/\rho=-\partial\phi_1/\partial t$. The linear equations can be satisfied by a velocity potential which has a sinusoidal dependence on time and the horizontal coordinate vector $\boldsymbol{x}=(x, y)$, and a hyperbolic depth dependence

$$\phi(x, y, z, t)=A\cosh k(z+h)\exp[\mathrm{i}(\boldsymbol{k}.\boldsymbol{x}-\omega t)] \tag{3.9}$$

where A is a constant, provided that the frequency ω and the magnitude of the horizontal wave number vector $k=|\boldsymbol{k}|$ satisfy the dispersion relation

$$\omega^2=gk\tanh kh. \tag{3.10}$$

In general, long (low-frequency) waves travel faster than short (high-frequency) waves, and the energy travels slower than the wave crests. The wave crests move with the phase speed $c \equiv \omega/k$, whereas the energy moves with the group speed $c_g = \partial\omega/\partial k$, the signal speed. When the depth is large compared with the wave length, (3.10) yields $c = 2c_g = g/\omega$, which is the dispersive 'deep water' limit. When the wave length is large compared with the water depth, (3.10) yields $c = c_g = (gh)^{1/2}$, the familiar nondispersive 'shallow water' limit. In the deep water limit for linear waves the fluid particles orbit in circles whose radii decrease exponentially with distance (depth) from the surface, and the particle speed is much smaller than the phase speed. The average energy per unit area for a linear sinusoidal wave of amplitude 'a' is equal to $\frac{1}{2}\rho g a^2$ and is half kinetic and half potential.

A very simple and useful physical picture which explains the existence of surface gravity waves was discussed by Einstein (1916) (see also Defant 1961, vol 2, p74, Rayleigh 1876). By considering the motion of a semi-infinite fluid flowing under a stationary solid wavy wall, he showed how the fluid speed can be adjusted so that there is a balance between the static and dynamic pressure forces along the wall, and then the wall can be taken off without disturbing the fluid. By observing the fluid from a reference frame moving with the speed of the fluid far away from the wall (which equals the phase speed), one has the usual picture of water waves propagating along the surface.

The basic fluid equations (3.1) and (3.2) can also be applied to the air, and then the air and water motions can be coupled by modifying the boundary condition (3.5) to include the air pressure. However, the main difficulty in describing the air motion mathematically, which has caused progress on the coupled problem to be slow, is associated with the fact that the air motion is intrinsically turbulent.

3.1.2. Statistical representation of the wave field. Although some aspects of wind wave generation, propagation, and decay can be discussed with single sinusoidal wave components, a more fruitful approach for a general formalism starts with a statistical description of the wave field. There are good reasons for starting with such a description of the sea surface. Within a storm area the forces which generate the waves involve the turbulent wind and are too complicated to be described in detail. Therefore the resulting surface displacement cannot be predicted exactly. Even outside the storm area the surface displacement may be a superposition of waves which have been generated by many independent storms throughout the ocean basin. Most wave records do not at all look like a single sine wave.

The configuration of the sea surface varies irregularly in both space and time. In the linear approximation it is useful to assume that the sea surface irregularities are locally homogeneous, stationary, and Gaussian (Longuet-Higgins 1952, 1962b, Pierson *et al* 1955, Hasselmann 1962a). This implies that average quantities are invariant under translations of space and time and that the first-order amplitudes are statistically independent for different wave number vectors. The Gaussian property follows from an application of the Central Limit Theorem to a sea surface which is a superposition of a large number of statistically independent wave components, provided that the processes which are changing the sea state have scales which are large compared with the wave scales. This is generally true since the time and space scales associated with the generation by storms and with energy variations within the wave field are large compared with the wave scales. Attempts to avoid the Gaussian assumption have so far not been very successful. For further discussion of the Gaussian assumption in the nonlinear case see Hasselmann (1966, 1967, 1968).

The usefulness of the Gaussian assumption is that the sea state can be described by a single statistical moment, the second moment. All higher moments can be related to the second moment, which in turn is directly related to the wave energy spectrum. The main theoretical result is then a statistical description of the energy balance of the wave field in terms of the energy spectrum. Certain forms of the energy spectrum can be computed from time series observations by using standard spectral analysis techniques. Thus, in principle, theory and observations can be brought together through the energy spectrum.

The formalism is as follows. Regard the wave field as a superposition of free waves. The displacement of the free surface $\xi(\boldsymbol{x}, t)$ is then represented by a Fourier–Stieljes integral, or more conveniently by the summation

$$\xi(\boldsymbol{x}, t) = \sum_{k} \{\eta_k \exp[\mathrm{i}(\boldsymbol{k}.\boldsymbol{x} - \omega t)] + \eta_k^* \exp[-\mathrm{i}(\boldsymbol{k}.\boldsymbol{x} - \omega t)]\} \tag{3.11}$$

where η_k is a random Fourier amplitude whose complex conjugate is η_k^*, and ω and $|\boldsymbol{k}|$ are related by (3.10).

A homogeneous, stationary wave field has the properties

$$\begin{aligned} \langle \eta_{k_1} \eta_{k_2} \rangle &= 0 \quad \text{all } \boldsymbol{k}_1, \boldsymbol{k}_2 \\ \langle \eta_{k_1} \eta_{k_2}^* \rangle &= 0 \quad \boldsymbol{k}_1 \neq \boldsymbol{k}_2 \\ \langle \eta_k \eta_k^* \rangle &= F(\boldsymbol{k}) \Delta \boldsymbol{k} / 2\rho g \end{aligned} \tag{3.12}$$

where the angle brackets denote an ensemble average, $\Delta\boldsymbol{k}$ is the wave number increment of the Fourier sum, and $F(\boldsymbol{k})$ is the continuous energy spectrum. The total mean wave energy per unit surface area is then

$$E = \rho g \langle \xi^2 \rangle = \iint F(\boldsymbol{k})\, \mathrm{d}\boldsymbol{k}. \tag{3.13}$$

In observational work the constant factor ρg is often left out, which only changes the units of the energy spectrum.

It should be noted that ensemble averages and the energy spectrum $F(\boldsymbol{k})$ are not the most natural descriptions of the sea surface from the point of view of observations. Ensemble averages cannot be carried out in practice and must be replaced by either space or time averages. According to the Ergodic Hypothesis, space and time averages are equivalent to ensemble averages for stationary and homogeneous fields.

The full two-dimensional wave number spectrum $F(\boldsymbol{k})$ has never been measured in the ocean, although various approximations to it or projections of it have been obtained. By far the largest number of wave observations have been made by a single instrument which measures, as a function of time, the wave elevation (or some parameter directly related to it) in a frame of reference fixed to the ocean bottom. From a time averaging operation on such a wave record one obtains the one-dimensional energy spectrum $F_1(\omega)$ as a function of frequency alone with no directional information.

If the autocorrelation function† $R(\tau)$ is defined by

$$R(\tau) = \langle \xi(t) \xi(t+\tau) \rangle \tag{3.14}$$

then the relationship between the energy spectrum $F_1(\omega)$ and the observable wave amplitude at a point $\xi(t)$ is given through the Fourier transform pair (Wiener 1960)

$$\begin{aligned} F_1(\omega) &= \int_{-\infty}^{\infty} R(\tau) \exp(-\mathrm{i}\omega\tau)\, \mathrm{d}\tau \\ R(\tau) &= (1/2\pi) \int_{-\infty}^{\infty} F_1(\omega) \exp(\mathrm{i}\omega\tau)\, \mathrm{d}\omega. \end{aligned} \tag{3.15}$$

† There is a method of computing spectra, called the 'fast Fourier transform' technique, which avoids the use of the autocorrelation function (see eg Bendat and Piersol 1971, p299).

The relationship between F_1 and F is obtained from the definition of the total energy (equation 3.13) and is

$$\int F_1(\omega)\,\mathrm{d}\omega = \iint F(\boldsymbol{k})\,\mathrm{d}\boldsymbol{k} = \iint F_2(\omega, \theta)\,\mathrm{d}\omega\,\mathrm{d}\theta \tag{3.16}$$

where $F_2(\omega, \theta)$ is another form of the energy spectrum which uses θ to indicate the angle of energy propagation in a selected coordinate system.

3.1.3. The radiative transfer equation. The radiative transfer equation (3.17) describes the energy balance of the wind wave field in terms of the energy spectrum of surface gravity waves. This equation, which formally summarizes all the various physical processes which can change the wave energy, is given by

$$\frac{\mathrm{D}F}{\mathrm{D}t} \equiv \frac{\partial F}{\partial t} + \dot{\boldsymbol{x}}.\frac{\partial F}{\partial \boldsymbol{x}} + \dot{\boldsymbol{k}}.\frac{\partial F}{\partial \boldsymbol{k}} = S \tag{3.17}$$

where the spectrum $F(\boldsymbol{k}, \boldsymbol{x}, t)$ is locally a function of the wave number $\boldsymbol{k}$, but is allowed to vary slowly as a function of $\boldsymbol{x}$ and t. The characteristic equations are

$$\begin{aligned} \dot{\boldsymbol{x}} &= \partial\omega(\boldsymbol{k}, \boldsymbol{x})/\partial\boldsymbol{k} \\ \dot{\boldsymbol{k}} &= -\,\partial\omega(\boldsymbol{k}, \boldsymbol{x})/\partial\boldsymbol{x} \end{aligned} \tag{3.18}$$

where the dot denotes the time derivatives. Equations (3.18) are equivalent to Hamilton's equations for a particle.

In (3.17) $\mathrm{D}F/\mathrm{D}t$ is the Lagrangian rate of change of the spectrum relative to a wave group moving along the ray paths determined by (3.18). If the equilibrium depth 'h' is a slowly varying function of horizontal position $\boldsymbol{x}$, the frequency ω in (3.18) is then a function of position as well as wave number (through equation 3.10). When the water depth is large (deep water limit) compared with all wavelengths, the frequency is no longer a function of position and the term $\dot{\boldsymbol{k}}.(\partial F/\partial \boldsymbol{k})$ in (3.17) vanishes and the rays are straight lines (in the absence of variable currents). When the depth is small (shallow water limit) compared with a wave length, the waves are refracted by the bottom topography and the rays are generally curved lines. The term $\dot{\boldsymbol{x}}.(\partial F/\partial \boldsymbol{x})$ in (3.17) vanishes if the spectrum is not a function of position. The term $\partial F/\partial t$ is the local time rate of change of the spectrum. Equations (3.17) and (3.18) apply to a plane ocean and were given originally by Gelci *et al* (1956) and in their present form by Hasselmann (1960); Groves and Melcer (1961) and Backus (1962) have given the generalization for propagation on a spherical earth.

The source function S in (3.17) represents the net transfer to or from the spectrum at the wave number $\boldsymbol{k}$ due to all interaction processes which affect the component $\boldsymbol{k}$. If S were zero the spectrum F would be conserved following a ray (Longuet-Higgins 1957). This situation is closely approximated for swell propagating across the ocean far outside storm areas and away from currents and coastal boundaries. In storm areas and near coasts S is not zero but contains a number of different terms which include linear and nonlinear processes of wave growth, dissipation, and redistribution of wave energy in wave number space. In general the source term is a function of the spectrum itself, so that equation (3.17) cannot be solved analytically for the spectrum except in certain special cases.

Following Hasselmann (1968) we assume that the source function S can be written as a superposition of a number of individual source terms S_i:

$$S = \sum_{i=1}^{n} S_i. \tag{3.19}$$

The first two terms in (3.19) are the best known and have the form

$$\begin{aligned} S_1 &= \alpha \\ S_2 &= \beta F(\boldsymbol{k}) \end{aligned} \tag{3.20}$$

where the functional dependence on the spectrum is explicitly exhibited. The coefficients α and β depend in a known manner on the properties of the wind field according to separate theories.

The source function S_1 represents the constant energy transfer to the wave field through turbulent atmospheric pressure fluctuations according to the theory of Phillips (1957). This wave generation mechanism is uncoupled in the sense that the developing wave field is assumed not to change the atmospheric pressure fluctuations which are forcing it to grow. If this mechanism operated alone the wave spectrum would grow linearly with time.

The source function S_2 represents the increasing transfer of energy to the wave field due to an instability in the coupling between the wave field and the mean boundary layer flow in the air according to the theory of Miles (1957). This is a coupled wave generation mechanism which would lead to an exponential growth of the spectral energy were it to act alone.

The source terms S_3 and S_4 have the form

$$\begin{aligned} S_3 &= F(\boldsymbol{k}) \int \gamma(\boldsymbol{k}, \boldsymbol{k}')F(\boldsymbol{k}')\, \mathrm{d}\boldsymbol{k}' \\ S_4 &= -\delta F(\boldsymbol{k}) + \int \epsilon(\boldsymbol{k}, \boldsymbol{k}')F(\boldsymbol{k}')\, \mathrm{d}\boldsymbol{k}'. \end{aligned} \tag{3.21}$$

The term S_3 is a nonlinear correction to Miles' (1957) theory, and the term S_4 represents the energy transfer due to interaction between waves and turbulence in the atmosphere. These terms were given by Hasselmann (1968) as part of his general weak interaction theory which includes all expansible interactions derivable from perturbation theory.

The source function S_5 is better known and has the form

$$S_5 = \int (T_1 F(\boldsymbol{k}')F(\boldsymbol{k}'')F(\boldsymbol{k}-\boldsymbol{k}'-\boldsymbol{k}'') - T_2 F(\boldsymbol{k})F(\boldsymbol{k}')F(\boldsymbol{k}''))\, \mathrm{d}\boldsymbol{k}'\, \mathrm{d}\boldsymbol{k}''. \tag{3.22}$$

This term represents the energy transfer among the various wave number components due to weakly nonlinear wave–wave interactions according to the theory of Hasselmann (1962a, b). The coupling takes place between groups of four wave components whose wave numbers and frequencies satisfy certain resonance conditions. The integration over all resonant groups leads to a redistribution of wave energy in wave number space and eventually tends to smear out energy peaks. The coupling coefficients T_1 and T_2 depend on algebraic combinations of the interacting wave numbers and frequencies.

The various other source terms are discussed more fully by Hasselmann (1968). One term represents the dissipation in shallow water due to turbulent bottom friction, and the functional form was given by the theory of Hasselmann and Collins (1968). Another term represents the dissipation of wave energy due to wave breaking, which is at present a poorly understood process and no functional form for it has been given. There may be other processes, as yet unknown, which will contribute to the total source function S.

Equations (3.21) and (3.22) show that the source functions depend on the entire wave spectrum and not only on the wave component $\boldsymbol{k}$. In particular S_3 and S_5 are nonlinear in the spectrum. Thus, in general, all components of the wave field are coupled. In order to determine the wave spectrum at some point in the ocean, the spectrum must first be determined simultaneously over the entire region of the ocean in which S is nonzero. This makes the wave prediction problem very difficult in principle, but in practice satisfactory predictions can be made by incorporating some empirical relationships.

3.2. Linear theories of wave generation

As mentioned in the introduction, Ursell's (1956) review sparked two independent and complementary theories of wave generation which both appeared one year later, one being Phillips' (1957) theory and the other Miles' (1957) theory. These two theories received a considerable amount of attention in the fields of oceanography and fluid dynamics, and a great deal of additional work was based on them. Summaries and discussions of these theories can be found in Phillips (1966), Kinsman (1965), and Hasselmann (1967, 1968).

3.2.1. Phillips' theory. Phillips' (1957) theory considers the generation of waves on initially still water by normal pressure fluctuations due to the onset of a turbulent wind. It is assumed that the waves do not modify the pressure force which generates them. Phillips found that the waves grow by a resonance mechanism when the speed and length of the atmospheric pressure fluctuations match those of the water waves. The waves continue to grow by this mechanism until the wave slopes become large enough that nonlinearities, which are neglected in the theory, become important.

In constructing his theory Phillips had Eckart's (1953) wave generation theory before him (§2.1). However, where Eckart had represented the pressure fluctuations as a specific collection of pressure spots of a given size and duration which moved with constant speed over a finite storm area, Phillips allowed the pressure field to evolve in a random way as it was being convected over the water surface by the mean wind.

Phillips' (1957) theory can be divided into two parts by considering the time from the onset of a turbulent wind to be either much less than (initial stage) or much greater than (principal stage) the time scale for the development of the pressure fluctuations. In the initial stage the most prominent waves generated are shown to be ripples (capillary–gravity waves) of wave length 1·7 cm which move in the two directions $\cos^{-1}(c/U)$ to that of the mean wind and thereby produce a rhombic pattern on the surface (c is the phase speed of the 1·7 cm waves, and U is approximately the mean wind speed at a height of one wave length above the surface).

The major growth of the gravity waves takes place in the principal stage of development. Here it is shown that the energy grows linearly with time and is proportional to the spectrum of the pressure fluctuations, which in Hasselmann's (1968) notation is

$$\frac{\mathrm{D}F(\boldsymbol{k})}{\mathrm{D}t}=\frac{\pi\omega^2}{2\rho g}F_{\mathrm{p}}(\boldsymbol{k}, -\omega) \tag{3.23}$$

where $F_{\mathrm{p}}(\boldsymbol{k}, -\omega)$ is the three-dimensional spectrum of the pressure fluctuations. Although the waves grow by a resonance mechanism, the energy grows linearly, not quadratically, with time. This is due to the fact that the pressure fluctuations are not phase locked to the waves, but the amplitude and phase of a pressure component wanders randomly relative to a wave component which has the same speed and wave length as the pressure component. Equation (3.23) is the main result of Phillips' theory.

Phillips arrived at the equivalent to equation (3.23) by starting with the linearized equations of motion (3.8) for the water, with the addition of the pressure term p/ρ in the dynamic boundary condition (the third equation in 3.8) to bring in the coupling with the air. The water is assumed to be inviscid, and since the waves are generated from rest by the action of normal pressures, the motion is irrotational.

If the surface displacement $\xi(\boldsymbol{x}, t)$ and the pressure $p(\boldsymbol{x}, t)$ are given the Fourier representation

$$\begin{aligned} \xi(\boldsymbol{x}, t) &= \sum_{\boldsymbol{k}} \xi_k(t) \exp(\mathrm{i}\boldsymbol{k}.\boldsymbol{x}) \\ p(\boldsymbol{x}, t) &= \sum_{\boldsymbol{k}} p_k(t) \exp(\mathrm{i}\boldsymbol{k}.\boldsymbol{x}) \end{aligned} \tag{3.24}$$

then the linear response of the wave component ξ_k to the forcing pressure component p_k is determined by the following equation (obtained from the elimination of the velocity potential in the second and third equations of 3.8):

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\xi_k + \omega^2\xi_k = \frac{-\omega^2}{\rho g}p_k. \tag{3.25}$$

By considering the pressure component $p_k(t)$ to be a stationary random function of time, equation (3.25) is the classical problem of an undamped harmonic oscillator driven by a random forcing function. The asymptotic solution to (3.25) for large times gives the result that the mean-square displacement of the water surface increases linearly with time. The solution also shows that a particular wave component grows in response to that component of the turbulent pressure which has the same wave number and frequency and moves at the same speed as that of the free wave. Equation (3.23) expresses these results in spectral form, assuming further that the fluctuations in pressure and surface displacement are statistically homogeneous with respect to horizontal position.

Phillips' theory is not considered to be the mechanism to explain the major growth of wind waves, because the observed pressure fluctuations are too small and because observed energy growth rates are more nearly exponential than linear. Subsequent measurements (Longuet-Higgins 1961) showed that the turbulent pressure fluctuations in the air are much smaller than Phillips originally assumed in evaluating his theory. However, Phillips' generation mechanism may be important in bringing the energy level of the waves from zero to the point where other mechanisms, such as an instability, can take over. Phillips' theory cannot explain the observed damping of waves which propagate against the wind.

3.2.2. Miles' theory. Miles' (1957) theory considers the generation of water waves due to shear flow instability in the coupled air–water system. The original theory was extended and developed in a series of papers by Miles (1959, 1960, 1962, 1965, 1967). Contributions to this theory were also made by Benjamin (1959) and Lighthill (1962).

Miles (1957) improved on the Kelvin–Helmholtz model (§2.1) by assuming that, in the absence of wave motion, the mean wind speed had a prescribed continuous variation with height above the water surface, in better qualitative agreement with observed wind profiles. Miles considered the mean shear flow in the air to be produced by turbulent processes, but except for this fact, neglected all effects of the turbulence on the interaction between the wind and the water. The motion of the air was then taken to be laminar, inviscid and incompressible. In addition to these assumptions the water motion was assumed to be irrotational as in previous studies, and the wave slope was assumed to be small enough to justify neglecting the nonlinear terms. In addition, any mean motion in the water, which might be induced by the traction of the shear flow in the air, was neglected. A pre-existing water wave induces a disturbance in the shear flow, and that part of the induced pressure disturbance which is in phase with the wave slope does work on the wave and causes it to grow. This coupled mechanism then results in an exponential growth rate for the wave energy.

The main result of the theory is summarized in Hasselmann's (1968) notation by

$$\frac{\mathrm{D}F(\boldsymbol{k})}{\mathrm{D}t}=\left(-\frac{\pi\omega\rho_{\mathrm{a}}}{2\rho g}\frac{\mathrm{d}^2U_{\mathrm{c}}}{\mathrm{d}z^2}\Big/\frac{\mathrm{d}U_{\mathrm{c}}}{\mathrm{d}z}\right)|W_{\boldsymbol{k}}^{\omega}|^2F(\boldsymbol{k}) \tag{3.26}$$

where ρ_{a} is the air density. The derivatives of the mean wind profile $U(z)$ are evaluated at the critical layer defined by $U_{\mathrm{c}}-c=0$, ie the height at which the wind speed equals the phase speed of the water waves. $W_{\boldsymbol{k}}^{\omega}$ is the response of the boundary layer to a periodic unit amplitude surface displacement of phase velocity ω/k.

The exponential growth rate of the energy spectrum is apparent from the form of equation (3.26). The growth rate is positive since normal wind profiles have negative curvature and positive slope. An essential feature of the solution is the ratio of the curvature to the slope of the mean wind profile evaluated at the critical height. This result comes from the solution of the inviscid Orr–Sommerfeld equation for velocity perturbations induced in the air by the waves. Since normal wind profiles are approximately logarithmic, the energy transfer decreases with increasing height of the critical layer (ie decreasing profile curvature). The larger the wave length, the faster the waves travel and the higher the critical layer is, and therefore the less effective their growth rate is by this mechanism. On the other hand, if the critical layer lies very close to the water surface in a laminar sublayer with a linear velocity profile, then the energy transfer would vanish according to (3.26). According to Miles' theory, waves neither grow nor decay if they travel either faster than the maximum wind speed or at angles greater than 90° to the wind (waves are *not* damped if they propagate against the wind). The Miles' mechanism is most effective for waves which travel in the same direction as the wind, which is expected from general stability considerations (Lin 1966, p27).

The mathematical details of the Miles' mechanism are somewhat difficult and the physical picture is not as clear as in the Kelvin–Helmholtz theory. The exact solution of the governing differential equation, the inviscid Orr–Sommerfeld equation, for typically observed wind speed profiles does not appear to be possible, and numerical methods must be used to evaluate the energy growth rate. It turns out that the energy and momentum are transferred to the wave entirely from the critical layer, at some distance above the wave, as a result of a singularity in the governing equation at the critical layer. A physical explanation of the energy transfer has been given by Lighthill (1962) in terms of vortex forces acting on fluid particles near the critical layer. Unfortunately, in contrast to Miles' (1957, 1959) original evaluations of his theory, later observations showed the growth rate of wind wave energy in the ocean to be much larger than could be accounted for by Miles' theory (§5.2). However, it is possible that some modification of Miles' original theory may yet yield better agreement between theory and observation (§3.8).

3.3. Nonlinear wave–wave interactions

As gravity waves continue to grow in an active wind field, the average wave slope continues to increase. The average wave slope is a measure of the nonlinearity of the waves, for the larger the slope, the larger the nonlinear terms in the basic equations (ie the surface boundary conditions in equations 3.4 and 3.5). Under continued wave growth the nonlinear terms will ultimately become so important that the linear theories of Phillips and Miles will no longer apply.

Work on the nonlinear wind wave problem expanded rapidly after a discovery made by Phillips (1960). By carrying out the conventional perturbation expansion (the wave slope being the small parameter) about the linear solution of a single sinusoidal wave, Phillips found that under certain conditions unsteady perturbations were possible at the third order in the expansion. The conditions for unsteady perturbations to exist, called resonance conditions, are

$$\begin{aligned} \omega_1 \pm \omega_2 \pm \omega_3 \pm \omega_4 &= 0 \\ \boldsymbol{k}_1 \pm \boldsymbol{k}_2 \pm \boldsymbol{k}_3 \pm \boldsymbol{k}_4 &= 0 \end{aligned} \tag{3.27}$$

where the frequency and wave number pairs $(\omega_i, \boldsymbol{k}_i)$ are those of free (primary) waves which individually satisfy the dispersion relation (3.10). The unsteady perturbation is interpreted to mean that there is a continuous flow of energy among four primary waves when (3.27) is satisfied.

The reason that the unsteady perturbation occurs at third and not second order in the expansion is that the resonance conditions (3.27) cannot be satisfied in general for only three primary waves, which is due to the functional form of the dispersion relation (3.10). The conditions (3.27) can be satisfied by three primary waves in certain trivial cases, but in these cases it turns out that the energy transfer vanishes. For some other types of waves with different dispersion relations a resonant energy transfer can take place among three wave components, such as for capillary waves (McGoldrick 1965), internal gravity waves (Kenyon 1968), Rossby waves (Kenyon 1967), and edge waves (Kenyon 1970).

Phillips' (1960) discovery was extended by the theoretical work of Longuet-Higgins (1962b), Benney (1962), Ball (1964), Bretherton (1964), and Benjamin (1967). The resonant interactions were verified for a few particular cases by laboratory measurements (§5.4). The effect of the resonant interactions on the entire energy spectrum of wind waves was derived by Hasselmann (1962a).

Hasselmann's statistical theory is summarized by (Hasselmann 1968) equations (3.17) and (3.22) as

$$\mathrm{D}F(\boldsymbol{k})/\mathrm{D}t = \int (T_1 F(\boldsymbol{k}')F(\boldsymbol{k}'')F(\boldsymbol{k}-\boldsymbol{k}'-\boldsymbol{k}'') - T_2 F(\boldsymbol{k})F(\boldsymbol{k}')F(\boldsymbol{k}''))\, \mathrm{d}\boldsymbol{k}'\, \mathrm{d}\boldsymbol{k}'' \tag{3.28}$$

where the coupling coefficients T_1 and T_2 contain the resonance conditions (3.27) as well as rather complicated algebraic expressions involving the wave numbers and frequencies of the interacting waves. Equation (3.28) expresses the time rate of change of the energy density at wave number $\boldsymbol{k}$ due to all possible interactions involving wave number $\boldsymbol{k}$ which satisfy the resonance conditions (3.27). The fact that the 'resonant' interactions produce a linear change with time in wave energy is due to incorporation into the theory of the classical response of a harmonic oscillator to stationary random forcing.

The energy transfer given by (3.28) conserves energy over the entire spectrum (there are also other invariants). If at some starting time a fixed amount of energy is present in the form of an initial spectrum, then the total energy will remain constant as the resonant interactions slowly redistribute the energy in wave number space. It can be shown that the energy transfer vanishes identically if the initial energy is distributed equally over all wave numbers (white spectrum), and it can be proved (Hasselmann 1966) that the interactions will tend irreversibly toward this distribution. Early numerical calculations (Hasselmann 1963) showed unexpectedly that in some cases the resonant interactions in a peaked spectrum can initially transfer energy to the spectral maximum. However, more recent calculations (Hasselmann *et al* 1973) show that if the starting energy spectrum has a very narrow peak, the wave–wave interactions will initially tend to broaden the peak.

The characteristic time scale T obtained from (3.28) for a significant energy transfer to take place is of the order of magnitude

$$T \sim T_0 \sigma^{-4} \tag{3.29}$$

where T_0 is a characteristic wave period, and σ is a typical wave slope. In an active wind field wave slopes may be of order 10^{-1} or smaller. Thus the interaction time is large compared with the wave period, showing that the nonlinear interactions are indeed weak (ie perturbation theory is justified). It is significant that the interaction time is of the same order as the development period of the waves or the duration period of storms, which is one way of indicating the possible importance of the resonant interactions for the total energy balance of the waves.

The resonant interactions by themselves cannot serve as the basis for a theory of wave generation (or dissipation), since they do not change the total wave energy. However, it is thought that these resonant interactions may play an important role in wave growth by redistributing to low frequencies the wave energy supplied by the wind to the high-frequency portion of the spectrum (§5). If the wind continues to feed energy into the waves and the resonant interactions cannot redistribute the energy fast enough, then the waves will ultimately break, as discussed further in §3.6.

3.4. Wave propagation

After being generated by a storm, wind waves can propagate for great distances over the surface of the earth, their travel being largely uninterrupted until they break and dissipate upon reaching a coast (almost no wave energy is reflected from the coast in the wind wave frequency range). The observational studies of Barber and Ursell (1948), Munk *et al* (1963) and Snodgrass *et al* (1966) have shown that wind-generated waves outside storm areas can travel as far as halfway around the world with very little attenuation. Molecular viscous dissipation is utterly negligible, and the level of turbulence in the ocean does not appear to affect the waves (Phillips 1959). Also the waves do not appear to be affected by propagating through zones of high wind such as the Trade Wind Belt. Nonlinear interactions do not appear to be important in scattering surface wave energy more than a few storm diameters outside an active generating area (Hasselmann 1963b). Finally, the conversion of surface wave energy into internal gravity wave energy by wave–wave interactions does not seem to be important for the energy balance of surface gravity waves (Kenyon 1968).

In fact, for propagation over large distances the waves seem to obey the linear theory of wave propagation from a limited initial disturbance, which was developed by Cauchy and Poisson in the early 1800s (see Lamb 1932, p394). This theory considers the elevation of the free surface at large distances and a long time after a disturbance, which is limited in space and time, has occurred on an otherwise still ocean. The theory leads quite naturally through the method of stationary phase to the concept of group velocity as the signal velocity of the wave energy. In practice, one can predict the arrival time of waves at the shore using the group velocity by knowing the position and time of the storm. Alternatively, by monitoring the directional arrival of waves at the shore one can deduce the space and time origin of the storm which produced the waves by using linear propagation theory. What one sees at the shore as the result of a single distant storm of short duration is the arrival first of low-frequency waves followed by waves of higher and higher frequency at successively later and later times, which is what one expects because the group velocity decreases with increasing frequency according to (3.10). The rate at which the wave frequency increases with time at the shore is inversely proportional to the distance to the storm.

Munk *et al* (1963) and Snodgrass *et al* (1966) recorded the arrival of waves off the California coast, inferred the origin of the storm, then checked the weather maps to see if the storms were where *the waves* said they should be. In most cases the agreement was satisfactory. However, one slight but systematic discrepancy was noted in both studies with regard to the direction of the storm as determined by the wave arrivals at the shore in California. They found that the direction of the storm as inferred from the measurements was typically a few degrees to the left of the storm position as inferred from the weather maps, and in fact some of the inferred storm positions were on land (Antarctica). This discrepancy cannot be explained by the effect of the earth's rotation (Backus 1962) nor by the earth's oblateness (Snodgrass *et al* 1966). An explanation of the discrepancy was offered by Kenyon (1971) in terms of wave refraction by ocean currents. Most of the waves recorded by Snodgrass *et al* (1966) and Munk *et al* (1963) were generated by storms in the region near Australia, New Zealand and Antarctica and therefore had to pass through the Circumpolar Current, which is a major ocean current flowing eastward around Antarctica.

Wave rays can be bent by ocean currents which vary in space by classical refraction laws (equation 3.18). The amount of bending predicted can be surprisingly large in certain ocean situations. In fact, waves which propagate toward a current with a phase velocity component in the same direction as the current can be totally reflected by the current if a certain critical angle is exceeded. The Gulf Stream is chosen as an example of the strongest of ocean currents to illustrate the effect. The Gulf Stream has a maximum speed of about 2 m s^{-1} and the speed decreases to zero on either side of the maximum, the total width of the current being about 100 km. If the critical angle to be exceeded for total reflection is defined as the angle between the wave ray and the normal to the current, then for the Gulf Stream refraction theory predicts that waves of period 8 s will have a critical angle of 50° and waves of period 16 s will have a critical angle of 60°. Since waves with these periods are well within the wind wave frequency range and these critical angles are not very large, the example shows that major ocean currents could have a significant effect on the propagation of surface gravity waves. Another result of the theory is that waves propagating against a shear current, such as the Gulf Stream, can be trapped inside the current (total internal reflection) provided the initial conditions are right. Unfortunately, as yet there are no ocean observations available to verify these rather remarkable predictions of classical ray theory.

Many of the properties of wave–current refraction can be summarized by the approximate formula for the radius of curvature R of the wave rays in a steady shear current (Kenyon 1971):

$$R = c_g/\zeta \tag{3.30}$$

where ζ is the vertical component of the vorticity of the current, and c_g is the group speed of the waves relative to the current. Equation (3.30) shows that the magnitude of the radius of curvature decreases with increasing wave frequency (decreasing group speed) and with increasing current vorticity. In other words, the refraction effects are largest for high frequencies and large current shears. The sign of the radius of curvature is given by the sign of the vorticity, which determines which way the rays bend for given current shears.

In contrast to the refraction of waves by currents the refraction of waves in shallow water due to variations in water depth is well known and easily visualized near beaches. As waves propagate into water of decreasing depth they reach the nondispersive shallow water limit in which the phase speed is given by $c=(gh)^{1/2}$. Considering a beach that has offshore bottom contours which are straight and parallel, a wave crest which is not initially parallel to the bottom contours will tend to become so due to the depth dependence of the phase speed. The swinging of wave crests parallel to the shore line as waves approach shore is nicely illustrated in aerial photographs (see eg Stoker 1957, p353). Two-dimensional variations in bottom topography act like lenses in focusing the wave rays and produce observable effects. A submerged hill or ridge will focus the wave energy on the beach, whereas a submarine canyon will have the opposite effect. Complicated topographic effects can be easily visualized by a conceptual analogue computer devised by Eckart (1950) which makes use of the ray–particle analogy. Surface gravity waves also can be trapped along coasts due to repeated reflections from the shore line and refraction due to increasing offshore depth, and these are known as edge waves (Eckart 1951). Edge waves have been observed in the ocean (Munk *et al* 1964), but little is known about their cause.

3.5. Wave dissipation

In the life history of ocean wind waves, dissipation is thought to occur mainly by wave breaking and primarily in two periods near the waves' birth in a storm and death on a beach. As mentioned in §3.4, measurements have shown that wind waves can propagate over great distances in the ocean with very little attenuation, indicating that frictional dissipation is negligible for the most part. Dissipation due to wave breaking takes place along with generation in a storm, and waves are ultimately destroyed by breaking on the beach. Wave breaking is probably a strong nonlinear process which is not open to attack by standard mathematical (perturbation) techniques. So far the effects of wave breaking on the energy spectrum under conditions of overall wave growth have been discussed only by means of empirical relationships as mentioned in §3.6.

The weak interaction of short and long gravity waves leads to an attenuation of the long waves according to the theories of Phillips (1963) and Hasselmann (1971a) (see §3.8). These theories are the only available ones which can explain the observed damping of swell in an opposing wind. The generation theories of Phillips (1957) and Miles (1957) do not work in reverse.

Dissipation could also occur as waves propagate into shallow water, before they break, due to interactions with the bottom and the turbulent currents in the bottom boundary layer. Hasselmann and Collins (1968) give a theory for the dissipation of the wave spectrum caused by the interactions of the waves with the turbulent bottom currents. Observations by Hasselmann *et al* (1973) are not in agreement with this theory, however. More recently Long (1973) has looked at the interaction of waves with an irregular bottom and has suggested that the scattering of surface waves by bottom irregularities may be important for the decay of swell near coasts.

Wave dissipation can also occur under certain conditions when waves encounter strong currents (Phillips 1966).

3.6. *Empirical relationships*

3.6.1. The equilibrium range. An empirical relationship was derived by Phillips (1958) which has been useful for describing the high-frequency range of the wind wave spectrum. The relationship predicts that at high frequencies the one-dimensional energy spectrum $F_1(\omega)$ should decrease with increasing frequency as the inverse fifth power of the frequency. Observations have apparently confirmed this relationship over a broad range of frequencies (Phillips 1966). The range of frequencies for which the relation holds is known as the 'equilibrium range'.

The inverse fifth power law for frequency was derived by Phillips (1958) on the basis of dimensional analysis assuming that the only two relevant parameters are the acceleration of gravity and the wave frequency. The physical reasoning behind this is as follows. In an active wind field the high-frequency waves grow so rapidly that their amplitude is ultimately limited by the stability of the water surface (ie when the downward acceleration of the water surface at the wave crest is comparable with the acceleration of gravity). When this occurs the waves have reached equilibrium under the given wind conditions in the sense that they cannot grow any higher without breaking. White capping and breaking waves are, of course, observed in a storm. The weakly nonlinear wave–wave interactions (§3.3) are apparently not very important during periods of rapid growth of the high-frequency waves. Phillips' frequency law is meant to hold for frequencies larger than that of the peak in the energy spectrum and lower than frequencies for which surface tension is important. There is a sizable frequency band which satisfies these conditions.

Although Phillips' concept of an equilibrium spectrum has some observational support, it can be criticized on theoretical grounds. One of the characteristic features of a wind wave spectrum is that the direction of propagation of the waves has a broad angular spread about the mean wind direction. The angular dependence, which may be a function of the wind speed, could be an important parameter in addition to the two parameters upon which Phillips based his spectral law. Also for waves generated by an offshore wind the wave spectrum might be expected to depend on the distance (fetch) of the waves from the shore (§5.2).

3.6.2. Fully developed spectrum. The idealized concept of a 'fully developed spectrum' is an old one and an appealing one, for it leads to useful empirical forms for the entire energy spectrum, including Phillips' equilibrium range. The basic idea is simple. Consider an ocean initially at rest. A steady wind suddenly commences to blow and continues to blow forever over an infinite ocean. Waves of short wave length and high frequency will grow fast and reach equilibrium quickly. Waves of successively lower and lower frequency will then grow to equilibrium, filling in Phillips' equilibrium range. The waves of very low frequency will grow to a certain extent, but since they travel faster than the wind, it is difficult for the wind to transfer energy to them (and in practice they outrun the storm anyway). One expects, and observes, a low-frequency cut-off as well as a high-frequency cut-off with a peak in the energy spectrum at about the frequency for which the wave phase speed equals the wind speed. Therefore one anticipates that after a sufficient length of time the entire energy spectrum will reach steady state.

Observed spectra show a much more rapid cut-off at low frequencies than at high frequencies, and this is modelled by a super-exponential form such as in the fully developed spectrum proposed by Pierson and Moskowitz (1964):

$$F_1(\omega) = \alpha g^2 \omega^{-5} \exp\left[-\beta(\omega_0/\omega)^4\right] \tag{3.31}$$

where $\omega_0 = g/U$, U is the mean wind speed measured 19·5 m above the sea surface, $\alpha = 8{\cdot}1 \times 10^{-3}$, and $\beta = 0{\cdot}74$. A discussion of other empirical spectra can be found in Walden (1963) (see also §5.5).

Empirical spectra such as that of Pierson and Moskowitz (1964) are useful both for summarizing observations and for estimating the importance of various physical effects on the energy spectrum. However, the concept of an equilibrium spectrum can also be challenged on theoretical grounds. For example, why could not the waves of very low frequency continue to grow, be it ever so slowly, as long as the wind blows, thus making a steady-state spectrum impossible? Arguments as to whether a fully developed spectrum is possible or does occur still go on. It has recently been suggested (Hasselmann *et al* 1973) that the existence of a peaked spectrum could be due to a self-stabilizing process associated with wave–wave interactions. However, as yet there is no simple physical explanation for why one might expect the wave–wave interactions to produce a peaked spectrum.

3.6.3. The directional spectrum. Much less is known about the two-dimensional or directional energy spectrum than the one-dimensional (frequency) spectrum. Present knowledge is based on a very few attempts to measure the directional spectrum, and a few empirical forms for the spectrum have been put forward to summarize these observations. So far theoretical guidance for the empirical forms of the directional spectra has been minimal.

Arthur (1949) first noticed that waves leaving a storm have quite a broad (±45°) directional distribution relative to the mean wind direction. The first empirical form for the directional spectrum $F_2(\omega, \theta)$ was proposed by Pierson *et al* (1955):

$$F_2(\omega, \theta) = F_1(\omega) H(\theta) \tag{3.32}$$

where the angular spreading function $H(\theta)$ was independent of frequency and normalized such that

$$\int_{-\pi/2}^{\pi/2} H(\theta)\, \mathrm{d}\theta = 1, \qquad H(\theta) = 0 \quad \text{for } |\theta| > \pi/2$$

($\theta = 0°$ is in the direction of the mean wind). A functional form for $H(\theta)$ based on more recent observations is given in §5.5.

At present there are no theories which predict the shape of the rather broad directional spectrum. The form of the directional spectrum should follow from the nature of the physical processes of wave generation. Of course, the wind itself does not blow constantly in one direction but can have rather large angular variations about the mean direction. The extent to which the directional variability of the wind contributes to the beam width of the waves is not known.

However, if the wind did blow steadily in one direction, one would expect certain qualitative features of the directional spectrum based on the theory of Phillips (1957). The prediction is that the spectral density will be high in two narrow ranges of azimuth centred on the theoretical resonance angles $\pm\cos^{-1}(c/U)$, where, as before, c is the wave phase speed and U the mean wind speed. At the resonance angle the component of wind velocity in the wave direction equals the wave velocity, a condition which is optimum for wave growth by random pressure fluctuations. As the wave phase speed approaches the wind speed (ie for decreasing frequencies), the angular separation between the two narrow beams should decrease to zero. However, these predictions are not easy to observe because normally the resonance effects would be swamped by the exponential growth rate associated with the major wave generation mechanism, the theory of which has still to be found.

3.7. Wave momentum

Most of the theoretical development of ocean wind waves has been cast in terms of the wave energy and particularly in terms of processes affecting the energy spectrum. However, another important property of surface gravity waves is that they also have momentum. The wave momentum is an observable quantity and is associated with a velocity called the Stokes drift velocity (Stokes 1847). The Stokes drift velocity is the fourth characteristic velocity associated with the propagation of surface gravity waves. For waves which are not too steep (slope less than one), this velocity is smaller than the velocity of the fluid particles, which in turn is smaller than the velocity of propagation of wave energy (group velocity), which in general is smaller than the velocity of propagation of the wave phase (phase velocity).

Stokes (1847) discovered that in a second-order perturbation expansion of equations (3.3)–(3.6) the fluid particles do not return exactly to their initial position at the end of a wave period, but are displaced slightly in the direction of wave propagation. This finite-amplitude effect can be expressed as a small, steady velocity which is directed parallel to the phase velocity. The Stokes drift velocity can be defined formally as the difference between the time-averaged Lagrangian (fixed particle) velocity and the time-averaged Eulerian (fixed position) velocity (Longuet-Higgins 1953). This second-order motion decreases much more rapidly with depth than the 'linear' particle motion for wave lengths much less than the water depth.

The Stokes drift velocity for a complete energy spectrum has been given by Kenyon (1969) as

$$\boldsymbol{U}(z)=\frac{1}{\rho}\int F(\boldsymbol{k})\,\frac{\boldsymbol{k}}{\omega}\left(\frac{2k\cosh 2k(z+h)}{\sinh 2kh}\right)\mathrm{d}\boldsymbol{k}. \tag{3.33}$$

The mean momentum per unit surface area $\boldsymbol{M}$ is related to the Stokes drift velocity and the energy spectrum by

$$\boldsymbol{M}\equiv\int_{-h}^{0}\rho\boldsymbol{U}(z)\;\mathrm{d}z=\int F(\boldsymbol{k})\,\frac{\boldsymbol{k}}{\omega}\,\mathrm{d}\boldsymbol{k}$$

by equation (3.33). This relation reduces to

$$\boldsymbol{M}=\frac{E}{c}\left(\frac{\boldsymbol{k}}{k}\right)$$

for a single sinusoidal wave component, showing that the momentum and energy are related through the phase velocity as noted by Starr (1959).

An evaluation of equation (3.33) using the empirical spectrum of Pierson and Moskowitz (equation 3.31) shows that the ratio of the Stokes drift velocity at the surface to the wind velocity 19·5 m above the surface could be between one and three per cent, depending on the exact form of the spectrum used (Kenyon 1969). Although the Stokes drift velocity has been measured in the laboratory (Longuet-Higgins 1960, Chang 1969), as yet there are no measurements which have satisfactorily isolated its existence in the ocean. Ursell (1950) and Hasselmann (1970) have pointed out that a steady Stokes drift velocity cannot exist in the ocean because there is no force which could balance the Coriolis force acting on the translating fluid particles. However, it is still possible that the Stokes drift velocity could exist in the ocean over a time scale which is small compared with the time scale associated with the Coriolis force ($2\pi/2|\mathbf{\Omega}|\sin\theta$, where $\mathbf{\Omega}$ is the angular velocity of the earth and θ is the latitude), but yet large compared with a wave period.

3.8. Recent developments

The field observations of Snyder and Cox (1966) and Barnett and Wilkerson (1967) made it clear that there might be serious difficulties in trying to apply Miles' (1957) generation theory to explain the major growth rate of ocean wind waves (see §5.2). There were two main reactions to this news. One reaction was to point out that the major inadequacy in the Miles' theory was the neglect of any interaction between the waves and the turbulence in the air. Attempts were then made to overcome this inadequacy, but so far these attempts have not been fully evaluated. The second reaction was to try either to come up with an entirely new theory to explain the major growth mechanism, or to re-examine older alternative mechanisms. No new complete theory has yet appeared, although a few new suggestions have been put forward.

One of the new possibilities began with the examination of the interactions between waves of widely different scales. The basic idea was to find out what energy exchanges could take place in a situation in which waves of small wave length were superimposed on waves (or currents) of much larger wave length. Since the wind generates waves of short wave length (ripples) very quickly, one could imagine short waves continuously feeding energy into the longer waves thus providing a generation mechanism for the longer waves.

In a series of papers Longuet-Higgins and Stewart (1960, 1961, 1962, 1964) had already explored the interaction between short gravity waves and long gravity waves (and currents). Working mainly from second-order perturbation analyses for particular cases they developed expressions, in terms of a quantity called 'radiation stress', for the energy and momentum exchanges for situations in which the interactions are weak (small wave slopes). An alternative approach was taken by Whitham (1967) using an averaged Lagrangian method (see also Bretherton and Garrett 1968, Bisshopp 1969, Whitham 1962). An asymmetry in the distribution of wave slope arises from the interaction such that the short waves become shorter and steeper at the crests of the long waves and longer and flatter at the troughs of the long waves. Unfortunately the weak interaction does not lead to energy transfer rates for the long waves which are comparable with those observed.

By using the radiation stress concept, Phillips (1963) predicted that the breaking of short gravity waves on the crests of long gravity waves would extract energy from the long waves and cause their energy to attenuate at a linear rate with time. On the other hand, Longuet-Higgins (1969a) came to the opposite conclusion that the breaking of short waves on the crests of long waves would cause the long waves to grow. However, Hasselmann (1971a) showed by considering both mass and energy transfer that the interaction of short and long gravity waves will always lead to a slow attenuation of the long waves. It appears, therefore, that the interaction between short and long gravity waves cannot explain the major growth rate of the long waves. On the other hand, the theories of Phillips (1963) and Hasselmann (1971a) are the only ones available which might be able to explain the observed decay of ocean swell under an opposing wind.

Another approach was taken recently by Stewart (1967) and Longuet-Higgins (1969b) to try to assess the importance of the tangential wind stress on the generation of gravity waves. Earlier Sverdrup and Munk (1947) were able to account for the order of magnitude of observed wave heights by assuming that all the energy communicated to the water by the tangential stress appeared in waves and none in currents. In later work (eg Phillips 1957) it seemed intuitively clear that tangential stresses could not be very effective in setting up the irrotational water motion of gravity waves. The basic idea explored by Stewart (1967) and Longuet-Higgins (1969b) is that the tangential stress of the wind acts unequally over the wave surface and causes vertical velocities due to convergences within a thin boundary layer in the water close to the surface. These vertical velocities then have the right phase relation to feed energy into the irrotational wave. The theory gives a linear time rate of increase in the wave amplitude which is proportional to the variable component of the wind stress divided by the product of the phase speed of the wave and water density. For normal ocean conditions the rate of increase of wave amplitude appears to be too small to account for the major input of energy to the waves, but nevertheless the tangential stress could make some contribution to the growth of wind waves.

A relatively large effort has recently been directed toward trying to modify Miles' (1957) theory to include interactions between the waves and the air turbulence. Miles (1967) was one of the first to attempt such improvements to his own theory and suggested that further progress would require some *ad hoc* hypothesis for the specification of the wave-induced turbulent Reynolds stresses. This suggestion was taken up by Davis (1969, 1970, 1972), who investigated the nature of the turbulent flow over a wavy boundary and how the flow could do work on the waves. Hasselmann (1967, 1968) had already included the interaction between waves and turbulence as part of his general weak interaction scheme. A more recent attempt to tackle the wave–turbulence interaction has been made by Manton (1972).

The evaluation of the wave–turbulence interaction is still going on, and it is not appropriate to critically review the work on this programme at this time. Numerical calculations of Hasselmann's (1967, 1968) interaction coefficients are still needed. The results of recent calculations of the wave–turbulence interactions by Long (1971) and Townsend (1972) are not in complete agreement, and the differences between them need to be resolved. One of the future hopes is that the interaction between turbulence and waves will lead to the explanation for the major growth rate of ocean wind waves. Although the evaluation of this difficult problem is the first priority, it is also likely to take some time. Therefore it is not out of place to encourage the development of some new theoretical ideas (§7.1). The wave–turbulence interaction may not be the only physical feature of possible importance for wave growth which has been left out of Miles' (1957) theory.

4. Techniques for measuring the wave field

Understanding the physical processes that effect wind waves requires adequate methods of quantitatively observing the wave field. The purpose of this section, then, is to describe modern observational methods, their advantages and drawbacks.

An observation and qualification are in order before we proceed. As we saw in

§2.2, there were few reliable methods of observing the sea surface prior to the early 1950s. This section indicates a virtual renaissance in our abilities to quantify the wave field. The qualification we wish to apply is the fact that only wave measuring methods are discussed. No attempt has been made to catalogue the great advances made in devices for sensing the turbulent atmospheric flow field—the 'other' part of the wave generation problem.

4.1. Measurement of the one-dimensional spectrum

4.1.1. Methods using sea surface elevation. Many different techniques have been developed to obtain the time history of sea surface elevation at a single point in space, the quantity required to compute the one-dimensional spectrum $F_1(\omega)$. Perhaps the most common method that is used is to insert a pole or rigid staff into the water and use it as one component of an electric circuit. This makes it possible to transform variations in sea level into fluctuating electrical currents. One such device is called the resistance wire wave staff. It is basically a length of insulated cable that has been wound with a fine, bare wire. The wire represents a resistance which can be shorted out by sea water. The result is a variable resistance inversely proportional to the depth of probe immersion.

Another type of staff in common use is the capacitance staff. This is simply insulated wire sealed at one end and inserted into the sea water (a conducting fluid). The insulator on the wire acts as a dielectric. The central conductor of the wire and the sea water form the two plates. Changes in sea surface elevation then lead to a variable capacitance that is directly proportional to the probe immersion. An excellent account of capacitance probes and the problems one can get into by using them has been given McGoldrick (1971). As one might expect there is also an inductance staff that works on much the same principle as the two described above.

The basic problem with all of these devices, however, is that they require a fixed platform to which they must be attached. This implies that they can only be used in shallow water or in the laboratory. Attempts have been made to use wave staffs in deep water, but the effort is difficult logistically.

Newer techniques for obtaining time histories of the sea surface profile involve the use of remote sensing devices, eg a radar, laser or acoustic beam. An infrared profiler has been mounted on a ship and used by DeLeonibus *et al* (1973) to obtain reasonably good estimates of the one-dimensional wave spectrum. However, since the ship is generally moving relative to the actual wave field, the induced Doppler problems make this a difficult technique to use. The method does, nevertheless, allow a direct measurement of the time history of sea surface elevation in the deep ocean.

More exotic techniques of measuring the same quantity involve the use of a laser or radar. Both devices have been mounted in aircraft (Barnett and Wilkerson 1967, Schule *et al* 1971) thereby providing the opportunity for measurement anywhere in the ocean. Using the time history of sea surface elevation that is obtained from a rapidly moving aircraft, one can compute the apparent spectrum of wave encounter F_{app}, which is related to the two-dimensional spectrum by

$$F_{\mathrm{app}}(\sigma) \simeq \int F_2(\omega, \theta) \left| \frac{\partial \theta}{\partial \omega} \right| \mathrm{d}\sigma$$

where σ is the apparent frequency. The relation between real frequency ω and apparent frequency σ is

$$\sigma = \omega - (\omega^2 V/g) \cos \psi$$

where V is the speed of the aircraft and ψ is the angle between the direction of aircraft motion and wave propagation. Since the speed of the aircraft is known, the transformation indicated above should be straightforward. The catch is, however, that one must know the directional properties of the wave field or be able to estimate them in order to carry through the transformation. It turns out that the estimate of the one-dimensional spectrum obtained from the aircraft is not particularly sensitive to the directional properties of the wave field. The spectral estimates obtained from aircraft compare quite well with more conventional ground-based observations.

4.1.2. Methods based on other properties of the wave field. There is a second class of devices that can obtain an *indirect* estimate of the time history of sea surface elevation. We will discuss two devices in this class; namely, pressure transducers and accelerometers. The former was discussed briefly in §2.2. It has been brought to a high state of perfection by Frank Snodgrass (eg Snodgrass *et al* 1966). The device is basically a vibrating wire attached to a rigid diaphragm. The diaphragm is exposed to the external sea water. Changes in pressure induced by wave motion then act differentially upon this diaphragm, changing the tension in the wire slightly. The resulting frequency changes are used to modulate a voltage which is then transmitted to shore via cable for recording. The pressure fluctuations at depth $p(z, t)$ are transformed into estimates of sea level fluctuation $\xi(t)$ by the relation from linear theory (§3.1 and Kinsman 1965)

$$p(z, t)=\rho g\xi(t)\,\frac{\cosh k(z+h)}{\cosh kh}. \qquad (4.1)$$

In its early history the 'vibratron', as this device is called, was rather sensitive to changes in sea water temperature. These difficulties have largely been taken care of in the intervening years by Snodgrass and his co-workers. Another of the major difficulties with the vibratron, however, has been that it must be linked to shore by a cable that leads through the inhospitable surf zone. This difficulty has been largely cleared up recently with a pressure transducer/telemetering buoy combination (Brown and Gaul 1967, Barnett 1971). The transducer works on much the same principle as described above, only now the frequency-modulated voltage is used to modulate radio transmission which is received on shore and converted eventually to estimates of sea surface elevation.

Pressure transducers have generally been used to measure waves in shallow water where the sensor could be fixed to the sea floor. In this circumstance the user must beware, for the conversion from pressure to sea surface elevation requires the application of some theoretical transfer function (equation 4.1). Linear theory is most often used. Unfortunately, reasonably large waves in relatively shallow water are not linear phenomena.

Accelerometer devices have also found some application as wave measuring devices since the vertical acceleration in a wave field, $\ddot{\xi}(z, t)$, is related to the surface elevation by linear theory such that

$$\ddot{\xi}(z, t)=\xi(t)\,\omega^2\,\frac{\sinh k(z+h)}{\sinh kh}. \qquad (4.2)$$

The most recent, and perhaps most exciting, advance in using an accelerometer as a wave measuring device has been made by a firm in Holland. Their buoy, called the 'Wave Rider', is an arrangement of an accelerometer, a radio telemetry link and a moored buoy. The buoy may be easily deployed in any place the scientist might wish. The key to their system is a clever mechanical arrangement that isolates the acceleration sensing device from the outer shell of the buoy and from lateral accelerations. The only vertical motions remaining to be sensed by the accelerometer are those due to waves.

4.1.3. Intercomparisons. With all of the different techniques of measuring the one-dimensional spectrum that have been described above, the inquisitive scientist might well ask: 'Do they give the same information?' A more sophisticated question would be: 'Since different techniques may measure different properties, can something new about the wave field be learned from the comparisons (assuming the instruments work properly)?' The ideal circumstance would be to put all of the instruments together in one spot and then compare their estimates of the one-dimensional wave spectrum.

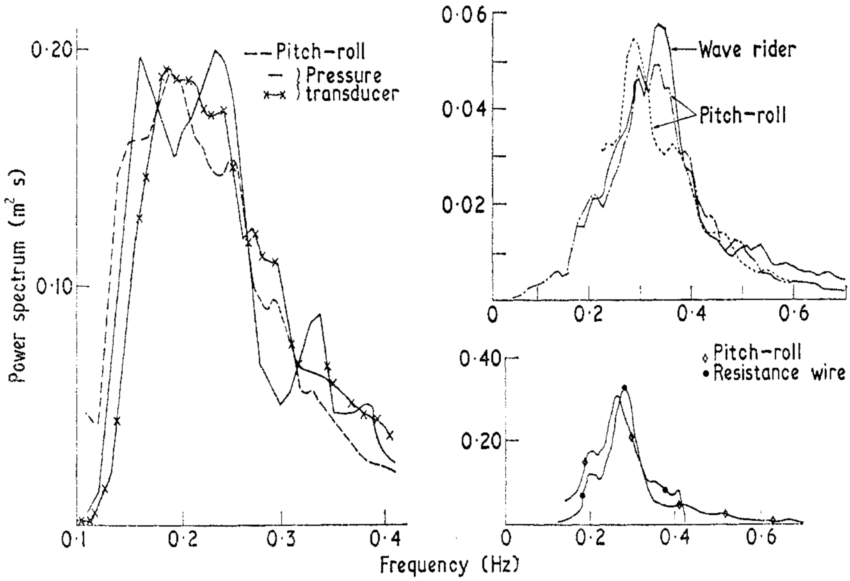

Figure 1. Intercomparison measurements of frequency wave spectra from JONSWAP. The instruments compared are the resistance wire wave staff, the pitch and roll buoy, the wave rider buoy, and the subsurface pressure transducer.

Just such an experiment has been carried out during the Joint North Sea Wave Project (JONSWAP, Hasselmann *et al* 1973). Simultaneous measurements of the wave field were made with pressure transducers, wave staffs and acceleration sensing systems.

The intercomparisons are illustrated in figure 1. It will be seen that over the main range of frequencies where significant energy occurs, the instruments compare well. At the lowest frequencies, the acceleration sensing devices experience the familiar and not unexpected 'red catastrophy'. This results from the double integration of the slow DC drift of the signal (equation 4.2). At high frequencies the pressure transducers experience a similar difficulty. This is due to the fact that they were far enough beneath the surface that they could not sense the higher-frequency waves (equation 4.1). The small amount of system noise that was present in conjunction with large correction factors resulted in erroneous spectral values.

4.2. *Methods of estimating the two-dimensional spectrum*

The two-dimensional spectrum ($F(\boldsymbol{k})$, $F_2(\omega, \theta)$, §3.1.2) is to first order a complete description of the wind wave fields. Its measure is thus of prime importance to research in wind waves. Unfortunately it is not a simple task to obtain either F or F_2.

One method of obtaining estimates of the two-dimensional spectrum is by using the British National Institute of Oceanography 'pitch and roll' buoy. This doughnut-shaped buoy, about 1 m in diameter, carries instrumentation capable of measuring its vertical acceleration, its pitch, roll and heading relative to geographic north. These four time histories are transmitted from the buoy to a nearby ship by means of hard line connection. It has been shown by Longuet-Higgins *et al* (1963) that the spectra and cross spectra of these time series can be related to the Fourier expansion of F_2,

$$a_n + ib_n = (1/\pi) \int_0^{2\pi} F_2(\omega, \theta) \exp(in\theta)\, d\theta \quad n = 0, 1, 2$$

such that the Fourier coefficients (a_n, b_n) are explicit functions of the spectra.

The buoy does have some drawbacks, however. While it is useful in obtaining mean wave direction, its estimate of mean wave spread (ie the second directional moment of the two-dimensional spectrum) is hampered by a lack of resolution, for only five Fourier coefficients are available to define the directional spectrum. A narrow 'beam' of wave energy is not well defined by so few components†. Also the buoy requires a nearby ship and rather delicate handling of that ship so the hard line connection between the two does not interfere with the buoy's pitch and roll. Finally, the buoys have the unfortunate habit of capsizing when the wind gets much over 25 knots (12 m s^{-1}).

A second way of estimating directional properties of the wave field is through the use of an array of sensors. This work was first pioneered by Barber (1954), who considered the array from the viewpoint of linear antenna theory. His work was expanded upon by Munk *et al* (1963). The array is used to provide simultaneous time histories of sea surface elevations from a number of different spatial locations. These time series are then cross-correlated to give estimates of the covariance function in both time and space. The Fourier transformation of this covariance function provides an estimate of the two-dimensional spectrum. This measurement technique has been used almost exclusively in shallow continental margins of the world's oceans and also wave tanks. The beauty of the method is that one can design the antenna (array) to be sensitive to particular components of the two-dimensional spectrum (cf Barber 1963).

Arrays of sensors are not without problems, however. Shortcomings include potential ambiguities in wave direction, resolution windows that are rather narrow in both frequency and direction space, and the requirement to have all of the sensors working simultaneously if one is to avoid serious degradation of the direction finding abilities of the array. It can also be shown that the response of the array to various incoming wave components is dependent not only upon the direction of approach of those components but also on the nature of the spectral beam width.

The performance of an array of wave sensors and the pitch and roll buoy have recently been compared. The occasion was the JONSWAP (see §5) during which a pitch and roll buoy was placed within an array of six wave staffs. The resulting comparisons of mean direction and RMS directional spread are shown in figure 2. The mean directions agree quite well. The variations in estimates of mean-square spread are almost

† The 'cloverleaf' buoy (Cartwright and Smith 1964) is a development of the 'pitch and roll' buoy that allows several more harmonics to be determined and hence theoretically allows better resolution.

precisely those expected from theoretical calculations. It is thus possible to correct spread estimates obtained by the array and in such a manner that they would be compatible with the estimates of the pitch and roll buoy.

A number of other methods have also been tried for obtaining estimates of the two-dimensional spectrum. Some appear quite promising, but none is in common use at this time. One successful technique was the SWOP experiment in which two aircraft flying at the same altitude obtained a stereographic photo of the sea surface. The laborious job of picking off sea surface elevations on a spatial grid in $\boldsymbol{x}$ space was

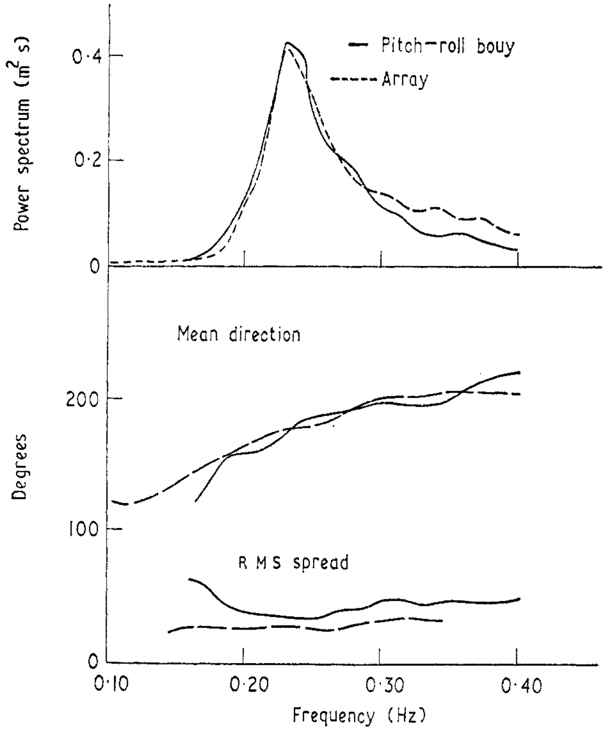

Figure 2. Intercomparison of frequency spectrum and directional parameters from a pitch and roll buoy and a linear array. Data taken during the JONSWAP experiment.

accomplished by hand. The Fourier transform of this two-dimensional field of relative wave height is essentially an estimate of the directional wave number spectrum $F(\boldsymbol{k})$. Considering the amount of work involved, this technique will probably not see widespread use. An excellent account of the experiment and the pitfalls of attempting to take stereographic photos of the sea surface has been given by Cote *et al* (1960).

Another technique of estimating $F_2(\omega, \theta)$ has involved the use of single photographs of the sea surface. The method was pioneered by Cox and Munk (1954), who used photos of the sun's glitter pattern to estimate fundamental properties of the surface wave spectrum. Stilwell (1969) has recently carried the analysis of such photos to a high level of sophistication. Under the proper light conditions the slopes of the various component waves have different effective reflectances. Therefore the negative of the sea surface photograph is composed of a number of light and dark areas related to the wave slopes. Using the optical Fourier transform methods, Stilwell was able to obtain from a photo the directional properties of the wave field. With extreme care, he reports being able to estimate the magnitude of the two-dimensional spectrum, eg the amount of energy that is on the sea surface.

Unfortunately, present optical techniques all have the drawback that they must be done under almost ideal atmospheric conditions (clear sky, no white caps) or in a laboratory. The methods, while promising, may be some time into use on an operational basis.

Perhaps the most exciting method of obtaining high-quality estimates of the two-dimensional wave spectrum is through the use of radar or microwave scattering from the wind waves. Crombie (1955) found experimentally that Bragg scattering is a primary mechanism involved in radar scattering by wind-generated waves. A large amount of theoretical work in the last several years by Wright and Keller (1971), Hasselmann (1971a, b), Barrick (1972) and a group at the Scripps Institution, eg Stewart (1971), has made it clear that the theoretical understanding of backscatter is sufficiently advanced to allow this electromagnetic technique to be used to measure $F_2(\omega, \theta)$. The theory essentially considers the interactions of the radio wave (i) as it scatters from an ocean wave (o) producing a backscattered wave (s). The relation between the wave numbers is

$$\boldsymbol{k}_{\mathrm{s}} = \boldsymbol{k}_{\mathrm{i}} \pm \boldsymbol{k}_{\mathrm{o}}$$

with the side condition being simply the application of the Bragg law

$$\boldsymbol{k}_{\mathrm{o}} = \pm 2\boldsymbol{k}_{\mathrm{i}}.$$

The second-order interaction involves the incident radio wave and *two* ocean waves interacting to produce a backscattered radio wave. Similar relations presumably exist at higher order.

Using these relations and others, Munk and Nierenberg (1972) have determined that the directional spectrum can be estimated with a single radar frequency looking over a relatively large patch of ocean. Considering the vector interaction rules stated above, it becomes clear that each small section of ocean in the large patch will provide information about one particular $\boldsymbol{k}_{\mathrm{o}}$. A composite picture of $F(\boldsymbol{k})$ in a region can thus be constructed.

4.3. *Joint air–sea measurements*

As we shall see in the following section, there is a distinct need for simultaneous measurements of wave field and atmospheric field parameters in the region just above the wave surface. This clearly cannot be accomplished with fixed instruments for they will be submerged half the time by the waves themselves. With this in mind, then, there is a definite requirement for a platform that follows the wave surface enabling scientists to measure certain parameters of both fields simultaneously.

The first effort in this direction was apparently made by Dobson (1971a,b). Working in a very shallow area off Vancouver Island, Dobson constructed a small surface following device that enabled simultaneous measurements of atmospheric pressure and wave height. There were some instrumental difficulties, however, particularly when a white cap inundated the surface float. A somewhat similar system has been developed in the laboratory by Shemdin and Hsu (1967). Their system involves a wave measuring device hooked into a servosystem which in turn moves a pressure sensing device up or down, attempting to keep it a constant distance above the sea surface. Improved versions of both of these are presently being developed at Johns

Hopkins University (M Peep 1971, private communication). The next breakthrough in obtaining information on the interaction of the atmospheric and the ocean wave field will almost certainly come from devices of this nature.

5. Observations and their relation to theory

5.1. Introduction

In this section we examine the data available to test the theories discussed in §3. A distinction will be made between observations taken in the laboratory and those obtained in the natural environment. The distinction is made since different ranges of c/U and turbulent intensity are generally encountered in the two environments. As we saw in §3, different generation and dissipation mechanisms may act selectively depending on the environment. Also it may be noted that the laboratory conditions are generally more controllable than standard geophysical experiments, and therefore it is possible to address, more precisely, discrete physical problems.

The amount of sophisticated instrumentation that has been brought to bear on the wind wave problem has increased tremendously in the last decade. It is perhaps for this reason that most of the significant experimental results have surfaced in the last five to seven years. This section will concern itself with the highlights of the programmes that gave rise to these results.

5.2. Observations of wave generation

5.2.1. Oceanic observations. There are few observations of wave generation in nature, and even fewer observations of wave/atmosphere fields during wave generation. In this section we divide the discussion into three segments: observations of wave growth; energy input to the wave field; and modification of air flow by the waves.

Wave growth. The first set of significant measurements of wave growth under the action of a known wind were reported by Snyder (1965) and later Snyder and Cox (1966). In this experiment a directional wave recorder (accelerometer) was towed at constant speed downwind from Eleuthera Island, Bahamas. This low-lying island essentially blocked all waves from affecting the measurement area except those waves being generated by wind blowing off the leeward coast. A singularity in the spectral transformation, relating the true frequency and direction of a wave component to its apparent frequency and direction observed from the towed platform, allowed Snyder and Cox to estimate the spectral intensity of that component having a group velocity equal to the towing velocity (about 3 m s^{-1}).

They obtained spectral growth curves for a single wave component ($\omega_0 = 1{\cdot}9$ rad s^{-1}) over a range of wind speeds between 4 and 10 m s^{-1}. From these data they quantitatively evaluated the relative importance of the wave growth theories of Phillips (1957) and Miles (1957). This was thought possible since only the initial wave growth was examined and thus nonlinear effects were assumed to be small. It was also assumed that the two mechanisms were the only ones operative. The results were thus expressed as two parameters 'α' and 'β', where from §3.2

$$\partial F_1(\omega_0)/\partial t = (\alpha + \beta F_1)_{\omega_0}.$$

The linear part of the growth (α) was shown to agree reasonably well with the theory of Phillips (1957), in spite of a lack of knowledge regarding the atmospheric turbulence field and the uncertainties in evaluation of the theory. However, the major portion of the growth (β) was shown to be an order of magnitude higher than that predicted by Miles' theory alone. Assuming the observed growth rates were due entirely to a linear input from the atmosphere, Snyder and Cox inferred a momentum transfer from the atmosphere to the wave field which was *several times greater* than the known total momentum loss from the atmosphere to the ocean.

A second and complementary set of measurements of wave growth came almost immediately from Barnett and Wilkerson (1967)†. These authors observed the growth of the *entire* spectrum off the east coast of the United States during conditions of strong (15 m s^{-1}), steady offshore wind. The airborne radar (laser) wave profiler (§4.1.1) was the instrument used to obtain the basic data. Only one storm was sampled on one upwind and one downwind flight path. However, the wind blew long enough to achieve a steady fetch-limited condition, ie $\partial F/\partial t = 0$, under which equation (3.17) reduces to (assuming no refraction)

$$V.\nabla_x F = \sum_i S_i.$$

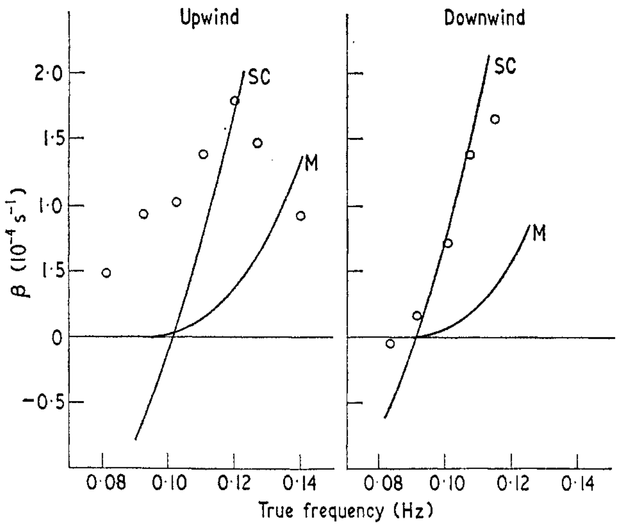

Figure 3. Exponential growth parameter data. Measurement versus theory. Curves marked 'M' are predicted by Miles (1957), while the curves SC are from the empirical relations suggested by Snyder and Cox (1966). The data points are from Barnett and Wilkerson (1967).

The source function was again linearized ($S = \alpha + \beta F_1$), and the values of α and β were compared with theory and the results of Snyder and Cox. The data generally confirmed the results of the latter authors: Phillips' (1957) theory seemed reasonable with several provisos, but the Miles' (1957) theory did not agree with the observations (figure 3).

Barnett and Wilkerson also discovered a previously unsuspected phenomenon which they termed the 'overshoot effect'. It had earlier been believed that a wave component grew first linearly with time (distance) then exponentially until it gradually

† This same type of experiment was later repeated by Schule *et al* (1971) and Ross *et al* (1970) with basically the same results.

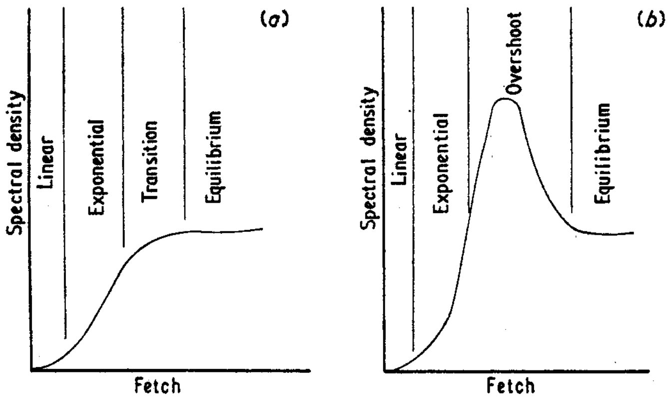

Figure 4. Schematic growth curves for selected frequency component of the wave spectrum. (*a*) Conventional growth curve based on early theories of wave generation, and (*b*) the observed growth curve clearly demonstrating the overshoot effect (after Barnett and Sutherland 1968).

reached an equilibrium value (figure 4*a*). However, the authors found that actual wave growth experienced a history shown in figure 4(*b*). The energy associated with the peak of the spectrum was found to be consistently higher by factors between 1·2 and 2 than the asymptotic, equilibrium level approached by the same frequency at larger fetches. This feature of wave growth indicated that nonlinear mechanisms might be active in the generation process.

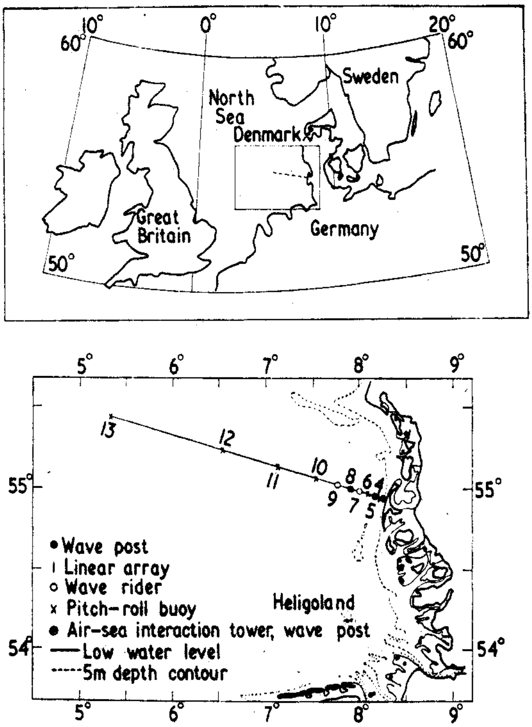

Figure 5. Location map for the Joint North Sea Wave Project (JONSWAP).

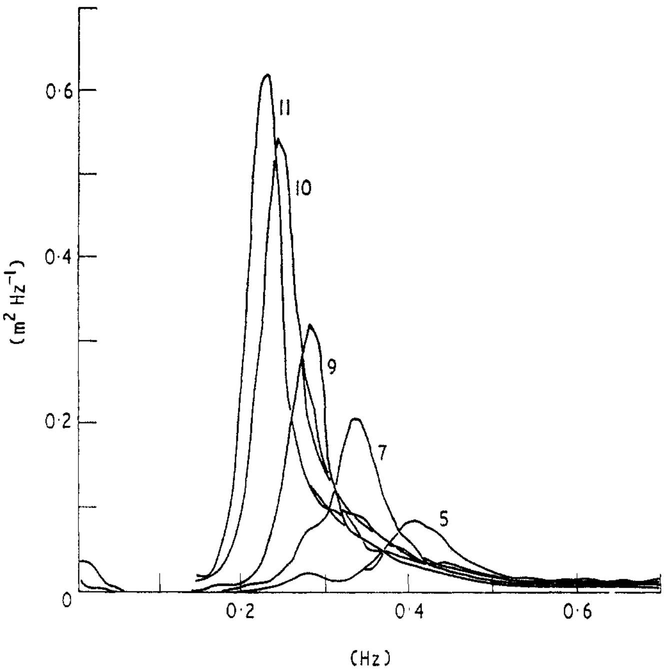

Figure 6. Evolution of the wave spectrum with fetch for offshore wind. Numbers refer to JONSWAP stations at which measurements were made (see figure 5).

The most complete set of measurements of wave growth thus far obtained in nature resulted from the JONSWAP experiment (Hasselmann *et al* 1973). Scientists from four countries cooperated to operate an array of up to 13 wave stations quasi-continuously for 10 weeks during 1968 and 1969. The array extended 160 km to the west of the Island of Sylt in the North Sea (figure 5). Under conditions of offshore (east) winds, the array provided excellent one- and two-dimensional fetch-limited spectra with which to investigate wave growth. The data from the best cases (well-defined, quasi-stationary offshore wind conditions) gave over 300 spectra from which the following results were derived.

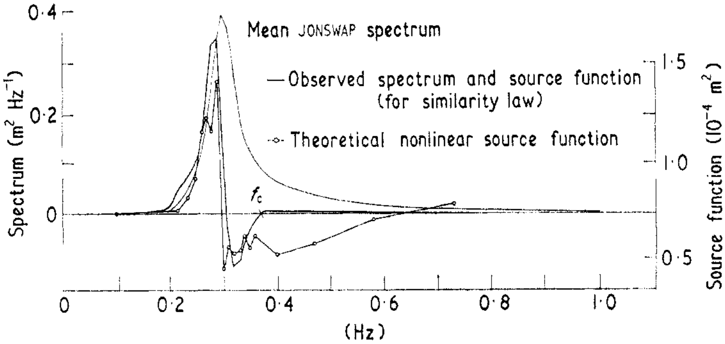

Figure 7. Mean JONSWAP similarity spectrum. Source function S computed from theory and similarity laws versus the theoretical prediction of nonlinear source function (after Hasselmann *et al* 1973).

(i) The source functions of growing wave spectra exhibit a characteristic plus/minus signature associated with the shift of the sharp spectral peak towards lower frequencies (figures 6 and 7). This two-lobed distribution is predicted quantitatively by the nonlinear energy transfer due to resonant wave–wave interactions (second-order Bragg Scattering, figure 7). This result seems to confirm the theory of Hasselmann (1962) and indicates that the wave–wave interactions play a major role in wave growth. Thus the evolution of the sharp spectral peak is found to be a self-stabilized feature of this process.

(ii) At short fetches the energy balance of the main part of the spectrum is governed by undetermined energy inputs from the atmosphere to the central part of the spectrum (S_{in}), the nonlinear transfer from this region to higher and lower frequencies (S_{nl}), and advection (figure 8). Dissipative processes (S_{ds}) apparently play a minor role in the

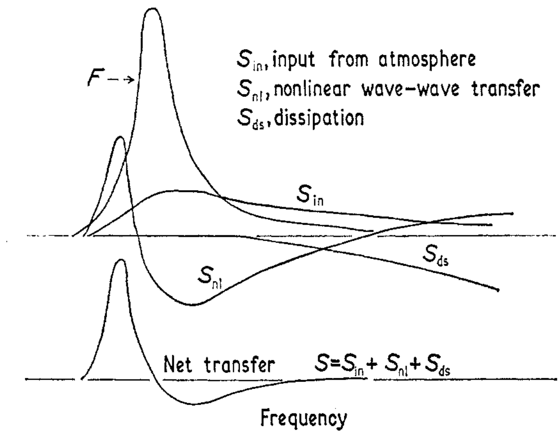

Figure 8. Schematic energy balance for the case of negligible dissipation in the main part of the spectrum from the JONSWAP.

region of maximum spectral energy. The nonlinear process accounts for the major portion of growth at the low-frequency spectral face via a redistribution of energy originally put in by the wind to the mid range of the spectrum. At *longer fetches* the situation is not as clear because of the unknown dissipation in the low-frequency part of the spectrum. In general, the data suggest that the scales of the spectrum adjust such that the wave–wave interactions continually balance the energy input from the wind.

(iii) For small fetches, approximately $80 \pm 20\%$ of the momentum transferred across the air–sea interface goes into the wave field. This general result is in agreement with Dobson's (1971a,b) direct measurements (below), although the frequency dependence of the input was not determined by JONSWAP. But the wave field acts as a 'sieve', for about 80–90% of the wave-induced momentum flux presumably passes directly into currents, perhaps via the nonlinear transfer to higher frequencies and subsequent dissipation; the rest remains in the wave field and is radiated away.

These estimates, and the importance of wave–wave interactions in wave generation partially remove the paradox introduced by Snyder and Cox (§7.2). It can now be seen that much of the observed growth could be accounted for by a *redistribution* of energy within the wave field rather than excessive energy inputs from the atmosphere.

At *large fetches* the assumption of zero dissipation in the lower-frequency regions yields a minimal atmospheric momentum flux to the waves of order 20% of that imparted to the ocean. However, up to 100% is possible if dissipation is important.

Energy input to waves. The above set of experiments gives rather clear estimates of the rates at which the energy spectrum grows. However, there are few estimates of the amount of energy actually being put into the wave field by the atmosphere. It is this latter information which is required to fully test the wind wave generation theories of §3.

The rate at which the atmosphere does work on the wavy sea surface is

$$W = \langle p \partial \xi / \partial t \rangle$$

where p is the atmospheric pressure and ξ is the distortion of the sea surface from its mean position; a consistent phase difference between the two signals results in net work on the wave field. The goal, then, is to obtain estimates of both p and ξ as functions of time (§4.4). In a rough wind sea this is a formidable task.

Early attempts at the difficult task of obtaining the necessary data (eg Longuet-Higgins *et al* 1963) were partly successful and stimulated future work. However, it was nearly ten years later that the first convincing estimates of $\langle p \partial \xi / \partial t \rangle$ were obtained by Dobson (1971a,b). He mounted a sensitive pressure transducer on a small buoy, constrained to move vertically up and down a wave staff. Considerable care was taken to see that this 'Lagrangian' measuring system did not physically interfere with the atmospheric flow field thereby introducing extraneous pressure signals. Apparently good data were obtained for six different situations with wind speeds ranging between 3 and 8 m s^{-1}.

The experiment provided measurements of the phase relation between p and ξ at the sea surface over a wide range of c/U. The measured phase shifts from 180° greatly exceeded those predicted by Miles' (1959) theory. The discrepancy is displayed in figure 9 in terms of the parameter β. Also shown for comparison is a curve summarizing the results of Snyder and Cox. This introduces an apparent paradox into the wave generation problem. Snyder and Cox's estimates describe the rates of wave growth, and Dobson's data indicate that all of this growth is accounted for by direct atmospheric inputs. But what of the JONSWAP data that suggest the major growth is due to wave–wave interactions redistributing to lower frequencies energy input to the spectrum at 'high' frequencies? A clear answer to this dichotomy does not appear possible with present data.

Dobson's data indicated that most of the atmospheric energy input to the wave field occurred for frequencies at or slightly above the spectral peak. This observation is consistent with the conclusion of JONSWAP *inferred* from wave growth data alone, although as mentioned above it is not consistent with the idea of principal generation due to wave–wave interactions. In addition, Dobson also found that approximately 80% of the available atmospheric momentum flux to the sea was going to the wave field, a result again supported by JONSWAP for short fetches. But is the momentum used in direct generation of the waves or input to relatively high frequencies for subsequent redistribution to lower frequencies?

In a complementary study, Elliott (1972) made measurements of pressure and wave height in which the pressure sensor was fixed to the wave staff at a constant elevation above the *mean* sea surface. The data showed phase differences between pressure and waves during active generation of about 135° as opposed to the 180° value expected from potential theory alone. The phase relation extended to a height of at least one wave length for each frequency considered.

There is an important discrepancy between the Dobson and Elliott experiments which is yet to be resolved. Dobson finds active generation occurring for waves travelling at, or slightly slower than, the local wind speed. Elliott found that active wave generation appears to occur only when the wind is greater than about *twice* the wave phase speed. This disagreement, viewed in the light of the paradox described above, dictates the need for additional field observations of the rate at which the atmosphere inputs momentum to the wave field.

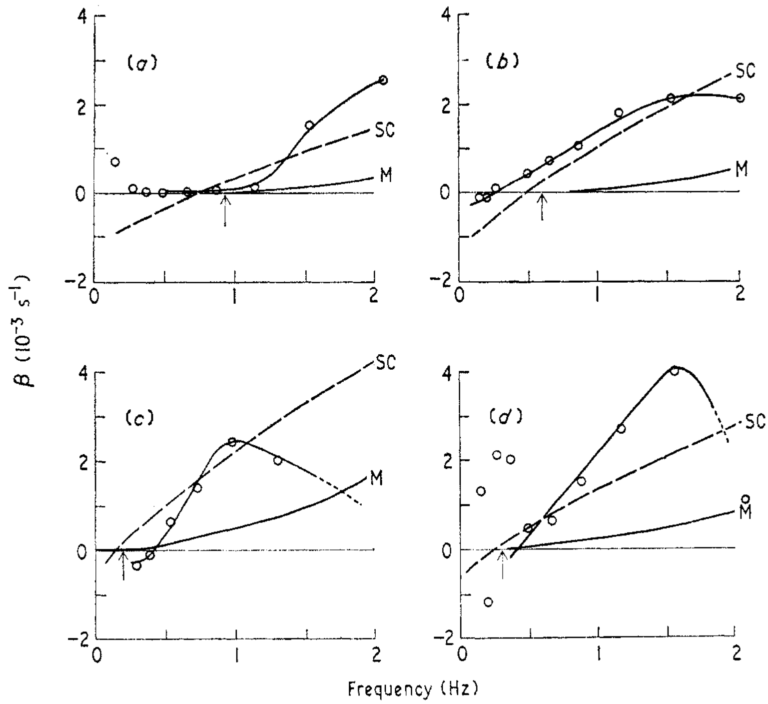

Figure 9. Exponential growth parameter data from the *direct* measurements of Dobson (open circles) compared with the observation of Snyder and Cox (SC) and the theory of Miles (M).

Air flow over a wavy surface. Recent theories of wave generation (§3.8) involve interactions between the mean air flow, the air turbulence and the wave field. Their verification will require extensive measurements of the three-dimensional spectrum of atmospheric turbulence at different levels above the rough sea surface. The technology to do this is being developed (eg Shemdin 1969).

However, initial measurements of the turbulence field suggest that theoretical verification may not come from field measurements. A group working under Mollo-Christensen at MIT has obtained data which indicate that the field of turbulent atmosphere flow is characterized by intermittency which itself is related to the wave field (Dorman 1971, Ruggles 1970, Mollo-Christensen 1970, Merceret 1972). Thus the act of wave generation may be intermittent to first order, and the generation of waves and turbulence is inseparable. These latter statements generally apply to small-scale features of both the atmospheric and wave fields. Hence one's view of wave generation may be critically dependent on the type of time–space averaging techniques employed in data analysis, for the averaging of nonlinear processes poses serious difficulties. On the brighter side, one can hope that the overall influence the processes the MIT group are studying is not of first-order importance to the energy balance of the spectrum at mid to low ranges of c/U, where the maximum energy is found.

5.2.2. Laboratory observations of wave growth. Laboratory studies of wave growth during the last fifteen years could almost justify a review paper by themselves. However, in the following section we attempt to state succinctly the sense of the results in three major areas, including several references and pointing out areas of potential disagreement. The three areas are: estimates of wave growth; attempts to validate theories of wave generation; and studies of the nature of air flow over a free water surface.

Wave growth. Well-defined measurements of spectral development have been obtained by Sutherland (1968), Mitsuyasu (1968) and Hidy and Plate (1966), among others. The first two authors obtained estimates of sequential growth that are in satisfying qualitative agreement with field observations discussed in the previous section. The major portion of the growth is exponential in nature, and the spectrum is limited at high frequency by an equilibrium form. The overshoot effect also is a prominent feature of the data (figure 10). The presence of a nonlinear generating mechanism is

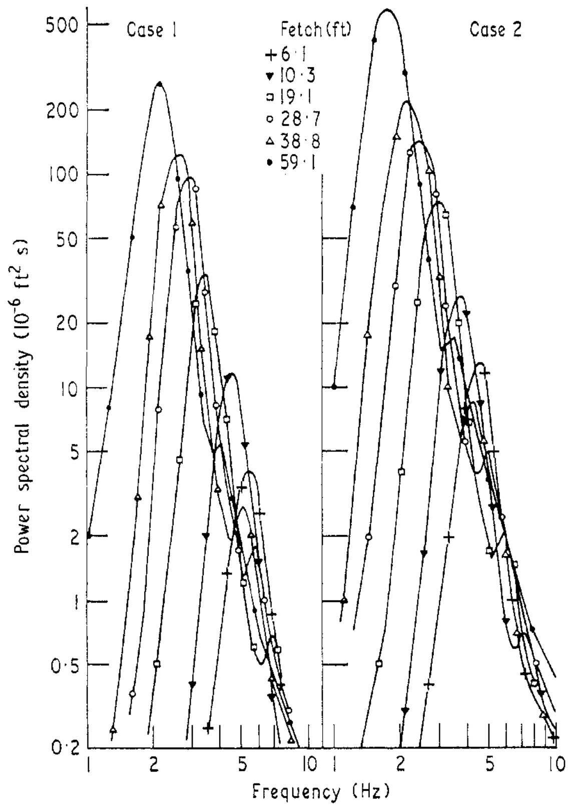

Figure 10. Power spectra of fetch-limited waves at different distances downwind (after Sutherland 1968). Note the pronounced overshoot effect.

thus strongly suggested. The latter authors, however, display sequential spectra that do not exhibit significant overshoots. This conflict does not appear to be caused by experimental set-up or data analysis. An explanation of this discrepancy, while in order, has not been offered.

Verification of theory. The major focus of the laboratory work over the past decade has been to verify Miles' (1959, 1962) theories of wave generation. The results have been, until recently, remarkably disparate.

Hidy and Plate (1966) observed waves generated by a fan blowing air over a large tank of water. Using wind velocity profiles measured with Pitot tubes and taking account of Doppler effects due to wind-induced water currents, they concluded that the combined Miles–Phillips theories could be used to predict observed wave growth to within a factor of two. The authors find numerous reasons to explain the factor of two, perhaps the most crucial of which is the fact that the wind profile is not that assumed in Miles' theory. Sutherland (1968), on the other hand, conducted essentially the same experiment with the opposite conclusion: 'The viscous Reynolds' stress theory . . . (Miles 1962) . . . is thus inadequate for predicting growth rates in situations where a spectrum of waves is involved.' His experimental wind profiles were found to differ slightly but significantly from the expected logarithmic profile, and it also seemed highly unlikely that the viscous sublayer called for by theory existed in the study.

Another approach to checking theory was adopted by Shemdin and Hsu (1967). Using a pressure sensor that followed the water surface†, they obtained measurements of aerodynamic pressure distribution at the interface between a wind field and a simple progressive wave‡. The instability theories for shear flow past a wavy boundary predict a phase shift such that

$$\langle p \partial \xi / \partial t \rangle \neq 0.$$

The authors found a phase shift that was in rather good agreement with Miles' predictions, although they too had difficulty in defining the details of the air flow required for an evaluation of theory.

However, Bole and Hsu (1969), working also with mechanically generated waves (in the same wind wave facility), conclude that the instability theory predicts rates of growth that are typically a factor of three less than actually observed. The latter authors attribute the discrepancy between their results and those of Shemdin and Hsu to instrumental and analytical difficulties experienced by the latter authors. The list of potential errors is impressively indicative of the difficulty of the measurement.

Other experimental studies by Hires (1968) and Wilson (1971) seem to substantiate the fact that even under specifically designed experiments the instability theory fails to account for the observations.

The problem of experimental verification of (linear) instability theories has been put in perspective by Stewart (1970). Working in a small, but carefully designed wind wave tank, Stewart measured the mean velocity field over monochromatic deep water waves with a hot-wire anemometer for a range of U/c between 0·4 and 3·0. The wave-induced perturbation velocity field and its associated Reynolds stresses were also measured. These observations were compared with the several linear theories which

† Recall Longuet-Higgins *et al* (1963) and Dobson (1971a,b) used a similar technique in the field.

‡ In the preceding paragraph the authors worked with a full spectrum of waves.

purported to predict the wave-induced perturbation velocities (Miles 1959, Davis 1970). The basic conclusion was that while the theories could predict the qualitative nature of the results, they could not consistently and quantitatively reproduce the observations.

Numerical experiments in evaluating the theories (Stewart 1970, Davis 1970) showed the results to be highly sensitive to flow field uncertainties; uncertainties comparable with experimental errors in the previously mentioned works. The experimental data also clearly demonstrated the importance of viscosity to the wave generation process. This factor, plus the resulting wave-induced Reynolds stresses, indicates rather strongly that the process of wave generation cannot be explained by a linear (shear flow) instability theory.

The major problems raised in this section thus seem explained by a combination of experimental inaccuracy, difficulty in evaluating theories, and the fact that wave generation must be explained by a more detailed theory than has heretofore been tested.

Air flow over a wavy surface. The basic problem now facing experimentalists is to determine how the atmospheric turbulence spectrum interacts with the mean flow field and the surface wave field.

Previous workers (eg Hidy and Plate 1966, Sutherland 1968) present data which suggest that the *mean velocity* profile over the rough wavy surface is logarithmic, although others disagree (Shemdin and Hsu 1967, Shemdin 1969). However, more recent work (Wu 1968, Stewart 1970) seems to confirm the logarithmic profile at least above the viscous sublayer that lies very close to the water surface. In other words, the wavy, free surface appears to the wind as a rough, unmoving surface (the 'law of the wall' applies).

An important contribution of Wu's work is the fact that the 'roughness' parameter (z_0) necessary to the logarithmic profile, ie $U(z) - U_1 \ln (a/a_0)$, is a function of surface wave height. As the waves increase to the point where wave breaking (white capping) occurs, the dependence of z_0 on wind undergoes a radical change. At higher wind velocities the surface roughness is proportional to the average height of the principal gravity waves. This result, plus the observations by Shemdin and Stewart that the profile over a wave crest differs from that over the trough, indicates an inseparable interaction between the two media.

All theories to date (except that of Jeffreys) require continuous flow above the wavy surface. Under conditions of high, short-crested waves and wave breaking this idealization seems unlikely to hold. Indeed, separation of the air flow to the leeward of a wave crest may explain the marked change in z_0 observed by Wu. A similar situation may account for the intermittency observed by Ruggles in nature (§5.1.1).

Laboratory data for the occurrence of separation† come from Shemdin (1969). The air flow in a reference frame travelling with his (mechanically generated) wave exhibited vortex motion. At wind speed less than the wave speed a high-pressure zone led the wave crest, indicating negative momentum transfer. The situation is just reversed for $U \geqslant c$. If this result can be confirmed, it suggests that wave generation is a strong interaction (as opposed to Hasselmann's weak interaction theory) that will be difficult to treat theoretically.

In summary, available data suggest that the wave-induced Reynolds stresses

† Any experienced sailor who has encountered high seas in a smallish boat could also attest to the occurrence of something like flow separation.

contribute significantly to the momentum budget of the wave field. Hence the interaction of surface waves with the turbulent flow above them appears to be an important force in the generation of the waves themselves. This process is at least partially represented by S_4 in the energy balance equation of §3.1.

5.3. Observations of wave dissipation

In this section we consider the mechanisms that dissipate wave energy in the open sea and in shallow water. The open ocean mechanisms can include wave breaking (white capping aided by nonlinear wave–wave interactions) and adverse wind action. Dissipation in shallow water is generally thought to be due to bottom friction, although, as we shall see, this assumption may not be valid. In view of the dearth of observations of wave dissipation, we combine results from both nature and the laboratory in the following discussion.

White capping. Anyone who has seen an aroused sea under the action of a 10–20 m s^{-1} wind would agree that white capping is a most obvious means of wave dissipation. Yet this mechanism has not yet been accounted for theoretically† or experimentally. The best we can do now is to maintain faith in the effectiveness of the mechanism and invoke it to explain the high-frequency characteristics of the wave spectrum. This is a deplorable state of affairs.

Opposing wind. Another obvious mechanism for wave attenuation is the action of an opposing wind field. Theoretical estimates of this effect are only now being made (§3.8). Observations with which to check the theory, while not numerous, are adequate to set some limits on the magnitude of the process. Dobson (1971a,b) used essentially the field experimental set-up described in §5.2.1 to make simultaneous observation of p, $\partial\xi/\partial t$ and U for waves propagating against the wind. With a very limited number of data, he found rates of attenuation comparable with rates of generation and 10^4 times greater than viscous damping alone. The sense of the momentum flux was such that the *wind* was receiving momentum from the waves. The observed values of wave attenuation were only 30% less than those predicted by Phillips (1963), who developed his theory by considering the interaction of turbulent and wave-induced Reynolds stresses. Yet Dobson seems to discount this agreement with theory on the grounds that 'turbulence levels were so low'.

Shemdin (1969) and Dobson (1971b) also observe attenuation for waves travelling with, but faster than, the wind. The former author, using laboratory data, attributes this effect to flow separation; the latter offers no explanation. Both authors show the generation–dissipation process to vary rather smoothly through the range of $c/U \gtrless 1$. Unfortunately the attenuation measurements are generally closely confined to the region $c/U \simeq 1$.

The most complete set of measurements of waves propagating against a wind field has been provided by JONSWAP. Several situations occurred where a quasi-stationary, *onshore* wave field was being opposed by offshore winds up to 13 m s^{-1}. The estimates of observed wave attenuation rates (Γ) as functions of wind speed are shown in figure 11. It was concluded that Γ was insensitive to either wind speed or the component of wind velocity parallel to the swell propagation direction.

† Hasselmann (1974) has recently offered a theoretical explanation of white capping and its effect on the energy balance of the wave spectrum.

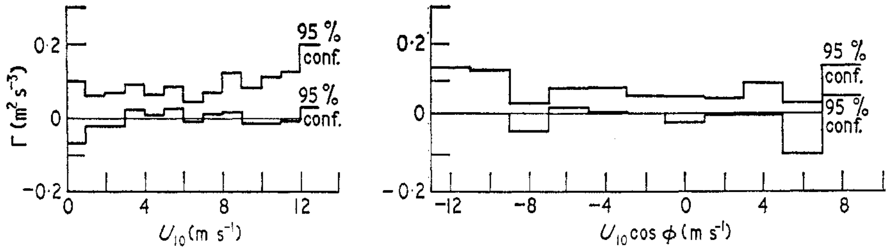

Figure 11. Decay parameter Γ versus wind speed and component of wind speed parallel to swell propagation direction. The stepped lines denote 95% confidence limits within which the data fall. These confidence limits are based on approximately 80 data points.

In summary, the effect of an opposing wind on a wave field is uncertain. The apparent difference between the results of Dobson and Shemdin must be reconciled with the voluminous JONSWAP data.

Swell attenuation. A unique set of wave attenuation observations has been provided by Snodgrass *et al* (1966) via their swell attenuation study (SAS). Waves (swell) were observed with pressure transducers along their great circle propagation path from New Zealand to Alaska. The decrease in energy along the path was expressed in terms of dB/degree of latitude. For frequencies below 0·07 Hz the attenuation was negligible (<0·02 dB deg^{-1}). At 0·08 Hz the rate was 0·15 dB deg^{-1}, and at higher frequencies the rate was too large to be accurately estimated with the station spacing used in the experiment. The results indicated that once outside the generating area, low-frequency wave energy (swell) travels the length of the Pacific (through all types of wind–wave conditions) with virtually no attenuation. The fact that several theories (Hasselmann 1962, Phillips 1966) predict this negative result is encouraging but hardly substantiation for their validity. Further, the results seem to refute the idea (Phillips 1963) that preferential white capping on a swell crest can lead to substantial wave attenuation, since no measurable attenuation occurred as the swell passed through the Trade Wind Zones.

Another major finding of SAS indicated that the rates of wave dissipation in the near-storm region were not inconsistent with that predicted by Hasselmann's wave–wave interaction theory. However, the experimental design precluded a substantiation of the theory. SAS did establish the fact that the attenuative effect of wave breaking and wave–wave interaction were roughly comparable.

Additional support for the potential dissipative effect of wave–wave interaction has been provided indirectly by Mitsuyasu (1964), and Mitsuyasu and Kimura (1965). A spectrum of waves was generated by wind in the first half of a large wave tank. The latter half of the tank experienced no wind. The redistribution of energy within the spectrum as the waves propagated through the calm area was measured and found to be dependent on the characteristics of the initial wave field. The shape of the 'attenuation' curves when plotted against wave frequency was in qualitative and quantitative agreement with approximate theoretical calculations. However, the wave–wave scattering theory requires that the integral of the source function over the frequency range 0 to ∞ be zero, ie energy be conserved. This condition was not obtained in Mitsuyasu's experiment. Evidently the theory has some shortcoming, or, equally likely, other energy transfer processes were operative in addition to the wave–wave mechanism. More will be said about the wave–wave interactions in §5.4.

Shallow water effects. Wave dissipation mechanisms of increasing practical importance are those which occur in shallow water, for it is well known that they actively effect the wave spectrum as it propagates on to the continental margins. Early estimates of this

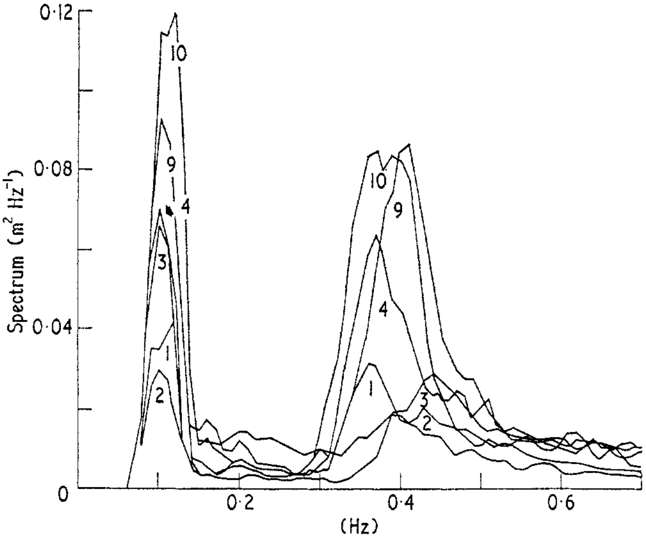

Figure 12. Examples of swell peaks observed during selected JONSWAP dissipation case. Numbers refer to wave stations (see figure 5). Note that the swell energy decreases towards shore with the exception of station 1 where the energy is presumably enhanced by shoaling effects.

mechanism were obtained by assuming a quadratic friction law and a friction coefficient of the order 10^{-2}. Hasselmann and Collins (1968) developed a theory of attenuation due to friction associated with turbulent bottom boundary currents which seemed in accord with the small number of data available to them (eg Walden and Ruback 1967).

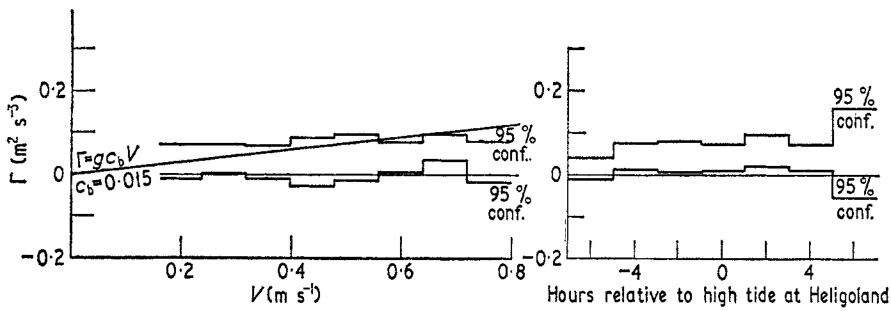

Figure 13. Swell decay parameter Γ versus current speed V and tidal phase. 95% confidence intervals are as before. The Hasselmann–Collins' (1968) theory calls for a decay rate that is linear in V (straight line in left-hand panel). The attenuation should experience modulation at tidal frequencies. This is not observed in the right-hand panel.

However, detailed analysis of the JONSWAP data, while in order-of-magnitude agreement with the previous observations, contradicts a significant prediction of the Hasselmann and Collins theory. A quadratic friction law should lead to a strong modulation of swell decay rates by tidal currents. Experimentally no such variation is found.

The JONSWAP observation stations were situated as shown in figure 5. The energy content of long waves moving onshore and feeling bottom could be monitored regularly with this experimental set-up. Other significant geophysical variables (eg current, wind, etc) were monitored simultaneously. A typical case study is shown in figure 12, where sequential spectra for six different stations are presented. The rate of attenuation (Γ) was computed for this and numerous other cases. The results are summarized in figures 13 and 14. The illustrations show that Γ had no first-order dependence on bottom current, tidal phase, swell direction, or swell frequency. There was some indication of a dependence on swell energy itself.

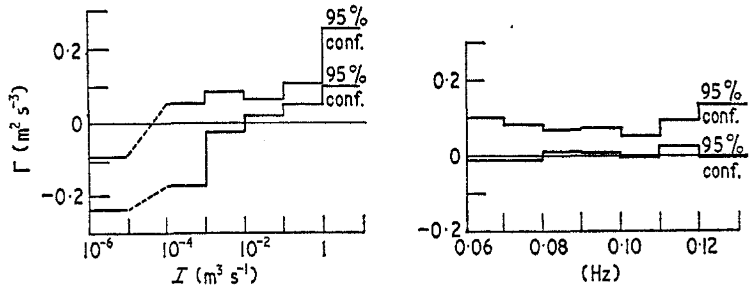

Figure 14. Decay parameter Γ versus wave frequency and energy flux *I*. The 95% confidence lines are defined as before. Note the apparent relation between decay Γ and energy *I*. Illustration after Hasselmann *et al* (1973).

The most plausible explanation for the above result has been offered by Long (1973), who shows that the observed attenuation could be produced by backscattering (Bragg interactions) with bottom irregularities of scale comparable with the wave length of the swell. The information on the spectra of bottom irregularities and swell field are not presently adequate to test this theory.

5.4. *Nonlinear properties of the wave field*

As the previous sections have shown, the nonlinear aspects of surface waves hold one of the keys to future progress in the field, even though the nonlinearities are quite weak. Experimental verification of nonlinear theories is generally not easy, but several wave studies have, nevertheless, apparently confirmed theoretical predictions.

Many authors (eg Phillips 1960, Hasselmann 1962, Longuet-Higgins 1962a,b) have considered the interaction between four intersecting waves. Under certain conditions (§3.3) a resonance and energy interchange can occur. Following an earlier suggestion of Longuet-Higgins (1962a,b), both Longuet-Higgins and Smith (1966) and McGoldrick *et al* (1966) performed a set of detailed laboratory experiments designed to test the interaction theory. Two different wave trains were generated mechanically along the adjacent walls of a square wave tank. By adjusting the wave frequency ratios to that required for resonance the authors were able to generate a third wave train whose original amplitude had been zero (two of four interacting waves required by theory were identical). The experiment confirmed the theoretical requirement for the existence of this third wave, while simultaneously confirming its expected growth rate and dependence of the growth rate on the amplitude of the original waves. McGoldrick (1970) has extended the study of nonlinear wave interaction to the case where *both* gravity and surface tension are important restoring forces. The theory was again found to be generally valid.

Complementary experiments reported by Benjamin (1967) also clearly suggest the applicability of the nonlinear theory for simple plane waves. The authors showed that a classical Stokes solution to the basic wave equations are, in a sense, unstable in the presence of small perturbations. The resulting 'wave–wave' interactions lead to a

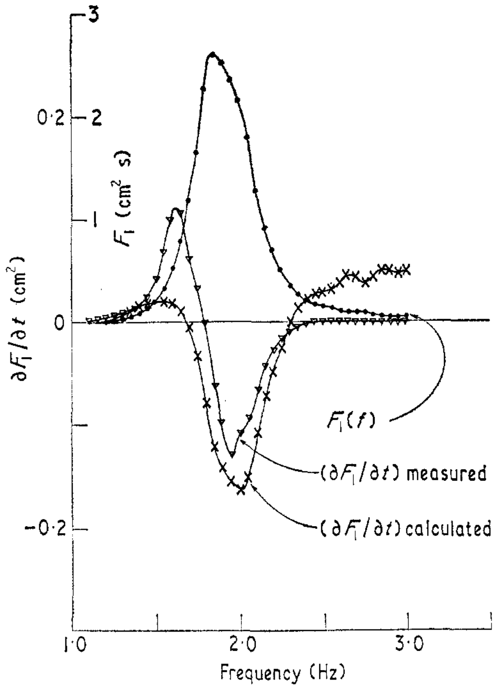

Figure 15. Wave spectrum and rate of energy transfer (after Mitsuyasu 1968). Theoretical transfer rates are calculated from nonlinear wave–wave interaction theory.

degeneration of the original wave form into a series of groups. The energy density associated with these groups may become so concentrated in physical space that wave breaking actually occurs.

In a fully aroused wind sea the nonlinear theory is more difficult and must be approached from a statistical point of view (cf §3.3). Attempts to directly verify the theory under these circumstances are difficult at best; however, two successful studies indicate that the weak wave–wave interactions do exist in the presence of a spectrum and that they are of first-order importance to the energy balance of the wave field.

The many experimental results discussed in the previous sections have all *suggested* that wave–wave interactions were operative. Barnett and Sutherland (1968) showed that the features of spectral development in both nature and laboratory could be scaled according to properties of the observed wave fields so as to be virtually identical. This result further suggested the existence and activity of nonlinear interactions.

Laboratory results on growth and decay (§5.3) of a random, wind-generated wave field by Mitsuyasu (1968) give additional though not conclusive proof of the generalized theory of Hasselmann (§3.3). Mitsuyasu obtained source functions that had the same characteristic plus/minus signature as those obtained by JONSWAP. He used a parameterization of the theoretical nonlinear source function to compare theory with his observations. An example of the agreement he obtained is shown in figure 15. The agreement is at first remarkable, for the parametric approximation to theory was done for energies and frequencies 1–4 orders of magnitude removed from those studied by Mitsuyasu. The self-similar nature of the wave–wave interactions, however, allows such a scaling to be successful.

The JONSWAP data previously discussed in §5.2.1 gave rather convincing proof of the efficiency and accuracy of the wave–wave theory (see figure 7). Until an equally appealing, alternative explanation for the above results is developed, the wave–wave interaction theory must be assumed to be approximately valid.

5.5. Major features of the wave field

Over the years, great attention has been focused on the description of certain aspects of the wave field. We have selected three of these features to review here briefly. These areas are: the directional properties of a wind sea; the asymptotic shape of the spectrum at high frequencies; and the shape of the spectrum at various stages of its evolution. Study in these areas, while not always directly applicable to understanding wave energy transfer process, has provided a set of background information which has helped to guide the selection of areas of research on wind waves. Unfortunately, perhaps too much effort has been devoted to these and other associated areas of study, thereby avoiding some of the more crucial questions that have been lying around unanswered for fifteen years (see §7).

Directional properties. The distribution of wave energy in $\boldsymbol{k}$ space for a fully aroused wind sea was first estimated with some accuracy during the SWOP project by Cote *et al* (1960) (see §4.2). Using several aircraft the authors obtained stereo photos of the rough sea surface under the action of a steady wind. After a tremendous effort the authors finally obtained two estimates of $\xi(x, y)$ over a small area of ocean. The resulting Fourier transformation gave an angular spreading function H of the form

$$H(\theta) = 1 + a_1 \exp(-b) \cos 2\theta + a_2 \exp(-b) \cos 4\theta$$

where $\theta = 0°$ is the direction of the wind, a_i are constants, and $b = \frac{1}{2}(\omega U/g)^4$. Basically the equation says that the waves are distributed as $\cos^2$ near the spectral peak, with the beam width broadening at higher frequencies.

A substantial advance in estimating $H(\theta)$ was made by Longuet-Higgins *et al* (1963) and Cartwright and Smith (1964). Using the measured tilt and acceleration of a floating buoy (§4.2), the authors fit the observations with a spreading function of the form

$$H(\theta) = \cos^s(\theta/2); \quad -\pi \leqslant \theta \leqslant \pi \quad \text{and} \quad s = s(\omega). \tag{5.1}$$

The values of s ranged from 1 at high frequencies to 10 at low frequencies. Subsequent measurements by Ewing (1969) with the same instrument have confirmed the general pattern.

A recent experiment by Tyler *et al* (1972) has used a unique measurement technique to further confirm the form of H. Radio waves in the 2–30 MHz frequency band were scattered off ocean waves with the back scatter signal being interpreted with first-order Bragg theory (Barrick 1972, §4.2). By moving the radio receiver the authors synthesized an antenna with an angular resolution of $\pm 3°$. The directional spread estimates were obtained and checked against a pitch and roll buoy of the type used by Longuet-Higgins *et al* (1963).

The resulting spread was fit by a function virtually identical to $\cos^s(\theta/2)$. The resulting estimates of s are plotted in figure 16 against $\mu(=U/cK)$, where K is von Karman's constant and the other symbols are as before. The data essentially confirm the results of Longuet-Higgins *et al.*

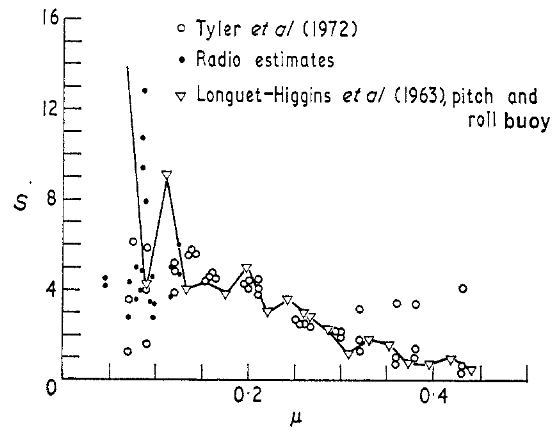

Figure 16. Summary plot of wind wave spread estimates. Pitch and roll buoy measurements of Tyler *et al* (1972) are shown by open circles. The Longuet-Higgins *et al* (1963) measurements (triangles) are connected by a line, and the radio measurements are indicated by full circles. See §5.5, equation (5.1) for definition of parameters S and μ.

Less accurate estimates of spread made under generation conditions during JONSWAP show distributions of wave energy with θ that are in agreement with the preceding results. The JONSWAP data also indicate that the spread could be scaled relative to the local spectral peak such that $H=H(\theta, f/f_{\mathrm{m}})$, where f_{m} is the true frequency of the peak $(=\omega_{\mathrm{m}}/2\pi)$ and $f=\omega/2\pi$.

In view of the above, we may conclude that the direction properties of a growing or fully aroused wind sea are roughly known to first order.

High-frequency shape. The slope of the spectrum for frequencies higher than that of the peak has attracted much attention, principally because the quantity is easy to measure and because there was a dimensional argument that predicted that the form of the spectrum in this region should be proportional to frequency to the inverse fifth power (§3.6). Burling (1959), Kinsman (1960) and numerous others have estimated the slope of this high-frequency tail and found it to be approximately -5 as predicted by Phillips (1958). However, the authors of these studies always find it necessary to invoke *different* values of the constant of proportionality, in order to obtain agreement between theory and observation.

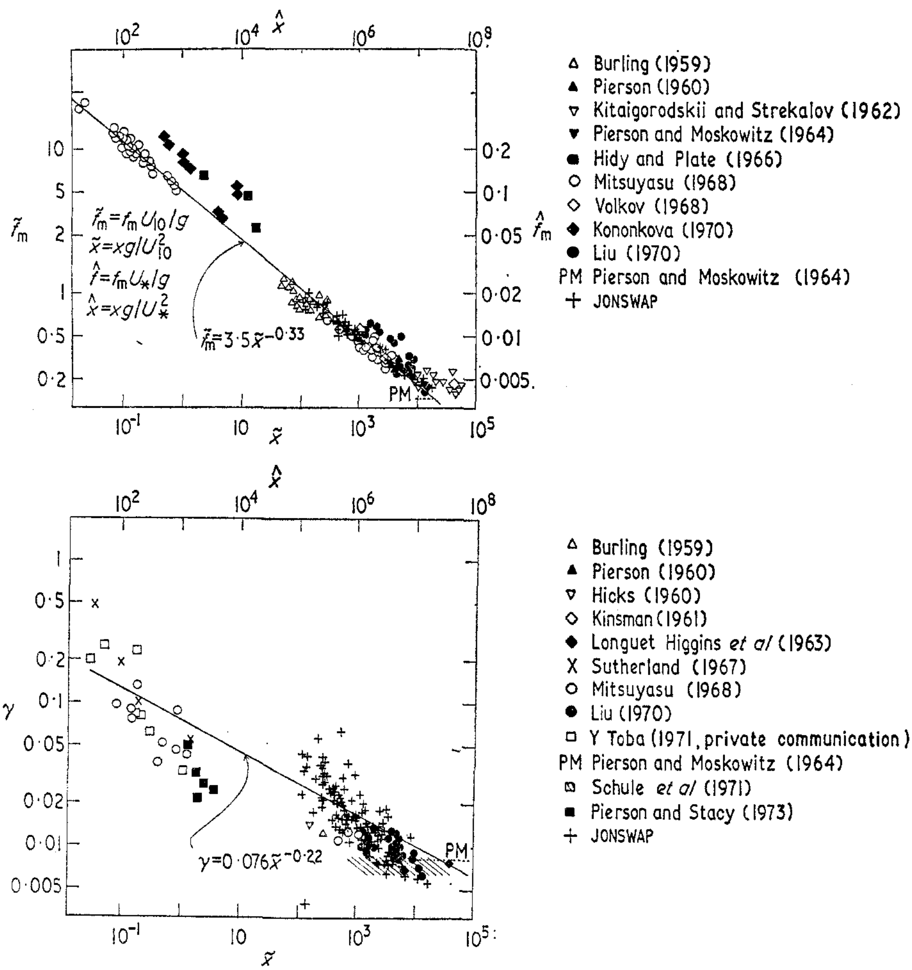

Figure 17. Upper panel: Frequency of maximum energy (spectral peak) versus fetch scaled according to Kitaigorodskii. Illustration after Hasselmann *et al* (1973). Lower panel: Phillips' 'constant' versus fetch scaled according to Kitaigorodskii. Small fetch data are obtained from wind wave tanks. Illustration after Hasselmann *et al* (1973).

The results of JONSWAP, illustrated in figure 17 with those of other workers, put the matter in perspective. The proportionality 'constant' is clearly variable, being dependent on nondimensional fetch ($=xg/U^2$). The full line is the theoretical prediction of Kitaigorodskii (1962) which will be discussed shortly.

The failure of the original theory stems from the fact that wave breaking was thought to be the only dissipative mechanism controlling wave growth. Within the frequency range f_m to $3f_m$ it was found that the energy balance results principally between the atmosphere energy input and the nonlinear energy transfer to higher and lower frequencies. Only at higher frequencies ($>3f_m$) does wave breaking come to the fore. Under these conditions a simple dimensional argument could not succeed.

Spectral shapes. In the mid and late 1950s it seemed that each worker in the field of ocean waves had proposed a different functional form for the 'fully developed' spectrum. This was the maximum or steady-state spectrum that characterized a particular wind speed. Once reached, the fully developed spectrum was independent of x or t provided the wind did not change (§3.6).

Neumann (1953) first proposed a 'fully developed' spectrum of the form

$$F_1(\omega) = \text{constant} \times \omega^{-6} \exp(-2g^2/U^2\omega^2) \tag{5.2}$$

from visual observations of wave height taken aboard merchant ships. Darbyshire (1955, 1959), Roll and Fischer (1956), Burling (1959) and Bretschneider (1959) all followed with different forms. The arguments that fill the literature over the functional forms, magnitude of constants and analysis methods used to estimate the fully developed spectrum make amusing reading (cf *Ocean Wave Spectra* 1963). In summary, it is fair to say that most authors were dealing with poorly defined data, analysed by then new techniques. Much of the confusion resulted from these sources.

Determined to settle the many disagreements, Willard Pierson set about a spectral analysis of wave data measured at British weather ships (§2.2). Great care was put into the data analysis and even greater care into the analysis of weather maps describing the wind fields that gave rise to the observed waves. These maps were a prime means of determining what was or was not a 'fully developed' spectrum. The functional form eventually arrived at (Pierson and Moskowitz 1964) after examining over 400 cases was given in equation (3.31). A key item in this definition is that the wind be measured at 19·5 m above the sea surface. The fact that other authors used winds at different levels of a logarithmic profile was found to be the source of some of the past discrepancy†. The reader should beware, however, for there has been no convincing demonstration that a 'fully developed' spectrum should exist.

As has been hinted in §5.2.1, the process of wave generation produces a self-similar spectrum that depends only on fetch, local friction velocity (wind speed), and gravity. This was predicted on dimensional grounds by Kitaigorodskii (1962) and confirmed by Mitsuyasu (1968, 1969), Liu (1971) and JONSWAP, among others. The field measurements suggest a functional form set forth by JONSWAP:

$$\begin{gathered} F_1(f) = \frac{\gamma g^2}{(2\pi)^4} f^5 \exp\left[-\frac{5}{4}\left(\frac{f}{f_\mathrm{m}}\right)^{-4}\right]\left[\Omega \exp\left(-\frac{(f-f_\mathrm{m})^2}{2\sigma^2 f_\mathrm{m}{}^2}\right)\right] \\ \sigma = \begin{cases} \sigma_\mathrm{a} & \text{for } f \leqslant f_\mathrm{m} \\ \sigma_\mathrm{b} & \text{for } f > f_\mathrm{m} \end{cases} \\ \Omega = \text{constant (see figure 18).} \end{gathered} \tag{5.3}$$

Note that true frequency $f\,(=\omega/2\pi)$ has been used in (5.3).

The dependence of γ on fetch has been discussed (figure 17, lower panel), while the dependences of f_m, etc are shown in figures 17 (upper panel) and 18 along with the results of other authors. A schematic of the spectrum is shown in figure 19. Under simple wind conditions it is evidently now possible to specify the form of the spectrum from knowledge of fetch and wind speed alone.

The fetch-limited spectral form is quite similar to the 'fully developed' form (equation 3.31) labelled E^{PM} in figure 19 *except* for the shape function (second square

† Which brings up several points we have carefully avoided in this article so far: At what elevation should the wind be measured? How does one define the 'mean' or 'effective' wind? See §7.

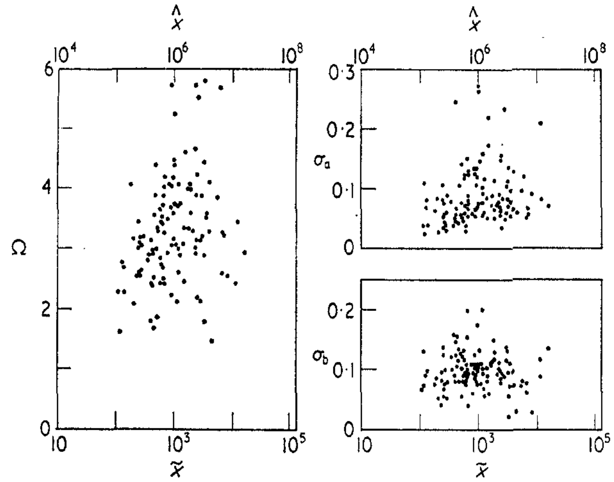

Figure 18. Shape parameters for the similarity spectrum observed during the JONSWAP project versus nondimensional fetch. The scatter in the parameters seems real and is most likely accounted for by variability in the wind field inducing similar variability in the wave field. See §5.5, equation (3.28) and figure 20 for definition of variables.

brackets in equation 5.3). This exception introduces a discrepancy between the available data that is presently unresolvable. If a fully developed spectrum exists, then there must be an attenuation mechanism that extracts energy from waves moving faster than the wind. This dissipative mechanism could balance the positive energy contribution by wave–wave interactions to the low-frequency region, thus allowing a fully developed spectrum. Such a condition is hinted at by a small number of data (eg Dobson 1971a,b); however, it is by no means established.

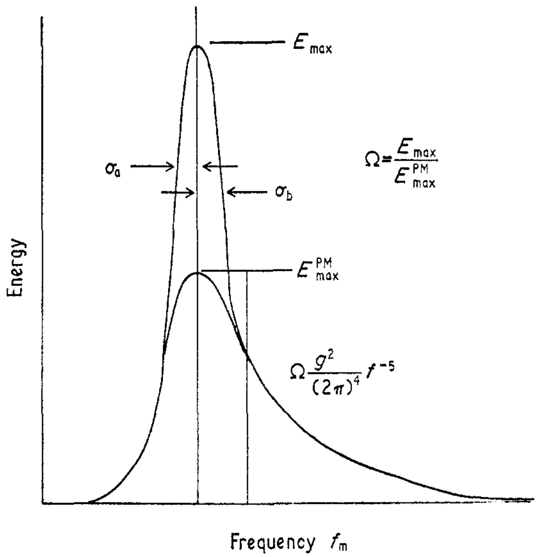

Figure 19. Schematic view of the best-fit fetch-limited spectrum obtained during the JONSWAP project. Definition of the free parameters and the functional fit is illustrated in the figure.

6. Wave prediction—the combining of theory and observation

6.1. Introductory remarks

Since 1955 progress in the prediction of wind waves has gone in two main directions. One group has attempted to establish prediction methods based on a concept of significant wave height† and easily observable properties of the spectrum. A second group has attempted to predict waves using the concepts of wave spectrum and the theories and observations that have been discussed in the last sections. The divergence between these groups, particularly in the last ten years, is remarkable. The former group, seeking to predict the significant wave height, has made virtually no use of the theory and observations described above. Their goal has been a simple, fast method of estimating the gross properties of the wave field. The present authors will devote their entire attention to wave prediction methods based on the concept of a wave spectrum and the more recent advances in theory and observations. The 'significant wave' prediction methods are seen to be simplifications of the more complex spectral methods.

Within the spectral school there have been two major areas of approach. These have recently coalesced so that it is now probably safe to say that those interested in predicting wind wave spectra have agreed on a general approach. In the late fifties and early sixties, however, such agreement was far from common. One group, led by Willard Pierson and members of his staff at New York University (NYU), took the philosophy that available theory was inadequate to describe wind wave processes and that the most fruitful approach would be to build prediction models based on data and a few 'solid' concepts such as the spectrum, group velocity, etc. Another group, led by the work of Gelci *et al* (1956), Groves and Melcer (1961) and Hasselmann (1960), took the approach that the wave prediction problem should be founded on firm theoretical ground. It was this latter work that led to the introduction of the radiative transfer equation as a basic framework around which to build a wave prediction model. As the reader might expect, the data-based models needed the theory to make them coherent, while the theoretical models needed considerable 'empirical brushing up' to make them produce reasonable results. Present wave forecasting models are, therefore, a combination of theory and observation. These two ingredients have been blended to provide reasonably good hindcasts (after-the-fact predictions) of the wave spectrum under a number of different and complex geophysical situations.

6.2. Numerical forecasts of wave spectra

First efforts along these lines were made by Baer (1962), who took the work of Pierson, Neumann and James (PNJ), §2.4, and programmed it for a large digital computer. Baer took on the then ambitious task of attempting predictions for the entire North Atlantic Ocean. For verification data he proposed to use spectra obtained from British weather ships some 300 miles off the English coast. Baer's problems were many, but two are most important in the context of this review. These are: the methods by which wave energy propagated over the surface of the ocean; and the methods by which the wind was allowed to generate waves.

In the Baer model, and later NYU models, a grid network was established over the North Atlantic with a spacing between grid points of approximately 200 nautical miles.

† The 'significant wave height' is proportional to the total energy of the spectrum (Longuet-Higgins 1952).

At each grid point a two-dimensional energy spectrum was defined by a discrete series of frequencies and directions of wave propagation. The approach to the propagation problem was simple and is best described in Baer's own words: 'In principle what was done was to keep track of how far past and to the side of an average gridpoint the wave components were. Then when the component had moved far enough to reach an adjacent gridpoint, the energy was jumped (to the adjacent gridpoint). Thus if it takes say three time steps for a particular component to travel to the adjacent gridpoint, on every third step the entire field will be jumped. That is to say, the energy from a gridpoint upstream replaces the energy at the adjacent gridpoint downstream.' This technique can lead to substantial errors in the rectangular coordinate system that was used in the Baer model. Nevertheless, it was a first approximation to solving the energy propagation problem.

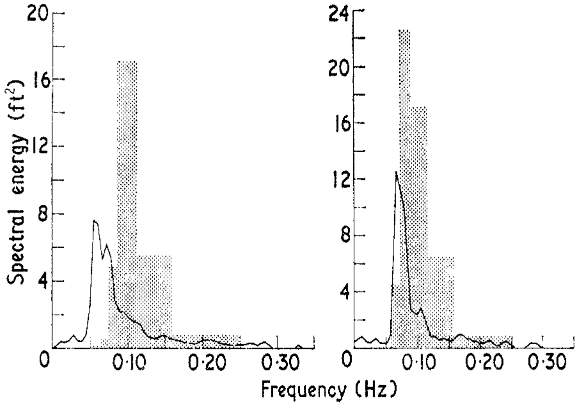

Figure 20. Comparison of the observed and hindcasted one-dimensional wave spectrum produced by the Baer (1962) model. The full curves represent the observed wave spectrum, while the speckled areas represent the hindcasted spectrum.

Wave growth was based on the concept of the Neumann spectrum (equation 5.2, §5.5) and on a table which described the maximum allowable growth of the spectrum in a selected time interval (two hours in this case). In practice the method worked like this: Given a partially developed Neumann spectrum and a given wind speed, the computer would enter the table and determine the total amount of energy to be added to the spectrum over the next time step. This energy would be distributed in both frequency and direction space so that the following criteria were satisfied: first, that the highest frequencies were fully developed before any energy was given to the low frequencies; and second, that when adding energy at a fixed frequency, the energy would be spread over direction in a method similar to the directional energy distributions observed during the SWOP experiment (§5.5). Such a generation technique led to a power spectrum that grew from high frequencies to low frequencies. It possessed a sharp, steep, forward face which is a common feature of most observed wave spectra.

The Baer model was used to hindcast wave spectra observed at the British weather ships. A typical comparison of hindcast and observation is shown in figure 20. These results obviously leave something to be desired. Nevertheless they represent the first attempt to forecast the wind wave spectrum numerically.

Over the next years, Pierson with numerous co-workers spent much effort in developing a better estimate for the fully developed spectrum (see §5). Also, the growth tables used by Baer were at first refined and then discarded in favour of one form of the radiative transfer equation (3.17), namely

$$\partial F_2(\omega, \theta)/\partial t = S_1 + S_2 + S_4 - S_6. \tag{6.1}$$

This transition to the theoretical frame provided by the radiative transfer equation is best described in the works of Pierson *et al* (1966) and Inoue (1967). The first three

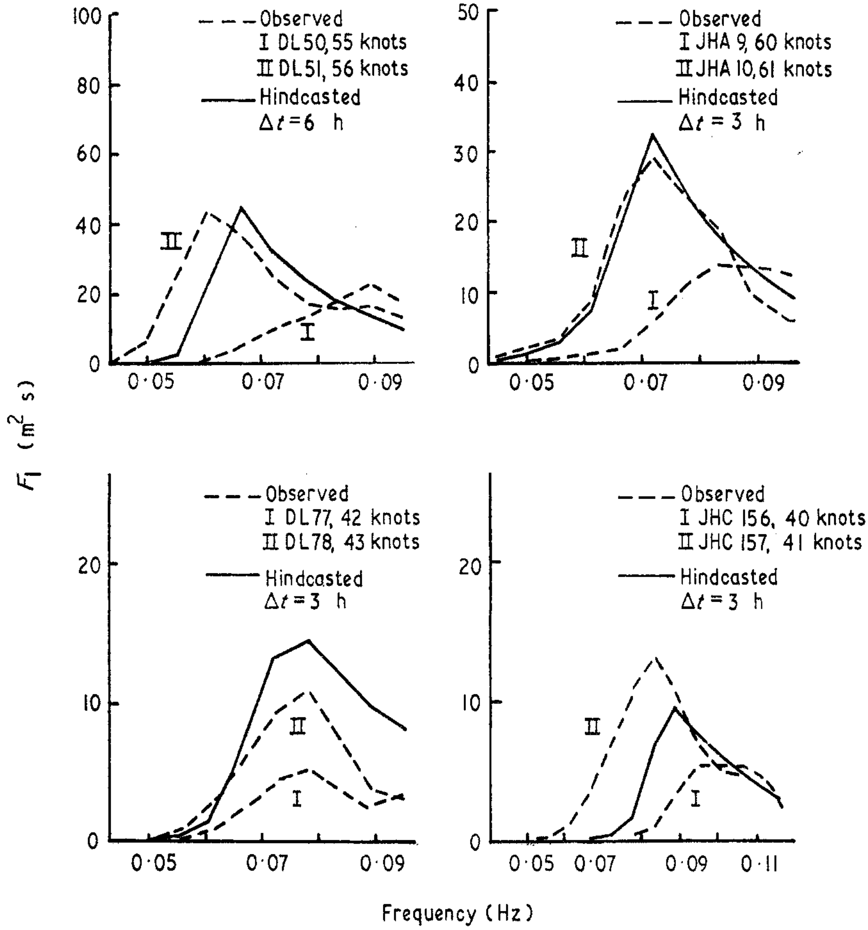

Figure 21. A comparison of the observed and hindcasted one-dimensional wave spectrum produced by the NYU model (Inoue 1967). The symbols I and II represent observations bracketing the time of the hindcast. The observed wind speeds are also given.

terms of equation (6.1) were to represent the combined theories of Phillips and Miles as amended by Phillips (1966). The numerical values used in equation (6.1) bear little resemblance to these theories, however, and the resulting wave predictions would be poor if the growth were not limited by the functional form of the fully developed spectrum (S_6). Effects of wave–wave interactions were explicitly neglected. Wave breaking and other dissipation processes are implicitly included by assuming a limited form for the fully developed spectrum.

The hindcast verifications found by Inoue are considered rather good (figure 21).

Unfortunately the comparisons are not entirely independent, since some of the verification data were used in construction of the model. Other independent checks, however, have given results in reasonable agreement with the numerical predictions.

6.3. *Prediction models based on the radiative transfer equation*

Realizing that the radiative transfer equation (3.17) described the energy balance of the wave spectrum, Gelci *et al* (1956) and a succession of his collaborators attempted to use it to hindcast ocean waves. This initial suggestion, augmented by Groves and Melcer (1961) and put in perspective by Hasselmann (1960), has eventually been adopted as the rational framework for ocean wave prediction.

The early prediction efforts of Gelci were hindered by lack of knowledge concerning the source function, and thus did not produce good results. However, the propagation problem was addressed using standard finite-difference techniques to solve the characteristic equation governing the spatial transfer of energy. This approach allowed the first rigorous account to be taken of energy propagation.

Barnett (1966, 1968) used finite-difference methods to solve the radiative transfer equation over the entire North Atlantic Ocean under the following assumptions:

$$\partial t/\partial F + V.\nabla_x F = S_1 + S_2 \pm S_5 - S_6$$

where S_1 is a version of Phillips resonance theory modified by the observations of §5.1, S_2 is an exponential growth term based entirely on the observations described in §5.1, S_5 is paramaterizations of the wave–wave interactions (the simplification was needed

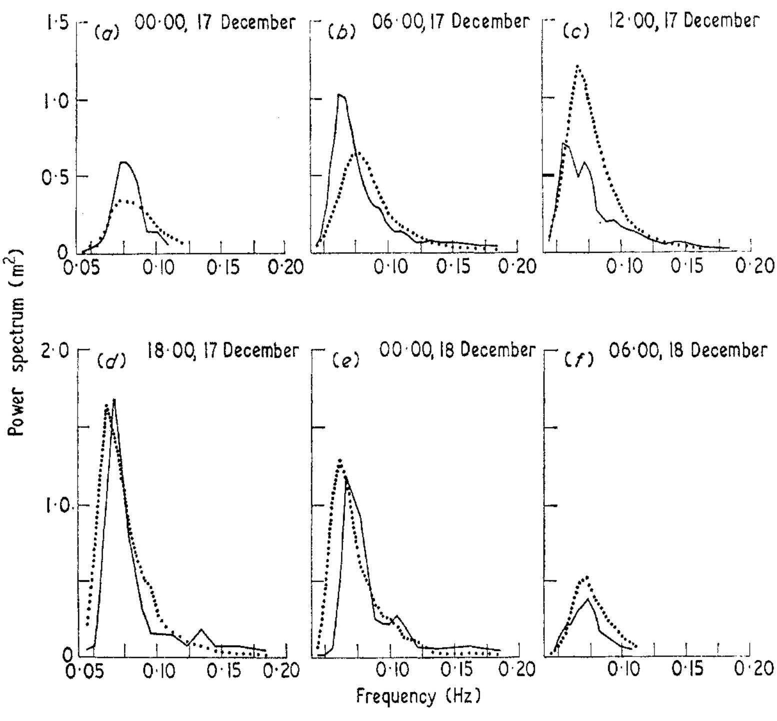

Figure 22. Predicted power spectra versus measurement. The full curves are observed, while the dotted curves are hindcast (after Barnett 1968).

since the full theoretical form of S_5 requires computational time that far exceeds feasibility), and S_6 is a representation of wave breaking (limiting growth) based on the equilibrium ideas of Phillips (§5.5). No assumption of a fully developed spectrum was made!

Solution of the resulting nonlinear integral differential equation was carried out for the same time period as the earlier hindcasts of Inoue and Baer. The results are shown in figure 22. It should be noted that the verification demonstrated in the figure are truly independent, as none of the verification data went into the model.

The same approach has been followed by Ewing (1971). His formulation utilized a more accurate parametric version due to Cartwright of the term S_5, and a higher-order finite-difference approximation for the advective term of (3.17). Ewing's comparisons were against the two-dimensional spectrum, the first such verifications attempted. The results (not shown) gave, in Ewing's words: '... adequate estimates of the significant height and one-dimensional wave spectrum. The standard deviation of all the computed estimates of $H_{1/3}$ compared to measurements is about 0·6 m. Reliable estimates of the two-dimensional wave spectrum were only achieved in a limited region at the high-frequency end of the spectrum.' This latter discrepancy was attributed to inadequate specification of the wind field.

6.4. *Prediction in shallow water*

Extension of the previous deep water models to shallow water has not progressed rapidly. Barnett *et al* (1969) successfully predicted the wave spectra for the shallow margins of the South China Sea. The effect of shallow water dissipation was added to the source terms described in §6.2 via the Hasselmann–Collins theory (§§3.5 and 5.3). Propagation and refractive effects in shallow water were included, with the assumption of parallel bottom depth contours. This allowed easy solution of the characteristic equations (3.18) governing propagation, and immediately eliminated the occurrence of caustics (ray path intersections) which greatly complicate the shallow water problem. The resulting hindcasts (not shown) compared well with both measured wave heights and spectral estimates from pressure recorders.

6.5. *Summary*

A basic framework for wind wave prediction has been found in the radiative transfer equations. The propagation of wave energy is properly accounted for in this framework by either of several schemes, each with differing amounts of error. Specification of the source function is a blend of theory and data, but generally leads to a nonlinear integral form of (3.17). However, forms of S so far proposed are certainly in need of improvement. The differing specifications of S seem to produce equally good hindcasts, indicating that at least a rough quantitative estimation of S is available under many different geophysical situations.

The basic problem now appears to lie in a better specification of the wind field, for the total energy in the wave field is known to be roughly proportional to the square of the local wind speed. A small error in wind (eg 10%) can thus lead to a 20% error in the total predicted energy.

The prediction of waves in shallow water and/or for hurricane conditions is just beginning. Much of the future advances in the area of wave prediction can be expected in these areas.

7. Summary and need for additional research

Great advances have been made over the last twenty years in understanding and observing the physical processes effecting wind waves. Yet, in spite of these results, we do not yet know the mechanism(s) by which the wind generates waves. The present state of knowledge, while light years ahead of that described by Ursell (1956), is in need of considerable improvement. In this section we will briefly summarize the previous sections and offer suggestions for future work in areas that appear to us to be particularly needy.

7.1. Theory

There are now several theories that can describe wave growth. Earlier theories, which did not account for the interaction of the atmospheric turbulence and wave fields, have been unable up to now to explain the observations of wave growth. Recent theories involving atmospheric turbulence in the wave generation process need to be evaluated numerically and tested experimentally. The nonlinear wave–wave interaction theory, while able to describe the observed growth of the wave spectrum over part of the frequency range, does not account for the manner by which the wind transfers energy to the wave field. There is also the strong, and apparently growing, feeling among some wave researchers that the generation process may be either intermittent or a 'strong interaction' involving separation of the air flow to the leeward of a wave crest. The theory of Jeffreys (1924) offers the only analytic description of the latter process. Additional theoretical work is badly needed to consider both the 'separation' and 'intermittancy' hypotheses.

The theory of wave dissipation is seriously defficient. Wave breaking is undoubtedly one of the primary mechanisms of dissipating wave energy. Wave breaking occurs during active wave generation and is visible through white capping. So far it has only been possible to include the effects of white capping on the energy spectrum by means of an empirical expression which is based on dimensional analysis†. The resulting frequency to the minus fifth power has been useful for describing the high-frequency range of the wave spectrum, but no further progress in our understanding of this process has occurred in the last fifteen years. Any attempts to describe the energy budget of the wave spectrum will surely be unsatisfactory until this situation is remedied.

As waves propagate into shallow water near coasts, before they break on the beach, there is a very short period of time in their life cycle in which dissipation can occur through interactions with irregularities of bottom topography and also through interactions with turbulent currents near the bottom. Theories for both of these processes have been formulated recently (Long 1973, Hasselmann and Collins 1968) and are still in the process of being evaluated through comparison with observations.

A theory for the propagation of wave energy has been available since the 1800s: the linear theory of propagation from a limited initial disturbance. This theory has been shown to work quite well for propagation of wind waves over great distances in the ocean. Once wind waves have left the storm area, their propagation is virtually undisturbed until they approach a coast, where the water depth becomes comparable with the wave length and the wave rays bend according to known refraction laws.

† An exception to this statement is the recent work of Hasselmann (1974).

Ultimately the waves break and dissipate their energy on the beach. However, over most of their propagation path, there are very few physical processes which can significantly alter the waves according to the observations which are available. For example, none of the following processes appears to be important outside of storm areas and away from coasts: the effect of the earth's rotation; molecular or turbulent viscous dissipation; propagation through strong wind belts; interactions with internal gravity waves; interactions with other surface waves; contamination of the surface by films or oil or other material. However, there is one process which can drastically alter the propagation of wind waves in the open ocean, and that is the interaction of the waves with major ocean currents. The simple theory describing this phenomenon in terms of well-known refraction laws is available. Waves can be totally reflected or internally trapped by currents under situations which would seem to be not very uncommon in the ocean. As yet there are no observations with which to check these rather remarkable predictions of refraction theory.

In general, relatively little work has been done on the interaction of waves and currents. In particular, there is no satisfactory theory available which can describe the observed simultaneous generation of waves and currents by the wind. The two problems of how much energy and momentum goes into waves and how much goes into currents when the wind blows still need a great deal of attention. Perhaps the two problems are intrinsically coupled and should not be studied separately.

7.2. Observation and instrumentation

Observations and instrumentation have been adequate to provide, with one exception (below), reasonable estimates of the growth of the wave spectrum under the action of a 'steady wind'. These relatively few sets of data have provided some crucial insight into the processes of wave growth and have been most useful in evaluating various wave generation theories. Some limited data are also available on the amount of energy and momentum put into the wave field under generation conditions. However, these data demonstrate serious inconsistencies when interpreted in the light of the growth observations or when compared with each other.

In regard to dissipation we have evidence to describe, although not explain, the effects of shallow water on the spectrum. The effect of an opposing wind field on the wave spectrum is only partially documented for swell conditions, but not documented at all for a fully arisen sea. There are no direct observations of dissipation due to white capping.

The characteristic shape and behaviour of the spectrum under generation circumstances are known and qualitatively explained. The high-frequency portion of the spectrum in most cases is well described, though not completely understood, in terms of a frequency power law. The directional properties of an actively growing spectrum are also reasonably well documented but not well explained.

In the areas of observation (and instrumentation) the most critical need is for data on the energy input from the atmosphere to the wave field. It appears that such observations must be accompanied by a simultaneous, quantitative description of the statistical properties of both the wave field and atmospheric field immediately above the sea surface. These measurements must be carried out for a wide range of atmospheric stability. The instrumental difficulties of accomplishing these objectives in the field appear tremendous but not impossible. On the other hand, such measurements in the laboratory appear quite feasible, although of unknown representativeness.

Additional wave growth measurements, plus the type of observations called for above, are badly needed for: (i) High wind speeds (>20 m s^{-1}) were blowing spray and foam transform the air–sea 'interface' into a 'boundary layer' of variable density. Under these conditions tangential forces (largely ignored to date) may contribute to wave growth via acceleration of spray which eventually returns differentially† to the wave field. (ii) Waves whose phase velocity is comparable with or faster than the 'local' wind velocity. Such waves generally are associated with the spectral peak (maximum energy), yet most of the available growth data relate to waves for which $c/U > 1$. Better documentation of the growth and atmospheric energy input (outflow?) at $c/U \geqslant 1$ will be vital to the testing of wave generation theories.

Observations of wave dissipation due to white capping are desperately needed. They will probably require estimates of $F(\mathbf{k})$ in order to separate out the effects of wave–wave interactions. A measurement technique to accomplish this simply and practically over reasonably large areas is not available. Perhaps more effort in the area of mapping the rough sea surface with electromagnetic radiation would ameliorate this problem. Assuming this done, and the wave–wave effects accounted for, the experimentalist is left with the formidable problem of accounting for energy dissipation due to white caps. A good quantitative method of describing white caps does not exist, let alone a means of estimating the energy they might dissipate. Obtaining this information, however, is vital to further progress in wind wave research.

The behaviour of wave trains propagating through a caustic point has recently been described in the laboratory, but additional information would be welcome. Unfortunately, no data are available to confirm directly the potentially large effects of major currents on the directional properties of the wave spectrum. Also, no direct measurements of the wave momentum associated with the Stokes drift velocity are available in the ocean at this time.

7.3. *Wave prediction*

In view of the above advances it is not surprising that substantial progress has been made in the problem of wind wave prediction. A basic framework for prediction has been found in the radiative transfer equation. The propagation of wave energy is properly accounted for in this framework by either of several schemes, each with differing amounts of error. Specification of the source function is a blend of theory and data, but generally leads to a nonlinear integral form of (3.17). However, forms of S so far proposed are certainly in need of improvement. The specifications of S, while somewhat different, seem to produce equally good hindcasts, indicating that at least a rough quantitative estimation of S is available under many different geophysical situations.

The basic problem now appears to lie in a better *specification* of the wind field, for the total energy in the wave field is known to be roughly proportional to the square of the 'local wind' speed. A small error in wind, say 10%, can thus lead to a 20% error in the total predicted energy. It seems also that a more sophisticated description of the wind field may be required. Perhaps wind stress determined from satellites or input information in the form of wind velocity, atmospheric stability/turbulent intensity will be required to improve predictions.

The prediction of waves in shallow water and/or for hurricane conditions is just beginning. Also, presently available wave prediction schemes do not account for the

† That is, on the upwind side of the wave profile.

potentially important effects of the interaction of waves with major ocean currents. Much of the future advances in the area of wave prediction can be expected in these areas.

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