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AXIAL FEEDBACK STABILIZATION OF FLUTE MODE IN A SIMPLE MIRROR REACTOR

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Abstract—Axial feedback stabilization of the flute mode in a mirror-confined plasma of density n_0 is considered. The instability is described using the usual low frequency slab model. The instability potential ϕ is sampled at various azimuths around the plasma circumference. The sampled potential is amplified, phase shifted in azimuth, and the resulting feedback voltage e is applied to a conducting endwall split into azimuthal segments. An external plasma of density n_x is present in the region between the confined plasma and the endwall. The admittance between the endwall and confined plasma is modeled to include the external plasma impedance and the sheaths at the endwall and mirror throat. For typical mirror reactor conditions, stabilization is obtained for exactly 90° azimuthal phase shift provided $(e/\phi)(n_x/n_0)$ is greater than $7p^2a_i^2/R_p^2$, where p is the azimuthal mode number, a_i the ion Larmor radius, and R_p the plasma radius. For $p=1$, $a_i/R_p\sim 0.01$, and $n_x/n_0\sim 10^{-3}$ so as not to degrade the reactor Q, the required gain e/ϕ for stabilization is modest, of order unity. By sampling the potential and its derivative, feedback stabilization is obtained over a wide range of azimuthal phase shift angles.

1. INTRODUCTION

THERE HAS recently been renewed interest in the use of a simple mirror configuration for a fusion reactor (MOIR, 1975). However, it is well known that a simple mirror is unstable to the flute mode (COENSGEN *et al.*, 1966). Various means of stabilizing this mode in a simple mirror reactor have been considered. Finite Larmor radius stabilization can stabilize the higher order angular modes, but is ineffective in stabilizing the $p=1$ mode (ROSENBLUTH *et al.*, 1962). Line tying stabilization due to the presence of external plasma is ineffective due to the external sheath impedance at the end walls (BABYKIN *et al.*, 1965; KUNKEL and GUILLORY, 1966). Feedback stabilization by means of voltages applied to plates radially surrounding the plasma (ARSENIN and CHUYANOV, 1968; THOMASSEN, 1971) and by means of variation of the confining magnetic field (GRAD and WEITZNER, 1969) has also been considered. We here consider the combination of line tying and feedback stabilization of the flute mode by means of currents which are injected at the conducting end wall sheath of a mirror confined, fusion plasma. The end wall is split into a number of pie-shaped segments, with feedback voltages applied to each segment. We show that with this technique, the requirements of feedback gain and external plasma density are modest for stabilization of the flute mode in simple mirror geometry.

2. DISPERSION EQUATION WITHOUT FEEDBACK

Figure 1 shows the axial distribution of electron and ion densities and the ambipolar potential which are considered here. The mirror-confined region of length l_p contains a hot plasma of density n_0, with ion and electron temperatures T_i and T_e. The external plasma extends for a length l_x on either side of the hot plasma, has a density n_x, an electron temperature $T_{ex}=T_e$, and an ion temperature T_{ix}. Both the mirror-confined and the external plasma electrons and ions are assumed to be Maxwellian. We treat the flute instability in a slab model as shown in Fig. 2, in which the effective acceleration $g=v_{th}^2/R_c$; where $R_c=l_B^2/R_p$ is the

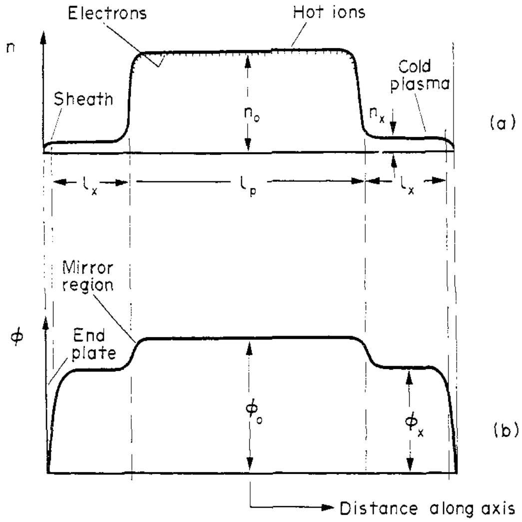

FIG. 1.—(a) Schematic of ion and electron densities near axis of mirror confined plasma with cold external plasma and sheaths at mirror throat and end plate. (b) Corresponding potential distribution.

field line curvature, v_{th} is the mean square velocity, and l_B is the mirror scale length. The wave is taken to vary as $\exp j(\omega t - ky)$. The guiding center approximation is used for the particle motion in the mirror region:

$$\mathbf{v} = \frac{m}{q}\frac{\mathbf{g}\times\mathbf{B}}{B^2} + \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{m}{q}\frac{1}{B^2}\frac{\mathrm{d}\mathbf{E}}{\mathrm{d}t}. \tag{1}$$

The continuity equation for each species in the mirror is written as

$$\partial n/\partial t + \nabla \cdot (n\mathbf{v}) = S, \tag{2}$$

where the source term S gives the current flow from the external plasma into the mirror region along field lines in response to the perturbed potential ϕ. The potential satisfies

$$\nabla^2\phi = -\sum_{\alpha} \frac{q_\alpha n_\alpha}{\varepsilon_0}. \tag{3}$$

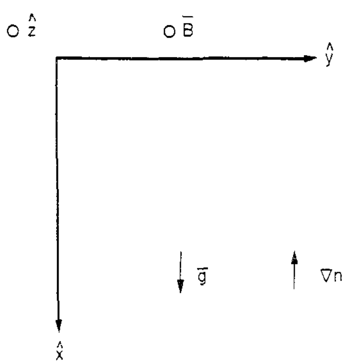

FIG. 2.—Slab model showing orientation of B field direction, density gradient, and gravitational force.

The effect of the external plasma appears only in the source term S. Letting $y(\omega)$ be the admittance per unit area of the external density region, and integrating (2) over the mirror volume, we find

$$S = -\frac{y(\omega)}{ql_p}\phi, \tag{4}$$

where l_p is the confined plasma length. Linearizing (1)–(4), we obtain the dispersion equation

$$1+\sum_\alpha \frac{\omega_{p\alpha}^2}{\Omega_\alpha^2} = \sum_\alpha \frac{1}{\omega+\omega_\alpha}\left(\frac{1}{kR_p}\frac{\omega_{p\alpha}^2}{\Omega_\alpha}+j\frac{y_\alpha}{\varepsilon_0 k^2 l_p}\right), \tag{5}$$

where $\omega_{p\alpha}^2 = q^2 n_\alpha/(\varepsilon_0 m_\alpha)$, $\Omega_\alpha = q_\alpha B/m_\alpha$, $\omega_\alpha = kg_\alpha/\Omega_\alpha$ and $R_p^{-1} = -n_\alpha^{-1}\,\mathrm{d}n_\alpha/\mathrm{d}x$ is the plasma radius. The usual flute instability for an electron–ion plasma with $T_i \gg T_e$ is obtained by putting $y_\alpha = 0$, $\omega_e \ll \omega_i$, and $1+\omega_{pe}^2/\Omega_e^2 \ll \omega_{pi}^2/\Omega_i^2$ in (5):

$$1 = \frac{\Omega_i}{kR_p}\left(\frac{1}{\omega+\omega_i}-\frac{1}{\omega}\right), \tag{6}$$

which has the solution

$$\omega = -\omega_i\left[\frac{1}{2}\pm\left(\frac{1}{4}-\frac{\Omega_i}{kR_p\omega_i}\right)^{1/2}\right]. \tag{7}$$

For small k we find

$$\omega = \pm j\left(\frac{g_i}{R_p}\right)^{1/2},$$

which is the usual result.

We put $1+\omega_{pe}^2/\Omega_e^2 \ll \omega_{pi}^2/\Omega_i^2$ and introduce $\varepsilon_i = a_i^2/l_B^2$, where $a_i^2 = \kappa T_i/(M\Omega_i^2)$, $\varepsilon_e = \varepsilon_i T_e/T_i$, $p = kR_p$, $\Omega = \omega/\Omega_i$ and the normalized sheath admittance

$$A(\omega) = \frac{y_e(\omega)\Omega_i R_p^2}{\varepsilon_0\omega_{pi}^2 l_p}, \tag{8}$$

where κ is Boltzmann's constant and all temperatures are given in units of degrees Kelvin. We then obtain the normalized dispersion equation

$$\Omega^2+\Omega[p(\varepsilon_i-\varepsilon_e)-jA/p^2]+\varepsilon_i+\varepsilon_e-\varepsilon_i jA/p-p^2\varepsilon_i\varepsilon_e = 0. \tag{9}$$

3. EXTERNAL PLASMA ADMITTANCE

The a.c. voltage drop across the external plasma is the sum of three effects: (a) the sheath impedance at the mirror; (b) the sheath impedance at the external wall; and (c) the impedance of the bulk of the external plasma. The sheath potential at the mirror is fixed by equating electron and ion currents

$$\Gamma_e = \Gamma_i + \Gamma_x. \tag{10}$$

Assuming Maxwellian electrons with $T_{ex} = T_e$, we have

$$\Gamma_e = \frac{n_0 v_e}{4} e^{-e\phi/\kappa T_e},$$
$$\Gamma_i \simeq n_0 \frac{l_p}{\tau_m}, \tag{11}$$

and

$$\Gamma_x = \frac{n_x v_e}{4},$$

where

$$v_e = [8\kappa T_e/(\pi m)]^{1/2}$$

and τ_m is the mean ion-scattering loss time. The sheath conductance/unit area g_m is given by

$$g_m \simeq -e\frac{\partial \Gamma_e}{\partial \phi} \simeq \frac{e^2}{\kappa T_e}\left(\frac{n_x v_e}{4} + \frac{n_0 l_p}{\tau_m}\right). \tag{12}$$

Even though $n_x \ll n_0$, $v_e \gg l_p/\tau_m$, such that the first term in (12) dominates.

The external impedance can be modelled by a lumped circuit as in Fig. 3, where R_m is the resistance at the mirror; R_x, L_x are respectively the resistance and inductance due to the external plasma; and G_w, C_w are the conductance and capacitance at the wall sheath. The circuit element modelling of the external plasma and wall sheath has been treated by KUNKEL and GUILLORY (1966) and is

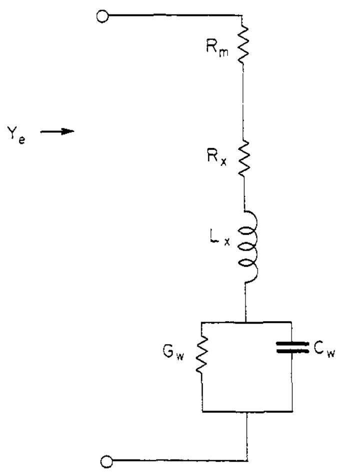

FIG. 3.—Equivalent circuit model for the mirror sheath, external plasma and endwall sheath.

given by:

$$R_x = \frac{\nu l_x}{\varepsilon_0 \omega_{px}^2 A_p} \tag{13}$$

$$L_x = \frac{l_x}{\varepsilon_0 \omega_{px}^2 A_p} \tag{14}$$

$$G_w = \frac{\varepsilon_0 v_{ix}}{4\lambda_{Dx}^2}(1 + j_0/j_i)A_p, \qquad j_0 < j_e - j_i \tag{15}$$

and

$$C_w = \frac{\varepsilon_0}{2\lambda_{Dx}}\left(l_n \frac{j_e}{j_0 + j_i}\right)^{-1/2} A_p, \qquad j_0 < j_e - j_i \tag{16}$$

where l_x is the external plasma length, ν is the effective collision frequency; ω_{px}, λ_{Dx}, v_{ix} are respectively the plasma frequency, electron Debye length and ion thermal speed in the external plasma; j_0 is the electron emission current at the endwall; j_e and j_i are the random electron and ion currents at the wall. For a non-emitting wall, $j_0 = 0$.

4. STABILIZATION CRITERIA WITHOUT FEEDBACK

If the sheaths are neglected ($R_m \to 0, G_w \to \infty$), and the external plasma is assumed collisionless ($\omega \gg \nu$), then the treatment of Babykin *et al.* (1965) is recovered. The external plasma resists the shorting of the instability potential only by the effect of finite electron inertia, represented by the inductor L_p in Fig. 3. Putting

$$\varepsilon_e \ll \varepsilon_i (T_e \ll T_i) \quad \text{and} \quad y_e = -j\varepsilon_0 \omega_{px}^2/(\omega l_x)$$

into (9) and expanding,

$$\Omega^3 - p\varepsilon_i \Omega^2 + (\varepsilon_i - G/p^2)\Omega + \varepsilon_i G/p = 0, \tag{17}$$

where

$$G = \frac{M}{m}\frac{n_x}{n_0}\frac{R_p^2}{l_x l_p}$$

In the usual case $\varepsilon_i = a_i^2/l_B^2 \ll 1$, and mode number p not too large, the condition for stability of (17) is $\varepsilon_i < G/p^2$, or

$$\frac{n_x}{n_0} > \frac{m}{M}\frac{l_p l_x}{l_B^2}\frac{a_i^2}{R_p^2}, \tag{18}$$

which is the result obtained by Babykin *et al.* (1965). This criterion is very favorable for stabilization, since a_i^2/R_p^2 is typically 10^{-2}–10^{-4}. Thus $n_x/n_0 \gtrsim 10^{-5}$–10^{-7} for stabilization, a condition easily met in a reactor with very little degradation in Q. The experimental results of Babykin *et al.* (1965), and of Coensgen *et al.* (1966), were reported to be in agreement with (18); however, the external plasma densities supposedly present seem much too low in view of the high neutral gas pressures and wall bombardment rates for both experiments. In addition, the sheath impedance is not negligible and in fact dominates over the external plasma impedance.

KUNKEL and GUILLORY (1966) have treated the case in which a wall sheath is present in addition to the external plasma bulk impedance. They neglected the sheath at the mirror. The sheath conductance G_w at the wall is much larger than the sheath susceptance ωC_w, provided $\omega \ll \omega_{pix}$, a condition usually met in practice. If R_w is compared to $(v_i/l_B)L_x$, we see that the sheath resistance dominates provided

$$\left(\frac{M}{m}\right)\frac{1}{1+j_0/j_i} \gg \left(\frac{\pi T_i}{8T_{ix}}\right)^{1/2}\frac{l_x}{l_B}, \tag{19}$$

a condition which is usually met in practice, even for an emitting wall. Except for very long connection lengths, and a highly collisional plasma, the resistance of the wall sheaths is the dominant source of voltage drop between the confined plasma and the wall. Setting $y_e = \frac{1}{4}\varepsilon_0 v_{ix}(1+j_0/j_i)/\lambda_{Dx}^2$ in (8), we find

$$A = (2\pi)^{-1/2}\frac{n_x}{n_0}\left(\frac{T_i T_{ix}}{T_e^2}\right)^{1/2}\frac{R_p^2}{a_i l_p}(1+j_0/j_i). \tag{20}$$

Dropping terms quadratic in ε ($p\varepsilon \ll 1$ for (1) to be valid), and noting $4\bar{\varepsilon}A/p \ll A^2/p^4 + 8\bar{\varepsilon}$ for $0 < A < \infty$, we obtain the unstable root in equation (9)

$$\Omega = -\tfrac{1}{2}p(\varepsilon_i - \varepsilon_e) - p\bar{\varepsilon}A(A^2 + 8\bar{\varepsilon}p^4)^{-1/2} - j\tfrac{1}{2}p^{-2}[(A^2+8\bar{\varepsilon}p^4)^{1/2} - A]. \tag{21}$$

The growth rate for zero sheath admittance $A = 0$ is, from (21), $\operatorname{Im}\Omega = -\sqrt{2\bar{\varepsilon}}$. For a reduction in growth rate by factor of F, (21) requires

$$A = \sqrt{2}(F - F^{-1})p^2\bar{\varepsilon}^{1/2}, \tag{22}$$

or

$$\frac{n_x}{n_0} = \sqrt{2\pi}(F - F^{-1})p^2\frac{a_i^2}{R_p^2}\frac{l_p}{l_B}\left(\frac{T_e^2}{T_i T_{ix}}\right)^{1/2}\left(1+\frac{T_e}{T_i}\right)^{1/2}(1+j_0/j_i)^{-1}, \tag{23}$$

which is in agreement with the result of KUNKEL and GUILLORY (1966). This result is discouraging, since F must generally be very large. For $j_0 = 0$ (non-emitting wall) and typical mirror reactor conditions, it is necessary that $F \gtrsim 10^5$ and this condition cannot be achieved with $n_x \lesssim n_0$ unless a_i/R_p is 10^{-3} or less.

5. AXIAL PROPORTIONAL FEEDBACK STABILIZATION

The possibility of stabilizing or greatly reducing the growth rate of the flute instability by applying feedback signals at the external end wall is now considered. Let $\omega_{pi}^2/\Omega_i^2 \gg 1$ and $y_i = 0$ in (5), the dispersion equation can be put in the form

$$j\omega C + Y_{NL} + 2Y_e = 0 \tag{24}$$

where

$$C = \varepsilon_0 k^2 l_p A_p \omega_{pi}^2/\Omega_i^2, \tag{25}$$

$$Y_{NL} = j\varepsilon_0 k^2 l_p A_p \frac{\omega_{pi}^2}{\Omega_i}\left(\frac{\omega_e}{\Omega_i} + \frac{\omega_i - \omega_e}{kR_p(\omega+\omega_i)}\right), \tag{26}$$

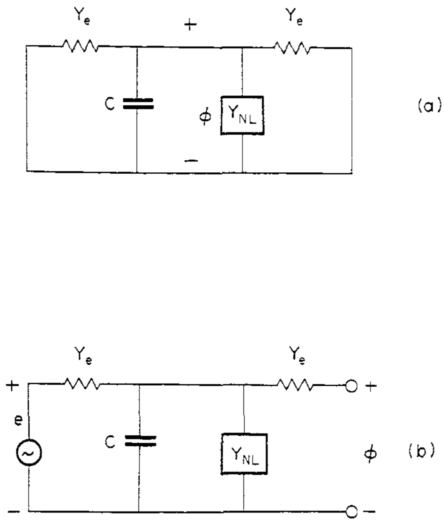

FIG. 4.—(a) Circuit model of the flute instability, including line-tying. (b) Circuit model for feedback stabilization of the flute mode at the ends.

Y_e is the admittance shown in Fig. 3, and the factor of two in (24) has been inserted to account for the sheath at both ends of the plasma. Equation (24) is modeled by the circuit shown in Fig. 4(a).

Let us consider using one end of the system to sense the instability potential ϕ and the other end to apply a feedback voltage e proportional to ϕ. The potential is sampled at a number of different azimuthal positions around the circumference of the plasma. The end wall is also split into a number of azimuthal segments. The sampled potential is amplified, phase shifted in azimuth, and correspondingly applied at the various azimuthal positions around the end wall circumference. We note that in principle, to control a flute mode with azimuthal mode number $p = kR_p$, it is necessary to sense the potentials on only two field lines spaced at an angle not commensurate with $2\pi/p$ around the circumference of the external end wall. A circuit model including the sampling and feedback voltages is shown in Fig. 4(b).

For the feedback voltage e we are considering, we have

$$e = \beta\phi = |\beta|\, e^{j\theta}\phi \tag{27}$$

where $|\beta|$ is an amplification factor and θ is the azimuthal phase shift. The effect of the feedback voltage is to modify the source term S in the continuity equation (2). In place of (4), we now have for the electrons

$$S_e = (\beta - 1)\frac{y_e(\omega)}{q_e l_p}\phi. \tag{28}$$

Proceeding as in Section 2, we obtain the same dispersion equation (9), with A replaced by

$$A' = (1 - \beta)A. \tag{29}$$

Using the transformation $\Omega = u - p(\varepsilon_i - \varepsilon_e)/2$, and $\bar{\varepsilon} = (\varepsilon_i + \varepsilon_e)/2$, we have the dispersion equation

$$u^2 - \frac{jA'}{p^2}u - \frac{jA'\bar{\varepsilon}}{p} + 2\bar{\varepsilon} - p^2\bar{\varepsilon}^2 = 0. \tag{30}$$

The two roots of (30) are:

$$u = +\frac{1}{2}\frac{jA'}{p^2} \pm \frac{1}{2}\left\{\left(\frac{jA'}{p^2} + 2p\bar{\varepsilon}\right)^2 - 8\bar{\varepsilon}\right\}^{1/2} \tag{31}$$

In general $Y_e(\omega)$ and thus jA' is complex, but we can phase shift the feedback signal in azimuth such that jA' is pure real. Choosing that value of θ which does this, we have the condition for stability

$$\left(\frac{jA'}{p^2} + 2p\bar{\varepsilon}\right)^2 > 8\bar{\varepsilon}. \tag{32}$$

This shows that marginal stability depends only on the mean driving force seen by ions and electrons. In the limit $p\bar{\varepsilon} \ll 1$, which is necessary for the validity of (1), stability is obtained for

$$|A'/p^2| > (8\bar{\varepsilon})^{1/2}. \tag{33}$$

As mentioned previously, the wall sheath conductance term is dominant compared to the mirror sheath or the external plasma admittance in the external admittance Y_e. Thus we can take the admittance as the wall sheath conductance (15). Then using (20) and (29), the required gain–density ratio product for stability is

$$|\beta - 1|\frac{n_x}{n_0} > (8\pi)^{1/2}p^2\frac{a_i^2}{R_p^2}\frac{l_p}{l_B}\left(\frac{T_e^2}{T_iT_{ix}}\right)^{1/2}\left(1 + \frac{T_e}{T_i}\right)^{1/2}(1 + j_0/j_i)^{-1}. \tag{34}$$

The stabilization mechanism is similar to the purely line tying result obtained by BABYKIN *et al.* (1965), in which the resistance of the sheaths was neglected and only the inductive impedance of the external plasma due to finite electron inertia was considered. It was found that a sufficiently large inductive admittance would stabilize the flute mode. In the present case, for a resistive sheath with $|\beta| \gg 1$, an azimuthal phase shift of $\pm 90°$ for the feedback signal produces an effective sheath admittance as seen by the mirror-confined plasma which is purely reactive (no resistive component). This admittance is proportional to the feedback gain, and for sufficiently high gain, the flute mode is stabilized by either an inductive or capacitive connection to the endwall.

To get a feeling for the stability requirement, consider stabilization of the $p = 1$ mode with a non-emitting wall. For simplicity we take $T_e \approx T_i \approx T_{ix}$ and $l_p \approx l_B$. For a typical low density mirror reactor, one has $a_i/R_p \approx 10^{-2}$, in which case (34) yields $|\beta - 1|(n_x/n_0) \gtrsim 7 \times 10^{-4}$. Since analysis has shown (MOIR, 1975, p. 14) that the reactor Q is not significantly degraded provided $n_x/n_0 \lesssim 10^{-3}$, it appears that modest values of the gain $|\beta - 1|$ are sufficient to stabilize the $p = 1$ flute mode. For the higher order modes, the required gain increases as p^2. As an example, for $n_x/n_0 \lesssim 10^{-3}$ and $|\beta-1| \approx 100$, mode numbers up to $p=12$ could in principle be stabilized.

For long, high density, linear mirrors, such as two-component (Post *et al.*, 1973) or multiple mirror (Logan *et al.*, 1974) systems, a_i/R_p is typically of order 0.1 and l_p/l_B may be as high as 100. This leads to the requirement $|\beta-1|(n_x/n_0) > 7$ for stability. However, the reactor Q for these systems is not significantly degraded even if $n_x/n_0 \approx 1$, so that the gain requirement for stabilization is still modest.

A drawback of proportional feedback stabilization is that complete stabilization is obtained only for azimuthal phase shifts of $\pm\pi/2$. To see this we consider the limit $1 \gg |jA'/p^2|^2 \gg 8\bar{\varepsilon}$, for which $|jA'/p^2| \gg 2p\bar{\varepsilon}$. Putting $jA' = |A'|\,e^{j(\theta-\pi/2)}$ and expanding the square root in (31), we obtain

$$\operatorname{Im}\Omega_1 = -\frac{|A'|}{p^2}\cos\theta \tag{35}$$

$$\operatorname{Im}\Omega_2 = +\frac{2\bar{\varepsilon}p^2}{|A'|}\cos\theta. \tag{36}$$

Thus one of the two roots is always unstable. Instability of the first root (35) is a consequence of positive feedback applied to the capacitor C of Fig. 4(a). Although unstable for $\theta \neq \pm\pi/2$, by choosing $\pi/2 < \theta < 3\pi/2$ and $|A'|$ sufficiently large, root (35) is stabilized and the growth rate of (36) can be reduced to a very low level.

6. PROPORTIONAL AND DERIVATIVE FEEDBACK

We now consider a situation where the feedback signal e is proportional to the sum of the perturbing potential and its time derivative, each phase shifted in azimuth. In the frequency domain

$$e = (\beta + j\omega\delta)\phi = (|\beta|\,e^{j\theta} + j\omega\,|\delta|\,e^{j\gamma})\phi \tag{37}$$

where $|\beta|$ and $|\delta|$ are amplification factors and θ and γ are respectively the azimuthal phase shifts for proportional and derivative feedback. In place of (4), the electron source term is then

$$S_e = \frac{y_e(\omega)}{q_e l_p}\,\phi[(\beta-1) + j\omega\delta]. \tag{38}$$

For simplicity, we assume the sheath impedance is purely resistive. From the form of the source term S_e, we see that derivative feedback corresponds to a reactive term although y_e is resistive, and this should have a stabilizing effect on the plasma. Proceeding as in Section 2, we obtain the modified dispersion equation:

$$\Omega^2(1-H) + \Omega\left[p(\varepsilon_i - \varepsilon_e) - j\frac{A'}{p^2} - p\varepsilon_i H\right] + 2\bar{\varepsilon} - j\frac{A'\varepsilon_i}{p} - p^2\varepsilon_i\varepsilon_e = 0 \tag{39}$$

where A' is given by (29),

$$H = A\Omega_i\delta/p^2 \tag{40}$$

is the normalized derivative gain, and A is the normalized sheath impedance (8).

The two roots of (39) are:

$$\Omega = \frac{-[p(\varepsilon_i - \varepsilon_e) - j(A'/p^2) - p\varepsilon_i H] \pm \{[(-jA'/p^2) - 2p\bar{\varepsilon} + p\varepsilon_i H]^2 + 8\bar{\varepsilon}(H-1)\}^{1/2}}{2(1-H)} \tag{41}$$

The interesting limit of (41) is $|\beta| \gg 1$, $|H| \gg 1$ and $p\bar{\varepsilon}\,|H| \ll |A'/p^2|$. In this case we have

$$\Omega = \Omega'[-1 \pm (1-R)^{1/2}] \tag{42}$$

where $\Omega' = jA'/(2p^2H)$ and

$$R = 8\bar{\varepsilon}p^4H/A'^2. \tag{43}$$

For $|R| \ll 1$, we find

$$\operatorname{Im}\Omega_1 = +\frac{|A'|}{p^2\,|H|}\cos(\theta - \gamma) \tag{44}$$

$$\operatorname{Im}\Omega_2 = +\frac{2p^2\bar{\varepsilon}}{|A'|}\cos\theta. \tag{45}$$

The two roots are stable in a finite diamond shaped region centered about $\theta = 0$ $\gamma = 0$, as shown in Fig. 5. There is a wide range of phase shifts over which complete stability is achieved.

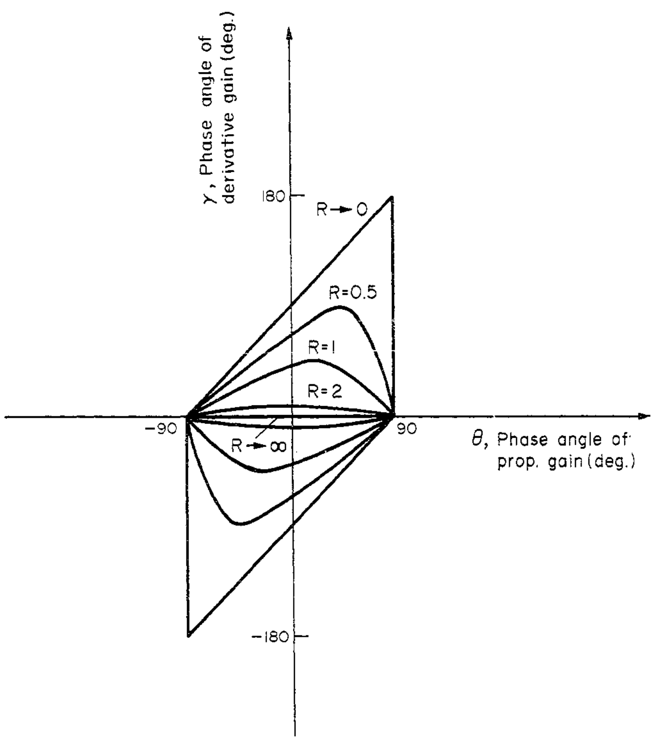

FIG. 5.—Stability map in γ, θ space with R as a parameter.

In the other limit $|R| \gg 1$, we find

$$\mathrm{Im}\,\Omega_{1,2} = (2\bar{\varepsilon}/|H|)^{1/2}\,[\pm\sin(\gamma/2) + |R|^{-1/2}\cos(\theta-\gamma)] \tag{46}$$

which shows complete stability only for $\gamma = 0$ and $-\pi/2 \le \theta \le \pi/2$, as shown in Fig. 5. The variation of the stable phase region for finite values of $|R|$ is also shown in the figure.

Derivative feedback acts so that, by sensing the rate of change of the perturbing potential, we anticipate its effect. By using both derivative and proportional feedback, we are applying two independent stabilizing feedback voltages to the plasma. Both derivative and proportional feedback are separately susceptible to instability due to small phase shifts in the feedback circuit or small reactive components in the line-tying admittance. Thus if we drop the derivative feedback term, or if we make the derivative feedback term too large, the plasma is stabilized only along a line in the θ–γ phase space, where the total admittance appears as a pure reactance. However, there is a region between these extremes where the plasma can be stabilized, with both modes decaying, and this stabilization is not adversely affected by small reactive components in the line-tying admittance or by small phase shifts in the feedback circuit.

As a design example, consider stabilization of the $p = 1$ mode and assume that ϕ' is obtained by means of an RC differentiator, for which $\delta = RC$ and $H = A\Omega_i\delta$. For a conventional low density mirror reactor, we take $T_iT_{ix}/T_e^2 \sim 1$, $R_p/l_p \sim 1$, $R_p/a_i \sim 100$ and $n_x/n_0 \sim 2\times10^{-4}$ to find from (8) that $|A| \sim 10^{-2}$. At $B_0 = 20$ kG, $\Omega_i \sim 10^8\ \mathrm{s}^{-1}$. A time constant δ of 10^{-4} s is easily achieved, yielding $|H| \sim 100$ from (40). For $\bar{\varepsilon} \sim 10^{-4}$ and $|R| = 1$, the required feedback gain is found from (29) and (43) to be $|\beta| \sim 30$, which is easily achieved.

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