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Plasma Physics, Vol. 15, pp. 1031 to 1038. Pergamon Press 1973. Printed in Northern Ireland

NONLINEAR DRAGGING OF PARTICLES DURING HIGH-FREQUENCY HEATING

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(*Received* 16 *October* 1972)

Abstract—The total longitudinal force which acts on a high-frequency heated plasma column is derived for any arbitrary absorption mechanism. This force arises from any departure from longitudinal symmetry (longitudinal plasma motion, a single travelling wave, etc.). It results in particle drifts and poloidal fields which may be relevant to toroidal plasma confinement. High-frequency power of order 10^5 W is sufficient to replace the driving electric field in a moderate scale tokamak.

1. INTRODUCTION

RECENTLY WORT (1971) has suggested the idea of the Direct Current tokamak in which the toroidal current is excited by a sufficiently strong electromagnetic wave travelling along the torus. The wave accelerates the resonant electrons owing to Landau damping and transit time pumping effects which results in the desired steady current. Similar currents (THONEMANN *et al.*, 1952; DEMIRKHANOV *et al.*, 1965; YOSHIKAWA *et al.*, 1966; KOTSARENKO *et al.*, 1967; AKULINA *et al.*, 1971; HIRANO *et al.*, 1971; OSOVETS *et al.*, 1972) have been observed in toroidal experiments and interpreted theoretically in some particular cases. The purpose of this paper is to point out some specific features of these phenomena and their relevance to experiments. Our analysis is based on the physical mechanism responsible, namely, the fact that an electromagnetic field excited by an external source transfers not only energy but also momentum to the plasma. This momentum influx (see Section 2) exerts the dragging force which acts on the plasma particles. A brief discussion of the corresponding particle motions is given in Section 3. Using some recent results from toroidal devices (BEREZHETSKII *et al.*, 1971; KOVRIZHNYKH, 1972), we present a possible interpretation of the unexplained part of the experiments reported by YOSHIKAWA and YAMATO (1966) in Section 4. Finally, approximate formulae are given in Section 5 for practical estimates of toroidal currents excited by HF heating.

2. TOTAL DRIVING FORCE

Let us consider a cylindrical surface $\mathscr{T}$ situated in vacuum along the z-axis; the transverse cross-section curve $\sigma \equiv \{f(x, y) = \text{const.}\}$ is arbitrary (but closed), and $z_1 \leq z \leq z_2$. The z-component F_z of the electromagnetic momentum flowing inward across $\mathscr{T}$ is determined by the well-known Maxwell stress tensor:

$$F_z = \frac{1}{4\pi}\int_{\mathscr{T}}(E_nE_z + H_nH_z)\,\mathrm{d}\mathscr{T}, \tag{2.1}$$

where $E_n = \mathbf{E}\cdot\mathbf{e}_n$, $E_z = \mathbf{E}\cdot\mathbf{e}_z$ etc., $\mathbf{e}_n$ and $\mathbf{e}_z$ are unit vectors in the directions of the outward normal to $\mathscr{T}$ and the z-axis. We assume the presence of a stationary field $\mathbf{E}_0$, $\mathbf{H}_0$ and of an oscillating (with frequency ω) field $\mathbf{E}_1$, $\mathbf{H}_1$ on $\mathscr{T}$. The time averaged value of F_z is then

$$\langle F_z\rangle = \frac{1}{4\pi}\int_{\mathscr{T}}(E_{0n}E_{0z} + H_{0n}H_{0z})\,\mathrm{d}\mathscr{T} + F_{\sim}, \tag{2.2}$$

where

$$F_{\sim} = \frac{1}{8\pi} \operatorname{Re} \int_{\mathscr{T}} (E_{1n}E_{1z}^{*} + H_{1n}H_{1z}^{*}) \, d\mathscr{T}, \tag{2.3}$$

the asterisk denoting the complex conjugate. The aim of the following procedure is to express (2.3) in terms of the normal component S_n of the Poynting vector,

$$S_n = \mathbf{e}_n \, . \, \mathbf{S} = \frac{c}{8\pi} \operatorname{Re} (E_{1\sigma}H_{1z}^{*} - E_{1z}H_{1\sigma}^{*}), \tag{2.4}$$

where $E_{1\sigma} = \mathbf{E}_1 \, . \, \mathbf{e}_\sigma$, $H_{1\sigma} = \mathbf{H}_1 \, . \, \mathbf{e}_\sigma$, $\mathbf{e}_\sigma = \mathbf{e}_z \times \mathbf{e}_n$ is the tangential unit vector of the curve σ. To simplify the algebra, we introduce the usual cylindrical system of co-ordinates r, φ, z with unit vectors $\mathbf{e}_r$, $\mathbf{e}_\varphi$, $\mathbf{e}_z$. Using Maxwell's equations ($\mathscr{T}$ is in vacuum) and the obvious relations $\mathbf{e}_n \, . \, \mathbf{e}_r = \mathbf{e}_\sigma \, . \, \mathbf{e}_\varphi$, $\mathbf{e}_n \, . \, \mathbf{e}_\varphi = -\mathbf{e}_\sigma \, . \, \mathbf{e}_r$, we have

$$E_{1n} = \frac{ic}{\omega}\left[\mathbf{e}_\sigma \, . \, \mathbf{e}_\varphi\left(\frac{\partial H_{1z}}{r\,\partial\varphi} - \frac{\partial H_{1\varphi}}{\partial z}\right) - \mathbf{e}_\sigma \, . \, \mathbf{e}_r\left(\frac{\partial H_{1r}}{\partial z} - \frac{\partial H_{1z}}{\partial r}\right)\right], \tag{2.5a}$$

$$H_{1n} = -\frac{ic}{\omega}\left[\mathbf{e}_\sigma \, . \, \mathbf{e}_\varphi\left(\frac{\partial E_{1z}}{r\,\partial\varphi} - \frac{\partial E_{1\varphi}}{\partial z}\right) - \mathbf{e}_\sigma \, . \, \mathbf{e}_r\left(\frac{\partial E_{1r}}{\partial z} - \frac{\partial E_{1z}}{\partial r}\right)\right]. \tag{2.5b}$$

Since

$$\mathbf{e}_\sigma \, . \, \mathbf{e}_\varphi \frac{\partial H_{1z}}{r\,\partial\varphi} + \mathbf{e}_\sigma \, . \, \mathbf{e}_r \frac{\partial H_{1z}}{\partial r} = \mathbf{e}_\sigma \, . \operatorname{grad} H_{1z} = \frac{\partial H_{1z}}{\partial\sigma}$$

and

$$\mathbf{e}_\sigma \, . \, \mathbf{e}_\varphi \frac{\partial H_{1\varphi}}{\partial z} + \mathbf{e}_\sigma \, . \, \mathbf{e}_r \frac{\partial H_{1r}}{\partial z} = \frac{\partial H_{1\sigma}}{\partial z},$$

we have from (2.5a)

$$E_{1n} = \frac{ic}{\omega}\left(\frac{\partial H_{1z}}{\partial\sigma} - \frac{\partial H_{1\sigma}}{\partial z}\right) \tag{2.6a}$$

and similarly from (2.5b)

$$H_{1n} = -\frac{ic}{\omega}\left(\frac{\partial E_{1z}}{\partial\sigma} - \frac{\partial E_{1\sigma}}{\partial z}\right). \tag{2.6b}$$

Consequently, equation (2.3) becomes

$$F_{\sim} = \frac{c}{8\pi\omega} Re\left\{i\int_{z_1}^{z_2}\oint_\sigma dz\, d\sigma\left(E_{1z}\frac{\partial H_{1\sigma}^{*}}{\partial z} + H_{1z}^{*}\frac{\partial E_{1\sigma}}{\partial z}\right)\right\}. \tag{2.7}$$

We first suppose that the spatial dependence of the field $\mathbf{E}_1$, $\mathbf{H}_1$ at $\mathscr{T}$ is given by a single eigen-mode propagating along the z-axis, viz.

$$E_{1j} = \mathscr{E}_j(r, \varphi)A(z) \exp\{i[k_z z + \varphi_j(r, \varphi)]\}, \tag{2.8a}$$

$$H_{1j} = \mathscr{H}_j(r, \varphi)A(z) \exp\{i[k_z z + \psi_j(r, \varphi)]\}, \tag{2.8b}$$

where $\mathscr{E}_j$, $\mathscr{H}_j$, A, k_z, φ_j and ψ_j are real quantities $j = n, \sigma, z$. Inserting (2.8) into (2.7) and assuming $A^2(z_2) = A^2(z_1)$, we obtain

$$F_{\sim} = \frac{k_z P}{\omega}, \tag{2.9}$$

where P is the power supplied across $\mathscr{T}$ by the HF field $\mathbf{E}_1$, $\mathbf{H}_1$:

$$P = \frac{c}{8\pi}\int_{\mathscr{T}} \mathrm{Re}\,(E_{1z}H_{1\sigma}{}^* - E_{1\sigma}H_{1z}{}^*)\,\mathrm{d}\mathscr{T}. \tag{2.10}$$

The procedure presented above can obviously be generalized to the case when the field $\mathbf{E}_1$, $\mathbf{H}_1$ is a super-position of modes of the type (2.8). If the frequencies of the modes (ω_m is the mth mode) differ from each other, then, due to time averaging,

$$F_{\sim} = \sum_m \frac{k_{zm}P_m}{\omega_m}, \tag{2.11}$$

where P_m is the power flow (2.10) corresponding to the mth mode. Of course, the presence of one mode may in general affect the power absorbed by another mode and, consequently, the quantities in the superposition (2.11) may be mutually dependent. Another situation arises when two or more modes with the same frequency are present in the same place. As seen from (2.7), equation (2.11) then holds only for the average over many wavelengths along the z-axis and, moreover, the amplitudes of these modes must not vary greatly with $z(|k_z| \gg |A^{-1}\,\mathrm{d}A/\mathrm{d}z|)$.

An alternative approach is to introduce the momentum influx per unit length

$$f_{\sim} = \frac{1}{4\pi}\oint_{\sigma} (E_{1n}E_{1z} + H_{1n}H_{1z})\,\mathrm{d}\sigma \tag{2.12}$$

instead of $F_{\sim}$ in (2.3), and to use the Fourier transform $(t, z) \to (\omega, k_z)$. Denoting the doublet (k_z, ω) by k and using the convolution theorem, we have

$$f_{\sim k} = \oint_{\sigma} \frac{d\sigma}{4\pi}\int \mathrm{d}k_1\,\mathrm{d}k_2 (E_{1n,k_1}E_{1z,k_2} + H_{1n,k_1}H_{1z,k_2})\,\delta(k - k_1 - k_2). \tag{2.13}$$

The relevant $k = 0$ component (i.e. the time and space average of $f_{\sim}$) is obtained after some algebra similar to that used previously (2.4)–(2.7):

$$f_{\sim 0} = \int \frac{k_z P_k}{\omega}\,\mathrm{d}k_z\,\mathrm{d}\omega, \tag{2.14}$$

where

$$P_k = \frac{c}{4\pi}\oint_{\sigma} (E_{1z,k}H^*_{1\sigma,k} - E_{1\sigma,k}H^*_{1z,k})\,\mathrm{d}\sigma. \tag{2.15}$$

The fact that the coefficient on the right hand side of (2.15) is twice that of (2.10) stems from the symmetry properties of the Fourier representation.

3. GENERAL DISCUSSION OF THE TOTAL FORCE

The results given in formulae (2.9), (2.11) and (2.14) are not of course surprising, and have an obvious quantum-mechanical interpretation. The number of quanta is $P/\hbar\omega$, the z-component of momentum of each quantum is $\hbar k_z$ (see also KAUFMAN, 1971). In the following discussion we return to the expression (2.2). The first term on the right vanishes, e.g. for the plausible assumptions $E_{0z} = 0$, $H_{0n} = 0$ on $\mathscr{T}$. In a quasi-stationary state, the time derivative of the electromagnetic momentum is negligible. If, moreover, the momentum flux passing the plane $z = z_1$ (inside the curve σ) equals that at $z = z_2$, then $F_\sim$ is the z-component of the total force acting on particles in the volume bounded by $\mathscr{T}$, $z = z_1$ and $z = z_2$.

Which particles absorb the momentum supplied by the HF field depends on the details of the dissipation mechanism. The particular case of Landau damping has been analyzed by means of the quasi-linear theory (YOSHIKAWA and YAMATO, 1966) and by energy considerations (WORT, 1971). The theory given in the preceding Section also includes the case of cold magnetized plasma with collisional dissipation (KLÍMA, 1967). Another illustrative example is the case of cyclotron absorption in homogeneous $\mathbf{H}_0$ and negligible E_{0z}, E_{1z}. The equation of motion of a resonant particle (STIX, 1962)

$$m \frac{\mathrm{d}v_z}{\mathrm{d}t} = \frac{k_z}{\omega - k_z v_z} \frac{\mathrm{d}}{\mathrm{d}t} \frac{m v_\perp{}^2}{2}$$

implies that

$$m \frac{\mathrm{d}v_z}{\mathrm{d}t} = \frac{k_z}{\omega} \frac{\mathrm{d}}{\mathrm{d}t} \frac{m v^2}{2}, \qquad v^2 = v_\perp{}^2 + v_z{}^2,$$

which is consistent with (2.9).

Let us consider a coordinate frame L' which moves with velocity $\mathbf{V}_0 \| \mathbf{e}_z$ in the laboratory frame L. Equation (2.9) remains valid in L':

$$F_\sim{}' = \frac{k_z' P'}{\omega'}, \qquad \mathbf{e}_z' = \mathbf{e}_z, \tag{3.1}$$

the dashes denoting quantities measured in L'. It is easy to show from the Lorentz transform equations and from (2.9) that the power flow P transforms equally with ω and $F_\sim$ transforms equally with k_z. If we neglect terms in $V_0{}^2/c^2$ in comparison with unity, we have

$$\omega' = \omega - k_z V_0, \qquad k_z' = k_z - \frac{\omega V_0}{c^2}, \tag{3.2}$$

$$P' = P\left(1 - \frac{k_z V_0}{\omega}\right), \qquad F_\sim{}' = F_\sim\left(1 - \frac{\omega V_0}{k_z c^2}\right). \tag{3.3}$$

These relations are useful for the analysis of a model with moving plasma given in the next Section. To show a connection with another branch of theory, we may choose L' as moving with the plasma and note that sign $(P'/P) =$ sign $(\omega - k_z V_0)$, which leads to the well-known principle of the travelling wave MHD generator. P' differs from P by just the value of $F_\sim V_0$, a relation used by JUNGWIRTH (1972) in the case of a collisionless, external field-free plasma.

4. TWO WAVES IN MOVING PLASMA

Consider a plasma column moving along its axis with velocity $V_z \mathbf{e}_z$. Two waves propagating in opposite directions ($+\mathbf{e}_z$ and $-\mathbf{e}_z$) are excited by a source (e.g. a Stix coil) with one fixed frequency ω_1 in the laboratory frame L. The dispersion in in the plasma rest frame L' can be written in terms of a 'refractive index' N_z':

$$k_z'(\omega') = \frac{\omega'}{c} N_z'(\omega'). \tag{4.1}$$

According to (2.11), the total driving force $F_{\sim}'$ in L' is the sum of the partial forces

$$F_{+}' = \frac{1}{c} N_{z+}'(\omega_{+}')P_{+}', \qquad F_{-}' = \frac{1}{c} N_{z-}'(\omega_{-}')P_{-}', \tag{4.2}$$

the subscripts $_{\pm}$ denoting quantities connected with the two waves in question. Since

$$\omega_{\pm}' = \omega_1 - k_{z\pm}V_z, \qquad k_{z-} < 0 \,, \tag{4.3}$$

we have

$$F_{+}' = \frac{1}{c} N_{z+}'(\omega_1)P_{+}'(\omega_1)\left[1 - k_{z+}V_z \frac{\mathrm{d}}{\mathrm{d}\omega_{+}'} \lg (N_{z+}'P_{+}')\right], \tag{4.4}$$

when the terms $\sim V_z^2$, V_z^3, ... are neglected. Similarly,

$$F_{-}' = \frac{1}{c} N_{z-}'(\omega_1)P_{-}'(\omega_1)\left[1 - k_{z-}V_z \frac{\mathrm{d}}{\mathrm{d}\omega_{-}'} \lg (N_{z-}'P_{-}')\right]. \tag{4.5}$$

The partial forces F_{+} and F_{-} in L follow from (4.4) and (4.5) by using the second of equations (3.3). Since $N_{z-}'(\omega_1) = -N_{z+}'(\omega_1)$ and $P_{-}'(\omega_1) = P_{+}'(\omega_1)$, the sum of F_{+} and F_{-} is

$$F_{\sim} = 2V_z \frac{N_z P}{c}\left[\frac{1}{cN_z} - k_z \frac{\mathrm{d}}{\mathrm{d}\omega} \lg (N_z P)\right], \tag{4.6}$$

where k_z, N_z, P and their derivatives are taken at $V_z = 0$ and $\omega = \omega_1$; P is the power absorbed from one wave. A necessary condition for the instability of the motion V_z can be formulated:

$$P\left[1 - cN_z k_z \frac{\mathrm{d}}{\mathrm{d}\omega} \lg (N_z P)\right] > 0. \tag{4.7}$$

Strong toroidal electric currents were observed when Landau damping ion cyclotron waves on electrons (YOSHIKAWA and YAMATO, 1966). It was not understood why those currents appeared when exciting, supposedly symmetrically, two waves propagating in opposite directions. The following interpretation is based on the recent appreciation (e.g. BEREZHETSKII *et al.*, 1971; KOVRIZHNYKH, 1972) that plasma transport along the torus arises as an inevitable consequence of diffusion.

In the case of ion cyclotron waves (STIX, 1962),

$$\frac{\mathrm{d}}{\mathrm{d}\omega} \lg N_z \simeq \frac{1}{2} \frac{1}{\omega_{ci} - \omega} > 0, \tag{4.8}$$

ω_{ci} is the ion cyclotron frequency. Since we are interested in the force $F_{\sim}$ acting on the electrons, P in (4.6) is the power absorbed by the electron Landau damping of one wave. According to the experimental conditions (YOSHIKAWA and YAMATO, 1966), we approximate that $P \sim$ the damping rate $\sim \exp[-(\omega/k_z v_e)^2]$; v_e is the electron thermal velocity and $(\omega/k_z v_e)^2 \simeq 3 \div 4$. Consequently, we have the following rough estimate:

$$\frac{\mathrm{d}}{\mathrm{d}\omega} \lg P \approx 2\left(\frac{\omega}{k_z v_e}\right)^2 \frac{\mathrm{d}}{\mathrm{d}\omega} \lg N_z. \tag{4.9}$$

The instability condition (4.7) is not satisfied here, i.e. the action of the wave with k_z antiparallel to V_z predominates. However, we may ask at what plasma velocity V_z

the value of $F_\sim$ in (4.6) is comparable with N_zP/c, the force corresponding to the presence of one wave. For $\omega/\omega_{ci} = 0{\cdot}93$, we find by using (4.8) and (4.9) in (4.6) that is should be $V_z \approx v_i =$ the thermal velocity of ions. Actually, longitudinal velocities of this order have recently been indicated by experiment (BEREZHETSKII *et al.*, 1971). The sign of V_z (and consequently the sign of $F_\sim$ in (4.6)) depends on the direction of the main magnetic field. To conclude this interpretation, we note that estimates (5.1) and (5.5) accord with the data of YOSHIKAWA *et al.* (1966) within a factor of 2 or 3.

It seems likely that similar phenomena play a role in the HF heating experiments reported by AKULINA *et al.* (1971). The singular behaviour of the toroidal current observed at the electron cyclotron resonance may be attributed to the acceleration $\sim(-v_\perp{}^2 \nabla_\parallel H_0)$ in a local mirror region of the external fields. It must be emphasized here that the present analysis is not generally valid for HF heating in the magnetic field of a Stellarator. The experiment of YOSHIKAWA *et al.* (1966) is an exception, since the HF field was essentially in the straight part of the Stellarator and, moreover, the absorption mechanism was not sensitive to the type of external magnetic field.

5. FORMULAE FOR PRACTICAL ESTIMATES

In view of the simple summation rules (2.11) and (2.14), the presence of one wave is assumed in this Section. According to Section 3, we assume that the dragging force acting on electrons is approximately equal to the total force $F_\sim$. (The opposite case has been included in WORT (1971).) We define the mean force f_e acting on an electron as $F_\sim$ divided by the total number of electrons in the volume considered. For cylindrical plasma (radius a, length $z_2 - z_1 = l$, concentration n_0), the equivalent longitudinal electric field is

$$E_{ez} = \frac{f_e}{e} \simeq \frac{k_zP}{\pi e a^2 l n_0 \omega}. \tag{5.1}$$

Following Wort's analysis, we assume that the force acting on the electrons is balanced by electron–ion collisions and use our results for the motion of particles along a torus. The steady toroidal drifts of electrons (V_{ez}) and of ions (V_{iz}) produced by the HF field are given by the following approximate equations:

$$f_e \simeq m_e\nu_e(V_{ez} - V_{iz}), \qquad V_{iz} \simeq f_e\tau_c/m_i, \tag{5.2}$$

where ν_e is the effective collision frequency of electrons and τ_c is the containment time of ions. The ratio

$$\frac{V_{iz}}{V_{ez}} \simeq \frac{\nu_e\tau_c}{\nu_e\tau_c + (m_i/m_e)} \tag{5.3}$$

may apparently approach unity, the steady ion velocity being

$$V_{iz} \simeq \frac{k_zP\tau_c}{\pi m_i a^2 l n_0 \omega}. \tag{5.4}$$

P is now the power absorbed in the toroidal plasma having major radius R and $l = 2\pi R$.

According to the first of equations (5.2), the total toroidal electric current $\sim(V_{iz} - V_{ez})$ is

$$I \simeq \frac{ek_z P}{2\pi m_e v_e \omega R}. \tag{5.5}$$

This estimate accords with the experiments of HIRANO *et al.* (1971). To obtain a given poloidal magnetic field $H_{0\varphi} \simeq 2I/ac$ at $r = a$, we require the following HF power to be absorbed in the plasma torus:

$$P \simeq \frac{\pi m_e c v_e R a \omega H_{0\varphi}}{ek_z}. \tag{5.6}$$

Let us insert the classical collision frequency (see e.g. BRAGINSKII, 1963) and introduce $q = (aH_{0z}/RH_{0\varphi})$. At the typical value of the Coulomb logarithm $\lambda = 18$, we obtain

$$P \simeq 3 \times 10^{-11} \frac{a^2 H_{0z} n_0 \omega}{q T_e k_z v_e}, \tag{5.7}$$

where, exceptionally, P is in watts and T_e is in electron volts (the remaining quantities are in Gaussian units). It is natural that, for the case considered by WORT (1971), formula (5.7) gives approximately the same result as presented there. Let us take parameters typical of existing tokamaks: $a = 7$ cm, $H_{0z} = 2 \times 10^4$ Oe, $n_0 = 10^{13}$ cm^{-3}, $q = 4$, $T_e = 10^3$ eV, $v_e = 2 \times 10^9$ cm/sec and $(\omega/k_z) \leqslant v_e$. The necessary HF power is $P \leqslant 10^5$ W, a quite plausible value.

We should like to comment on the effect of ion drift discussed in WORT (1971). According to (5.5) and (5.2) the toroidal current depends on the ion motion via the quantities P and ω/k_z. Provided the ion drift given by (5.4) is not inconsistent with the plasma containment it is unnecessary to stop the ions from drifting.

6. CONCLUSION

The simple expressions for the dragging force deduced in Section 2 are valid for any absorbing (radiating) medium and for arbitrary mechanism of absorption (radiation). During HF heating of a plasma cylinder or torus, longitudinal electric currents and plasma motions can arise as a consequence of any violation of the longitudinal ($\pm z$) symmetry. The poloidal magnetic field produced by these currents changes more or less the magnetic surfaces, which must be considered or controlled from the standpoint of plasma containment. On the other hand, it seems quite realizable to replace the driving electric field in a moderate scale tokamak by HF dragging. For this purpose, the heating system must ensure sufficiently strong absorption on the electrons and a suitable distribution of this absorption throughout the plasma cross section. The latter requirement can be satisfied by using an appropriate multi-wave HF system.

Although the theory evolved for a straight cylinder has been used for toroidal models, one may expect specific peculiarities in a rigorous "toroidal" theory.

Acknowledgements—The author is indebted to the participants of the meeting held at the Institute on 5 October 1972 for the review of the paper. He also thanks V. Kopecký for discussions at the early stage of the work.

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