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REVIEW ARTICLES

The measurement of the optical transfer functions of lenses

The measurement of the optical transfer functions of lenses

K. ROSENHAUER AND K.-J. ROSENBRUCH

Physikalisch-Technische Bundesanstalt, Braunschweig, Germany

Contents

Abstract. After a short survey of the theoretical relations between the optical transfer function and the image aberrations and diffraction, the basic requirements for the measurement of optical systems are discussed. The necessary parts of such measuring devices are then described: these are the optical part, the electronics and data processing. The large number of measuring methods described in the literature are considered in three groups, namely (i) direct methods, (ii) methods using the Fourier technique and (iii) indirect methods.

After a description of some of the methods of each group, their common factors and their differences, their advantages and disadvantages as to costs and measuring time, and their sources of error as well as the attainable accuracy are discussed in detail. Further applications, which are strictly outside the theoretically valid scope, for example partially coherent or polychromatic light, are treated. The application of the methods for testing photographic emulsions is then described. Finally, the problems of quality criteria derived from optical transfer function curves are discussed.

1. Aberrations and resolving power

The image-forming rays in optical systems do not meet at one point (i.e. they are not homocentric) because of aberrations, so that a point object does not form a point image but a diffusion patch. This patch is caused both by aberrations and diffraction of light. The aberrations occurring with monochromatic light are generally called spherical aberration, astigmatism, comatic aberration, curvature of field and distortion. There are also the chromatic aberrations which are distinguished as (i) the chromatic aberration of single-image points, meaning that the foci or image points for the different wavelengths do not coincide, and (ii) the chromatic error of magnification caused by the different focal lengths of the various wavelengths producing different image sizes for the individual wavelengths (Herzberger 1958, p. 299).

It is rather difficult to judge the combined effect of all aberrations from those of single aberrations. Moreover, the distribution of the energy over the entrance pupil of the lens is generally not considered. A measuring method which integrates over all aberrations is therefore preferred. The known forms of optical test-bench represent such methods, and usually the resolving power of the optical system for selected objects is determined. Such test objects include two-line targets, three-line targets, sector stars, or other figures, isolated letters or gratings. The resolving power for such objects can be observed directly by the eye, or photographed and then visually analysed. The results then depend not only on the quality of the optical system but also on the particular properties of the observer's eye and on the photographic material used (Reckmeyer 1934, Roeder 1941, Hansen 1942).

2. Optical transfer function

In order to avoid this shortcoming, new techniques and measuring methods have been used in recent years. The optical system can be compared to an electrical circuit where the object corresponds to the input and the image to the output. A sine wave, in the optical case a sine distribution of intensity, will appear again as a sine wave in the output if the transmitter is linear. As transfer theory was well developed for telecommunication systems, it was easily applied to optical systems. In this way an objective measure for image quality was found for incoherently illuminated objects. It is, in principle, relatively easy to calculate and measure this function. The limits of application and the differences between transfer theory in telecommunications and in optics are described by Born and Wolf (1964, p. 459).

An optical system cannot, of course, be characterized by only one transfer function, but requires several functions, the number of which depends on the parameters of the lens, for example, the f number, focus position, image angle, wave length of light, etc. The measurement of the optical transfer function represents an appreciable step forward compared with the measurement of the resolving power, for this is a threshold value with no absolute significance. Moreover, it is generally not sufficient to consider only a pair of separated object points, but it is necessary to treat the image of the whole object. The quality of an optical system has then to be defined in such a way that it specifies how far the image is similar to the object, not only geometrically but also with regard to the contrast between neighbouring points. The optical transfer function is distinguished from the electrical transfer by the fact that, in the latter case, the signals to be transmitted are temporal, whereas they are spatial (in one or two dimensions) in optical transfer theory. With this change, one may consider any optical system as a transmitter, or filter, from the object to the image.

3. Qualities of the optical transfer function

The characteristic transfer qualities of an optical system are described by the so-called optical transfer function, sometimes called the frequency response function. It is determined by aberrations and diffraction ignoring the influence of stray light and often neglecting variations of transparency over the pupil. If there are further incoherent image processes, such as photographic emulsions, images on ground glass, projections, etc., the total transfer function can be determined by the product of the functions of these linear processes. The product function then determines the resulting image quality.

Any distribution of intensity can be represented by a two-dimensional Fourier integral or series. This means that one can regard any given intensity distribution in the object plane as the superposition of sinusoidal intensity distributions. Vice versa, one can deduce the corresponding frequency spectrum from any intensity distribution. On these grounds one-dimensional periodic sinusoidal distributions of intensity are employed to find the transfer factors for the different sine-wave structures by calculation or experimentally. The optical transfer function then indicates how the total frequency spectrum of the object, which is limited in all cases by a cut-off frequency, is transferred to the image, both as to amplitude and phase of modulation, by the optical system.

The bandwidth of spatial frequencies transferred by the optical system and the degree of contrast modulation are determined by the numerical aperture and the aberrations respectively. From Fourier analysis of images one knows which frequencies must be transferred for a particular use, and hence the required degree of correction of the aberrations. Herein lies the practical importance of transfer theory for the optical designer.

The mathematical bases of the transfer theory for optical systems have been known for some time, and will be reviewed briefly. Duffieux (1946) showed that the Abbe theory of the image formation of coherently illuminated objects may be usefully restated in terms of Fourier analysis. These bases were further developed by Schade (1948), Maréchal (1947) and Hopkins (1953).

The optical imaging process can be described by means of a convolution integral for the incoherent illumination. It is, however, required that the image formation be linear and isoplanatic. Linear means that there are linear mathematical relations between the intensity of object and image. The so-called condition of isoplanatism appears to be sufficiently satisfied for corrected optical systems.

4. Basic principles of the theory of optical transfer

Using normalized coordinates for object plane and image plane as well as for the entrance and exit pupil, the intensity distribution of the image $B'(u'v')$ can be written as a convolution integral of the object intensity distribution $B(u,v)$ and the point spread function $G(uv, u'v')$:

$$B'(u',v') = \frac{1}{2\pi}\iint B(u,v)\, G(u'-u, v'-v)\, du\, dv. \tag{1}$$

The form of the point spread function, determined by the aberrations, is proportional to the square of the amplitude distribution in the image of a point source:

$$G(u',v') = |F(u',v')|^2. \tag{2}$$

Under certain conditions, the amplitude distribution can be represented as a two-dimensional Fourier integral according to the diffraction theory of Kirchhoff. This takes the form

$$F(u',v') = \text{const.} \iint f(x',y') \exp\{2\pi i(u'x'+v'y')\}\, dx'\, dy' \tag{3}$$

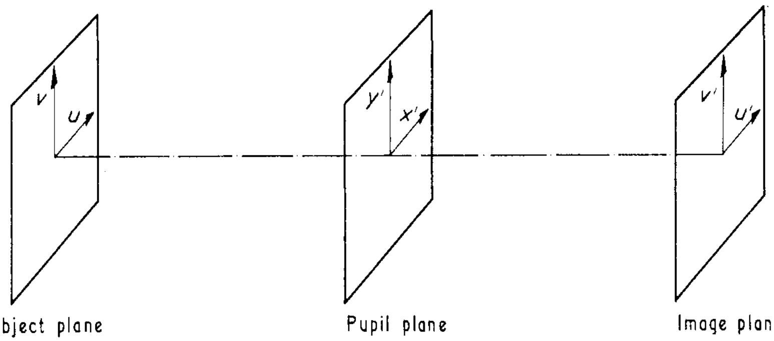


Figure 1. Definition of coordinates: u, v in object plane; x', y' in pupil plane; u', v' in image plane.

where the function $f(x',y')$ is called the pupil function. Apart from a constant it is the inverse Fourier transform of the amplitude $F(u'v')$. The same relations define the Fourier spectra of the intensities. Thus

$$G(u',v') = \iint g(s',t') \exp\{2\pi i(s'u'+t'v')\}\, ds'\, dt' \tag{4}$$

$$g(s',t') = \iint G(u',v') \exp\{-2\pi i(s'u'+t'v')\}\, du'\, dv'. \tag{5}$$

Putting these equations in the convolution integral, we obtain

$$B'(u',v') = \iint g(s't')\, b(s',t') \exp\{2\pi i(s'u'+t'v')\}\, ds'\, dt' \tag{6}$$

$$b'(s',t') = g(s',t')\, b(s',t'). \tag{7}$$

The optical transfer frequency is now defined as the inverse Fourier transform of the point spread function $G(u'v')$:

$$g(s',t') = \frac{b'(s',t')}{b(s',t')}. \tag{8}$$

Corresponding to the treatment of circuit theory one represents the object function and the image function by means of the Fourier transformation in frequency space. There are, however, fundamental differences between the two cases of incoherent image formation and electrical circuit theory. For incoherent objects, an optical system necessarily always acts as a low-pass filter with perfect response for zero frequency. This follows from the fact that the input and output in the optical case, being intensities, are necessarily described by positive and real functions.

It stands to the credit of Hopkins (1953) to have recognized that the so-called optical transfer function (8) is a measure of the image quality of extended objects, for this can be expressed as the autocorrelation of the pupil function:

$$g(s', t') = \frac{1}{2\pi} \iint f(x', y') f^*(x' - s', y' - t')\, dx'\, dy'. \tag{9}$$

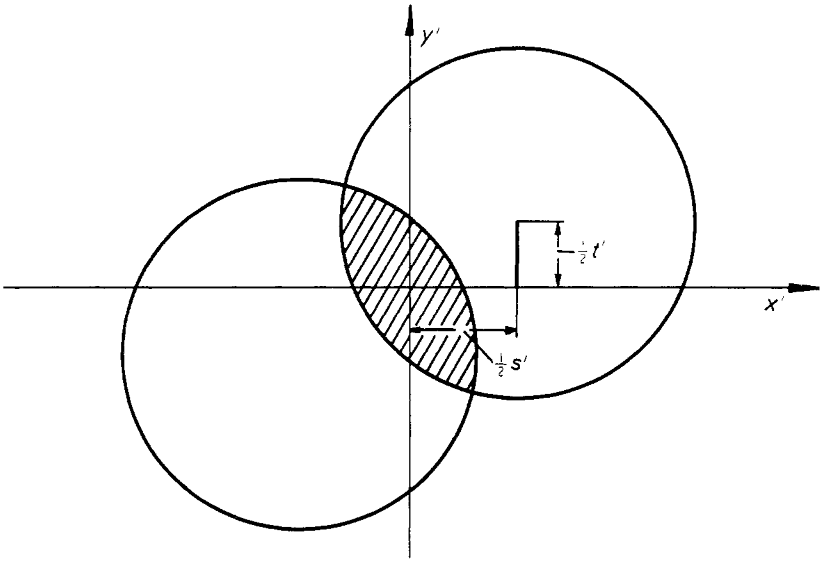


Figure 2. Region of integration: the hatched area, which is the overlapping part of the two sheared pupils, is the region of integration.

For calculation of the optical transfer function from this formula, it is only necessary to know the wave function. This can be measured or calculated from the construction data of the optical system. If the Fourier transformation is not used, one would first have to calculate the point spread function from the wave aberration, then calculate the image intensity by convolution, and only then determine the optical transfer function. It is now advantageous to introduce a relative transfer function normalized by the optical transfer function of zero frequency:

$$D(s', t') = \frac{g(s', t')}{g(0, 0)}. \tag{10}$$

The transfer function is generally complex and can therefore be expressed in a real and an imaginary part, or in modulus and phase. According to a formula due to Hopkins (1955), one can shear the pupil coordinates x' and y' by an amount $\frac{1}{2}s'$ and $\frac{1}{2}t'$ so that the normalized optical transfer function becomes

$$D(s', t') = \frac{\iint f(x' + \frac{1}{2}s', y' + \frac{1}{2}t') f^*(x' - \frac{1}{2}s', y' - \frac{1}{2}t')\, dx'\, dy'}{\iint |f(x', y')|^2\, dx'\, dy'}. \tag{11}$$

This integral means that the region of integration is the region common to two relatively displaced pupils, centred on the points $(\pm\frac{1}{2}s', \pm\frac{1}{2}t')$. If the system is free from aberration, $W(x',y') = 0$, and the optical transfer function is then equal to the area of the region common to the displaced pupils. For a circular pupil

$$D(s') = \pi\{2 \arccos \tfrac{1}{2}s' - \sin(2 \arccos \tfrac{1}{2}s')\}. \tag{12}$$

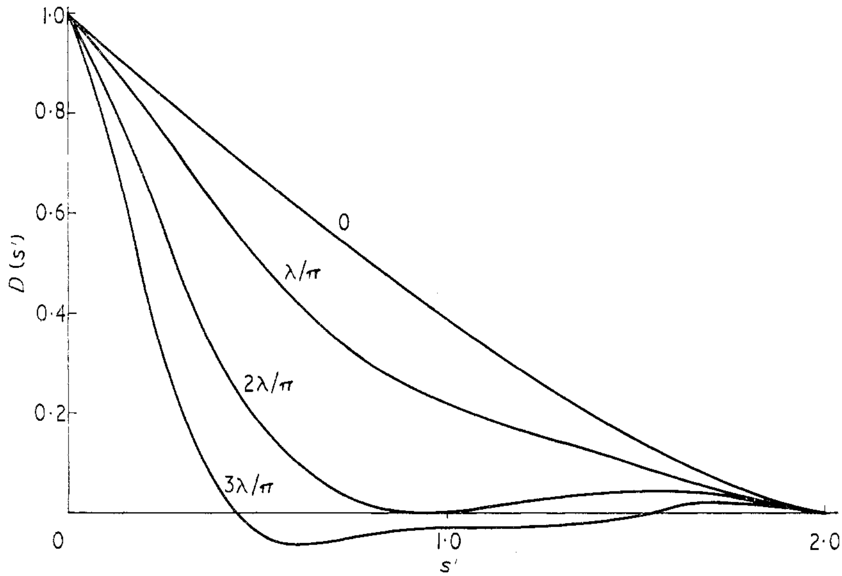


Figure 3. Optical transfer function for a lens with different amounts of defocusing. The values on the curves give the size of wave aberration; 0 gives the curve for a lens free of aberration.

If we assume uniform amplitude over the wave fronts, the pupil function is then of the form

$$f(x',y') = \exp\left\{\frac{2\pi i}{\lambda} W(x'y')\right\} \tag{13}$$

where $W(x'y')$ is the aberration of the wave front from an ideal spherical wave centred on the image point. Using (13) equation (11) can also be written

$$D(R) = \frac{\iint_A \exp(2\pi i/\lambda)\, W(x'+\tfrac{1}{2}bR, y') - W(x'-\tfrac{1}{2}bR, y')\, dx'\, dy'}{\iint_a dx'\, dy'} \tag{14}$$

where λ is the wavelength of light, b is the radius of the ideal spherical wave originating from the image point, R is the spatial frequency in lines/mm, $s' = \lambda bR$, A is the integration area equal to the region common to two displaced pupils, centred on the point $(\pm\frac{1}{2}\lambda bR, 0)$, and a is the region of the pupil. For simplicity the direction of the spatial frequency is taken perpendicular to the direction of x'.

5. Methods for measuring the optical transfer function

The main advantage of using the optical transfer function for the specification of image quality is that there are several measuring methods available for obtaining those functions rather easily. They can then be used to control the quality of practical optical systems.

In the last fifteen years many methods have been published of which only some can be described here. They have been selected for the very different principles used, in order to show the great variety of techniques. In the references at the end of this article about thirty publications are cited which explain the different methods and contain detailed descriptions.

5.1. *Mechanical and optical demands*

For measuring the optical transfer function a stable holding device is necessary for the test lens, permitting precisely defined adjustments and displacements, to allow measurements on the lens in the desired image planes at different field angles and in different azimuths. The demands for precision and stability of the mechanical construction of the testing device are different according to focal length and aperture. For example, a displacement of the image plane by 3 μm causes a change in the optical transfer function of 0·10 for a lens with an aperture of f/2 equivalent to a numerical aperture of 0·25. For larger apertures the error increases.

Since the theoretical basis of the optical transfer function is only valid for incoherent object illumination, this requirement must be satisfied in practice. This means that a sufficiently large aperture should be used for the illumination system by means of ground glass or other diffusers. Moreover, the aberrations vary with the object distance so that the desired object distance must be employed, or simulated by an optical auxiliary such as a collimator with sufficiently long focal length. Using auxiliary optics, care must be taken not to disturb the conditions for incoherence nor to change the optical transfer function of the test lens by aberrations in the auxiliary system. Since calculation of the influence of such auxiliary devices on the measured result is often rather difficult, one should avoid them, or ensure that they are of such high quality compared with the test lens that they need not be taken into account.

5.2. *Basic components of the systems*

The fundamentals of all optical transfer function measuring methods consist of the light source, the test object, the test lens in its holding device and an image receiver with electronic detection devices. The main difference between the various methods lies in the form of the test object and the extent of the role of the electronic devices.

6. Direct measuring methods: grating test methods

The optical transfer function is obtained in the *direct* measuring method using as the object a grating with sinusoidal transparency, which is illuminated incoherently and with the grating period variable over the spatial frequency region of interest. The test lens images the test object in the desired image plane. The intensity distribution in the image, which is also sinusoidal, can be measured by means of a very narrow slit with a photomultiplier and electronic detection. From the intensity distribution J the contrast in the object or the image is defined as

$$K = \frac{J_{\text{max}} - J_{\text{min}}}{J_{\text{max}} + J_{\text{min}}}$$

where J_{max} is the intensity at the maxima, and J_{min} the intensity at the minima of the test object.

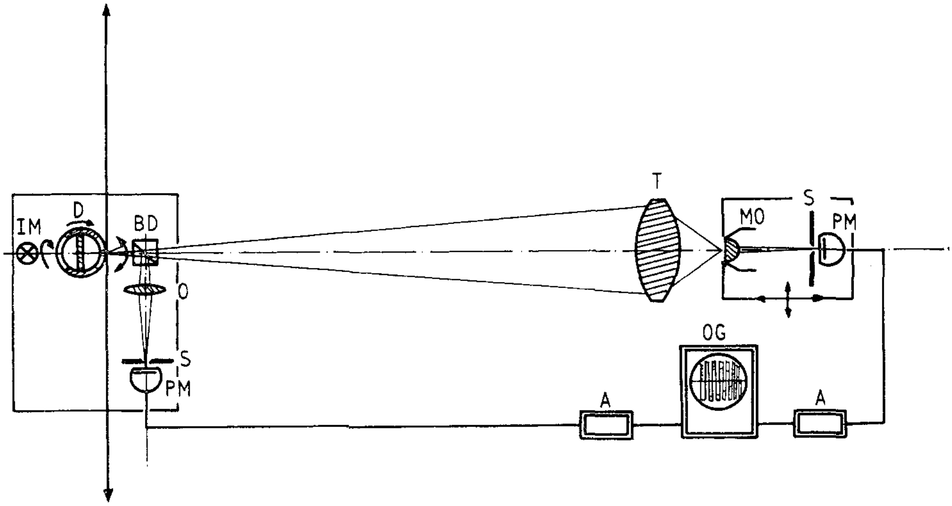


Figure 4. Direct-scanning optical transfer function measuring method: IM, illumination unit with light source, condenser, filter and ground-glass plate; D, rotating drum with grating as test object; BD, beam splitter; T, test lens; MO, microscope objective; S, slit; PM, photomultiplier; O, ideal lens; A, amplifier; OG, oscilloscope.

The optical transfer function is the quotient $K_{\text{image}}/K_{\text{object}}$. If we change to a different spatial frequency, different target gratings are required which are put on a drum in sequence (Rosenhauer and Rosenbruch 1957). In this case the drum rotates and the scanning slit can remain stationary in the image field, the scanning movement being provided by the rotation of the drum. The spatial grating images are thus transformed by the photomultiplier into temporal electrical signals. The signal coming from the multiplier can be displayed on an oscilloscope and then be seen on the screen in its original spatial form (see figure 5 (plate)). In the linear transfer region of the multiplier, the envelopes of the oscilloscope images are the optical transfer functions, provided that K_{object} has a constant value and that the number of periods of the different spatial frequencies and their succession around the drum are such that a linear scale of spatial frequencies is given. As the different spatial frequencies correspond to different temporal frequencies in the oscilloscope, the amplifier and multiplier are required to have sufficiently wide band characteristics. Greater difficulties, however, lie in producing a test object with sinusoidal transparency and different spatial frequencies with the contrast $K_{\text{object}} = 1$. Since the test grating is scanned in only one direction, other methods are used in which masks of varying height in the direction perpendicular to the scanning direction are employed, which, integrated with a correspondingly long slit in the image plane in this direction, produce effectively sinusoidal transparency (Shannon and Newman 1963, Herriot 1958, Ingelstam 1959).

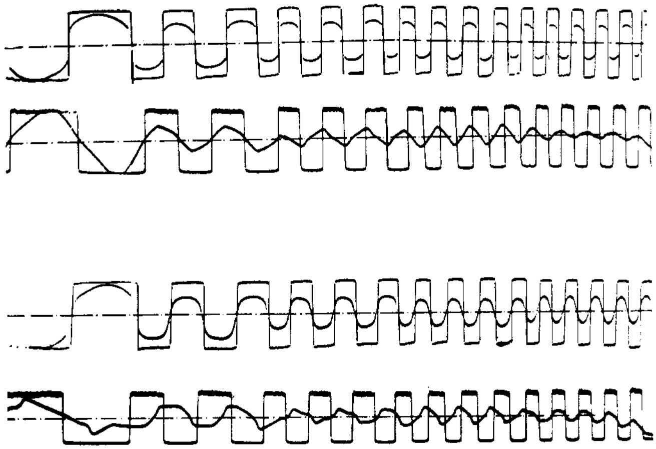


Figure 6. Temporal stretched display of the oscilloscope (see figure 5): upper curve, a good lens at 10° field; second curve, a low-performance lens at 10° field; the third and fourth curves give the results for the same lens at 14° field.

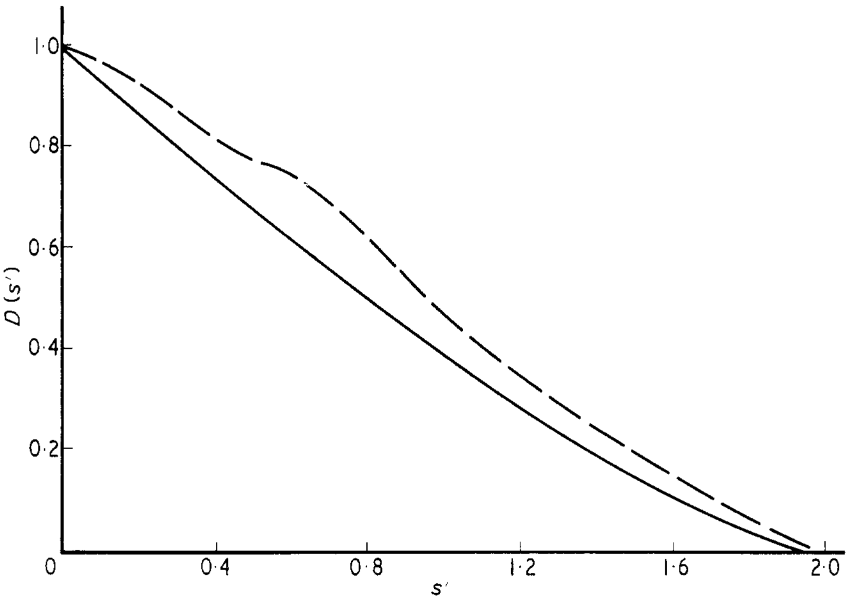


Figure 7. Optical transfer function for an aberrationless lens for sine-wave (full curve) and square-wave (broken curve) test gratings.

The same test objects are also applied to test the quality of the sound tracks recorded on sound film. Instead of sine-wave gratings square-wave gratings are sometimes used. The influence of harmonics on the results is taken into account by calculation, using a formula due to Coltman (1954). This calculation can be avoided by using a star sector with light and dark sectors of equal distances. By scanning the image of such a rotating star on concentric circles using a small aperture, the spatial frequency increases in inverse proportion to the scanning radius, but the electrical voltage produced in the multiplier behind the scanning hole has a constant temporal frequency independent of this radius (Lindberg 1954).

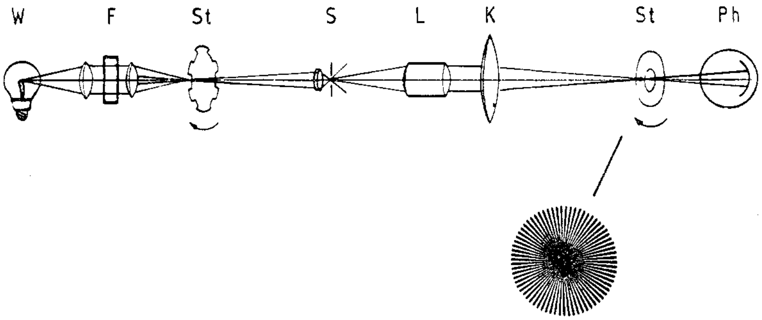


Figure 8. Sector star optical transfer function measuring method: W, light source; F, collimating and filtering unit; St, rotating sector with very low spatial frequency to normalize the optical transfer function; S, microscope objective to illuminate a small hole; L, test lens; K, collimator; St, sector star; Ph, photomultiplier (Ingelstam 1959).

The higher harmonics can easily be filtered out by suitable electric filters, so that only the first harmonic wave of the signal is detected. This has precisely the same effect as if the test object were sinusoidal. The same result can also be achieved by moving a test grating on which the different spatial frequencies are arranged side by side in inverse proportion to the spatial frequencies by means of a cam (Murata 1959). In this case, the multiplier also receives a constant temporal frequency independent of the changing spatial frequency, which can easily be transformed into a sine wave.

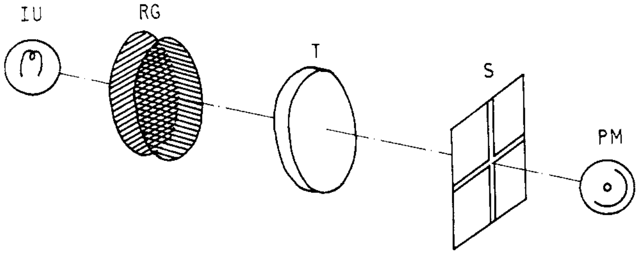


Figure 9. Optical transfer function measuring method with moiré pattern: IU, illumination unit; RG, the two rotating gratings, which produce the moiré fringes; T, test lens; S, scanning hole; PM, photomultiplier.

A nearly sinusoidal intensity distribution with continually changing spatial frequencies can also be produced by crossing two square-wave gratings with high spatial frequency. The spatial frequency of the resulting moiré pattern depends on the angle of crossing. If we put the two gratings on two disks rotating in opposite directions, moiré patterns are formed at the middle of the plane of the crossed disks

(Lohmann 1957, Lohmann and Bothe 1959, Förstner and Köhler 1960). The spatial frequency changes according to the distance between the centre and the slit.

Care must be taken to ensure that the spatial frequency of the test grating is suitably chosen with regard to the test lens. For the lowest spatial frequencies the optical transfer function factor should always be unity, and for the highest chosen spatial frequencies it should lie near to zero. If this is not possible with one test object, several test charts should be employed. Often, a special measurement is needed for the lower spatial frequencies in order to normalize the optical transfer function.

7. Indirect measuring methods

7.1. *Slit and edge methods*

Besides the direct measuring methods, it is possible to obtain the optical transfer function using test objects of any form by Fourier analysis of the object and image,

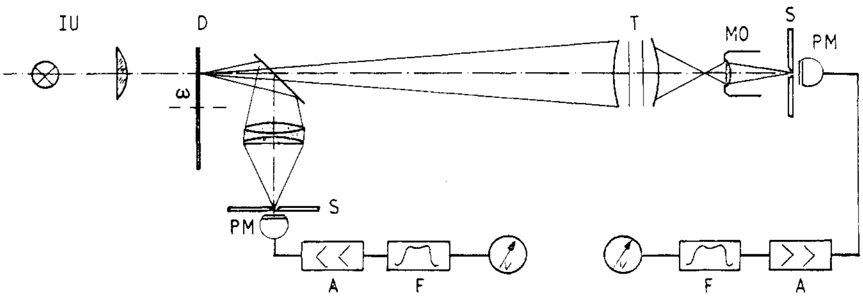


Figure 10. Optical transfer function measuring method by scanning the edge image with wide-band filters: IU, illumination unit; D, rotating sector disk; T, test lens; MO, microscope objective; S, scanning hole; PM, photomultiplier; A, amplifier; F, wide band filter; V, voltmeter. Two modifications are available: (i) fixed disk frequency, changing filter frequency; (ii) changing disk frequency, fixed filter frequency.

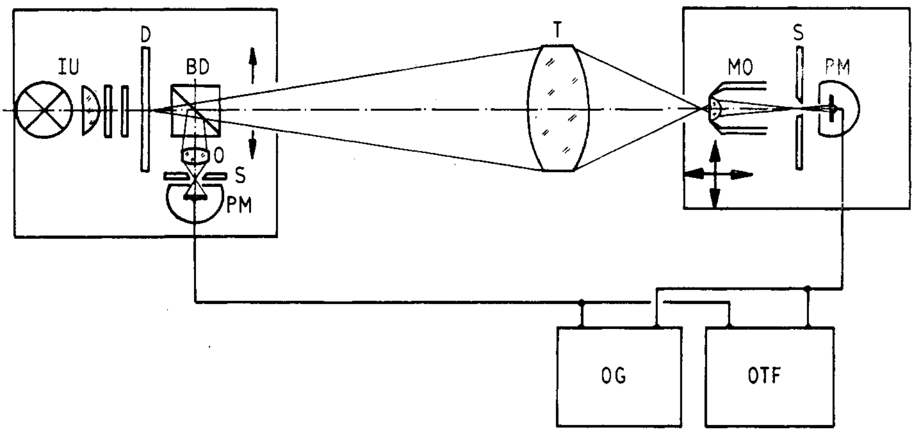


Figure 11. Optical transfer function measuring method by scanning the edge image with narrow-band filters: OG, oscilloscope; OTF, electronic detector and plotter. Other symbols as in figure 10.

and hence obtaining the amplitudes and phases of the single Fourier components. Simple objects, easily constructed and frequently described, are slits and edges. The frequency spectra of an edge, or a rectangular-wave grating of low frequency with a mark-to-space ratio of 1 : 1 and a bright-to-dark contrast ratio of about

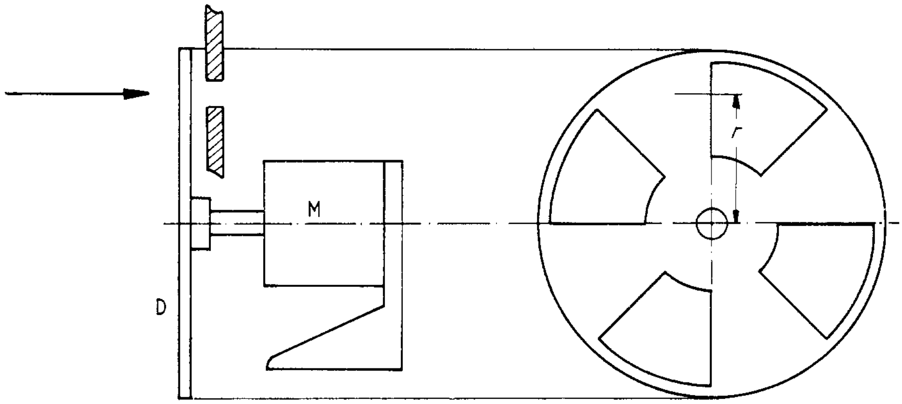


Figure 12. Front and side view of sector disk (figure 11): M, driving synchronous motor; *r*, radius of the scanning circle given by the position of the scanning hole.

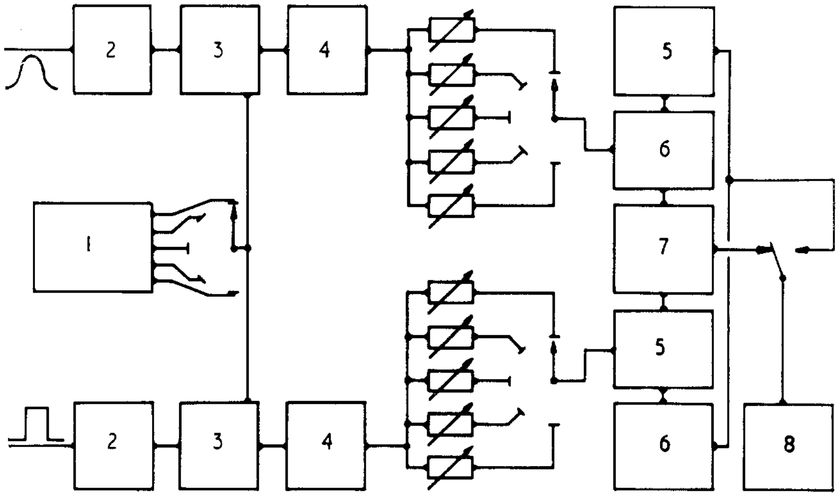


Figure 13. Block diagram of electronic circuit of figure 11 (two-channel analyser): 1, quartz frequency generators; 2, preamplifiers; 3, mixing stage; 4, 60 kHz filter, followed by adjustable resistors; 5, amplifier; 6, logarithmic amplifier; 7, phase meter; 8, plotter.

1 : 1000, are known. The optical transfer function can also be obtained by electronically analysing the image distribution which the test lens forms of an edge or of a very narrow slit; the amplitudes and phases of the output are then compared with those of the input (see figures 11, 12, 13 and 14 (plate)). These methods, which have been published with several modifications, avoid the use of complicated test charts but they need more complicated electronic systems for the Fourier analysis than the method described above (Rosenhauer and Rosenbruch 1957, Rosenbruch

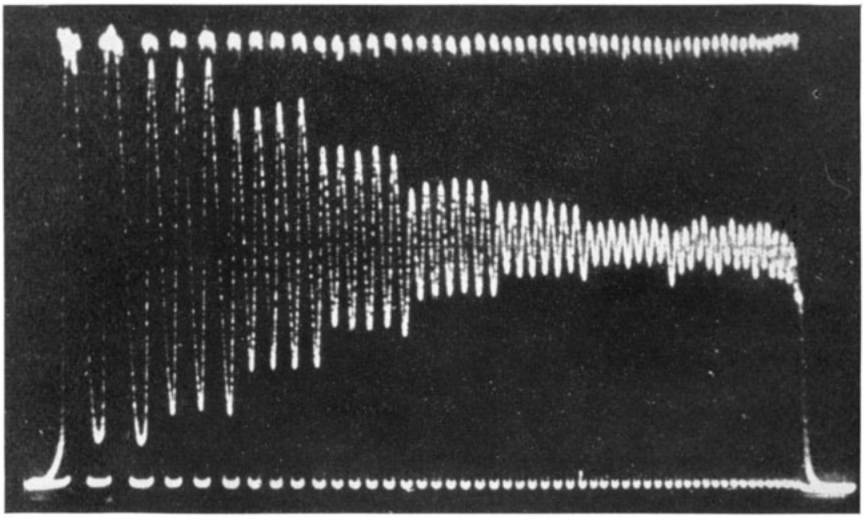
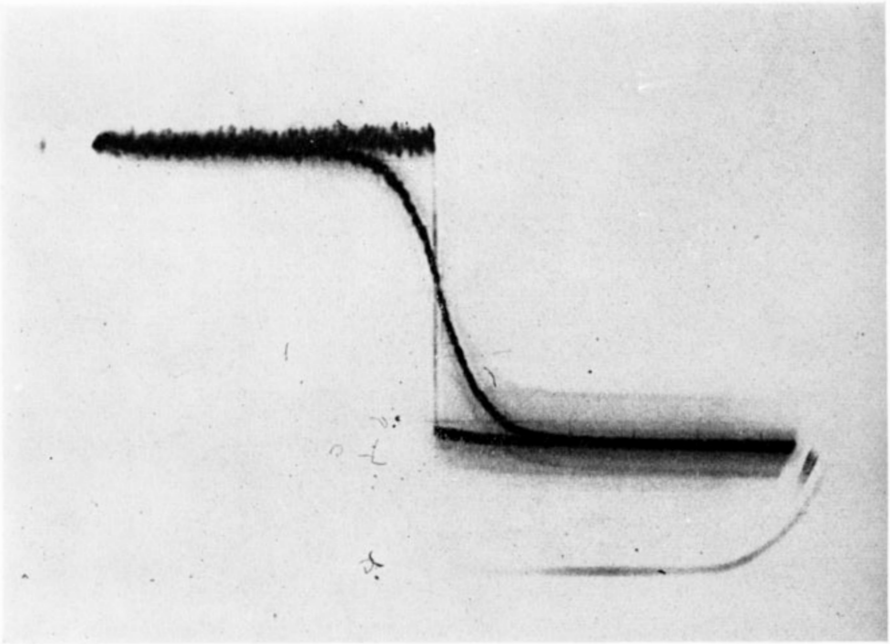
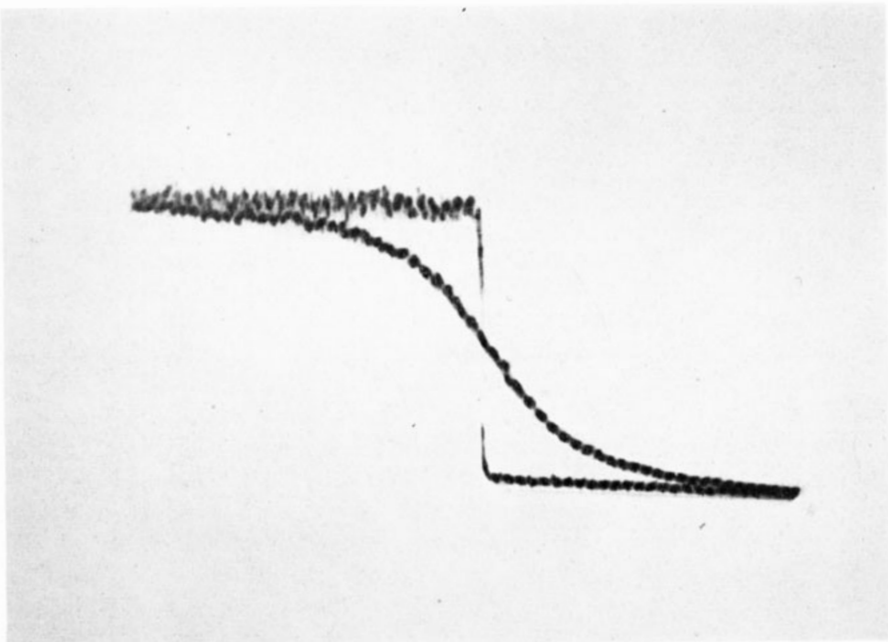


Figure 5. Oscilloscope display. A square-wave test object, the spatial frequency of which becomes smaller stepwise, is imaged by the test lens T and an ideal lens O (see figure 4).



(*a*)



(*b*)

Figure 14. Oscilloscope display (figure 11) of an edge image for a nearly ideal lens, and (*a*) a real focused and (*b*) defocused lens.

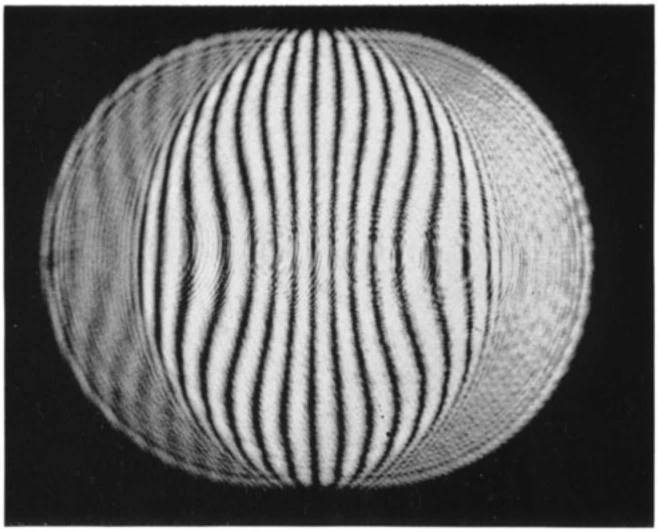


Figure 16. Sheared pupil with spherical aberration and defocusing produced by a plane-parallel plate interferometer.

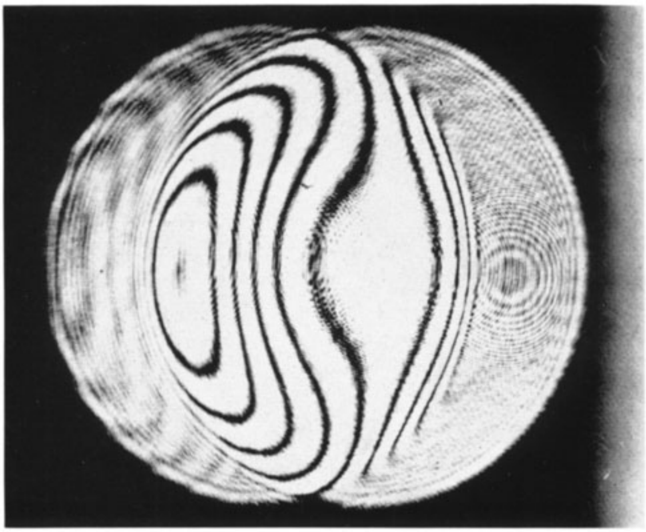


Figure 17. Sheared pupil with comatic aberration, symmetric (spherical) aberration and astigmatism.

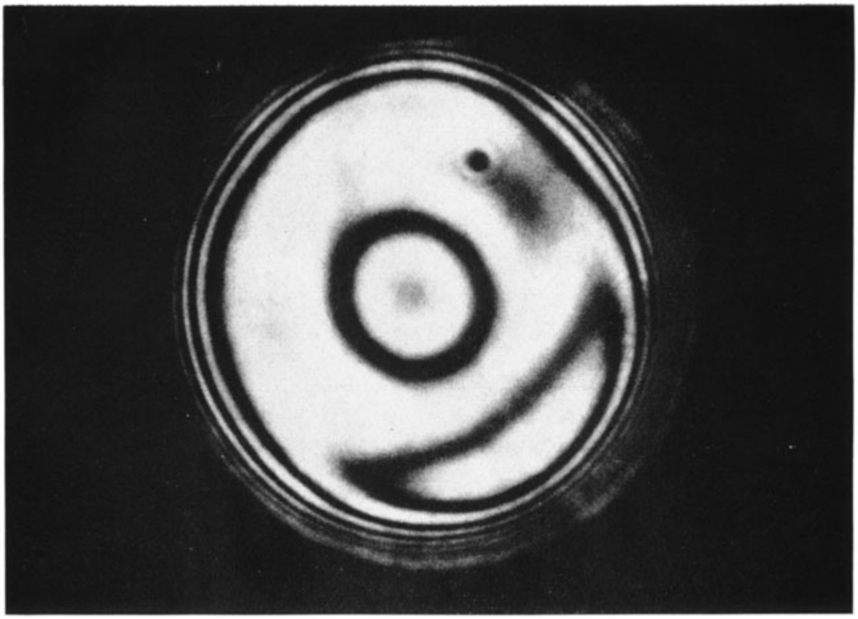


Figure 21. Wave aberration photographed from a Twyman–Green interferometer. Spherical aberration and comatic aberration are due to slight decentring.

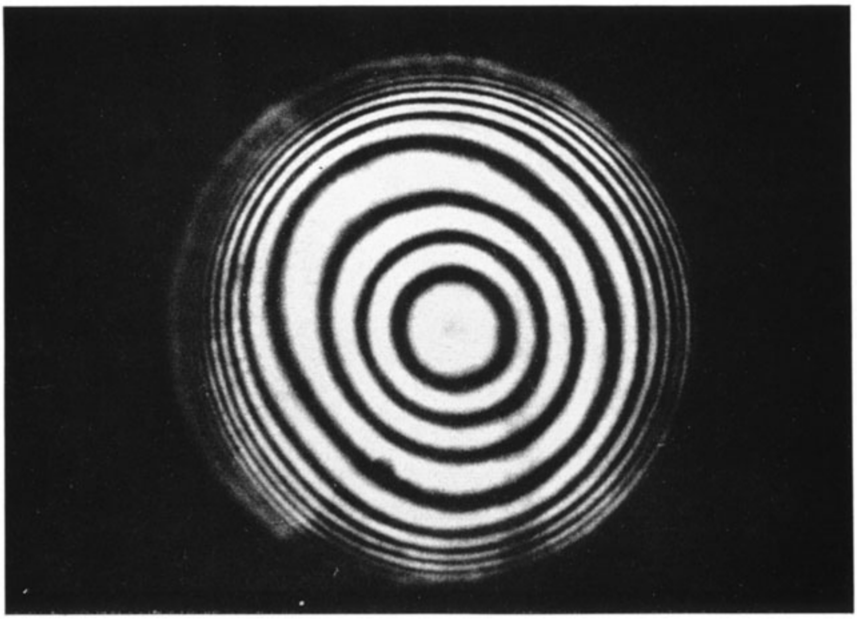


Figure 22. As in figure 21, but with additional defocusing.

and Rosenhauer 1964, H. D. Polster 1955, Rep. Perkin-Elmer Co., No. 413, Birch 1958, Sayanagi 1958).

7.2. *Interference methods*

A third group of methods measures the optical transfer function not in the image plane but in an image of the exit pupil, as first proposed by Hopkins (1955). By means of a shearing interferometer, two wave fronts are sheared against each other by an amount $S' = \lambda f R$ where λ is the wavelength of the light, f is the focal length of the test lens, or the radius of the ideal spherical wave for finite image

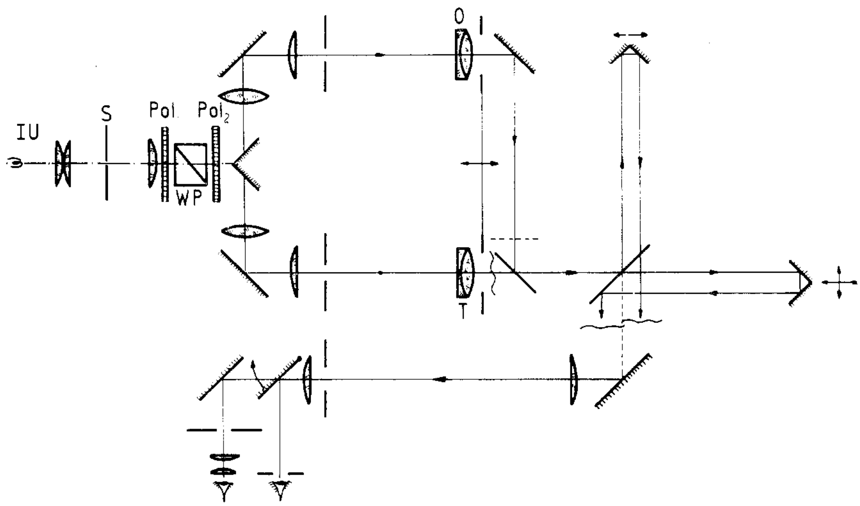


Figure 15. Shearing interferometer as optical transfer function measuring device: IU, illumination unit; S, slit; Pol_1 and Pol_2, polarizers; WP, Wollaston prism; O, ideal lens; T, test lens.

distance, and R is the spatial frequency. The overlapping region of the two sheared pupils has interference fringes determined by the path difference

$$W(x'+\tfrac{1}{2}s',y')-W(x'-\tfrac{1}{2}s',y')$$

where $W(x',y')$ is the wave aberration (see figures 16 and 17 (plate)). The total intensity in the overlapping part of the pupils is found to be proportional to the transfer factor.

Michelson's interferometer was first applied to the measurement of the optical transfer function in this manner by Baker (1955) and was later further developed for automatic operation by Kelsall (1959). As shown in figure 18 the wave front to be tested is passed into the interferometer. The mirrors in the two arms are fixed and the shear S' is introduced slowly by rotation of plane-parallel plates inserted in the optical paths. On the other hand, the phase difference is introduced quickly by moving the prism backwards and forwards. When the average brightness is normalized to unity, the output of the photomultiplier gives a record of $D(s')$ directly. The phase difference between the photocurrent and the reference signal, which is electrically generated, is detected by a phase meter.

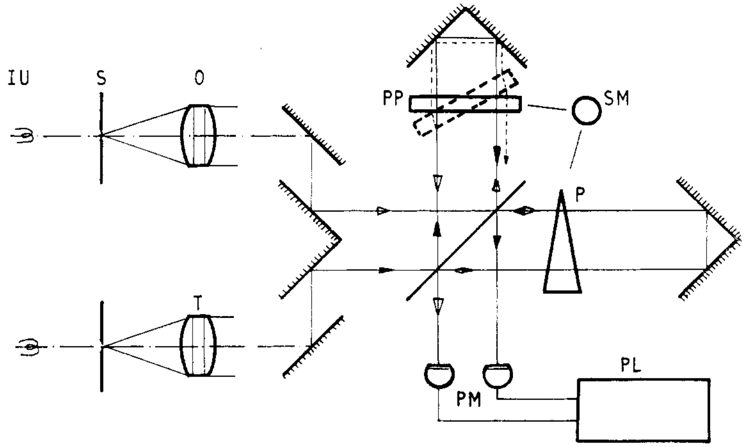


Figure 18. Optical transfer function measuring interferometer of Kelsall: IU, illumination unit; S, slit; O, ideal lens; T, test lens; PP, plane-parallel plate; P, prism; S, synchronous motor to drive the prism to modulate the light and to drive the plane-parallel plate to vary the spatial frequency; PM, photomultiplier; PL, plotter.

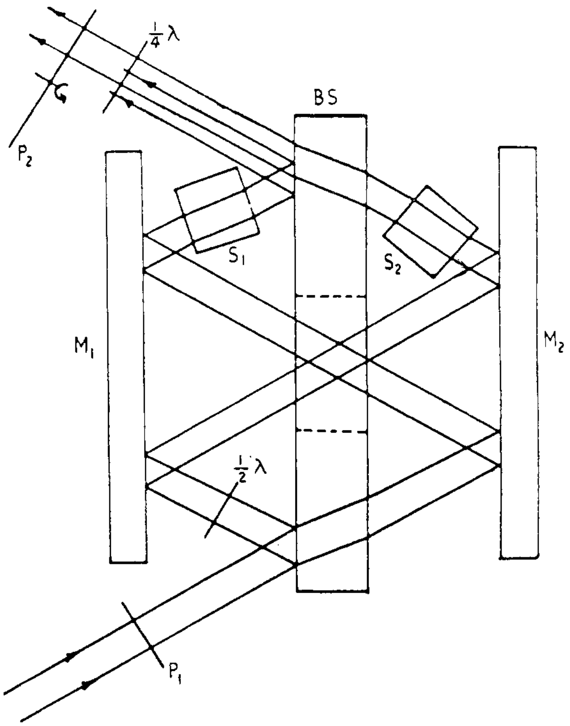


Figure 19. Optical transfer function measuring interferometer of Montgomery: M_1 and M_2, mirrors; BS, beam splitter; S_1 and S_2, shear plates; P_1 and P_2, polarizers. Collimated beam from test lens enters from bottom left (Montgomery 1966).

Since interferometers with two arms are sensitive to mechanical shocks or vibrations which cause slight tilting between the wave fronts, a new shearing interferometer was developed by Montgomery (1964, 1966). It has a rigid interferometric unit and a device for self-compensation tilting (figure 19). The collimated light from the test lens enters the interferometer unit and is split into two beams at one end of the beam splitter. The optical path of one beam is changed by the linear displacement of the prism to give the phase difference between the two beams. The shear of the wave front is introduced slowly by the rotation of the

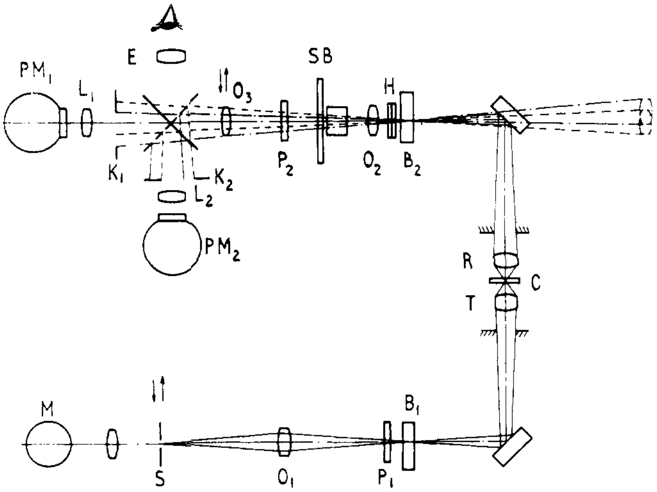


Figure 20. Optical transfer function measuring polarizing interferometer of Tsuruta for testing microscope objectives: M, light source; O_1, O_2 and O_3, lenses; B_1 and B_2, identical Wollaston lenses; T, test lens; R, reference lens; C, coverglass; P_1 and P_2, crossing polarizers; H, half-wave plate; SB, Soleil–Babinet compensator; E, eyepiece; K_1 and K_2, aperture diaphragms corresponding to the effective light source (if the light is not incoherent); L_1 and L_2, light collectors; PM_1 and PM_2, photomultipliers (Tsuruta 1963).

plane parallel plates. Since both light beams in the interferometer are reflected from each of the side mirrors and also from the beam splitter, relative tilting between the two wave fronts leaving the interferometer is self-compensated even if the three mirrors are not perfectly adjusted to be parallel.

Another type of interferometer which is used for measurement of the optical transfer function is the polarizing shearing interferometer. The wave front from the lens under test is split into ordinary and extraordinary beams by means of a birefringent double-image prism between two crossed polarizers. The shear of the two beams is determined by the birefringence of the material and the geometry of the prism. As the two beams pass along a common path to produce the interference fringes, the tilting between them is compensated. In addition, this interferometer is extremely stable against shocks and vibrations, because the optical path difference is produced by retardation in the solid birefringent prism.

In these autocorrelation methods, a slit is imaged at infinity by the lens under test and the aberrated wave front is put into the interferometer, which then acts as a Fourier analyser of the slit image. On the other hand, interference fringes produced by an interferometer can be used as a sinusoidal object for the measurement of an optical system (Tsuruta 1963). In the latter case, the interferometer acts as a generator of sine-wave objects, that is the measurement is based essentially on the direct method described before. These two methods with interferometers are equivalent to each other and differ only in the direction of light travel. The advantage of these methods lies in showing the aberrations and the optical transfer function at the same time; their disadvantage is the restriction to monochromatic light.

The method by which the shearing of the exit pupils is produced is not important for the measurement. However, any type of differential interferometer is well suited for this technique. Some methods are unsuitable owing to the introduction of more aberrations during the measurements compared with those occurring in normal use. For instance thick plane-parallel plates should not be allowed in convergent beams of light.

7.3. *Determination of geometrical and optical aberrations*

Finally, methods may be mentioned in which the geometric–optical aberrations are measured and the optical transfer function is calculated using the formulae mentioned above. The aberrations may be measured by means of any of the well-known geometric–optical methods described by Hartmann (1908) and Wetthauer (1921) or the methods of Ronchi (1928), Väisälä (1922) and others.

In order to calculate the transfer function the image errors must be presented in the form of the wave-aberration function. Different procedures have been used to find this from practical methods.

It is possible to find the wave aberration directly with the Twyman–Green interferometer or with a similar interferometric method (see figures 21 and 22 (plate)). On the other hand, it is also possible to find the wave aberration by integrating the geometric–optical aberrations, which may be measured experimentally, or which may be found from ray tracing. Sometimes it is possible to establish some characteristics of the optical system, when the standardized wave-aberration coefficients are known, without having to calculate the transfer function in detail which involves a relatively large amount of calculation. It may then be possible to vary the aberrations systematically so that, for all regions of interest, these wave-aberration coefficients are known; the form of correction of the system is chosen by linear interpolation between these coefficients to arrive at a suitable system for the purpose in hand. The optical transfer function is chosen as a so-called 'function of merit' for the corresponding spatial frequencies. Nevertheless, systematic investigations of the relations between the aberrations and the image characteristics have then to be undertaken (Rosenhauer and Rosenbruch 1965, Rosenhauer, Rosenbruch and Sunder-Plassmann 1966).

8. Calculation of the optical transfer function (general solution)

Several methods have been developed for the numerical calculation of the optical transfer function for any given spatial frequency, especially for computers.

An accurate method due to Hopkins (1957) has been employed by Goodbody (1958) whose results are used in some of the following examples. Barakat (1962) has proposed a method which employs the Gauss quadrature in conjunction with Legendre polynomials, and uses non-equidistant mesh size for the integration.

The methods mentioned above require digital computers, but these calculations have also been performed using analogue computers. These can be fast, and use exact mathematical relations without approximations. Such computers permit the study of the relation between the shape and magnitude of the spherical aberration and the optical transfer function (Rosenhauer, Rosenbruch and Siems 1963, Rosenhauer, Rosenbruch and Sunder-Plassmann 1966). There is the advantage of studying how to optimize a system in a short time by systematic variation of the values of the aberration coefficients set on a potentiometer. This method, of course, has a restricted mathematical precision, in a practical case, of about 1–2%.

9. Comparison of the various methods

The majority of the methods described for measuring the optical transfer function differ considerably with regard to the time required for the measurement, and the cost of electronic and other equipment. Very often, for example, a following data plotter can produce the optical transfer function curve directly on a linear spatial frequency scale.

For use in laboratories and factories it is usually of importance to have either a relatively high accuracy of measurement irrespective of the time required or a rather quick measurement that can easily be made on a large number of test lenses. For this reason, the various methods mentioned and their variants, not mentioned here but described in the literature, are often suitable for the special circumstances. As the measurement of the optical transfer function is a kind of microphotometry, the precision of measurement is restricted by the highest spatial frequency and the lowest possible light intensity that can be measured. By the choice of intense light sources, sensitive photomultipliers and electronic amplifiers with large signal-to-noise ratios, the limits reached can be widely different. The measurement of the optical transfer function of lenses in unusual spectral regions (e.g. in the infra-red and for the smallest lens apertures, say f/32) often requires a restriction of the spatial frequency range or a limited precision of measurement for the reasons mentioned. In addition to the accidental errors, which are present in every method owing to variations in the special constants of the apparatus and which affect reproducibility, there are always systematic errors which falsify the measured result. These last may sometimes be known, for example, the width of the scanning slit used in front of the multiplier, the frequency response of the electronic amplifier and the spectral distribution of the light source. These may be taken into consideration completely in the calculation as corrections for errors in the measurement. Sometimes, however, such influences remain unknown, and comparison of measurements of the optical transfer function of the same lens by different methods shows differences greater than the limits of error for the measuring apparatus in question allow. Carefully designed equipment generally allows a reproducibility of the contrast measurement of the optical transfer function to about $\pm 0{\cdot}02$.

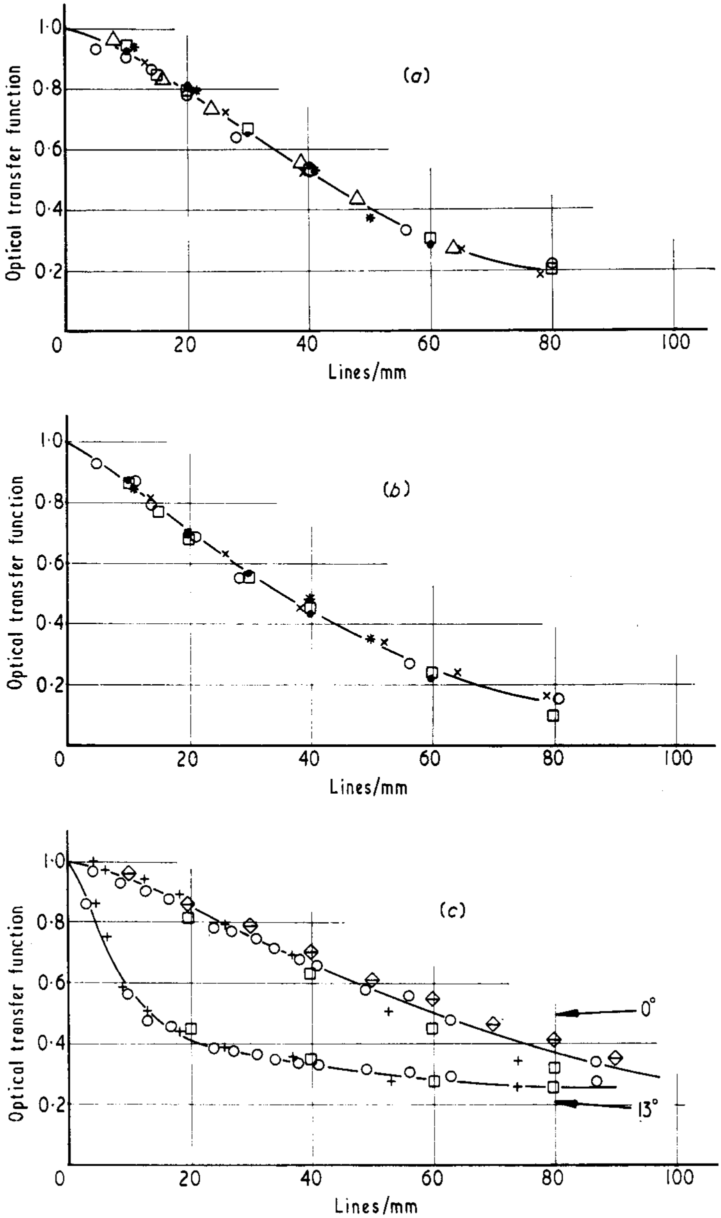


Figure 23. Comparison of different measuring devices on a lens: ○ edge image scanned by narrow-band filter, + edge image scanned by wide-band filter, □ square-wave grating, calculated for sine wave, $*$ sine-wave grating, ● sector star method, × slit image, mechanically analysed, △ calculated by analogue computer from spherical aberration, ▽ calculated by digital computer from spherical aberration. (*a*) Optical transfer function curve of a lens, focal length $f = 50$ mm, aperture f/2, best axial focus; (*b*) optical transfer function curve of the same lens as in (*a*) but $-0{\cdot}03$ mm away from best focus; (*c*) optical transfer function curves of a lens, focal length $f = 52$ mm, aperture f/2, best axial focus 0°, and for this image plane the meridional optical transfer function curve for a 13° field.

On some lenses, which have been measured with eight different optical transfer function measuring methods (in different laboratories), the authors have discovered contrast variations of up to 0·07 at the same spatial frequency. The greatest differences appeared in measurements of extra-axial image points, these being especially sensitive to slight inaccuracies of alignment. We are inclined to believe that the main sources of errors in measurements using well-designed equipment are not to be found in the optical and electronic part, but in the mechanical precision, stability of alignment fixings, holding devices of the test pieces and the mechanical stability of the test pieces themselves. Optical transfer function curves have been observed which often give appreciably different values and are never reproducible. This variation resulted from moving parts, such as occur in zoom lenses and interchangeable lenses. Sometimes larger errors of centring in lenses may give a variation with direction between the test object, lens and image receiver, which can easily cause significant variations in the measured values when this direction is in error even by a few minutes of an arc. These effects are greater the more complicated and the more corrected the optical system is, as then even the very smallest error may often reduce the value of the optical transfer function considerably. For this reason, it is not surprising that the optical transfer function, which may be calculated from the lens data, deviates more from the optical transfer function measurements than the limit of error would seem to allow.

In considering the errors of optical transfer function measurements, account has to be taken of which differences in optical transfer function may be recognized under normal conditions of use. Theoretical and practical research in connection with the criteria of image qualities has been carried out by many authors. Of course, this depends on the objects used, on the sharpness of vision and on the training of the observer, and on many additional factors for which a detailed explanation would be too lengthy here. But it may be concluded from the many studies published that the area under the optical transfer function curve, from zero up to the largest spatial frequency necessary for the observation and recognition of the object in question, must be modified by about 10% if any difference is to be seen as an image variation. If we bear this in mind, it can be said that the accuracy of the present optical transfer function measuring methods is sufficient, even under unfavourable conditions, to make the measured optical transfer function suitable for characterizing the image qualities of lenses.

10. Application of the optical transfer function methods to other regions

Measurement of the optical transfer function has become widespread in recent years, since it may be measured with sufficient accuracy by a variety of methods and because the final image of a cascade of imaging systems can be obtained simply. The optical transfer function of two cascaded imaging systems is obtained by multiplication of the single optical transfer functions. This considerable advantage shows the superiority of the use of the optical transfer function in comparison with all other test methods. However, the application of this simple rule of multiplication is only valid if the following conditions are fulfilled: (i) linearity; (ii) isoplanatism of the image. Linearity in transferring the light intensities is accomplished if the reproduced object is illuminated incoherently or self-luminously, that is by means of light waves having no stable phase relations. The condition of isoplanatism, that is the independence of the transfer qualities with respect to the position of the objects in the space, is well realized using corrected optical systems for a sufficiently large region.

Nevertheless, deviations from complete incoherence or from the condition of linearity may occur in some reproducing systems, and partially coherent illumination is obtained (Steel 1958). However, when the illumination is partially coherent, the system is linear neither in amplitude nor intensity, and it has been customary to use both amplitude and intensity in the analysis and to treat the problem in two stages; amplitude theory is applied for each point on the source and the results, converted to intensity, are summed over the source. A further integration with respect to the frequency may be required if the source is not monochromatic.

A theory of image formation which is linear for sources of any degree of coherence has been developed by Steel (1958). In this treatment the condition of linearity is rescued and a large number of the optical transfer function measuring methods described may be used with only small variations to measure a function quite similar to the optical transfer function (Menzel and Haina 1964). Optical systems working under such conditions include microscope lenses, especially in phase-contrast microscopy, projection lenses and enlarging lenses.

A further deviation from strict linearity which is important for the optical image of a cascaded system is given by the photographic emulsion. Owing to the advantages of using the optical transfer function in the estimation of the image quality of optical systems, the methods have been extended to treat the photographic process linearly. In a photographic emulsion, linearity has been achieved by separating the conversion of the incident light intensity into two steps of which the first one is linear.

The first step describes the scattering process which the light undergoes in the photographic layer. This scattering is linear with the incident intensity and may be described by a transfer function exactly analogous to the optical transfer function of lenses (Frieser 1955). The second step, which is the conversion of this scattered light intensity into developed silver grains by chemical processes, is not linear. During the development additional physico-chemical effects appear which are not linear, for example the Eberhard effect and neighbourhood effect. The first step essentially describes the microdistribution of the image, but the conversion of the incident light intensity to a developed image in the photographic layer is described by the so-called Hurter and Diffrill curve in cases where neighbourhood effects do not play a part.

Profiting by these considerations, it is possible to specify a transfer function curve for emulsions, also, and to determine this indirectly by the optical transfer function measuring methods. A test pattern is reproduced by a lens on the photographic emulsion which is to be tested, and the resulting image is scanned with a microdensitometer. Knowing the characteristic, or Hurter and Diffrill, curve, the transfer curve for the combined emulsion and lens is obtained, and from this, having measured the optical transfer function of the lens, the optical transfer function of the photographic emulsion is found (Ingelstam and Hendeberg 1959).

11. Optical transfer function for polychromatic light

It is often of interest to know how the overall optical transfer function of the image-forming system results from the monochromatic optical transfer functions, the spectral sensitivity of the receiver and the spectral distribution of the light source. Theoretically the optical transfer function is defined only for monochromatic light but there are various methods of combining these to calculate the polychromatic optical transfer function. Such a procedure is that described by Rosenhauer and Rosenbruch (1965). These calculations, although troublesome, are always necessary in those cases when the image system is to be used for spectral regions where suitable light sources and receivers are not available, as shown in the direct method of optical transfer function measurement described above. The polychromatic optical transfer function is then calculated from the data of the monochromatic optical transfer functions for given focal planes and wavelengths, and using the chromatic aberration, the spectral intensity distribution of the light source, the spectral sensitivity of the receiver and the spectral transmittance of the lens.

It is, of course, possible frequently to accomplish this spectral integration process by measuring the optical transfer function itself using suitable light sources and appropriate filters and receivers.

12. Different representations of the optical transfer function curves

As shown above, the optical transfer function may be determined by many methods. Its physical and mathematical relation to the aberrations is used in some of the measuring methods. For this reason a number of measuring methods can give at the same time experimental proof of the exactness of the theoretical connection of image errors and the optical transfer function. Whereas the image errors themselves are useful for the optical designer in the analysis of an optical system, the optical transfer function gives an integral measure of how these errors and diffraction influence the images formed. Using Fourier analysis and synthesis it is easy in principle to determine exactly the image of any object desired.

Even in a corrected optical system the isoplanatism condition holds only for small regions, the image quality changing with the angle of the field and with the image distance. Moreover, the optical transfer function, being an integral measure for all image errors, changes if any one image error is varied. Image errors which are easily varied in practice are defocusing and the dependence of the image errors on the azimuth. This naturally leads to the fact that for a complete description of the image qualities of a lens not only one function but a whole system of optical transfer function curves is necessary. Since a large number of curves is often difficult to comprehend, other methods of presenting the optical transfer function values are often adopted. For example, using a coordinate system, on the abscissa of which are recorded the field angles and on the ordinate the defocusing, the lines of the same contrast for a fixed spatial frequency and a fixed azimuth are registered. These figures are similar to the contours of mountains on a map and are sometimes called 'contrast solids'. In most cases such figures are made for some fixed spatial frequencies and one obtains at a glance a general view of how the image quality varies as a function of the field angle and the focal plane. Such a diagram, easily derived from the optical transfer function curve, may be produced by a suitable device directly from the lens, as shown by Murata and Matsui (1956).

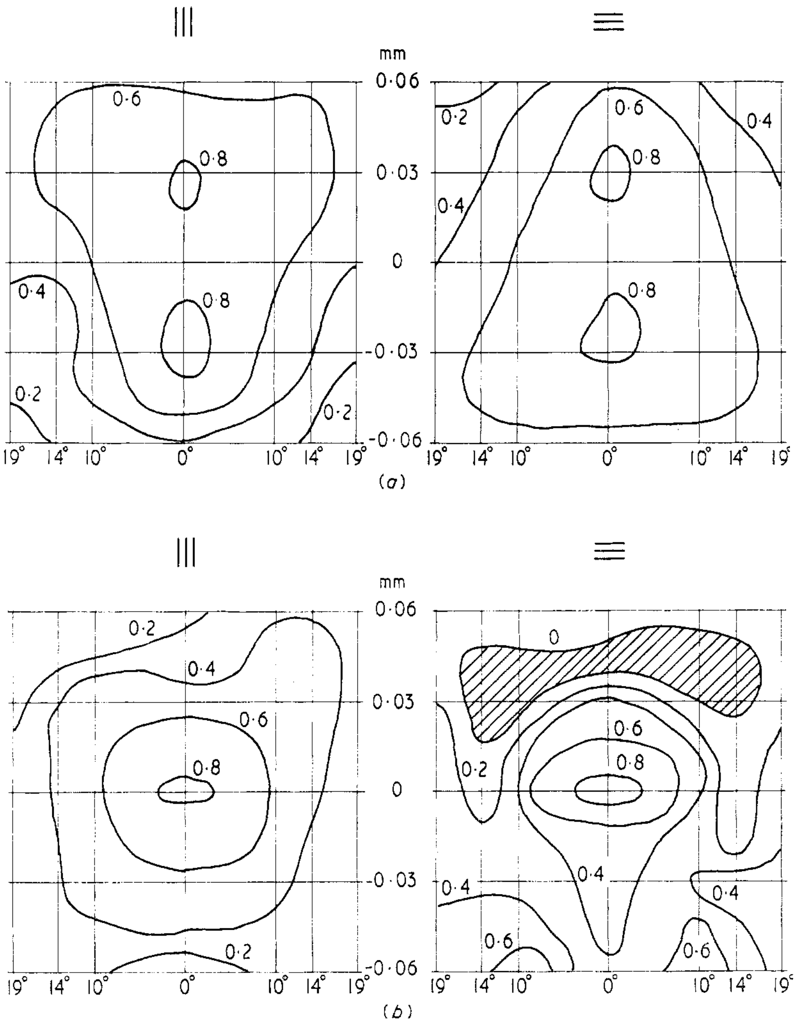


Figure 24. 'Contrast solids' for spatial frequencies of (*a*) 20 lines/mm and (*b*) 50 lines/mm for meridional and sagittal azimuths of a photographic lens. The numerical values on the curves mean the contrast transfer factor which is reached at this curve. The hatched region in (*b*) marks the reversal of sign of the contrast transfer.

13. Image-quality criteria

For some time it has often been attempted to derive image-quality criteria directly from the optical transfer function curves (Linfoot 1960). Generally an integrated average value for the system of the optical transfer function curves is then given for standardized objects and receivers. Although clear and easily comparable, such a criterion gives only one value instead of a group of values, so that much of the specific information is lost by such a simplification. Such criteria can sometimes lead to wrong conclusions regarding the true image quality if they are inadmissibly generalized. Whereas the optical transfer function contains all the information on the image system, independently of the object and image receiver, all composite image-quality criteria are only convenient ratings if the actual object and the image receiver to be used in practice are considered. In this connection it may be useful to summarize the various methods of specifying an optical system, each of them of special importance, insufficient in itself but indispensable to the required purposes. These are as follows:

(i) The constructional data of lenses include the radii of the lenses, the refractive indices and the dispersion of the glasses used, the glass thickness, the lens separations and the diameters of apertures. Using these data ray tracing and optical transfer function calculations are possible, but practice shows that the actual systems manufactured often show considerable deviations from their theoretical properties, especially on complicated systems where errors are unavoidable in manufacturing. The reliable comparison of systems of different construction with respect to the image quality is not possible from the construction data directly.

(ii) Image errors or aberration of lenses are experimental curves showing the measured values of, for example, spherical aberration, coma, astigmatism, curvature of field, distortion, chromatic aberration, etc. They render possible a direct comparison of systems of very different construction with respect to the image, but only when the values of one or two image errors are different. The optical transfer function and the image quality can be calculated from these data. The adherence to the constructional data in manufacturing and production errors can be determined by comparing measured and calculated aberrations.

(iii) Optical transfer function curves of lenses describe the imaging behaviour of the system. Equal values of different systems mean equal image quality. No information can be derived regarding the construction data or the amount of aberration. The comparison between the measured optical transfer functions and those which result from calculation using the constructional data directly shows the influence of the errors of manufacture on the image.

(iv) Criteria on quality describe the image behaviour of the lens with respect to specific objects and specific receivers. Equal values only allow conclusions regarding the two lenses used with the same objects and receivers. Information regarding the optical transfer function curves is generally impossible to obtain from such criteria. The mathematical relations mentioned for calculating such criteria from optical transfer function curves, and with given objects and receivers, are frequently empirically tested for some special cases, but they are very often to a large degree arbitrary.

This summary shows that the different indices used to characterize lenses, i.e. those given in (i) to (iv) above, can only be calculated in one direction, and that their significance increasingly departs from the primitive constructional data of the lens, but that they increasingly approach a description of the purpose of the system, that is to say the image. The optical transfer function has a unique experimental status as it is the last step of the description and is determined solely by the lens.

The development of the various methods of measurement described above indicates not only that new and convenient methods for testing are available, but that they have also deepened our understanding of the physics of image formation.

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