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# On the fusion triple product and fusion power gain of tokamak pilot plants and reactors

**A.E. Costley**

Tokamak Energy Ltd, Culham Innovation Centre, D5, Culham Science Centre, Abingdon, OX14 3DB, UK

E-mail: Alan.Costley@physics.org

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**Abstract**

The energy confinement time of tokamak plasmas scales positively with plasma size and so it is generally expected that the fusion triple product, $nT\tau_E$, will also increase with size, and this has been part of the motivation for building devices of increasing size including ITER. Here $n$, $T$, and $\tau_E$ are the ion density, ion temperature and energy confinement time respectively. However, tokamak plasmas are subject to operational limits and two important limits are a density limit and a beta limit. We show that when these limits are taken into account, $nT\tau_E$ becomes almost independent of size; rather it depends mainly on the fusion power, $P_{fus}$. In consequence, the fusion power gain, $Q_{fus}$, a parameter closely linked to $nT\tau_E$ is also independent of size. Hence, $P_{fus}$ and $Q_{fus}$, two parameters of critical importance in reactor design, are actually tightly coupled. Further, we find that $nT\tau_E$ is inversely dependent on the normalised beta, $\beta_N$; an unexpected result that tends to favour lower power reactors. Our findings imply that the minimum power to achieve fusion reactor conditions is driven mainly by physics considerations, especially energy confinement, while the minimum device size is driven by technology and engineering considerations. Through dedicated R&D and parallel developments in other fields, the technology and engineering aspects are evolving in a direction to make smaller devices feasible.

Keywords: tokamaks, pilot plants, fusion reactors, steady state operation

S Online supplementary data available from stacks.iop.org/NF/56/066003/mmedia

## 1. Introduction

The principal goal of fusion research is to produce plasma with a sufficiently high value of the fusion triple product, $nT\tau_E$, for energy releasing fusion reactions to occur under controlled and reproducible conditions. For fusion gain from a deuterium–tritium (DT) plasma, $nT\tau_E$ must be $\geqslant 1 \times 10^{21}$ m$^{-3}$ keVs with $T$ in the range 10–20 keV, and for reactors $nT\tau_E$ must be $\geqslant 3 \times 10^{21}$ m$^{-3}$ keVs [1]. For tokamak plasmas, $\tau_E$ scales positively with plasma size and so it is generally expected that $nT\tau_E$ will increase with size, and this has been part of the motivation for building devices of increasing size: the International Thermonuclear Experimental Reactor (ITER) with a plasma volume ~800 m$^3$ [2], currently under construction in France, is the largest example. However, tokamak plasmas are subject to operational limits and two important limits are a density limit and a beta limit, where beta, $\beta$, is the ratio of the plasma pressure to the magnetic pressure. The density limit is ~ $I_P/a^2$ [3] and the beta limit is ~ $I_P/aB_T$ [4], where $a$, $B_T$, $I_p$ are the minor radius of the plasma, the toroidal magnetic field at the plasma centre and the plasma current respectively. Experiments with many tokamaks have established the scaling of $\tau_E$ with the main plasma and device parameters, and also the scalings of the density and beta limits. We have combined these scalings using a simple analytical approach and also using a system code based on an established tokamak physics model. We find that these limits substantially reduce the size dependence of $nT\tau_E$: instead we find that $nT\tau_E$ depends mainly on the fusion power.

A parameter closely related to $nT\tau_E$ is the fusion power gain, $Q_{fus} = P_{fus}/P_{aux}$, where $P_{fus}$ is the fusion power and $P_{aux}$



 

is the power added to heat the plasma and/or to drive current. Not surprisingly in view of the finding on $nT\tau_E$, we find that this parameter is independent of size too. This result is consistent with that found in an earlier investigation [5]. These findings have considerable implications for the design of fusion pilot plants and power plants. We illustrate and discuss them with reference to performance calculations for a device similar in size to the Joint European Torus (JET), which is the largest tokamak currently in operation.

The paper has four main sections. In section 2 we investigate the impact of the operational limits on the fusion triple product and in section 3 we examine the impact on the fusion power gain. The implications of our findings for the design of pilot plants and reactors are discussed in section 4. A summary and conclusions are given in section 5. The paper is supported with online supplementary material (stacks.iop.org/NF/56/066003/mmedia), which is referred to at appropriate places in the text.

## 2. Fusion triple product

Experimental scalings for energy confinement time in tokamak plasmas show dependences on several different plasma and device parameters. For the initial analysis we take just the main dependences which are typically of the form

$$(\tau_E)_{\text{scaling}} \propto I_p R^2 n_e^{1/2} / A^{1/2} P_L^{1/2} \tag{1}$$

where $R$ and $P_L$ are the major radius and total power loss respectively. $A = R/a$ is the plasma aspect ratio. $n_e$ is the electron density and in plasma with charge neutrality $n_e \propto n$. By definition $\tau_E = W/P_L$, where $W$ is the stored energy. We assume cylindrical geometry, so $W \propto nTR^3/A^2$ and $P_L \propto nTR^3/A^2\tau_E$. Under some conditions the achieved confinement time can exceed the scaling confinement time and this is usually handled by including an enhancement factor, $H$: $\tau_E = H(\tau_E)_{\text{scaling}}$. Substituting the expression for $P_L$ in equation (1) and solving for $\tau_E$ gives $\tau_E \propto H^2 I_p^2 RA/T$. Hence $nT\tau_E \propto H^2 n I_p^2 RA$. For operation at a fixed fraction of the density limit, $n \propto I_p/a^2 \propto I_p A^2/R^2$ and so $nT\tau_E \propto H^2 I_p^3 A^3/R$. A key parameter of tokamak plasmas is the safety factor, $q \propto B_T R/A^2 I_p$. Hence $nT\tau_E \propto H^2 B_T^3 R^2/A^3 q^3$. In order to avoid hard disruptions, $q$ must be kept >2 and $A$ is typically ~2–3, and so to increase $nT\tau_E$ it is necessary to increase $B$ or $R$ or a combination of both. Two routes to fusion power can be discerned and are traditionally considered in the field: a route that uses large size, moderate field devices, and one that uses high field, small size devices; apparently at this point in the analysis both are available. Devices of both types have been envisaged: for example ITER [2] is an example of the former and Ignitor [6] is an example of the latter.

For plasma generating fusion power, however, the analysis can be taken further. It is well established that for such plasmas, there is a link between current, field and power due to the beta limit: $P_{\text{fus}} \propto \beta^2 B_T^4 V \propto \beta_N^2 B_T^4 R^3/A^4 q^2$ where $\beta_N = \beta/(I_p/aB_T)$ is the normalised beta, and so $B_T \propto A P_{\text{fus}}^{1/4} q^{1/2}/\beta_N^{1/2} R^{3/4}$. Hence:

$$nT\tau_E \propto \frac{H^2 P_{\text{fus}}^{3/4}}{\beta_N^{3/2} q^{3/2} R^{1/4}} \tag{2}$$

We see immediately that the explicit dependence on size is weak. It is notable that $nT\tau_E$ does not depend on $A$ and so this result is generic for all tokamaks. We note, also, that there is no explicit dependence on $B_T$. The earlier noted two routes to fusion have effectively been reduced to one; that is one where increasing the fusion power is the key requirement. Since the point of fusion is to produce fusion power this is not a disadvantage—it is a positive synergy. On the other hand, since $nT\tau_E$ is directly related to the fusion power gain this finding means that the power gain and power produced—two parameters of critical importance in reactor design—are directly coupled. There are other important implications from equation (2) but before identifying and discussing those we check the precision of the equation.

The analysis has used simplified expressions for the key parameters such as plasma volume, area and $(\tau_E)_{\text{scaling}}$, and has ignored important potentially influencing phenomena such as the self-driven plasma current and plasma radiation. Tokamak Energy Ltd has developed a system code that uses accurate expressions for the main plasma parameters and includes these effects [5]. Equation (2) has been tested using the code. In brief, the basic parameters of a reference plasma are selected and the variation of $nT\tau_E$ is determined as each of $H$, $P_{\text{fus}}$, $R$, $\beta_N$ and $q$ is varied. By adjusting the code input parameters the operation is held at fixed fractions of the density limit and the $\beta_N$ limit: for the density, we take 0.8 of the Greenwald density [3] and for the beta limit, we take 0.9 of $\beta_N(\max)$ where $\beta_N(\max) = 9/A$, which is a conservative limit based on other work [7]. A simple non-linear power law dependence is assumed and the exponent of each parameter is determined by a curve fit. The methodology adopted and the detailed results obtained are described in the online supplementary material (stacks.iop.org/NF/56/066003/mmedia).

A key feature in the testing is the empirical scaling employed. The code is implemented so that any empirical scaling can be used. Figure 1 shows an example of the results obtained. Here the dependence of $nT\tau_E$ on $R$ is determined for a plasma with $A = 3.2$, elongation $k = 2.17$, operating at $P_{\text{fus}} = 500$ MW with $\beta_N = 2.52$ and $q = 3.49$. For these results the empirical scaling developed from the ITER ELMY $H$-mode database, i.e. IPB98y2 [8] is used and $H$(IPB98y2) is taken as 1.5. In order to compare the results with the predictions of equation (2) it is necessary to normalise the results of the equation to those from the code and that is done at $R = 2.46$ m. Despite more than a factor of six change in major radius and approximately two orders of magnitude change in plasma volume there is very little change in $nT\tau_E$. The agreement between the code and the equation is good.

A second result is shown in figure 2 where the R dependence of $nT\tau_E$ is plotted for different values of $P_{\text{fus}}$. We see that $nT\tau_E$ increases with $P_{\text{fus}}$ as expected from equation (2).

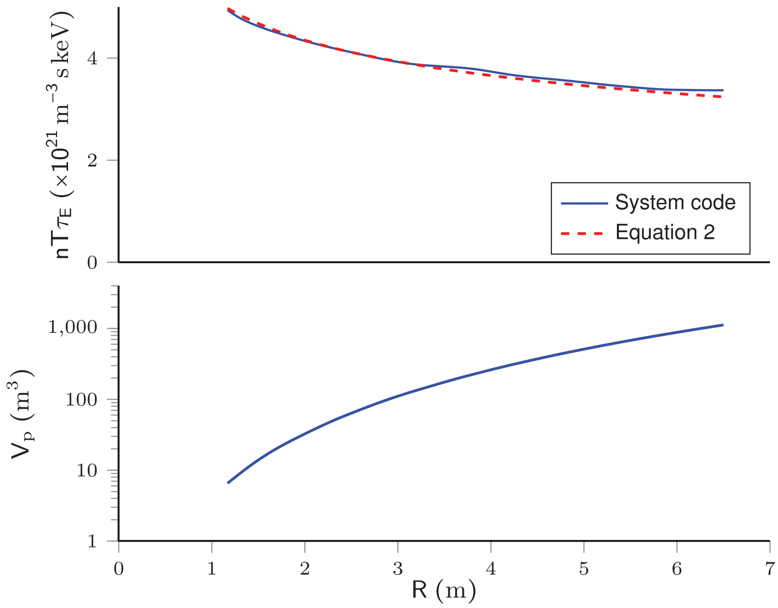


**Figure 1.** Upper: $nT\tau_E$ versus major radius from equation (2) and from the system code for one particular set of conditions ($P_{fus} = 500$ MW, $A = 3.2$, $H$(IPB98y2 = 1.5) normalised to the output of the system code at $R = 2.46$ m. Lower: Plasma volume versus major radius.

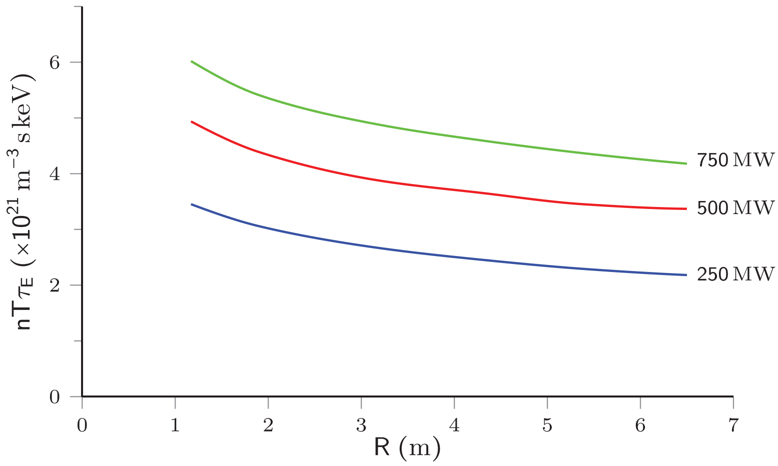


**Figure 2.** $nT\tau_E$ versus $R$ from the system code for $A = 3.2$, $H$(IPB98y2) = 1.5 and three different values of $P_{fus}$ (250, 500, 750 MW).

When expressed in dimensionless physics variables, the IPB98y2 scaling is inversely proportional to beta ($\beta^{-0.9}$) but dedicated experiments have shown that $\tau_E$ is essentially independent of beta. Alternative beta-independent scalings have been developed based on the same ELMy *H*-mode data; for example, the scaling developed by Petty *et al* [9]. The precision of equation (2) has also been tested using this scaling.

The exponents for each of the main parameters of $nT\tau_E$ determined with the tests described in the supplementary material for both the beta-dependent IPB98y2 scaling and the beta-independent Petty(2008) scaling are shown in table 1. The exponents from equation (2) are added for comparison. Despite the simplifications in the analysis, equation (2) is a good representation of the principal parametric dependences that determine $nT\tau_E$ for both scalings.

Designers of tokamak pilot plants and reactors usually strive to increase $\beta_N$ but equation (2) shows that an increase in $\beta_N$ actually leads to a decrease in $nT\tau_E$; a counter intuitive result. Further, if we recast equation (2), we see that

$$P_{fus} \propto \frac{(nT\tau_E)^{4/3}\beta_N^2 q^2 R^{1/3}}{H^{8/3}} \qquad (3)$$

There is a limit to $\beta_N$, for example the Troyon limit [4], and so at fixed $nT\tau_E$, that is at fixed fusion gain, there will be a limit to the total amount of fusion power that can be produced. Typically, reactors have an optimum fusion gain, which is a balance between the need for power gain and the need to input power for plasma control, and so, in effect, the beta limit restricts the maximum fusion power that can be produced: the inverse $\beta_N$ dependence therefore tends to favour lower power reactors. Further, and also unexpectedly, we see from equation (2) that operation at high $q$, which gives enhanced resilience against disruptions, is punitive to fusion gain, and so there is a trade-off in relation to q as well. The critical plasma performance aspect in this discussion is the energy confinement characterised by the $H$ factor: it is the confinement that links the fusion gain to the total fusion power and the dependence is strong.

**Table 1.** Exponents of the parameters of equation (2) determined with the Tokamak Energy System Code with two different empirical scalings derived from the data in the ITER ELMy *H*-mode database [8].

| Parameter | Equation (2) | IPB98y2 scaling [8] | Petty (2008) [9] |
|---|---|---|---|
| $H$ | 2.0 | 2.56 | 1.63 |
| $P_{fus}$ | 0.75 | 0.54 | 0.68 |
| $R$ | −0.25 | −0.22 | 0.07 |
| $\beta_N$ | −1.5 | −1.70 | −1.0 |
| $q$ | −1.5 | −1.37 | −0.52 |

The role of size is clear from the above results but the role of field is less obvious. Increasing $B_T$ at fixed $P_{fus}$, which can be achieved by leaving the current and density, leads to a reduction in $\beta_N$ and an increase in $q$. Both changes improve plasma stability. On the other hand, increasing $B_T$ at fixed $\beta_N$ and $q$ corresponds to operation at higher current, and that enables operation at higher density which in turn leads to an increase in $P_{fus}$ and hence $nT\tau_E$. From the analysis above, $P_{fus} \sim B_T^4$ and from equation (2) $nT\tau_E \sim P_{fus}^{3/4}$and so $nT\tau_E \sim B_T^3$. This strong dependence of $nT\tau_E$ on $B_T$ is confirmed in the testing with the system code; demonstration results are presented in the online supplementary material (stacks.iop.org/NF/56/066003/mmedia).

## 3. Fusion power gain

From the point of view of producing net fusion power, the fusion power gain, $Q_{fus}$, is the key parameter. For a DT plasma there is a simple relationship between $Q_{fus}$ and $nT\tau_E$ as can be readily derived. For a fusion plasma the energy confinement time $\tau_E \propto nTV/P_L$, where $V$ is the plasma volume, so $nT\tau_E \propto n^2T^2V/P_L \propto P_{fus}/P_L$. The loss power $P_L = P_{aux} + P_\alpha$, where $P_\alpha$ is the heating power from the fusion alphas. For the DT fusion reaction, $P_\alpha = P_{fus}/5$. $Q_{fus} = P_{fus}/P_{aux}$, so $(nT\tau_E)^{-1} \propto (P_{aux} + P_\alpha)/P_{fus} \propto 1/Q_{fus} + 1/5$ and hence $Q_{fus} = 5KnT\tau_E/(5 - KnT\tau_E)$ , where $K$ is a constant. Strictly, this expression only applies to steady state operation because the term, d$W$/d$t$, has been ignored in the expression for $P_L$. Substituting $nT\tau_E$ from equation (2) yields

$$Q_{fus} = \frac{5KH^2P_{fus}^{3/4}}{5\beta_N^{3/2}q^{3/2}R^{1/4} - KH^2P_{fus}^{3/4}} \quad (4)$$

This is the same result as that found previously (equation (1) in [5]). The significant common results are that the dependence on size is weak while the dependences on $P_{fus}$ and on normalised confinement time are strong and positive.

The impact of size on performance can be illustrated by calculating the dependence of $Q_{fus}$ on $H$ at fixed $P_{fus}$ for devices of different size. This is done in figure 3 where $Q_{fus}$ versus $H$(IPB98y2) is plotted for a JET size device ($R = 2.96$ m, $a = 0.95$ m) and an ITER size device ($R = 6.35$ m, $a = 1.85$ m). The values of the fixed plasma and device parameters are given in table 2. It is important to note that the current and field are not fixed during the scans and, in particular, are not limited at the design values of the actual JET and ITER devices. In both cases, $P_{fus}$ is held constant at 200 MW and at each point in the scan the plasma temperature and toroidal field are adjusted so that the operation is at 0.8 of the density limit and 0.9 of the beta limit. The plot confirms that the dependence on size is weak: the fusion gain is almost the same even though the major radii differ by a factor ~2 and the plasma volumes differ by a factor of ~8. The strong positive dependence of $Q_{fus}$ on the $H$ factor, as expected from equation (2), is evident.

The performance of the JET size device is worthy of further investigation since experimental fusion results from JET are already available albeit at low values of $Q_{fus}$ [10, 11]. Using the system code, $Q_{fus}$ versus $P_{fus}$ is determined for fixed

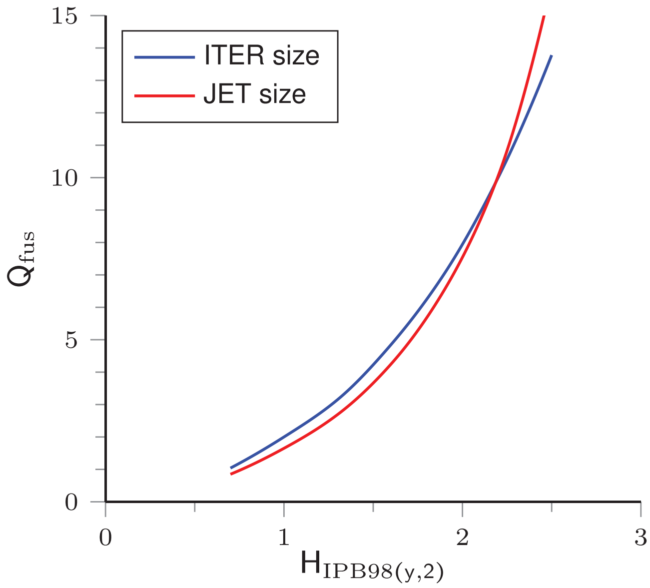


**Figure 3.** $Q_{fus}$ versus $H$(IPB98y2). Calculations for a JET size device and an ITER size device at $P_{fus} = 200$ MW. At each point in the scan the plasma temperature and toroidal field are adjusted so that operation is at 0.8 of the density limit and 0.9 of the beta limit.

**Table 2.** Values of the fixed device and plasma parameters used in the scans shown in figure 3.

| Parameter | JET size device | ITER size device |
|---|---|---|
| Major radius (m) | 2.92 | 6.35 |
| Minor radius (m) | 0.95 | 1.85 |
| Aspect ratio | 3.07 | 3.43 |
| Elongation | 1.85 | 1.80 |
| Triangularity | 0.2 | 0.5 |
| Plasma wall gap (m) | 0 | 0 |
| Plasma volume (m$^3$) | 95.7 | 756.9 |
| BetaN | 2.64 | 2.36 |
| Density profile exponent, $S_n$[a] | 0.5 | 0.5 |
| Temperature profile exponent, $S_T$[a] | 1.5 | 1.5 |
| Wall reflectivity for electron cyclotron radiation | 0.6 | 0.6 |
| Helium fraction from thermalized alpha particles | 0.02 | 0.02 |
| Impurity fraction | 0.01 | 0.01 |
| Ionic charge of impurity | 10 | 10 |
| Effective ionic charge, $Z_{eff}$ | 1.94 | 1.94 |

Density and temperature profiles are assumed to be of the form:
[a] $n(x) = n_0(1 - x^2)^{sn}, T(x) = T_0(1 - x^2)^{sT}$.

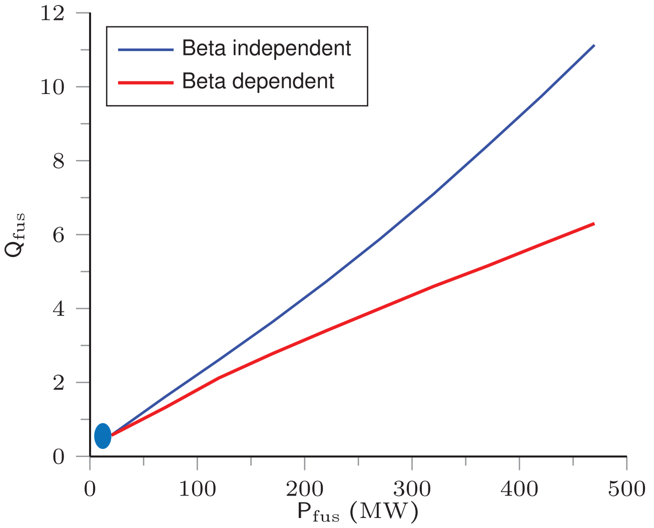


**Figure 4.** $Q_{fus}$ versus $P_{fus}$ for a JET size device under the assumptions of beta-dependent and beta-independent scaling. For the former $H$(IPB98y2) = 1.4 and for the latter $H$(Petty2008) = 1.15. The approximate, short pulse, operating range that JET has already achieved is shown as a filled oval for comparison.

*H* factor and, as before, at each operating point the plasma temperature and the toroidal field are adjusted so that the plasma is operating at 0.8 of the density limit and 0.9 of the beta limit. To carry out the scan we have to choose a confinement scaling and here we use both the IPB98y2 scaling and the beta-independent scaling [9]. Beta-independent scalings are arguably more appropriate because they give consistency between the results of multi-device scaling experiments and those of single device investigations of the beta dependence of the energy confinement time. They imply that as $\beta_N$ increases the $H$(IPB98y2) factor increases and indeed this has been observed on JET and other large tokamaks [12]. On the basis of those experiments we use $H$(IPB98y2) = 1.4 for a tokamak operating at the intended $\beta_N$ of 2.64. As a point of comparison we note that $H$(IPB98y2) = 1.4 is used for predictions of steady state operation of ITER [13]. At low fusion powers, in the region in which JET actually operated, $H$(Petty2008) = 1.15 gives the same confinement time as $H$(IPB98y2) = 1.4 and so that value is taken for $H$(Petty2008). The results of the power scan are shown in figure 4.

During the power scan the major plasma parameters such as the toroidal field and plasma current are changing. The values of the main plasma parameters and of some key device parameters are shown in figure 5. At approximately mid-range in the power scan (~250 MW), the results indicate significant fusion gain $Q > 3.5$ with values of $I_p$, $B_t$, $T$ and wall load ($n_w$) of approximately 6.5 MA, 6.5 T, 21 keV, and 1.2 MW m$^{-2}$ respectively. The transported power minus the radiated power, $P_{div}$, divided by the major radius is an indicator of the power load in the divertor. At mid-point in the scan $P_{div}/R$ ~ 25 MW m$^{-1}$ in the case of the beta-dependent scaling and ~20 MW m$^{-1}$ for the beta-independent scaling. These $P_{div}/R$ values are similar to those expected in ITER. Clearly in these optimised

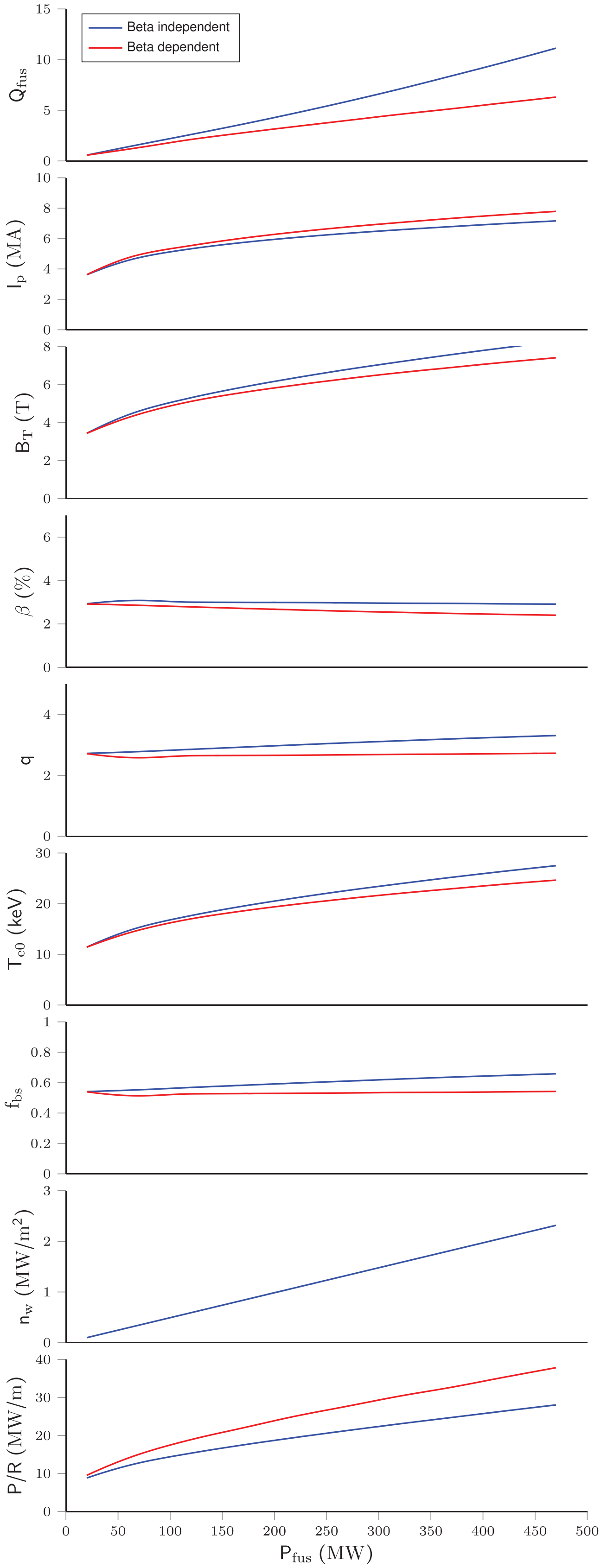


**Figure 5.** Variation of some of the key plasma and device parameters occurring in the power scans shown in figure 4.

conditions the fusion performance of a JET size device can be significant, especially if the energy confinement time scales independently of beta as indicated by the single device experiments. The wall load and $P_{div}/R$ values are within the ranges currently being envisaged for much larger DEMO devices (for example [14]). It is important to emphasise, however, that the device considered here, while being the same size as JET is not the actual JET device, which has current and field limitations that would not allow such a high $Q_{fus}$ performance.

## 4. Implications for the design of pilot plants and reactors

The weak dependence of $nT\tau_E$ and $Q_{fus}$ on size implies that it is not possible to compensate for low confinement by increasing device size. At first this might seem a negative result. However, it implies that small devices can, in principle, perform as well as large devices and so it is potentially positive. Smaller devices cost less and would enable more rapid development steps, and thereby would accelerate the development of fusion power. The dependence of $nT\tau_E$ and especially $Q_{fus}$ on $P_{fus}$ is advantageous because, of course, the point of fusion is to generate power, so high power is needed in any case, and if that leads to a high $Q_{fus}$ there is a beneficial synergy. On the other hand, the direct coupling between $P_{fus}$ and $Q_{fus}$ could be a constraint on design options. The strong dependence on $H$ is positive since that is a parameter that it may be possible to enhance, for example by modifying the edge recycling: this has been shown to lead to $H$(IPB98y2) factors >2 [15, 16]. Further, there are indications that the energy confinement time of low aspect ratio, highly elongated, spherical tokamaks (ST) has a relatively strong scaling on toroidal field potentially leading to high $H$ factors if moderate field STs can be constructed [17, 18]. Thus achieving sustained $H$ factors ~1.5–2 by one or more of these means could be realistic and would lead to high $Q_{fus}$ operation at lower values of $P_{fus}$, and potentially make even smaller, high $Q_{fus}$, devices possible.

The dependence on fusion power puts the design emphasis on the power production and handling capabilities and both of these depend on the employed engineering, technologies and materials. These aspects are evolving continually in a favourable direction. An example is the advent of high temperature superconductors (HTS), which are now approaching a cost and scale that will enable the production of magnets for fusion devices. Such conductors would be smaller and require less space for shielding, would operate at lower cryogenic power, and would be capable of operating in a higher magnetic field. A small tokamak where all the magnets are made from HTS has already been constructed and steady state operation of the magnets has been successfully demonstrated [19]. A high fusion performance, $Q_{fus}$ ~ 13, JET size device utilising HTS magnets has been designed at a quite detailed level [20]. New shielding materials are constantly under development (for example [21]), as are designs for divertor configurations capable of handling high power density loads (for example [22, 23]). In combination these developments could make relatively small, high performance devices feasible on a near term time scale.

## 5. Summary and conclusions

In summary, a simple analytical derivation has shown that for tokamak plasmas the fusion triple product has only a weak dependence on size when the operational density and beta limits are taken into account. The results of the analytical derivation have been compared with results obtained using an established system code and good agreement has been obtained. In effect, the size scalings in the operational limits cancel the positive size scaling in the fusion triple product that would otherwise be present from the size scaling of the energy confinement time. In consequence, the fusion power gain also has only a weak dependence on size. The key parameter is shown to be fusion power. The implications for the design of pilot plants and reactors are significant and potentially positive. Providing sufficient fusion power and in-vessel power handling capabilities can be achieved, relatively small devices can have a high fusion gain. Such devices would open the possibility of a much faster development path and also, perhaps, lead to fusion reactors based on multiple modules rather than one large power unit. Engineering methods, technologies and materials under active development, such as high temperature superconductors, improved neutron absorbing materials and high power density divertor concepts, have the potential to make the engineering of such devices feasible on a relatively short time scale.

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