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A simple watt balance for the absolute determination of mass

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Abstract
A watt balance is an electromechanical device that allows a mass to be determined in terms of measurable electrical and mechanical quantities, themselves traceable to the fundamental constants of physics. International plans are well advanced to redefine the unit of mass, the kilogram, in terms of a fixed numerical value for the Planck constant. A watt balance is one of the devices now being used to measure the Planck constant, but after redefinition will be used to make practical realizations of the kilogram. In this paper we describe a simple 'homemade' watt balance working at the level of about 10% which demonstrates the principle with a view to demystifying the proposed new definition of the kilogram to be based on the Planck constant.

Introduction

A significant overhaul of the present International System of Units (SI) [1] is expected to be implemented within the next five years with the aim of achieving a system of units based on the constants of nature. The biggest change will concern the unit of mass, the kilogram, which will no longer be defined by the mass of a single artefact standard, but instead will be defined in terms of a fixed numerical value for the Planck constant. The ampere, kelvin and mole will be defined by specifying exact numerical values for the elementary charge, e, the Boltzmann constant, k_B, and the Avogadro constant N_A, respectively. The present definitions of the unit of time, the second, and the unit of length, the metre, will remain essentially unchanged. The second will still be defined by specifying the exact numerical value of the frequency in hertz which is exploited by the caesium 'atomic clock' and the metre will still be defined by specifying the exact numerical value of the speed of light in metres per second. Note that at present the kilogram is the only unit based on an artefact and that it also affects the definitions of the ampere (which includes the specification of a force) and the mole (which refers to a mass of 1 kg).

An essential prerequisite to the new SI [2] is that these four constants must be measured experimentally in the present SI. The exact numerical values specified in the new SI will be chosen to be consistent with the present experimental values, thereby ensuring a seamless transition from the present to the new SI.

This paper describes how the new definition of the kilogram can be implemented by taking advantage of electromagnetic phenomena. As mentioned above, the current definition of the kilogram affects the ampere and the mole. It is therefore not surprising that a second method for implementing the new kilogram definition is also being pursued, based on counting the atoms in a single crystal [3]. These methods are complementary.

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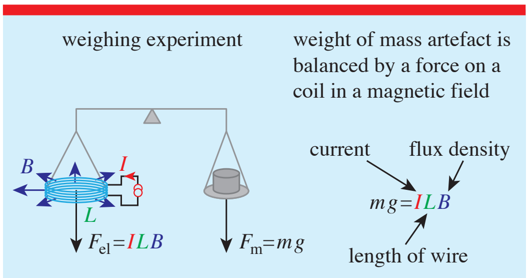

Figure 1. Schematic diagram of a watt balance. The weight *mg* is balanced by an electromagnetic force produced by current *I* flowing in a length *L* of wire placed in a magnetic field of flux density *B*. Adapted with permission from the BIPM. Courtesy of the BIPM.

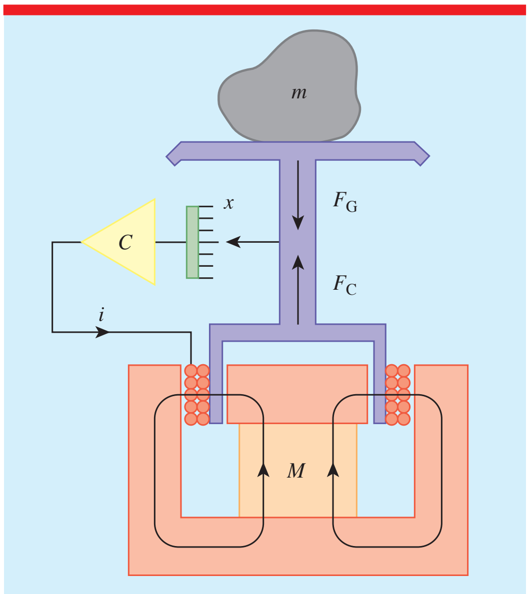

Figure 2. Schematic diagram of a commercial balance, with the cylindrical magnet assembly shown in cross section. The downward weight of an object of mass *m* is balanced by the upward electromagnetic force produced by current flowing through a wire coil placed in the magnet gap. Adapted with permission from Mettler Toledo AG.

Since 1889, the kilogram has been defined as the mass of a single object, the international prototype of the kilogram, kept at the International Bureau of Weights and Measures (Bureau International des Poids et Mesures, BIPM) at Sevres in France and used infrequently. By redefining the kilogram in terms of a fixed numerical value for the Planck constant, the only remaining base unit of the SI to be defined in terms of a material artefact will become fixed to a constant of nature. In this paper we describe a simple version of one of the devices currently being used to determine the value of the Planck constant, but which after redefinition will be used to make practical realizations of the kilogram; it is known as a watt balance. The major national metrology institutes, such as the National Physical Laboratory (NPL) in Teddington, UK and the National Institute of Standards and Technology (NIST) in Washington, DC, USA, have spent decades building and developing watt balances intended to operate at the levels of a few parts in 10^8. The principle of operation of a watt balance is, however, very simple and can be used to demonstrate how a mass can be measured in terms of the Planck constant. The purpose of this 'homemade' version is to demystify the watt balance for students and, indeed, for everybody. To help get a 'feel' for how a watt balance operates, we have constructed an inexpensive model which we describe below. While ours is many orders of magnitude away from the accuracies required in real metrology, it is simple to operate, demonstrates the principle behind any watt balance, and is so basic that ample room for improvement remains for enterprising students. It was actually made in the home of one of the authors, demonstrating that, in principle, anyone can make the link between the Planck constant and a mass piece placed on a simple watt balance.

The principles of a watt balance

The usual description of a watt balance begins with figure 1, in which the weight, mg, of a test object of mass m is balanced by an electromagnetic force ILB:

$$mg = ILB. \tag{1}$$

Here g is the acceleration of gravity, I is the current flowing through the electrical coil, B is the magnetic flux density of a strong magnetic field and L is the length of wire in the coil that is exposed to B. Contrast this with figure 2, which is a schematic diagram from a well-known balance manufacturer to illustrate the principle of servocontrol. There is no need to belabour the similarity between these two figures. So why is the electronic balance shown in figure 2 not

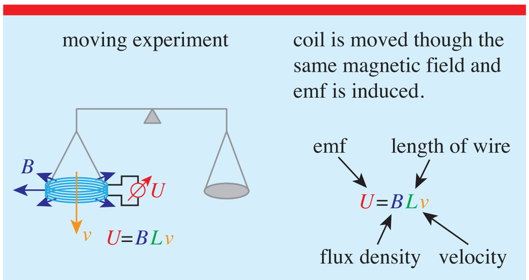

Figure 3. Schematic of a second watt-balance mode used to determine BL from measurements of voltage (emf) induced at the ends of the coil as it moves at velocity v through the magnetic field. Adapted with permission from the BIPM. Courtesy of the BIPM.

a watt balance? The reason is that it requires calibration by a weight piece (not shown) called the 'standard', whose mass is already known. This standard must in turn be calibrated against another and another up to the national standard kept in an institute such as the NPL whose national standard must finally be calibrated against the current international prototype of the kilogram kept at the BIPM. By contrast, the goal of a watt-balance experiment is to determine the mass of any suitable object in terms of h, not in terms of another mass. Going back to equation (1), we will obviously need the value of g, which can be measured to high accuracy using special gravimeters, but for our purposes is already known to be 9.8 m s^{-2}. However, the measurement of LB is problematic. In addition, h is nowhere to be seen. The solution implemented in watt-balance experiments is to carry out a second measurement using the same balance and this is shown in figure 3.

In figure 3, the mass to be measured has been removed and the electrical coil is now connected to a high-impedance voltmeter. The coil is moved vertically at a constant velocity, v, so that a voltage, U, is induced on the ends of the coil. The simple equation describing this is shown in figure 3, from which the troublesome term BL is seen to equal something that is amenable to measurement:

$$BL = U/v. \tag{2}$$

From equation (1) we know that $LB = mg/I$, so that in our geometry

$$m = IU/(gv). \tag{3}$$

A device whose operation is described by equation (3) is known as a watt balance because the unit in which IU and mgv are both measured is the watt.

The connection between m and h is already assured by using equation (3), provided that the electrical measurements of voltage and current are traceable to quantum electrical standards, as outlined in the next two paragraphs.

In the new SI [2], voltage measurements will be traceable to the ratio h/e, where e is the elementary charge ($-e$ is the charge carried by an electron). How this can be done is a remarkable story that starts with a Nobel prize for the discovery of the Josephson effect [4]. The effect has been used by the NPL and many other institutes throughout the world since 1990 to disseminate consistent values of voltage for the calibration of voltmeters. When the new SI takes effect, the value of h/e will have a fixed numerical value. Similarly, resistance measurements in the new SI will be traceable to h/e^2—another remarkable story and another Nobel prize for the discovery of the quantized Hall effect [5]. This effect has been used by the NPL and many others, again since 1990, to disseminate consistent values of resistance for the calibration of standard resistors. When the new SI takes effect, h/e^2 will have a fixed numerical value. A perfectly reasonable question to ask at this point is how can we fix (that is, define) the numerical value of a constant when it is supposed to be constants whose values are fixed by nature? For an explanation of what it actually means to fix the numerical value of a fundamental constant and more background to this whole question of units based on constants of nature, see [6, 7].

To appreciate how a watt balance connects m to h, the following is all one needs to know. The electrical current shown in figure 1 is determined from Ohm's law, $I = U'/R$, requiring voltage measurement across the leads of a stable resistor (of known resistance) that is part of the circuit. For our watt balance, we use a calibrated multimeter so that both U and U' are traceable to h/e and R is traceable to h/e^2. All parameters on the right-hand side of equation (3) will thus have their traceability to some combination of the frequency that defines the second, the speed of light in vacuum and the Planck constant [2]. What happened to the elementary charge, e? It conveniently drops out because the measurement of $U'U/R$ has been traced back to $(h/e)(h/e)/(h/e^2) = h$, i.e. the measurement of m is traceable to h. (The unit of h is kg m^2 s^{-1}, an indication that auxiliary measurements of length and time are also needed to measure a mass in kilograms. The important point is that these auxiliary measurements are independent of h and/or e.)

How to make a watt balance

We searched for a relatively inexpensive, ready-made device that resembles figure 2 and discovered that a loudspeaker designed for low frequencies (i.e. a woofer) would be suitable (figure 4). The tricky business of making a coil-and-magnet assembly is already done. The 'spider' shown in figure 4 is a spring that is linear over a small distance, but a spring nevertheless. The remaining elements of the loudspeaker serve to guide the coil vertically when it moves through the narrow gap of the magnet assembly. To make sure that the magnitude of m that we would like to measure (about 200 g) and the magnitudes of U, I and v that we are able to measure easily are not asking too much of the loudspeaker, we needed some engineering details. The information required is among the Thiele/Small small-signal parameters [8] reported in loudspeaker catalogues. The most important design parameter for us is BL (equal to LB) because, even though we must measure it ourselves as shown in figure 3, the approximate value of this parameter predicts the values expected for mg/I and U/v. The design parameter BL is often given in publications describing watt balances, but it is also a Thiele/Small parameter for loudspeakers, and for much the same reasons. For our loudspeaker (B&C model 6NDL38) [9], the parameter BL is specified by the manufacturer to be 9.5 T m (which, as shown above, is equivalent to 9.5 N A^{-1} and 9.5 V (m/s)$^{-1}$).

With a suitable loudspeaker selected, it was then necessary to find a convenient way to integrate it into a watt balance. The speaker coil was made to move up and down by means of a lever arm driven by a vertical movement from a piston of a LEGO model of an internal combustion engine. The stroke of the piston was about 9 mm and the lever arm was such that the

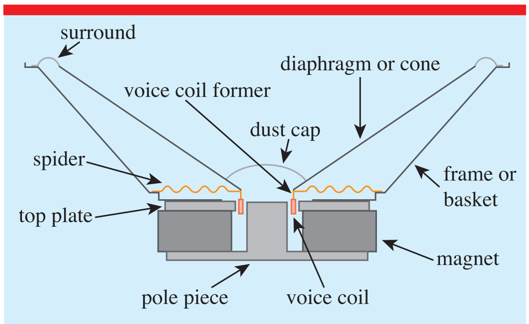

Figure 4. Loudspeaker cross section. Adapted from figure taken from Wikimedia commons.

loudspeaker coil moved by about 1.25 mm. The engine was driven by an electric motor through a reduction gear so as to give a rotation rate of about 60 revolutions per minute. The vertical movement of the coil, driven through the lever linkage from the up-and-down motion of the piston, was measured by a dial-gauge micrometre. The voltage which appeared at the loudspeaker input as the coil moved up and down was recorded on an oscilloscope. The set-up is shown in figure 5. From the distance moved, the rate of rotation of the motor, and the voltage produced we can deduce the voltage corresponding to a particular velocity of the coil, thus v and U in equation (2).

To measure the mass, m, the lever was uncoupled from the piston and the dial gauge set to read the vertical rest position of the loudspeaker coil. The resolution of the dial gauge was about 0.01 mm. The 'unknown' mass was then suspended from the lever arm at a convenient position, a distance d_1 from the lever fulcrum, to give a movement of a few millimetres to the loudspeaker coil, whose vertical axis is a distance d_2 from the lever fulcrum. The ratio d_2/d_1 was needed because a lever with unequal arms changes equation (1) to

$$mg = ILB(d_2/d_1). \tag{4}$$

(In figure 1 it is assumed for heuristic reasons that (d_2/d_1) is exactly 1. Accurate watt balances, such as that shown in [10], are designed to circumvent the need for a lever.) The electric current, I, was adjusted until the dial gauge indicated that the loudspeaker coil was again at the initial rest position. Adjusting I manually takes the place of a more elegant servocontrol

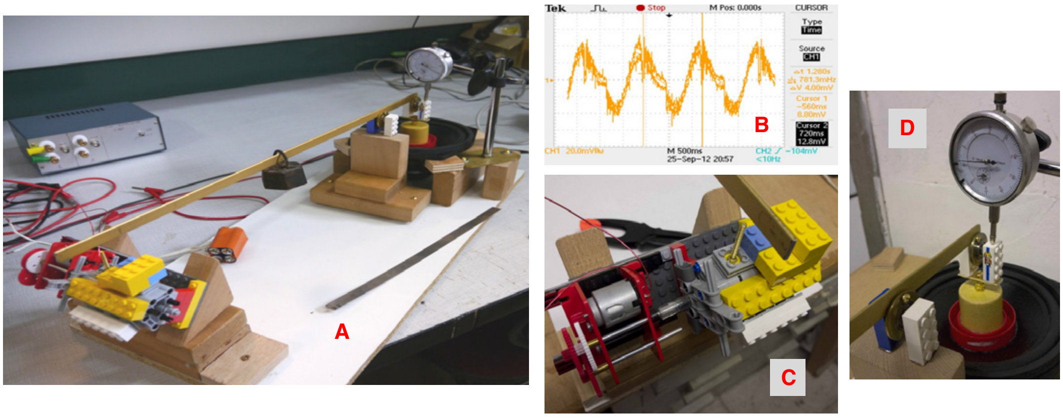

Figure 5. (A) The watt balance configured in the weighing mode. The 200 g test mass is suspended from the lever a distance d_1 from the fulcrum; the dial-gauge micrometre is reading the current position of the coil with respect to its position without the test mass. (B) Oscilloscope trace made during several cycles of the moving mode. During the moving mode, the test mass is removed and the lever is connected to the motor drive; a detail of the drive mechanism is shown in (C) and of the lever arm/loudspeaker assembly in (D).

system hinted at in figure 2, but serves the same function. The current can be determined from Ohm's law by voltage and resistance measurements, as described above.

Thus, from the measured values of v, U, I, (d_2/d_1) and taking the acceleration due to gravity, g, as 9.8 m s^{-1} we can deduce the mass. The accuracy is limited mainly by the lack of uniformity and smoothness of the motion of the pistons, but is sufficient to demonstrate the principle. The dimensionless ratio (d_2/d_1) is needed for our simple watt balance, but is not a feature of any watt balances actually operating in national institutes or the BIPM.

The apparatus shown in figure 5 was built in fact at the home of one of the authors (TJQ) with the LEGO parts assembled from a larger kit by another (LQ). The selection of the loudspeaker and the subsequent design calculations were carried out by RD. The electrical measurements were made with the help of Nick Fletcher in the laboratories of the BIPM.

We were gratified that signals of the predicted magnitudes were readily seen the first time we activated our watt balance. Mass measurements were as linear as could be expected from the precision of our measurements, as could be deduced by moving the test mass to different positions along the lever. Doubling the velocity v also had no measurable effect on the determination of m. Because we know that m should be 200 g (the new SI will not cause discontinuities with the current one), we can deduce that our set-up has a systematic error of order 10%. This is small enough to support our hypothesis that watt balances pioneered at places like the NPL and NIST are just better, very much better, versions of our own.

The purpose of this demonstration was to show that the new definition of the kilogram, although it apparently brings in some esoteric physics, is in fact simple to understand and provides a practical example of metrology suitable for high-school and undergraduate laboratories. It will be obvious, and a challenge to anyone interested, that with modest additional effort and access to a mechanical workshop a much more precise version could easily be built.

Acknowledgments

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Note. Our watt balance has been selected as an exhibit at the 2013 Royal Society Summer Science Exhibition.

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Lucas Quinn is Terry Quinn's grandson, aged 11 years and an expert in Lego construction.

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