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MEASUREMENT OF PLASMA TEMPERATURE BY ABSORPTION OF RESONANT LASER RADIATION

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Abstract—This paper proposes a method of evaluation of plasma temperature by measurement of its transmission coefficient with respect to radiation that has a resonance with one of the plasma emission lines. The optical thickness of this line can be calculated if the Saha–Boltzmann relation is valid and this is very sensitive to temperature and very insensitive to electron density. The laser used as radiation source must be powerful enough to prevent the self emission of the plasma interfering with the measurement, but not so powerful that it disturbs the population densities in the plasma. Limits on the laser power are calculated and there is a margin of 10^7 for most experimental plasmas. Calculations show that this method may be used to measure the temperature of an argon plasma if it lies between 15,000°K and 20,000°K to an accuracy of 1 per cent using an AII laser operated at λ4880 Å wave length. No knowledge of the electron density is required except that it should lie between $4 \cdot 10^{16}$ cm^{-3} and $2 \cdot 10^{20}$ cm^{-3}.

1. TRANSMISSION OF THE LASER BEAM

THE optical depth of a plasma at a frequency ν situated within one of its emission lines can be expressed, following ATHAY and THOMAS (1961) as

$$\tau(\nu) = \frac{h\nu}{4\pi} B(1, 2) \int_a^b n(1) \left[1 - \exp\left(-\frac{h\nu}{kT}\right)\right] \phi(\nu)\, \mathrm{dx}. \tag{1}$$

In this expression $\phi(\nu)$ is the profile of the emission line, $(b–a)$ is the geometric thickness of the plasma in the X direction, $n(1)$ is the population density of the lower energy state and $B(1, 2)$ is the Einstein intensity absorption coefficient. The exponential factor is the contribution due to stimulated emission which, for temperatures and frequencies of interest here, is much less than unity. Collision domination of the plasma is assumed in order that the emission and absorption profiles should be the same and also to permit the use of Boltzmann statistics for evaluation of the stimulated emission contribution.

If a beam of radiation from an external source containing frequencies within the profile $\phi(\nu)$ is shone through the plasma in the X direction then the intensity emerging at plasma boundary $x = b$ is

$$I(\nu, b) = I(\nu, a) \exp\left[-\tau(\nu)\right] \tag{2}$$

where $I(\nu, a)$ is the external source intensity incident on the plasma boundary at $x = a$. A detector situated at some coordinate $x \geqslant b$ and aligned to receive the beam emerging from the plasma will receive from the external source a power $E(b)$ that is the integral of equation (2) over the solid angle of acceptance and over the frequency distribution of the external source. If this source is a laser then its frequency range can be very small; for example BENNETT *et al.* (1966) have measured the frequency width of the AII λ4880 Å laser line and found it to be the order of 250 Mc/s. This

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should be compared with the width of 11,500 Mc/s for $\phi(\nu)$ obtained for the same spectral line when emitted by a collision dominated argon plasma at a temperature of $T = 20{,}000°K$ and an electron density of $N_- = 10^{17}\ \mathrm{cm}^{-3}$. A laser beam can also have an extremely small beam cross section and small beam solid angle. Consequently if the plasma is homogeneous in T and N_- over the beam cross section at any co-ordinate x then the exponential function of equation (2) can be taken outside the frequency and solid angle integrations to give

$$E(b) = E(a) \exp\left[-\tau(\nu_L)\right] \tag{3}$$

where $E(a)$ is the power of the laser beam incident on the plasma boundary at $x = a$ and ν_L is the laser frequency, assumed somewhere within the plasma line profile $\phi(\nu)$. The transmission coefficient of the plasma with respect to the laser beam follows directly from equation (3) as

$$\Gamma = E(b)/E(a) = \exp\left[-\tau(\nu_L)\right] \tag{4}$$

and this parameter can be measured by the detector as a function of time if the plasma is transient and the laser is continuous.

2. EVALUATION IN TERMS OF PLASMA DENSITY AND TEMPERATURE

If the collision domination is valid right down to the ground state of the plasma emitting species then $n(1)$ can readily be related to the total number density of this species by Boltzmann statistics. Furthermore this total number density can be related to the plasma electron density N_- by means of the Saha equations and the equation of electrical neutrality. Therefore $n(1)$ can be expressed in the form

$$n(1) = \gamma N_- \frac{g(1)}{U(T)} \exp\left[-\frac{E(1)}{kT}\right] \tag{5}$$

where $E(1)$ and $g(1)$ are respectively the excitation energy and statistical weight of the lower energy level and $U(T)$ is the partition function which is a comparatively weak function of temperature and almost independent of electron density. The parameter γ is the ratio of the total number density to the electron density. If the relevant energy transition is from the singly ionised species and if also the plasma is composed of only one chemical element then γ is unity provided doubly ionised and more highly ionised species are of negligible abundance. When multiple stage ionisation becomes important γ can assume very small values. Recent computations by CHOWDHURY (1969) for an argon plasma show that for temperatures less than 20,000°K this parameter lies in the range $0{\cdot}643 \leqslant \gamma \leqslant 1{\cdot}000$ for electron densities in the range $10^{16} \leqslant N_- \leqslant 10^{21}\ \mathrm{cm}^{-3}$. Substitution of equation (5) into equation (1), with the assumption that the plasma is homogeneous in the X direction along the laser beam, yields with some rearrangement,

$$\tau(\nu_L) = \frac{h\nu_L}{4\pi}(b-a)B(1,2)\frac{\gamma g(1)}{U(T)}\left[1 - \exp\left(-\frac{h\nu_L}{kT}\right)\right] N_-\phi(\nu_L)\exp\left[-\frac{E(1)}{kT}\right]. \tag{6}$$

Most experimental plasmas are sufficiently dense and low in temperature for the emission lines in the visible spectrum to suffer Stark microfield broadening to a larger extent than thermal Doppler broadening. If the upper energy level is non-degenerate then the microfield due to the electron impacts produces a line profile that is dispersive in shape about the line central frequency ν_0

$$\phi(\nu) = \frac{w}{\pi} \frac{1}{w^2 + (\nu - \nu_0)^2} \tag{7}$$

where w is the profile half-half width. Naturally if the laser frequency ν_L is to lie within this profile then it is almost imperative that both the laser and the plasma transitions take place between identical energy levels. However ν_0 and ν_L do not exactly coincide because there is a small Stark shift d produced by the plasma

$$\nu_0 = \nu_L + d. \tag{8}$$

The parameters w and d are calculable from broadening theory and can be expressed

$$w = \omega N_-; \qquad d = \delta N_- \tag{9}$$

where ω and δ are independent of electron density and are both comparatively weak functions of temperature. Substitution of equations (8) and (9) into (7) gives

$$N_-\phi(\nu_L) = \frac{\omega}{\pi} \frac{1}{\omega^2 + \delta^2}. \tag{10}$$

This product is not entirely independent of electron density as the right hand side of equation (10) would indicate because a small amount of ion broadening takes place in addition to the electron broadening and a slight density dependence results. ROBERTS (1967) has recently evaluated the parameters ω and δ for the AIIλ4880 Å transition and, with ion broadening included, the product $N_-\ \phi(\nu_L)$ is found to vary at constant temperature by less than ± 8 per cent for $10^{16} \leqslant N_- \leqslant 10^{21}$ cm^{-3} for any temperature in the range $10{,}000 \leqslant T \leqslant 40{,}000$°K. The weak temperature dependence of equation (10) is evidenced by a variation at constant density of less than ± 25 per cent in the product $N_-\phi(\nu_L)$ for $10{,}000 \leqslant T \leqslant 40{,}000$°K for any density in the range $10^{16} \leqslant N_- \leqslant 10^{21}$ cm^{-3}.

The foregoing discussion of the various terms that constitute equation (6) show that, for the case of a AIIλ4880 Å laser probing an argon plasma for which $T \leqslant 20{,}000$°K and $10^{16} \leqslant N_- \leqslant 10^{21}$ cm^{-3}, the optical depth $\tau(\nu_L)$ is almost independent of electron density and has a temperature dependence dictated by the final exponential term. In the case of singly ionised argon the lower energy level $E(1)$ is of such a value that this temperature dependence is very strong indeed, in fact the exponential term approximately doubles its value for each additional 1000°K of temperature in the region $10{,}000 \leqslant T \leqslant 20{,}000$°K. Table 1 shows the results of the calculations including figures for higher temperatures where the value of γ falls far below unity and becomes strongly density dependent. The error attributed to $\tau(\nu_L)$ is ± 10 per cent and is entirely the result of a ± 10 per cent theoretical error in the line width parameter ω. The much larger error of ± 20 per cent in the line shift parameter δ has a negligible effect since the dispersive profile is fairly flat in its central region. Recent measurements by BENNETT *et al.* (1964) and BAKOS *et al.* (1966) enabled $B(1, 2)$ to be evaluated to an accuracy of ± 2 per cent. Figure 1 shows the transmission coefficient Γ from equation (4) for the argon plasma throughout the temperature range where the density dependence is smallest. The ± 10 per cent error associated

TABLE 1.—OPTICAL DEPTH $\tau(\nu_L)$ OF A ONE CENTIMETRE GEOMETRIC DEPTH OF HOMOGENEOUS ARGON PLASMA EVALUATED AT λ4880 Å

This parameter is expressed as a function of N_- and T in the form a/b where $\tau(\nu_L) = a \times 10^b$

T 10^3 °K \ N_- cm^{-3}	10^{16}	10^{17}	10^{18}	10^{19}	10^{20}	10^{21}
10	3·26/−5	3·24/−5	3·21/−5	3·13/−5	3·00/−5	2·77/−5
11	2·37/−4	1·87/−4	1·85/−4	1·81/−4	1·74/−4	1·60/−4
12	7·96/−4	8·04/−4	7·92/−4	7·81/−4	7·51/−4	6·94/−4
13	2·77/−3	2·74/−3	2·69/−3	2·66/−3	2·56/−3	2·38/−3
14	7·82/−3	7·78/−3	7·67/−3	7·59/−3	7·31/−3	6·79/−3
15	1·91/−2	1·89/−2	1·88/−2	1·84/−2	1·79/−2	1·66/−2
16	4·10/−2	4·12/−2	4·08/−2	4·03/−2	3·89/−2	3·63/−2
17	8·02/−2	8·16/−2	8·09/−2	8·01/−2	7·39/−2	7·22/−2
18	1·39/−1	1·48/−1	1·47/−1	1·46/−1	1·41/−1	1·32/−1
19	2·11/−1	2·49/−1	2·53/−1	2·52/−1	2·43/−1	2·27/−1
20	2·67/−1	3·92/−1	4·08/−1	4·05/−1	3·93/−1	3·66/−1
25	1·18/−1	8·01/−1	1·99/0	2·33/0	2·32/0	2·14/0
30	2·46/−2	3·03/−1	2·11/0	5·53/0	6·67/0	6·17/0
35	2·09/−3	7·97/−2	9·85/−1	5·46/0	1·10/+1	1·06/+1
40	1·57/−4	1·18/−2	3·53/−1	3·26/0	1·10/+1	1·27/+1

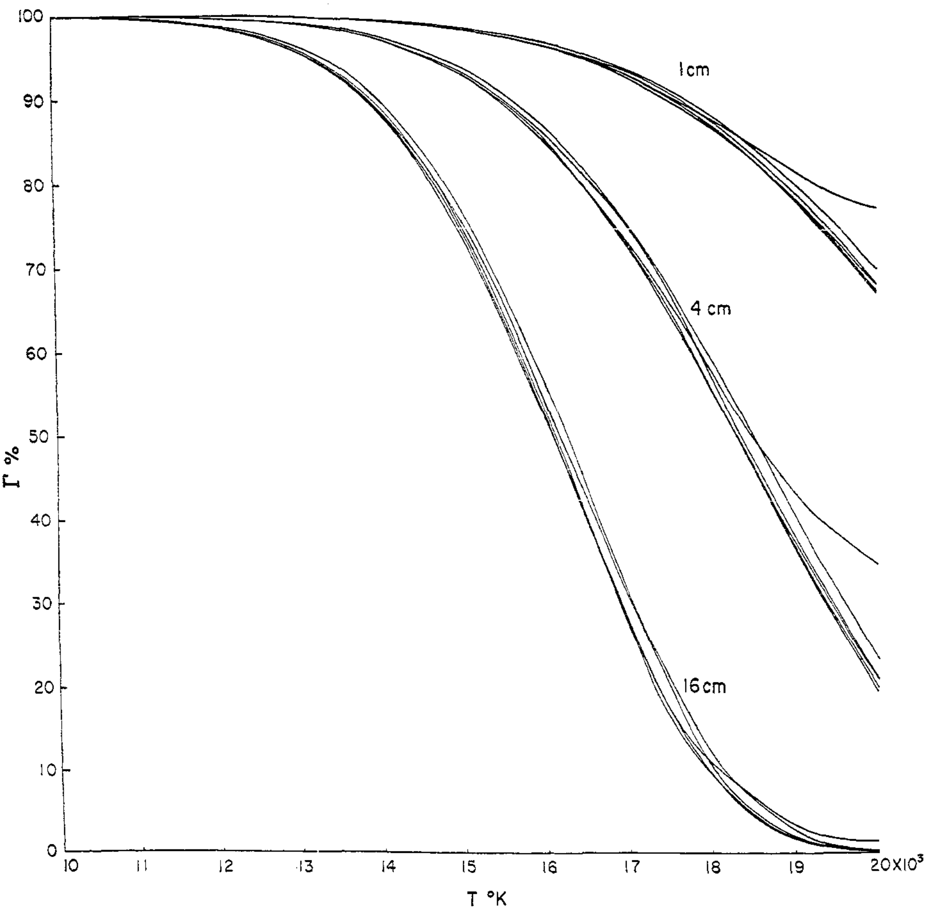

FIG. 1.—Percentage transmission Γ of a homogeneous argon plasma at λ4880 Å as a function of temperature T and geometric depth. Each family of curves is drawn for electron densities N_- of 10^{16}, 10^{17}, 10^{18}, 10^{19}, 10^{20} and 10^{21} cm^{-3} from the data of Table 1.

with Table 1 produces an error of ± 7 per cent in the transmission Γ over the central region of the linear section of the curves and this is equivalent to a temperature error of as little as ± 1 per cent.

3. LIMITATIONS TO THE LASER POWER

The lower limit to the laser power is determined by the tolerable amount of plasma radiation superimposed on the transmitted laser beam whereas the upper limit is determined by the violation of plasma collision domination due to the laser energy absorbed.

It is not readily possible to provide sufficient dispersion to resolve the transmitted laser radiation from the spontaneous radiation emitted by the plasma over the frequency range embraced by $\phi(\nu)$ and within the solid angle of acceptance. A convenient upper limit to the plasma radiation received by the detector is the Planck function $P(\nu_L, T)$ multiplied by the line profile width w and by the solid angle of detector acceptance $\Delta\Omega$. A good measurement of beam transmission can be made if the transmitted laser power received by the detector is ten times this upper limit to plasma power. Therefore the power of the laser at its source, assuming a pessimistically low value of 10 per cent for the transmission Γ, should satisfy

$$E(a) > 10^2 P(\nu_L, T)\Delta\Omega w. \tag{11}$$

Absorption of the laser beam by the plasma decreases the net radiative depopulation rate of the upper energy level (2). Numerically this change in population rate is equal to the number of net absorbing transitions per second and for a homogeneous plasma this change is greatest where the laser beam enters the plasma. Expressed as a decrease in radiative depopulation rate per atom in the lower energy level (1) the expression is

$$C = \frac{1}{4\pi} B(1, 2)E(a)\left[1 - \exp\left(-\frac{h\nu_L}{kT}\right)\right]\phi(\nu_L). \tag{12}$$

A suitable criterion for collision domination of an energy level is that the collisional population rate of that level should be at least ten times the radiative depopulation rate of that level. This collisional rate can be evaluated, following GRIEM (1964), as a function of T and N_- of the plasma. Since an upper limit to laser power is being sought it is sufficient to regard the change in radiative population rate of equation (12) as the entire radiative depopulation rate. Evaluation of equations (11) and (12) and the collisional rate for the AIIλ4880 Å transition over the temperature range presented in Fig. 1 shows that the total laser power $W(a)$ must satisfy

$$2\,.\,10^{-21}N_- < W(a) < 4\,.\,10^{-32}N_-^2 \text{ W}.$$

This calculation assumes a laser beam diameter of 2 mm and a beam divergence of 10^{-5} sterad.

4. VALIDITY OF THE THEORY

The approximations made in the foregoing analysis fall into two categories; those concerning the line profile and those concerning plasma collisional domination.

The assumptions of the theory of electron impact broadening are many and are fully discussed by ROBERTS (1967). For the AIIλ4880 Å transition there are two assumptions that determine the limits of the regime of validity. Firstly the theory assumes that the free electrons suffer little change in motion on perturbing the emitting particle. This condition places a lower limit on the plasma temperature, $T \gg 7 \,.\, 10^{3}$°K. Secondly it is assumed that perturbing energy levels are non-degenerate and this condition places an upper limit on the electron density, $N \ll 2 \,.\, 10^{20}$ cm^{-3}. As yet no theoretical or experimental investigation has been made to evaluate these conditions quantitatively. Thermal Doppler broadening is always present in a plasma and it influences the line profile $\phi(\nu)$ predominantly at low electron densities. If the small amount of ion broadening is neglected then the effects of Doppler broadening can be evaluated quantitatively using the Voigt profiles. For the AIIλ4880 Å transition it is found that the values of $\tau(\nu_L)$ in Table 1, evaluated in absence of Doppler broadening, are less than 10 per cent in error at 20,000°K providing $N_- \gg 4 \,.\, 10^{16}$ cm^{-3}.

In the previous analysis the collisional domination of energy levels right down to the ground state of the emitting species is assumed in order that equation (5) should be valid. According to GREIM (1964) the criterion for this to be so is that the radiative population rate of the ground state from the resonance level should be a factor of ten less than the corresponding collisional rate. For a singly ionised argon species this requirement is $N_- \gg 2 \,.\, 10^{13} T^{1/2}$ cm^{-3}. However Greim argues that this requirement can be relaxed by a factor of ten if considerable self absorption takes place. This is certainly true in the present case which, in essence, deals with strong self absorption of a non-resonance line. For temperatures up to 20,000°K this relaxed requirement is already satisfied by the requirement of negligible Doppler broadening.

Consequently it would appear that the results presented in Fig. 1 are valid for an argon plasma that has an electron density in the range $4 \,.\, 10^{16} \ll N_- \ll 2 \,.\, 10^{20}$ cm^{-3}.

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REFERENCES

ATHAY R. G. and THOMAS R. N. (1961) *Physics of the Solar Chromosphere*, p. 95. Interscience, New York.
BAKOS J., SZIGETI J. and VARGA L. (1966) *Phys. Lett.* **20**, 503.
BENNETT W. R., KINDLMANN P. J., MERCER G. N. and SUNDERLAND J. (1964) *Appl. Phys. Lett.* **5**, 158.
BENNETT W. R., BALLIK E. A. and MERCER G. N. (1966) *Phys. Rev. Lett.* **16**, 603.
BURGESS D. D. and COOPER J. (1965) *Proc. phys. Soc.* **85**, 1261.
CHOWDHURY S. S. (1966) *J. quantve. Spectros. & Radiat. Transf.*
GRIEM H. (1964) *Plasma Spectroscopy*. pp. 145–152 McGraw-Hill, New York.
ROBERTS D. E. (1967) Ph.D. Thesis, Chap. 5, London.