Methods of image reconstruction from projections applied to conformation radiotherapy

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Abstract. The problem of optimizing the dose distribution for conformation radiotherapy with intensity modulated external beams is similar to the problem of reconstructing a 3D image from its 2D projections. In this paper we analyse the relationship between these problems. We show that the main image reconstruction methods, namely filtered backprojection and iterative reconstruction, can be directly applied to conformation therapy. We examine the features of each of these methods with regard to this new application and we present first theoretical results.

1. Introduction

Conformation radiotherapy is a treatment technique with external beams that aims to match exactly the high-dose region in the target volume and the prescribed target contour (Takahashi *et al* 1961). This therapy method has gained some acceptance since its introduction by Takahashi in 1961. Today it is usually realized using multiple field or gantry rotation techniques for high-energy photon beams. In every position of the gantry, the incident beams are shaped to conform with the target contour by a multileaf collimator or by a fixed set of irregularly shaped collimators. With this technique, conformation therapy can be performed for targets that are convex in planes perpendicular to the gantry rotation axis. However, for several applications treatment volumes with concave regions would be more desirable, for example for the irradiation of the para-aortic lymph nodes (Nemeth and Schlegel 1987). For such cases other sophisticated treatment techniques have been developed. As examples we should mention the biaxial rotation technique (Morita *et al* 1974) or the tangential rotation technique (Rossmann 1955). These techniques are very well suited for a small number of treatment cases, but they are not flexible enough to be generally applied in the conformation irradiation of irregularly shaped targets.

A totally new method for the treatment of such targets has recently been introduced by Brahme (1988). Here not only the shape of the collimator is adapted to the target contour in every position of the gantry, but the intensity is also varied within every beam opening. In this paper we compare this new method of radiation treatment to the method of reconstructing a 3D image (rather, a density distribution of the tissue) from its projections, which is known from computer tomography (CT).

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Both of these methods are closely related: the projections in CT correspond to the intensity modulation functions (IMF) in conformation therapy (figure 1) and the density distribution corresponds to the dose distribution in the patient. The problem in conformation therapy is the following: given the prescribed dose distribution in the patient, calculate the IMF so that the resulting dose distribution comes closest to the prescribed distribution. This is indeed the inverse problem to CT: given the measured projections, calculate the tissue density distribution so that its projections come closest to the given projections.

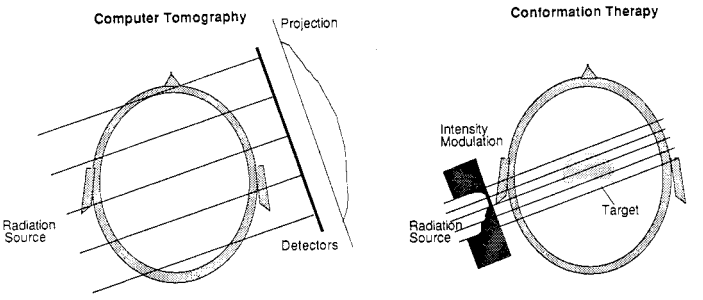

Figure 1. Schematic sketch comparing (*a*) computer tomography and (*b*) conformation radiotherapy. The beam geometry is assumed to be parallel.

The similarity between CT and conformation therapy has been noted in several recently published papers and notes (Brahme 1988, Webb 1989, see also Jones 1990, Webb 1990). However, the methods for the calculation of the IMF used by these authors are different from the methods which are usually used in CT. Brahme uses a 2D convolution technique and Webb uses the method of simulated annealing.

In this paper we present new methods for the calculation of the IMF. These methods are based on two algorithms which are well established in CT, and which allow a very fast calculation and optimization of the IMF. Due to the analogy of CT and conformation therapy, these algorithms can be transferred to this new application without significant changes.

2. Methods

There are two main algorithms used in CT. One is the 'filtered backprojection' and the other is the 'iterative reconstruction technique' (IRT) (Brooks and Di Chiro 1976). We discuss both of these methods in this paper. We call the method of filtered backprojection the method of *filtered projection*, because the projections are calculated and not the superposition of the backprojections, as is the case in CT. However, the application of the method of filtered projection to conformation therapy is based on some assumptions and approximations and there are some limiting factors such as the restriction of the IMF to non-negative values. For these reasons, the resulting dose distribution cannot be regarded as optimal and the IMF that are calculated with this method are only used as an initial guess for a further iterative optimization process. Here we define an optimization criterion and additional constraints. This optimization problem is solved with an iterative optimization technique (IOT) which is similar to the IRT in image reconstruction.

For practical reasons it is desirable that in conformation radiotherapy the number of beams (i.e. the number of IMF or projections) should be as small as possible. However, the required resolution of the dose distribution is usually small, as well. By comparing calculated dose volume histograms for practical cases using 3–15 intensity modulated beams, which are equispaced in 0–2π, we find that about seven or nine beams give satisfying results in all cases. Figure 2 shows such a comparison for the 'horseshoe' target (see section 3). The dose distributions are optimized with the methods described here, i.e. first filtered projection and then iterative optimization. Using more beams generally results in better distributions. However, using much more than seven or nine beams does not improve the dose distributions significantly, so that the higher expense does not seem to be justified. Our number of beams is a little higher than that of Brahme who uses about five beams. However, these numbers are very different from those by Webb, who reports that at least 32 beams are necessary (Webb 1989). This great discrepancy can partly be understood by the fact that, in contrast to Webb, we always use an odd number of beams because this results in a better fit of the dose distributions to the target contour (Brahme 1988). From our experience, a dose distribution resulting from an irradiation with, for example, nine equispaced beams is very similar to an irradiation with 18 equispaced beams for 15 MV photons. This is due to the fact that opposing fields improve the distributions only very slightly.

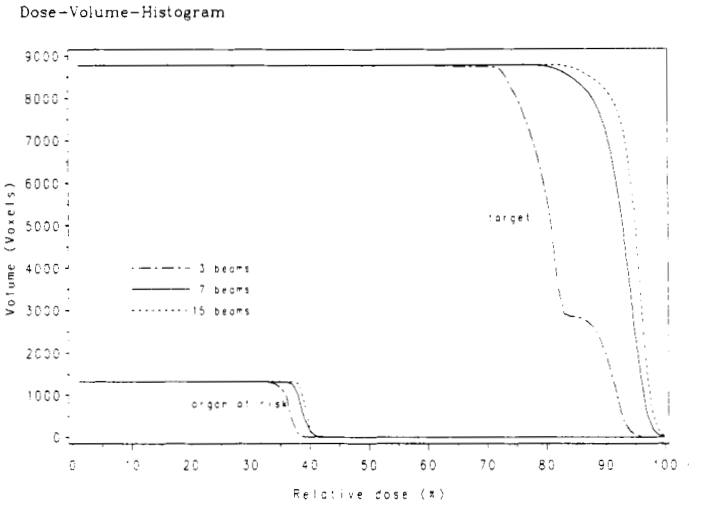

Figure 2. Comparison of optimized dose volume histograms for three (— · —), seven (——) and 15 (· · · · · ·) intensity modulated beams for the 'horseshoe' target (see section 3 for more details).

We should note that the term '3D function' (image or dose distribution) is used with different meanings in the following two sections. Dealing with filtered (back)projection (section 2.1) we treat a 3D function as a set of 2D functions and a 2D projection is replaced by a set of 1D line projections. In this section the beam geometry is assumed to be parallel (figure 1). This approximation is allowed because the method of filtered projection is only used to produce an initial guess. In the section dealing with optimization (section 2.2) we remove this approximation and regard real 3D volume images and 3D dose distributions. Throughout this paper the term 'projection' stands for the summation of density/dose values along the beam direction. In the case of CT the projection is thus the logarithm of the detector signal.

2.1. Filtered projection

The photon beams used in conformation therapy are usually produced by linearly accelerated electrons with an energy of $\geq$ 6MeV in a bremsstrahlung process. Thus the resulting photons also have high energies and the following features apply to them.

(i) The attenuation coefficient in the tissue is very small ($\leq 0.05\,\mathrm{cm}^{-1}$), which results in a flat depth–dose profile. (The dose build-up effect will not be considered here, because for the moment the target–skin distance will be assumed to be larger than the build-up depth).

(ii) The scattering is mainly in a forward direction and beam widening due to scatter is a second-order effect.

(iii) The sensitivity of the dose distribution to inhomogeneities of the tissue is usually relatively small. Thus in a crude approximation the inhomogeneities can be disregarded.

As a consequence of these characteristics the depth–dose distribution of a pencil beam is very narrow and its height is approximately constant, at least within a relatively small region such as the target volume. In this section we disregard scattering and inhomogeneities totally. Thus the process of irradiating a volume with intensity modulated beams can be regarded as smearing back the intensity values through the tissue along the beam direction for all beams. This is basically the same as the back-projection process in CT, where this smearing back is performed in a computer to obtain the image.

The next logical step is to transfer the other steps of image reconstruction to conformation therapy. Figure 3 shows a comparison of the corresponding steps. The actions which are performed in a computer are shown as areas shaded in grey. The projection step can easily be performed in a computer. The remaining question is: how must the filter function be chosen to be adequate for conformation therapy? We answer this question by first recalling the derivation of the filter function in image reconstruction.

The method of filtered backprojection was first advanced in radioastronomy by Bracewell and Riddle (1967). It is based on the well known central slice theorem, which states that the 2D Fourier transform of a 2D function on a line through the origin of the frequency domain can be obtained by projecting the 2D function onto a line and Fourier transforming the projection. Now the backprojection (irradiation) process is equivalent to filling the frequency domain with lines at different angles. However, this method emphasizes the low frequencies, because more lines contribute to low-frequency points in the frequency domain than to high-frequency points. For this reason the projections have to be high pass filtered before backprojection (irradiation).

For the ideal case where there is an infinite number of projections available, the filter function is simply given by the absolute value of the spatial frequency, $|\omega|$. If the number of projections is finite, the filter has to be limited by an additional low pass filter, i.e. the resolution of the resulting image is limited.

Since only a very small number of beams is required in conformation radiotherapy the consequence is that in this application the filter function has to be limited to very low frequencies. We use the discrete filter function

$$H(k) = \begin{cases} |k| \exp\left(-k^4/k_0^4\right) & \text{for } k \neq 0 \\ 1 & \text{for } k = 0 \end{cases} \tag{1}$$

where k denotes a discrete variable which is related to the spatial frequency ω and the maximum tumour width in the projections, W, by $\omega = k/W$. The value of

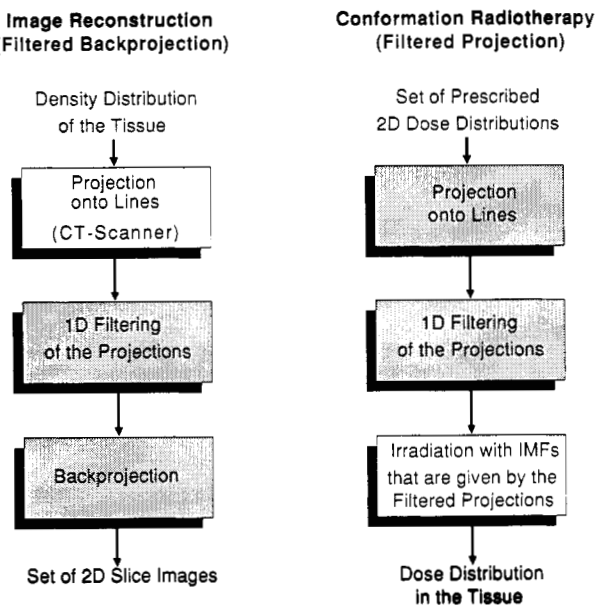

Figure 3. Comparison of the main processes of (*a*) image reconstruction by filtered backprojection and (*b*) conformation therapy.

k_0 determines the cut-off frequency. It is related to the number, N, of beams by $k_0 \simeq N2/\pi$ (Brooks and Di Chiro 1976). If the irradiation has to be performed with less than five beams, the filtering process no longer makes sense, because k_0 becomes too small.

Setting the filter function smoothly to zero by the low pass $\exp\left(-k^4/k_0^4\right)$ gives better results than simply cutting off the filter for frequencies greater than k_0. The value of one for the DC component ($k = 0$) preserves the DC value of the projections. This takes into consideration the positivity constraint of the IMF and improves the dose homogeneity in the target. Figure 4 shows the filter for a cut-off value of $k_0 = 6$ which we use for an irradiation with nine equispaced beams. If the beams are not equispaced, the cut-off value has to be calculated individually for each projection, depending on the local 'beam density'.

After filtering we obtain the IMF from the filtered projections by cutting off any negative values. Figure 5(*b*) shows the resulting dose distribution for an irradiation with nine beams. The merit of the filtering process becomes obvious by comparing this result to figure 5(*a*), which is obtained by applying the projection method without filtering. Although, in contrast to CT, only non-negative values of the IMF are allowed, this does not alter the shape of the isodose lines significantly. This is due to the fact that negative values of the filtered projections mainly appear within regions of high gradients in the projections, i.e. mainly at the field edges. As a consequence, the dose at the target boundary is not rapidly set to zero, but falls off smoothly.

The filtered projection method forces the isodose lines to follow the shape of the target. However, the target is not totally irradiated with at least 80% of the maximum dose. Thus further optimization is necessary.

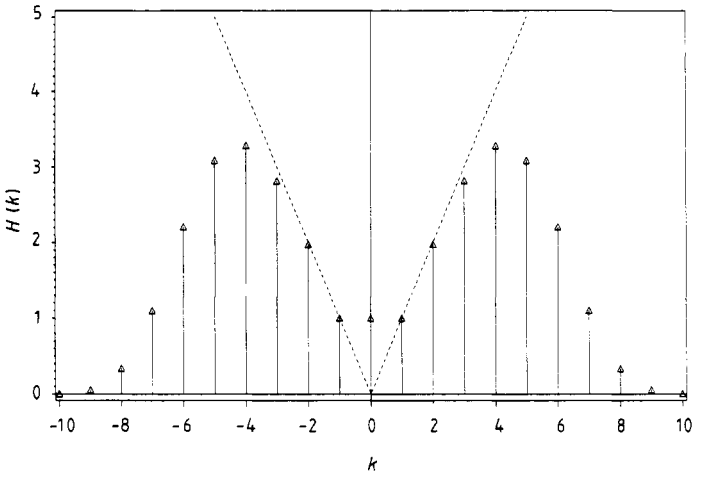

Figure 4. The filter function in the frequency domain. The dashed curve is the $|k|$ function. The cut-off value is $k_0 = 6$.

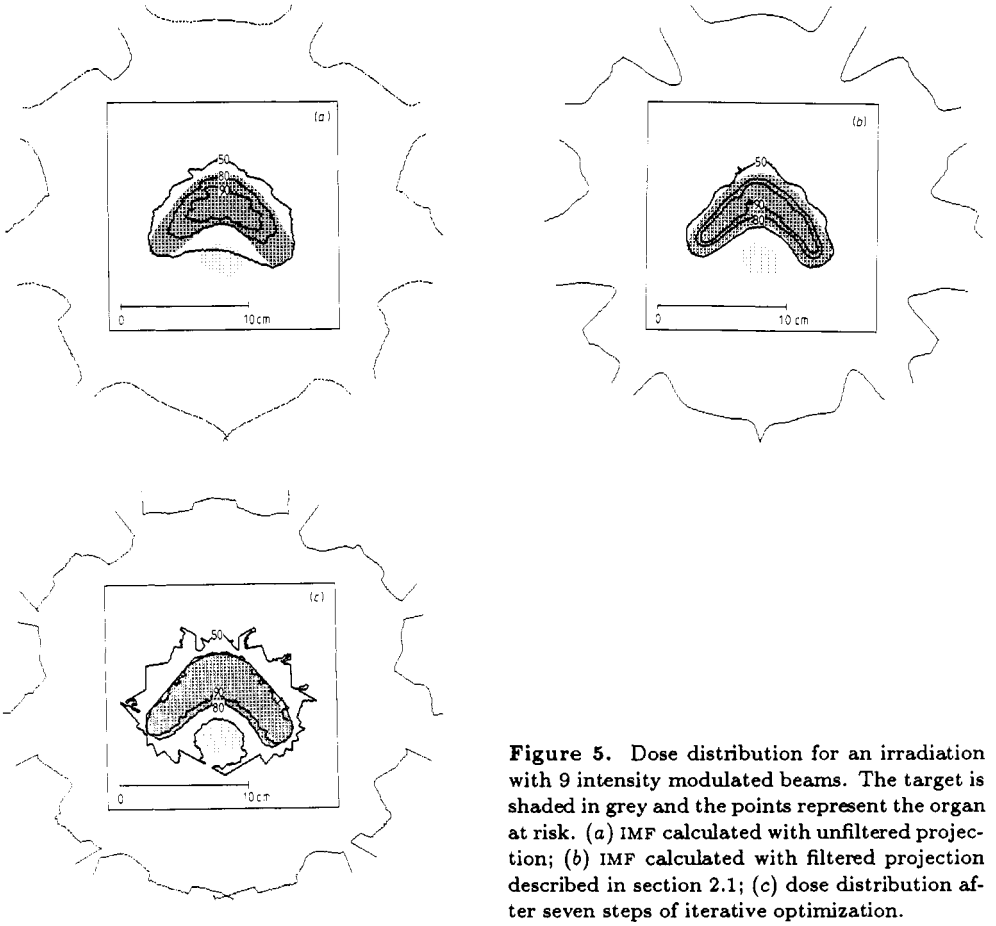

Figure 5. Dose distribution for an irradiation with 9 intensity modulated beams. The target is shaded in grey and the points represent the organ at risk. (*a*) IMF calculated with unfiltered projection; (*b*) IMF calculated with filtered projection described in section 2.1; (*c*) dose distribution after seven steps of iterative optimization.

2.2. Iterative optimization

2.2.1. Optimization criteria. The definition of the criteria which a 'good' treatment plan has to fulfil is one of the central problems in radiation therapy planning. Many researchers in this field have defined such criteria (Hope *et al* 1967, Redpath *et al* 1976). We hold the following four criteria to be the most important ones.

(i) The dose applied to the target should be very close to the prescribed dose.

(ii) The dose should be homogeneously distributed across the target.

(iii) The dose to particular organs at risk which are sensitive to radiation should be less than a tolerable maximum value.

(iv) In the tissue surrounding the target, the dose should be low.

Today these criteria are frequently more quantitatively defined using 'dose volume histograms'. We should emphasize that the criteria should be regarded as preliminary. They are used in this paper to demonstrate the abilities of our algorithm. The algorithm is so flexible that almost any other additional criterion can be incorporated.

In order to obtain a mathematical formulation of the problem we have to define an 'objective function' which is minimized during the optimization process. Further criteria can be included as additional constraints. We define an objective function which takes into consideration the first two criteria. The third criterion represents a constraint and the fourth criterion is fulfilled automatically by the conformation therapy technique, which fits the shapes of the incident beams to the target contour and thus spares the surrounding tissue.

2.2.2. Mathematical formulation. The mathematical definition of our objective function, F_1, is given by

$$F_1 = \sum_{i \in T} (d_i - p)^2 \stackrel{!}{=} \min. \tag{2}$$

where d_i is the calculated dose in the tissue and p is the prescribed dose. The summation is taken over all target points. Thus F_1 is the mean square deviation of the calculated dose and the prescribed dose in the target. This definition of the objective function involves the first two criteria above. It has also been used by several other authors (Starkshall 1984, Redpath *et al* 1976, McDonald and Rubin 1977, Legras *et al* 1986, Webb 1989).

We want to express F_1 as a function of the IMF which are to be optimized. Let $\boldsymbol{x}_k$ be a vector whose components are the intensity values within the opening of beam k, i.e. $\boldsymbol{x}_k$ is the IMF for beam k. Then let $\boldsymbol{x}$ be a vector which is composed of these $\boldsymbol{x}_k$:

$$\boldsymbol{x} = (\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_N)'$$

where N is the number of incident beams (typically seven or nine). Now the dose at every point in the tissue can be expressed as a linear combination of the components of $\boldsymbol{x}$:

$$\boldsymbol{d} = \mathbf{D}\boldsymbol{x}.$$

$\mathbf{D}$ is a 'dose calculation matrix' whose component, D_{ij}, is the contribution of pencil beam j to tissue point i.

D appears here for the purpose of simplification of notation but one should note that in most practical applications it will be difficult to handle this matrix in a computer. Let us for example assume that we wish to optimize an irradiation with nine incident beams each consisting of 1000 pencil beams. If the sampling is chosen so that the relevant 3D tissue area consists of 10 000 voxels, **D** will have the dimensions 10 000 × 9000. This is a large amount for any of today's computers, even if many matrix elements are zero. Thus in practice the required elements of **D** have to be calculated with a dose calculation algorithm during run time of the optimization (the same problem appears in image reconstruction with the 'weighting matrix' (Brooks and Di Chiro 1976)).

Equation (2) can now be written as

$$F_1(\boldsymbol{x}) = \|\mathbf{T}(\mathbf{D}\boldsymbol{x} - \boldsymbol{p})\|^2 \tag{3}$$

where **T** denotes the 'target operator' which extracts only the target points from all tissue points. **T** is a diagonal matrix where $T_{ii} = 1$ if $i \in T$, and $T_{ii} = 0$ otherwise.

As mentioned above, we have to consider some optimization constraints. The most important constraint is the limitation of the dose in sensitive organs at risk to a tolerable maximum value:

$$d_i \leq u_i \qquad i \in R. \tag{4}$$

R is a set of indices that refer to points in such organs at risk and u_i is the upper dose limit for these points. We take account of these constraints by formulating corresponding 'penalty functions' rP. These penalty functions are then added to the objective function, F_1. They are defined such that the minimization of the resulting objective function, $F = F_1 + rP$, leads to results, $\boldsymbol{x}$, which fulfil the constraints for an increasing sequence of values of r (Künzi and Oettli 1969). Our definition of a penalty function for the constraints given by equation (4) is as follows:

$$rP(\boldsymbol{x}) = r\|\mathbf{R}(\mathbf{D}\boldsymbol{x} - \boldsymbol{u})\|^2$$

where r is a parameter which controls the strength of the constraint. **R** is again a diagonal matrix whose elements are given by

$$R_{ii} = \begin{cases} 1 & \text{if } i \in R \text{ and } d_i > u_i \\ 0 & \text{otherwise.} \end{cases}$$

Thus $P(\boldsymbol{x})$ is positive if and only if the constraints are not fulfilled.

Besides the medical constraints there are the physical constraints which require that all of the intensities have to be non-negative. Thus our optimization problem is defined by

$$F(\boldsymbol{x}) = \|\mathbf{T}(\mathbf{D}\boldsymbol{x} - \boldsymbol{p})\|^2 + r\|\mathbf{R}(\mathbf{D}\boldsymbol{x} - \boldsymbol{u})\|^2 \stackrel{!}{=} \min$$

subject to

$$x_i \geq 0 \qquad i = 1, 2, \ldots, MN$$

where M is the number of pencil beams per beam. For the solution of such problems there exists a huge variety of iterative algorithms which are more or less similar to the Newton iteration:

$$\boldsymbol{x}(t+1) = \boldsymbol{x}(t) - \gamma \left(\nabla^2 F(\boldsymbol{x}(t))\right)^{-1} \nabla F(\boldsymbol{x}(t)).$$

The gradient $\nabla F(\boldsymbol{x})$ in our application is given by (omitting the factor 2):

$$\nabla F(\boldsymbol{x}) = \mathbf{D}'\mathbf{T}(\mathbf{D}\boldsymbol{x} - \boldsymbol{p}) + r\mathbf{D}'\mathbf{R}(\mathbf{D}\boldsymbol{x} - \boldsymbol{u}).$$

The inverse of the Hesse matrix $\nabla^2 F(\boldsymbol{x}) = \mathbf{D}'\mathbf{T}\mathbf{D} + r\mathbf{D}'\mathbf{R}\mathbf{D}$ cannot be calculated in an acceptable time because of the huge size of $\mathbf{D}$. Due to this, we approximate the Hesse matrix by a diagonal matrix $\mathbf{S}$ whose diagonal elements are those of the Hesse matrix:

$$S_{jj} = \sum_{i \in T} D_{ij}^2 + r \sum_{i \in R, d_i > u_i} D_{ij}^2.$$

This matrix is easy to invert by inverting its diagonal elements. $\mathbf{S}^{-1}$ can be regarded as a scaling matrix for the gradient. Thus we obtain the iteration equation:

$$\boldsymbol{x}(t+1) = \boldsymbol{x}(t) - (1/N)\mathbf{S}^{-1}\left(\mathbf{D}'\mathbf{T}(\mathbf{D}\boldsymbol{x}(t) - \boldsymbol{p}) + r\mathbf{D}'\mathbf{R}(\mathbf{D}\boldsymbol{x}(t) - \boldsymbol{u})\right). \quad (5)$$

If any component of $\boldsymbol{x}(t+1)$ becomes negative after the update, it is set to zero, i.e. $\boldsymbol{x}(t+1)$ is projected onto the constraint set of non-negative numbers. The optimization algorithm defined by equation (5) is known as the 'scaled gradient projection algorithm'. Note that the normalization constant, γ, has been set to $1/N$.

Our optimization problem is a convex problem, since it is easily shown that the objective function, F, is a convex function and that the set of constraints is a convex set. For such problems it can be proved that the iteration algorithm equation (5) converges to a minimum solution of the optimization problem for a constant value of r (Bertsekas and Tsitsiklis 1989). However, it cannot be guaranteed that the solution is unique. If it is not, the iteration algorithm can be shown to converge to a solution which is closest to the initial guess (Youla and Webb 1982). From this point of view it becomes clear that it is important to investigate the calculation of the initial guess.

To account for the medical constraints we increase r at every iteration step. This process is not critical, because the constraints are already approximately fulfilled for relatively small values of r. We also obtained good results by setting r to a constant value of 30 for every iteration step.

2.2.3. Comparison to IRT. The result above (equation (5)) is very similar to iteration equations which are used in image reconstruction (Rosenfeld and Kak 1982, Brooks and Di Chiro 1976) where the corresponding technique is called 'simultaneous iterative reconstruction technique' (SIRT). In those applications the vector of the image intensities stands on the left hand side of the iteration equation and the difference on the right hand side is that between measured projections and calculated projections. The structure of the equations including scaling and normalization is the same for both applications. Thus in our application we refer to well established methods. The advantage of using a penalty function for the constraints is the similarity of the terms containing $\mathbf{T}$ and $\mathbf{R}$ on the right-hand side of equation (5). The constrained optimization can therefore be performed in the same way as the unconstrained optimization.

The question of when to stop the iteration is not simple in image reconstruction (Brooks and Di Chiro 1976), nor is it in the case of conformation therapy. Based on investigations in image reconstruction which came to the result that 5–10 iteration steps are necessary, we use at least five iteration steps in conformation therapy. If then the dose at any point in the target is still less than 80% of the maximum dose, we proceed with more iterations, until the 80% criterion is fulfilled. If a satisfying dose distribution in the target cannot be achieved, the constraints have to be relaxed.

3. Results

Today's radiotherapy techniques using external beams are usually not well suited for the irradiation of irregularly shaped targets with concave regions. To demonstrate the abilities of the methods described above we apply these methods to a model treatment planning for which we use a target with an extended concave region (figures 5). Such a target is typical for the irradiation of the para-aortic lymph nodes or for the treatment of carcinoma of the oesophagus. We make the situation even more difficult by placing an organ at risk within the concave region.

For this study we use a dose calculation algorithm which is based on a depth–dose calculation by Schoknecht (1968). So far we do not consider tissue inhomogeneities. Figure 5 shows a dose distribution for a 2D slice which we calculated with this algorithm. The target is shown as a grey shaded area and the points represent the organ at risk. The dose calculation has been performed within the square field and the IMF are displayed on a circle around the isocentre. In figure 5(*a*) the IMF are calculated with the projection method without filtering, while figure 5(*b*) is obtained by applying the filtered projection. From these figures it is obvious that with the method of filtered projection one can achieve dose distributions in which the high-dose region is well fitted to the target contour. However, a large part of the target receives a dose of less than 80%, which is not satisfactory.

For a further improvement of the distribution we apply the iterative optimization technique. The constraint is the limitation of the dose in the organ at risk to a maximum of 40% of the maximum dose in the target. Figure 5(*c*) shows the result after seven steps of iterative optimization. For every iteration step the penalty value, r, has been increased by 5, beginning with $r = 5$. Now the 80% isodose fits the target contour almost exactly, which means that the homogeneity of the dose in the target has improved significantly. The dose in the organ at risk has increased slightly, but it is still spared and the constraint is fulfilled. A comparison of the dose volume histograms for the initial dose distribution obtained by the filtered projection method and the optimized distribution is given in figure 6. The calculation time for one iteration step in this 2D slice was about 15 s on a VAXstation 3200.

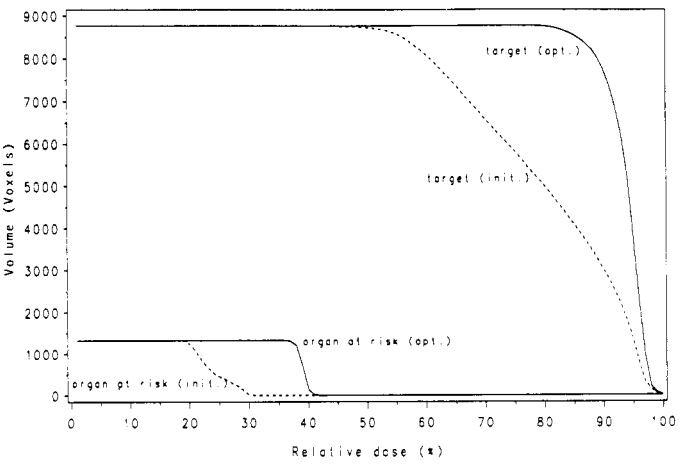

Figure 6. Dose volume histograms for the initial dose distributions of figure 5(*b*) (– – –) and figure 5(*c*) (——).

4. Concluding remarks

We have shown that the methods of image reconstruction from projections can be very promisingly applied to conformation therapy. In the cases studied so far, which incorporate complex shaped targets such as a 'horseshoe' target or the target used by Brahme, satisfying dose distributions can be produced by just seven or nine beams. Only for extreme situations, where more sensitive organs at risk are even closer to a complex shaped target with concave regions, might the number of beams have to be increased. We use the filtered projection method to obtain an initial guess for further optimization. However, if more effort were put into the calculation of the filter function, one might achieve more than an initial guess.

We should mention that although the dose distributions presented in this paper are only 2D, we are able to optimize 3D dose distributions. In doing this we take into consideration the beam divergence in all directions, i.e. the 3D optimization is not simply replaced by a set of 2D optimizations. The calculation time for one 3D iteration step is about 3 min on our VAXstation 3200, depending on the size of the target. Thus the total optimization time is between 20 and 30 min. Inhomogeneities are not taken into consideration and scattering effects are only approximately calculated with the equivalent field method. We are currently working on a more sophisticated dose calculation algorithm for this application.

The problem of achieving the intensity modulation in practice was not within the scope of this paper. Several techniques can be applied for this purpose. Among these are the use of a set of beam compensators, the scanning of a thin pencil beam and the complex motion of a multileaf collimator. We will report about our efforts in this field in a further publication.

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Résumé

Methodes de reconstruction d'images a partir de projections appliquées a la radiothérapie de conformation.

Le problème de l'optimatisation de la distribution de doses pour la radiothérapie de conformation avec modulation de l'intensitè des faisceaux externes est similaire au problème de la reconstruction d'une image tridimensionnelle à partir de ces projections bidimensionnelles. Dans ce papier les auteurs analysent la relation entre ces problèmes. Ils montrent que les principales méthodes de reconstruction d'images, c'est à dire la reconstruction par rétroprojection filtrée et la reconstruction itérative peuvent être directement appliquées à la radiothérapie de conformation. Les auteurs examinent les caractéristiques de chacune de ces méthodes pour cette nouvelle application et présentent les premiers résultats théoriques.

Zusammenfassung

Methoden der Bildrekonstruktion aus Projektionen angewandt in der Konformationstherapie.

Das Problem der Optimierung der Dosisverteilung bei der Konformations-Strahlentherapie mit intensitätsmodulierten externen Strahlen ist dem Problem der Rekonstruktion eines 3D-Bildes aus seinen 2D-Projektionen ähnlich. In der vorliegenden Arbeit wird die Beziehung zwischen diesen Problemen analysiert. Es wird gezeigt, daß die wichtigsten Methoden der Bildrekonstruktion, die gefilterte Rückprojektion und die iterative Rekonstruktion, direkt bei der Konformations-Therapie verwendet werden können. Die Merkmale dieser beiden Methoden werden untersucht unter Berücksichtigung dieser neuen Anwendung und die ersten theoretischen Ergebnisse werden vorgestellt.

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