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To cite this article: R.M. Kulsrud *et al* 1986 *Nucl. Fusion* **26** 1443

PHYSICS OF SPIN-POLARIZED PLASMAS

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ABSTRACT. If the nuclear spins in a fusion reactor are oriented, or polarized, in an appropriate manner, then the nuclear reactions are modified in such a way as to enhance the performance of the reactor. The methods by which the spins can be polarized and the various difficulties connected with these methods are discussed. The processes by which the polarization can be lost owing to various physical processes in a confined plasma are assessed. The different polarization modes of the plasma ions are applicable to the different nuclear reactions, D–T, D–^{3}He and D–D. The benefits for reactor performance of these modes are discussed. It is concluded that on the basis of current knowledge, the possibility of achieving polarized spin in a fusion plasma cannot be ruled out and that the physics of such plasmas is well worth pursuing both for its intrinsic interest and for its benefit to fusion. Further, detailed calculations of the modification of the D–T nuclear reaction produced by spin polarization are given, as well as calculations of the depolarization rates for physical processes in a confined plasma. Finally, a kinetic description of a polarized plasma is outlined which generalizes the normal kinetic description of a plasma.

1. INTRODUCTION

In a letter published in 1982 [1] it was pointed out that the nuclear reactions in a fusion device can be modified by taking advantage of their dependence on the nuclear spin of the reacting particles, since the nuclear spins need not be randomly oriented in a plasma but can be oriented relative to the local magnetic field. The calculations presented in that letter show that for appropriately oriented nuclei the time it would take to disorient them by binary collisions is at least five orders of magnitude longer than the nuclear burning time and therefore it was concluded that such a modification of the nuclear reactions is feasible. Polarization should be accomplished by first orienting or polarizing the nuclei outside the fusion device where they are in an atomic state. This can be accomplished by various standard means. Once the nuclei are polarized, they should be introduced into the reactor as fuel that has already been polarized. The nuclei can be introduced into the fusion device by standard means, such as gas puffing, injection of energetic neutral beams or pellet injection. It is believed that these polarized nuclei are not depolarized during this phase.

So far, only very small quantities of hydrogen and its isotopes have been polarized and generally their polarization level was not high. The main application was for accelerator beams and targets. Regarding the feasibility of polarization, a key question is whether polarized nuclei can be produced in sufficient quantities as required for a fusion device, i.e. whether atoms can be polarized at a rate comparable to that at which plasma ions are recirculated in a fusion device. This rate is roughly a million times larger than the polarization rates currently achieved. A further question is whether the polarization state of the nuclei can be increased to almost 100%. Finally, there is the question of costs. Because of engineering progress in the physics of polarizing nuclei it is believed that the costs of the polarization process would be reasonable. The new techniques are discussed in Section 2.

The benefits to be gained by polarizing the plasma nuclei are discussed below.

First, it is well established both theoretically and experimentally that the D–T cross-section can be increased by almost exactly 50% when all spins of both nuclei are completely polarized so that they are oriented parallel to the confining magnetic field. In this case the reaction products – alpha particles and neutrons – are emitted at an angle θ with respect to the magnetic field with a probability of $\sin^2\theta$. On the other hand, when all deuterons are polarized so that they are perpendicular to the magnetic field (the m = 0 state) while the tritons are unpolarized, the cross-section is unchanged and the probability distribution of the emitted particles is proportional to $(1 + 3\cos^2\theta)$. Although the latter polarization mode leads to no increase in reactivity, it does lead to some control over the direction of the neutrons and alpha particles. Because of their smaller perpendicular motion, the alpha particles are easier to confine in this mode. For some purposes this mode may be more desirable than the first mode which enhances the reactivity.

Second, it is believed on theoretical grounds that the behaviour of fusion reactions between D and ^{3}He is almost the same as that of reactions between D and T. This is not certain, however, since the D–T reactions proceed almost entirely through a single state of the compound nucleus ^{5}He, whereas the D–^{3}He reactions may proceed through two states of the compound nucleus ^{5}Li. Although one of these states certainly predominates, the other state, whose contribution is relatively small, may lead to a dependence of the nuclear reactivity on spin which is not so clear as in the case of D–T reactions and the gain in cross-section achieved by polarization of the nuclei may be less than 50%.

Third, the dependence of the D–D reaction on the spins of the reacting particles is not well known. It has been hoped that it would be possible to modify the D–D cross-section, reducing it by a large factor, by polarizing the spins of the interacting deuterons parallel to each other. This is the mode which occurs naturally with enhanced polarization for D–^{3}He (and D–T). Since in a D–^{3}He fusion device the only reaction producing neutrons is the D–D reaction, strong suppression of the D–D reaction would lead to a nearly neutron-free fusion device. The total absence of neutrons would have definite advantages. However, although the dependence of the D–D cross-section on nuclear spin is uncertain, there are some new findings which lead us to believe that such a suppression is unfortunately not possible.

On the other hand, a polarization mode in which all deuterons are polarized so that their spins are oriented perpendicular to the confining magnetic field does increase the reaction cross-section, possibly by a factor as large as two. This result could be of considerable importance in the future when pure deuterium fusion reactors are considered.

Regarding the feasibility of employing polarized nuclei in a fusion device, a second key question is to what extent nuclei will remain polarized in the hot fusing plasma long enough so that a substantial gain is actually achieved. In Refs [1] and [2] it was proved that binary collisions only lead to a low level of polarization. In general, there is considerable recycling of the nuclei so that they can leave the plasma and interact with the wall. This interaction very likely leads to a mean depolarization of the nuclei in the plasma. Also, several new depolarization processes which occur in the plasma itself have been suggested. Thus, to find out whether polarization is feasible, it is important to investigate carefully all of these depolarization processes.

The present paper gives a brief survey of work on spin polarized plasmas published after the appearance of Refs [1, 2] which first stimulated interest in the subject of nuclear polarization. The current status and the importance of polarization for nuclear fusion research is evaluated in Section 2. In Refs [1, 2] the findings regarding the effect of nuclear spin on the nuclear reaction rates were presented without proof. In the present paper the details of the nuclear reaction rates are set forth in Section 3 so as to enable the reader interested in polarization to evaluate the different polarization modes which could be useful in the development of a fusion device. The details of the various depolarization mechanisms are given in Section 4 where explicit formulas for depolarization rates in the plasma are presented. If nuclear polarization is to become attractive, it is necessary to employ a kinetic description of its evolution which includes all nuclei and their spins. Such a kinetic approach now exists and it is described in Section 5. Section 6 gives tentative conclusions regarding the usefulness of employing nuclear polarization on the basis of present knowledge of the physics of polarized plasmas.

2. RECENT WORK ON SPIN POLARIZED PLASMAS

Since 1982, a number of papers have appeared which report research on spin polarized plasmas. In this section we briefly survey some of the results presented in these papers and discuss their implications for the feasibility of using spin polarized plasmas to enhance the performance of a nuclear fusion device.

This research work can be grouped as follows: (1) Production methods of spin polarized nuclei in large quantities, (2) investigations of the dependence of the various relevant nuclear cross-sections on spin orientation, (3) investigations of ways to improve the performance of fusion devices by employing polarization, (4) investigations of the processes by which the nuclei in a fusion device can be polarized and of their depolarization rates, (5) development of a rigorous kinetic theory of polarization, and (6) proposals for experimental tests of depolarization rates in laboratory plasmas. These items are discussed in the following subsections.

2.1. Production methods of spin polarized fuel

Four spin polarization methods are currently being considered: (a) optical pumping, (b) cryogenic methods using molecular formation, (c) cryogenic methods employing Boltzmann equilibrium, and (d) polarization of energetic neutral beams.

Each of these methods has the potential to polarize hydrogen isotopes (and ^{3}He) at sufficiently high rates to be of interest for fusion applications. On the other hand, for the polarization of high energy beams in accelerators the well known Stern–Gerlach methods [3] have been considered, but the intensities achieved are too low to be of interest for fusion applications. Also the methods used to prepare polarized targets for interaction with these high energy beams, such as dynamic polarization, yield solids with a degree of polarization which is too low for fusion applications. We therefore do not consider them in this section.

(a) Optical pumping methods

Optical pumping methods were developed in the 1950s [4] and were early applied to the polarization of hydrogen nuclei [5, 6]. At that time the light sources for optical pumping were very weak so that only minor quantities of polarized atoms could be produced. Later, tunable dye lasers have been employed as sources of much more intense light. These have been used to polarize nuclei of noble gases such as xenon [7] and it has been possible to produce large numbers of highly polarized nuclei in short times. Noble gases are much easier to polarize because they occur in the atomic state and interact weakly with properly chosen walls.

However, before 1982, lasers were not used for the polarization of hydrogen isotopes because these are molecular gases and must be decomposed to atomic nuclei before they can be effectively polarized; also, hydrogen isotopes interact rather strongly with most walls. It is these two facts which render the polarization of hydrogen nuclei in large quantities difficult. Since at that time an important application was not envisaged, the polarization of hydrogen nuclei was not attempted. In 1982, the possible importance of polarization for fusion became known and it was attempted to polarize dense hydrogen gas by laser light. Experiments for the investigation of the physics commenced in 1983 at Princeton [8]. These experiments elucidated the complementary nature of the two necessary steps – breakdown of the hydrogen molecules into atoms and choice of an appropriate wall material to avoid rapid depolarization. If the wall material is inappropriate, a rather dense buffer gas, usually molecular hydrogen, is needed to slow down the diffusion of the atomic hydrogen to the walls. On the other hand, the wall substances which have a low interaction with the proton, deuteron and triton spins tend to be damaged by the discharge employed for molecular breakdown. The only possible solution of this problem seems to be to perform the breakdown into atoms in one tube and to transport the atoms to a second tube with favourable walls where optical pumping polarizes the atoms to a high degree.

In experiments where breakdown was carried out in a single tube, hydrogen was polarized to more than 70%, but at densities of 10^{12} cm^{-3}. Unfortunately the funding of these experiments was discontinued before the method using two tubes could be tried out. Only if funding from other sources can be obtained will it be possible to test the feasibility of the two-tube method.

It is important to estimate the additional cost of producing fuel for a fusion device in polarized form. For such an estimate the following points should be considered. The general idea of optical pumping is to inject spin into the nuclei by first circularly polarizing the photons of the laser light. When these photons are absorbed, generally by the relatively low density of the alkali atoms that are mixed with the hydrogen, about one half of the angular momentum of the light photons is transferred to the alkali electrons. The alkali electrons quickly transfer their angular momentum to the hydrogen atoms where it is shared between the electrons and protons (or other hydrogen isotopic nuclei). This process continues until both the electrons and protons have absorbed the spin angular momentum and become totally polarized. Since hydrogen atoms are initially unpolarized, $(1/2)\,\hbar$ units of angular momentum is needed per hydrogen atom. Thus, if there are no losses, one photon of the laser light is required to polarize one hydrogen atom. The energy of such a photon, which is usually in the red region of the spectrum (7921 Å if the alkali atoms are rubidium), is about 1.5 eV. To fuel a reactor of 1 GW, with recycling, about 10^{21} atoms$\cdot$s^{-1} are needed. This gives 240 W for the required input power of the laser. Regarding the cost, this amount of power is of course totally negligible. The main cost is the investment cost for the laser. The cost of the 1 W laser employed by Knize et al. [8] was of the order of 10^5 dollar; by linear scaling, the cost of a 240 W laser would be 24 million dollar. If inefficiencies in the polarization process are taken into account, the cost of this process will be higher. Without recycling the cost would be ten times higher. It is expected, however, that by the time such powerful lasers will be needed, they will be much less expensive. The feasibility of employing polarization for fusion devices can be evaluated by balancing the cost against the savings achieved (see Section 2.3).

The main purpose of the current experiment is to determine the physics for the polarization of hydrogen. It is probably not feasible to prepare the hydrogen in a vessel and then to transport it directly into the reactor since the time-scale for depolarization on the wall is of the order of seconds. A rapid flow of hydrogen gas past the region of polarization and into the fusion device will be necessary. The flow velocities need not be so high that the polarized atoms can be kept polarized, provided there is a sufficiently large magnetic field which keeps the spins aligned between the region of polarization and the fusion device. The conditions for this are well known and should be easy to satisfy.

Different conditions would apply if the polarized atoms were frozen to pellets for pellet injection. During the freezing process the gas must be kept in a strong magnetic field and out of contact with the wall. The possibility of freezing optically polarized hydrogen isotopes has not been seriously investigated.

(b) Cryogenic methods

Another very effective way of polarizing hydrogen isotopes is to reduce the temperature of the atoms to a few degrees Kelvin and to separate atoms of a particular electron spin by the use of a strong magnetic field and then by molecular formation on the walls [9, 10] to remove atoms with unwanted nuclear spin. This method has been used by Greytak and Kleppner [11] who have achieved densities of polarized hydrogen of more than 3×10^{16} cm^{-3} at these low temperatures. Clearly, this method is appropriate for obtaining frozen pellets which can be used for injection into magnetically confined plasmas or even for laser fusion. It is possible that the polarization of these pellets is more durable than that characteristic of hydrogen gases, so that it is conceivable that these pellets can be prepared and stored. On the other hand, with this method considerably more energy is required per polarized atom, since for the separation of each polarized hydrogen atom the recombination of one hydrogen molecule is required. To keep the hydrogen at cryogenic temperatures, an energy of 4.5 eV must be extracted with thermodynamic efficiency at temperatures in the range of 2–4 K. Thus, energies of 450 eV per polarized atom are required. The total power for a reactor of 1 GW, involving 10^{21} polarized atoms per second, is of the order of 100 kW. Even this amount of power (or a power ten times higher for the case without recycling) is small compared to the reactor output.

Until recently, experiments on cryogenic polarization were carried out only for the lightest isotope of hydrogen. The attractiveness of these experiments makes it desirable to perform similar experiments for deuterium. However, it was found that deuterium interacted strongly with the refrigerator walls which were helium coated and was absorbed by the helium, which is not the case with hydrogen [11]. This absorption was unexpected and posed new problems. Unfortunately, before this matter could be investigated, the funding for this experiment was also terminated.

(c) Cryogenic methods employing Boltzmann equilibrium (polarization at very low temperatures)

With magnetic fields of moderate strength the interaction energy between the magnetic moment of the nuclei and the magnetic field is negligible compared to κT, where κ is the Boltzmann constant and T the temperature. Thus, thermal equilibrium produces only very weakly polarized hydrogen or hydrogen isotopes. However, with very strong magnetic fields (of the order of 10 T) and very low temperatures (of the order of tens of milli-Kelvin), thermal equilibrium does lead to strong polarization of hydrogen isotopes, which in this case are in molecular form. These fields and temperatures are fairly easily achieved. However, the drawback here is the very long time needed to achieve thermal equilibrium at these low temperatures. These processes have been investigated experimentally by Hornig [12]. Molecular hydrogen was cooled to a very low temperature and it was found that the nuclear spins quickly achieved thermal equilibrium; this is due to the mechanism of interchange of the spin angular momentum with the orbital angular momentum of orthohydrogen, which is present even at such low temperatures [13]. However, when the hydrogen molecules are brought to higher temperatures the polarization is rapidly lost because the orthohydrogen atoms keep the spins in thermal equilibrium also at higher temperatures. The only possible recourse is to wait until orthohydrogen is converted to parahydrogen at these very low temperatures. Then the kinetic temperature of hydrogen can be raised without the spin staying in thermal equilibrium. Spontaneous relaxation of orthohydrogen takes one or two months at these temperatures. Hornig [12] has explored this polarization method for some years in an attempt to prepare polarized targets for studies in high energy physics. However, this method could also be applied in the preparation of polarized fuel for fusion devices. The key problem is to find methods of speeding up the relaxation of orthohydrogen to parahydrogen at low temperatures. Studies directed at fusion application of this method could not be continued because again the funding for them was terminated.

Thus, each of these three polarization methods has encountered a separate difficulty that could block its application to nuclear fusion. However, each of these difficulties could be overcome by research and experimentation. If it were possible to take up these experiments again, solutions would presumably be found.

(d) Polarization of energetic neutral beams

A different method of polarization has been proposed by Anderson et al. [14]. It involves the direct polarization of energetic neutral beams which are similar to those used for heating the plasma. With this method the electrons are kept in a thin gaseous target of alkali atoms, such as caesium, which are polarized by optical pumping. As the beam passes through this target, the electrons in the beam interchange spin with the electrons of the alkali atoms and their spin becomes equal to that of the nuclei. For neutral beams of 5 keV, surface densities of the order of 10^{17} cm^{-2} in the screen are predicted to lead to nearly completely polarized beams. As the polarized neutral beam enters the fusion device the atoms are ionized so rapidly that no depolarization of the nuclear ions can occur and the plasma becomes completely polarized. This scheme is particularly attractive for an early test of polarization since it seems to offer the easiest method of injecting polarized atoms into a fusion device. The drawback here may be certain effects of the thin target on the neutral beam, such as scattering, which could lead to degradation of the beam.

2.2. Nuclear reactions

Three reactions of interest for fusion are D–T, D–^{3}He and D–D. Section 3 sets forth the calculation of the spin dependence of the D–T reaction. The results of previous investigations were presented in Ref. [1]. The only relevant new result for the D–T reaction is that essentially one state of ^{5}He contributes more than 99% of the reactions, as shown by measurements of the energy dependence of the unpolarized D–T cross-section [15]. This means that the enhancement by polarization is very close to 50% for fully polarized D and T atoms. Concerning the spin dependence of the D–^{3}He reaction, no further results have become available. However, it appears that this reaction is not so simple as the D–T reaction. This is because a second resonant state of the compound nucleus ^{5}Li – a state which is close to the main contributing state of ^{5}Li – contributes substantially more than 1% to the cross-section. Thus, the enhancement by polarization may be less for D–^{3}He reactions than for D–T reactions. Further, it is possible that the angular dependence of the reaction products for D–^{3}He is not so pronounced as that for D–T.

The authors of Ref. [1] were very uncertain about the nature of the spin dependence of the D–D reaction. In particular, the question was put whether it was possible to suppress the cross-section by aligning the D–D spins parallel to each other. There have been no further experiments on the D–D reaction, but some progress has been made theoretically and it seems that this suppression of the cross-section is not possible.

The experiments of Ad'yasevich and Fomenko [16] on the differential scattering of a polarized beam of 290 keV deuterons by an unpolarized target led to explicit values for the matrix elements of the various spin dependent cross-sections. The analysis of these matrix elements in Ref. [2] showed that when the spins were aligned, the D–D cross-section was suppressed by about a factor of twenty below the spin averaged cross-section. If these matrix elements were not energy dependent, then the same suppression would occur at all energies. An analysis by Hale and Dodder [17] of experiments at other energies showed that there would be very little suppression. A theoretical calculation by Hofmann and Fick [18], based on assumed nuclear potentials, verified the latter results. An explanation for these discordant results was proposed by Hale [19] who surmised that the matrix elements were the sum of two different contributions arising from two different types of nuclear forces. These contributions could cancel in the relevant matrix element at

290 keV bombarding energy or 145 keV centre-of-mass energy. Thus, at this energy, both the cancellation and the suppression would be strong. However, when averaged over various energies, the suppression would be weak. It is of interest that this centre-of-mass energy is about equal to the peak of the major contribution to the D–D fusion cross-sections at thermal temperatures of 50 keV and that 50 keV is quoted as the appropriate temperature for a D–^{3}He fusion device.[1]

On the other hand, the results of all these calculations show that other polarization choices for the deuterons can lead to an enhancement of the D–D reaction by a factor of between 1.5 and 2. This will be of interest when a pure D–D fusion device is contemplated.

2.3. Gain from employment of polarization in fusion devices

We assume here that the plasma in a fusion device can be fully polarized and will remain fully polarized, and we discuss what benefits can be obtained. We limit ourselves to D–T reactors.

We have three different polarization modes: (a) The 'enhanced mode', in which the D and T nuclei are polarized parallel to the confining magnetic field and parallel to each other. For this mode the plasma reactivity is increased by 50% relative to the reactivity of an unpolarized plasma while the reaction products are emitted roughly perpendicular to the magnetic field. (b) The 'unenhanced mode', in which the D nuclei are polarized perpendicular to the magnetic field and the T nuclei are unpolarized. In this mode the reactivity is the same as in an unpolarized plasma, but the reaction products are emitted roughly parallel to the magnetic field. (c) The 'suppressed mode', in which the D and T atoms are polarized along the magnetic field, but antiparallel to each other. In this mode the reactivity is reduced by a factor of two and the reaction products are emitted in the same manner as in the 'unenhanced mode'.

The mode most likely to be of benefit to a fusion device is the enhanced mode. We assume that the emission of the alpha particles perpendicular to the field does not appreciably decrease the alpha particle confinement during the transfer of the bulk of the alpha particle fusion energy to the plasma. The alpha particle heating is thus increased by 50%.

The increase in alpha particle heating will make the startup and ignition of the plasma easier, since less external power is needed to raise the plasma to ignition temperatures. (This could also be advantageous for the proposed ignition experiment, the CIT.) The enhancement of the alpha particle energy reduces the temperature at which ignition occurs at a fixed density or, alternatively, it lowers the density needed for ignition at a given temperature. This means that less external power is required, even if the additional alpha particle power available near the critical temperature is not taken into account. The actual savings depend on the density and temperature dependence of the loss rates. If the required density is lowered by 1.5 through the reactivity enhancement, then the external power could be lowered by at least the same factor.

The possible gain from the employment of polarization during the main fusion cycle (after startup) depends very much on the limiting factors which control the cost of the reactor. If the reactor is not marginal, i.e. not limited by some specific process, then the gain to be achieved by increasing the amount of nuclear power should be modest. Some reduction of either the size or the magnetic field strength could be achieved without reducing the output power. It has been estimated that the actual savings in the cost of electricity would be from 5 to 10%.

On the other hand, if the reactor is sufficiently marginal in its operation, then the application of polarization could lead to large savings and could even make fusion possible. If the confinement time is the limiting parameter, then the confinement time necessary to achieve ignition could be lowered by a factor of 1.5 when polarization is used. If the critical plasma beta is important, then the beta at which ignition occurs could be reduced by using polarization. The amount of reduction in beta is determined by the dependence of the loss rates on density and temperature. The possible reduction ranges between 1.5 and $(1.5)^{1/2}$ for various loss models.

If wall damage is a critical element in the cost of fusion, then some savings can be achieved by employing the unenhanced mode of polarization. This mode shifts some of the wall damage from the inner wall to the outer wall, as shown in Ref. [2].

The above considerations indicate that if polarization could be made to work, then it would be possible to select the proper mode of polarization to improve the performance of a fusion device, whatever the limiting physical factors are.

Finally, with respect to inertial confinement fusion, More [20] has shown that if pellets could be polarized,

[1] Recently published calculations [33] show that the reaction $D + D \rightarrow n + {}^3He$ can be suppressed by polarization by a factor of about 15. Thus, a neutron-free D–^{3}He may be possible.

they should remain polarized during the implosion. Since the energy in the driver of the implosion – the laser or the ion beam – is proportional to the inverse cube of the nuclear cross-section [21], the energy required for ignition of a polarized pellet is less by a factor of $(1.5)^{+3} \approx 3.4$. Thus, the cost of the driver could be reduced by more than a factor of two by employing polarization of the pellet in the enhanced mode.

2.4. Depolarization

In Ref. [1] it was pointed out that binary collisions do not significantly depolarize a fusion plasma. The possibility of depolarization by waves resonant with the precession frequency was mentioned and it was stated that even very low amplitude magnetic fluctuations resonant with the precession frequency could lead to depolarization. Section 4 presents detailed derivations of wave depolarization rates as well as all depolarization rates for the different mechanisms of collisional depolarization. Since the appearance of Ref. [1], other depolarization mechanisms have been suggested. In Ref. [1] it was also suggested that interactions of the nuclei with the solid walls containing the plasma could be very important processes for depolarization. Some theoretical progress has been made in investigations of such interactions. These results are reviewed in this subsection.

Lodder [22] pointed out an important depolarization mechanism for the deuteron spin. This process hinges on the fact that the precession rate of the deuteron spin about a magnetic field is 0.86 times the gyration rate of the deuteron in the same field. This near resonance means that as the deuteron circulates about the magnetic field lines, the shear of the magnetic field forces the spin to oscillate in direction by a relatively large amount. When collisions are included, a random walk of the spin direction occurs which leads to depolarization. Lodder made a rough calculation of the depolarization rate for this process and showed that it could be serious. We include a derivation of this process in Section 4 and find that Lodder ignored several factors of two in his estimate, all of which can reduce the depolarization rate. Thus, this depolarization is not so rapid as Lodder supposed and it is not serious. It should be pointed out that Kritsch [23] also mentioned this process, but did not estimate its rate. (It is an important process in accelerators with polarized particles.)

In a series of papers, Coppi and co-workers (among whom are two of the present authors) [24–27] exhibited a possible self-limiting process on the polarization of D and T. This self-limiting process, which operates only in the enhanced mode, is associated with the anisotropic distribution of the alpha particles intrinsic to the polarization process itself. This anisotropic distribution of alpha particles can interact resonantly with those magnetosonic waves whose frequencies are near the D or T precession frequencies. This interaction drives the waves unstable and the associated magnetic fluctuations depolarize the nuclear spin of D or T. However, most of these waves escape from the inhomogeneous plasma too rapidly to be amplified. Nevertheless, the authors found one set of these waves trapped in a tokamak plasma. Further examination of these waves near the deuteron frequency showed that, because of the inhomogeneity of the magnetic field and the plasma present in a tokamak device, these waves were strongly damped during the electron transit time and would probably not be excited. On the other hand, an examination of waves near the triton precession frequency (which is 5.94 times the deuteron cyclotron frequency) showed that electron transit time damping was negligible for these waves, but that there was some moderately strong cyclotron damping. (Because of the inhomogeneous magnetic field, waves are cyclotron damped at different harmonics in different parts of the region of their existence.) The conclusion drawn from these papers was that if fourth-harmonic damping occurred, the waves would not be excited. The condition for excitation and the resulting depolarization of the tritons (in the enhanced mode) was that the aspect ratio of the tokamak must be larger than four [27].

It is generally known that interactions of the plasma nuclei with the surrounding walls represent the greatest danger to polarization. Because of recycling, depolarized nuclei re-enter the plasma, mix with the polarized ions and reduce the net polarization. It is easy to show that if the ions merely recombine on the surface of the wall and are ionized again after re-entering the plasma, then there is little depolarization because of the very strong magnetic field. (Also of interest here are the 'molecular tumbling depolarization phenomena' [28], which are however not discussed here.) The physics of depolarization during recycling has been surveyed by Greenside et al. [28]. They showed that the most important process was penetration of that part of the wall in contact with the plasma by the naked hydrogen nuclei. These nuclei penetrate this surface many hundreds of atomic layers and take some time to diffuse back out of the wall. During this time the nuclei are subject to fluctuating fields due to free or unpaired electrons, and subject to magnetic moments of fixed nuclei Doppler shifted by the diffusional motion of the hydrogen isotope nuclei in question, or subject to similar magnetic moments of other hydrogen isotope nuclei recycling through the wall. The authors of Ref. [28] pointed out that if the walls were made of metal, the depolarization time would be very short, of the order of milliseconds. For such walls it is clear that the plasma cannot be kept polarized under recycling conditions.

It is also pointed out in Ref. [28] that graphite is a much better wall material regarding the maintenance of polarization since there are no unpaired electrons and ^{12}C has no magnetic moment. The depolarization time of single hydrogen nuclei embedded in graphite is 1–5 s. However, if many hydrogen nuclei are simultaneously embedded in the graphite, they can depolarize each other in a shorter time. This would be the case with a fusion plasma wall. Thus, with a graphite wall the depolarization could be slow enough to be acceptable. To ascertain this, it is necessary to perform experiments with actual polarized plasmas. (Amorphous graphite would be better in this case because it permits a more rapid escape of nuclei from the wall.)

2.5. Kinetic theory of polarization

Recently, the authors of the present paper developed a kinetic theory of polarized plasmas [29] which yields a complete statistical description of the plasma nuclei, taking into account their spins. This approach would be of value in a scientific investigation of polarized plasmas. A detailed description of this approach is given in Section 5.

2.6. Experimental tests of depolarization rates

Because of the benefits to be obtained from polarized plasmas, it is important to perform experiments as early as possible in order to find out whether polarized plasmas can be realized.

A possible test involves the radioactive beta decay of tritium. The beta decay electrons are emitted at an angle that is correlated with the spin of tritium. The angular emission is proportional to $1 + a \cos\theta$, where θ is the angle between the direction of the emission and the direction of the tritium spin, and a is approximately 0.1. Thus, if a small quantity of polarized tritium, of the order of 10^{16} atoms, is gas puffed into an ordinary magnetic mirror filled with normal plasma, then it should be possible to determine tritium polarization as a function of time by counting the relative numbers of decay electrons emerging from either end of the mirror. The half-life of tritium is 12 years and thus 2.6×10^4 decays will occur per millisecond. If half of the decays are in the loss cone, the number of electrons emerging from the end of the mirror in the direction of polarization is $2.6 \times 10^3\ \mathrm{p \cdot ms^{-1}}$, where p is the degree of polarization. If all electrons are detected, the error of measurement, Δp, is about $1/(2.6 \times 10^3)^{1/2}$, i.e. about 2%. Thus the evolution of p should be measurable on this time-scale. The same measurement could also be made in closed devices such as tokamaks. However, in this case, only decays in the edge region would be measurable, so the detectable fraction of decay electrons would be smaller by a factor of the order of 100 and the attainable accuracy of measurement of p would be lower.

3. NUCLEAR PHYSICS WITH POLARIZED SPIN

3.1. The D–T reaction

First we discuss the effect of spin on the $D(T, n)^4He$ reaction. Its large cross-section makes it a natural choice for first-generation fusion reactors. The dependence of the reaction on spin at low energies is fairly clear. When a D nucleus and a T nucleus collide and when these nuclei penetrate the Coulomb barrier, their energy is very close to that of an excited state of the compound nucleus ^5He, which is an intermediate step in the D–T reaction. This nucleus is unstable, but its excited states have lifetimes which are long enough to be considered as quantum states with definite angular momentum and parity. An excited state of ^5He lies 107 keV above the energy of the unbound D and T at zero kinetic energy. This excited state has angular momentum J = 3/2 and even parity. If the colliding D–T system has angular momentum J = 3/2 and even parity (the orbital angular momentum is an even multiple of $\hbar$), then the excited ^5He nucleus is formed with high probability and decays to the state ^4He + n, with the release of 17 MeV. If the colliding nuclei have different angular momentum or parity, the chances of a reaction occurring are smaller by two orders of magnitude. In fact, recent experiments show that the fraction of reactions occurring through this excited state is greater than 0.99 [15].

A simple estimate shows that for ion energies typical of a fusion reactor, only those collisions are of interest which have impact parameters small enough that the relative orbital angular momentum is $\ell = 0$ or 1. Therefore, we may restrict ourselves to collisions in the $\ell = 0$ state. (The nucleus has some chance of penetrating the Coulomb barrier with $\ell = 1$ [30]. However, because of the even parity of the resonant state, we may ignore these events.) In the $\ell = 0$ state the total angular momentum is obtained by adding the spin angular momenta of the D and T nuclei. Since the spin of D is $\hbar$ and that of T is $(1/2)\hbar$, the total angular momentum is $J\hbar$, where $J = 1/2$ or $3/2$.

The statistical weight of the $J = 3/2$ state is four and that of the $J = 1/2$ state is two, so if the D and T nuclei are both randomly polarized, the probability for the colliding system to be in the $J = 3/2$ state is two-thirds. An immediate consequence of this is that if it is ensured that the nuclei come together with $J = 3/2$, the effective cross-section is 3/2 times the unpolarized cross-section. One way to accomplish this is to align all spins of both the D and T nuclei along B.

We now consider the detailed dependence of the effective nuclear cross-section on the spin orientations of the D and T nuclei. Let $|m_2, m_3\rangle$ represent the eigenstate in which the magnetic quantum number of D is m_2 and that of T is m_3. The spins of the D and T nuclei are in general uncorrelated; therefore we write

$$|\chi_D\rangle = \sum_{m_2} a_{m_2}|m_2\rangle \tag{1a}$$

$$|\chi_T\rangle = \sum_{m_3} b_{m_3}|m_3\rangle \tag{1b}$$

$$|\chi\rangle = |\chi_D\rangle\,|\chi_T\rangle = \sum_{m_2 m_3} a_{m_2} b_{m_3}|m_2 m_3\rangle \tag{1c}$$

for arbitrary D or T. $|a_{m_2}|^2 = d_{m_2}$ denotes the probability that the deuteron is in the m_2 state; correspondingly, for the triton, $|b_{m_3}|^2 = t_{m_3}$. The nuclear cross-section depends only on the J-variable and the relative energy, so the probability of a nuclear reaction may be written as

$$\sigma = \sum_{J=1/2,\,3/2} \sum_{m=-J}^{J} \sigma_J |\langle J, m|\chi\rangle|^2 \tag{2}$$

where $\sigma_{3/2} \gg \sigma_{1/2}$, and $|J, m\rangle$ is the J, m state of the colliding particles. Substituting Eq. (1c) into Eq. (2) we have

$$\sigma = \sum_{J=1/2,\,3/2} \sum_{m_2 m_3} d_{m_2} t_{m_3} |\langle J, m_2 + m_3|m_2, m_3\rangle|^2\, \sigma_J \tag{3}$$

The quantities $\langle J, m_2 + m_3|m_2, m_3\rangle$ are the Clebsch–Gordon coefficients for addition of angular momentum. We require those for which $\vec{J}\hbar = \vec{s}_D + \vec{s}_T = (\vec{J}_1 + \vec{J}_2)\hbar$, where $\vec{s}_D$ and $\vec{s}_T$ are the spin operators for D and T. These are given in Ref. [31] (page 76, Table 1). Making use of these coefficients, we find that Eq. (3) reduces to

$$\sigma = (a + \tfrac{2}{3}\,b + \tfrac{1}{3}\,c)\,\sigma_{3/2} + (\tfrac{1}{3}\,b + \tfrac{2}{3}\,c)\,\sigma_{1/2} \tag{4}$$

where

$$a = d_1\, t_{1/2} + d_{-1}\, t_{-1/2} \tag{5a}$$

$$b = d_0(t_{1/2} + t_{-1/2}) = d_0 \tag{5b}$$

$$c = d_1\, t_{-1/2} + d_{-1}\, t_{1/2} \tag{5c}$$

The conclusions given in the Introduction concerning the effective nuclear cross-section can be verified using Eq. (4), but neglecting $\sigma_{1/2}$. If the ions are unpolarized, the d's are 1/3, the t's are 1/2, and from Eqs (5a–c), $a = b = 1/3$, so the unpolarized cross-section $\sigma_0 = (2/3)\,\sigma_{3/2}$. For case (a), $a = 1$, we have $\sigma_a = \sigma_{3/2} = (3/2)\,\sigma_0$. For case (b), $b = 1$, $\sigma_b = (2/3)\,\sigma_{3/2} = \sigma_0$. For case (c), $c = 1$, $\sigma_c = (1/3)\,\sigma_{3/2} = (1/2)\,\sigma_0$.

When the compound state of ^{5}He breaks up into ^{4}He and n, it has definite angular momentum and parity so that the direction of emission of alpha particles and neutrons will have a definite distribution. The final state of the small number of reactants from the non-resonant $J = 1/2$ state is not known. We simply assume isotropic emission of its products with unpolarized neutrons. The final state vector for the orbital angular momentum and spin of the neutron relative to the alpha particle for $J = 3/2$ is

$$|\psi_f\rangle = |\ell s, m_\ell m_s\rangle \langle \ell s, m_\ell m_s|Jm\rangle (\sigma_J d_{m_2} t_{m_3})^{1/2}$$

$$\times \langle Jm|m_2, m_3\rangle \qquad (6)$$

where ℓ is the orbital angular momentum, s is the spin quantum number of the neutron (s = 1/2), and m_ℓ and m_s are the magnetic quantum numbers. The alpha particle has zero spin. Since the compound state has even parity, $\ell = 1$ is not allowed. The coefficients $\langle \ell s, m_\ell m_s|Jm\rangle$ may be obtained from Ref. [31] (page 72, Table 2). Expressing the final eigenstates in terms of spherical harmonics $Y_{\ell m}$, we have

$$\psi_f = \left(\frac{\sigma_{3/2}}{5}\right)^{1/2} \left\{ |-\tfrac{1}{2}\rangle \left[2Y_{22}(d_1 t_{1/2})^{1/2} + Y_{21}(d_1 t_{-1/2})^{1/2} + 2Y_{20}\left(\frac{d_0 t_{-1/2}}{3}\right)^{1/2} \right] - |\tfrac{1}{2}\rangle \left[Y_{21}(d_1 t_{1/2})^{1/2} + Y_{20}\left(\frac{2d_1 t_{-1/2}}{3}\right)^{1/2} + Y_{2,-1}\left(\frac{2d_0 t_{-1/2}}{3}\right)^{1/2} \right] \right\}$$

The spherical harmonics $Y_{\ell m}$ are given in Ref. [31] (page 52). The states $|1/2\rangle$, $|-1/2\rangle$ denote the spin eigenstates of the neutron with magnetic quantum numbers 1/2, −1/2, respectively. The differential cross-section for the neutron, independent of its spin state, is

$$\frac{d\sigma}{d\Omega} = \sum_{m_s = \pm(1/2)} |\psi_f|^2 = \frac{1}{2\pi} \left\{ \sigma_{3/2} \left[\frac{3}{4}\sin^2\theta\, a + \frac{1}{4}(1 + 3\cos^2\theta)\left(\frac{2}{3}b + \frac{1}{3}c\right) \right] + \sigma_{1/2}\left(\frac{1}{12}c + \frac{1}{6}b\right) \right\} \qquad (7)$$

Upon integration over $d\Omega$ we obtain Eq. (4).

A second quantity of interest is the difference $|\psi^+|^2 - |\psi^-|^2$ of the differential cross-sections for neutrons polarized with $m_s = 1/2$ and $m_s = -1/2$:

$$\frac{d\sigma^+}{d\Omega} - \frac{d\sigma^-}{d\Omega} = |\psi^+|^2 - |\psi^-|^2$$

$$= \frac{\sigma_{3/2}}{2\pi} \left\{ \frac{3}{4}[d - (2e + f)] \cos^2\theta \sin^2\theta - \frac{3}{4} d \sin^4\theta + \frac{(2e + f)}{12}(3\cos^2\theta - 1)^2 \right\} \qquad (8)$$

where

$$d = d_1 t_{1/2} - d_{-1} t_{-1/2}$$

$$e = d_0(t_{1/2} - t_{1/2}) \qquad (9)$$

$$f = d_1 t_{-1/2} - d_{-1} t_{1/2}$$

By forming either the sum or the difference of Eqs (7) and (8), the probability of the emission of a neutron into a specified solid angle with definite m_n is determined. For example, if $d_1 = t_{1/2} = 1$, the differential cross-section for $m_n = 1/2$ is $(3/16\pi)\,\sigma_{3/2}(1 - \sin^4\theta)$, and that for $m_n = -(1/2)$ is $(3/16\pi)\,\sigma_{3/2}(1 + \sin^2\theta)^2$. At 90°, the neutron is completely polarized with $m_n = -1/2$. Equations (4), (7) and (8) correspond to Eqs (1), (2) and (3) of Ref. [1]. In Ref. [1], f represents the fraction of D–T reactions that are resonant if the plasma is unpolarized. Thus, $f\sigma_0 = \sigma_{3/2}$ and $(1-f)\,\sigma_0 = (1/2)\,\sigma_{1/2}$. With these definitions, Eq. (4) reduces to Eq. (1). However, Eq. (2) of Ref. [1] is slightly incorrect; it should be as follows:

$$\frac{d\sigma}{d\Omega} = \frac{f\sigma_0}{2\pi} \left[(3/4)\, a \sin^2\theta + (2/3b + 1/3c)\frac{(2/f - 1 + 3\cos^2\theta)}{4} \right] \qquad (8a)$$

which agrees with Eq. (7). Equation (8) reduces to Eq. (3) of Ref. [1] for $\theta = 90°$.

3.2. The D–^{3}He reaction

The D(^{3}He, p)^{4}He reaction is entirely analogous to the D(t, n)^{4}He reaction since ^{3}He and T are mirror nuclei. A $3/2^+$ resonant state of ^{5}Li exists, 407 keV above the energy of free D and ^{3}He nuclei; this contributes very strongly to the reaction at low energies. The spin dependences of the nuclear cross-sections and the angular distributions of the reactor products depend on the existence of such a state. All results on the enhanced cross-sections for different polarization states of the D and T nuclei as well as the angular distribution of neutrons and alpha particles are applicable to the D–^{3}He reaction. The enhanced cross-sections for D–^{3}He are given by Eq. (4) and the differential cross-sections for protons and alpha particles are given by Eqs (7) and (8). In these equations, $\sigma_{3/2}$ and $\sigma_{1/2}$ now are D–^{3}He cross-sections and the quantities a through f refer to the spin distribution of the pairs of D and ^{3}He spins. However, in the energy level diagram of ^{5}Li there is another energy level which is much closer to the $3/2^+$ level than that for ^{5}He, so $\sigma_{1/2}/\sigma_{3/2}$ is of the order of 0.10–0.15. Thus, the enhancement is somewhat smaller and the distribution of emission products could be more isotropic than that for the D–T reaction.

4. DEPOLARIZATION MECHANISMS

In this section we assume that the spins of the nuclei in a thermonuclear reactor have been prepared to be in a given polarization state and we calculate the rate at which this polarization will change. For example, we consider the case in which at every point in a D–T reactor all the D nuclei are in the state $m_2 = 1$ and all the T nuclei are in the state $m_3 = 1/2$, relative to local axes, with the z-axis along B. As these nuclei move with their thermal motion they will meet changing directions and strengths of the magnetic field. If the desired polarization state is to be maintained, the nuclei must continue to be in the states $m_2 = 1$ and $m_3 = 1/2$ relative to the instantaneous axes at their positions. If the magnetic field changes slowly in space, this will always be the case. However, abrupt changes either in the motion of the nuclei or in the magnetic field direction will lead to a gradual change in their polarization which eventually produces a random distribution of the spin states. The various ways in which depolarization occurs are considered in Section 4.1 and the rates of depolarization are estimated.

4.1. Large-scale inhomogeneous fields

In all reactors the equilibrium magnetic field $\vec{B}$ varies in direction and magnitude over macroscopic lengths. Consider a particle of spin $\vec{s}$ and treat its kinetic motion classically. $\vec{B}$ at the position of the particle may thus be regarded as a known function of time. Then the Schrödinger equation for the spin state vector ψ is

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi \tag{10}$$

where

$$H = -\vec{\mu}\cdot\vec{B} = -g\frac{e}{2m_p c}\,\vec{s}\cdot\vec{B}(t) \tag{11}$$

where $\vec{\mu}$ is the magnetic moment, g is the gyromagnetic ratio, which is 0.86 for D and 5.94 for T, and $\vec{s}$ is the spin vector operator. At any time t, take local Cartesian co-ordinates x, y, z, with the z-axis along $\vec{B} = B\vec{b}$. The axes $\vec{x}$ and $\vec{y}$ are ambiguous. The rate of rotation of the threefold axes can be written

$$\vec{\omega} = \vec{b} \times \frac{d\vec{b}}{dt} + \omega_z \vec{b} \tag{12}$$

(ω_z is related to the orientation of the ambiguous local $\vec{x}$ and $\vec{y}$ directions and can be chosen in any convenient manner.) The spin state ψ can be transformed to the rotated co-ordinates at time t by the unitary transformation

$$|\psi'\rangle = U|\psi\rangle \equiv \exp\left(-\frac{i}{\hbar}\int_0^t \vec{s}\cdot\vec{\omega}(t')\,dt'\right)|\psi\rangle \tag{13}$$

If $|\psi\rangle$ represents the components of the spin state in terms of the eigenstates of $\vec{s}\cdot\vec{b}(0)$, then $|\psi'\rangle$ represents the components in terms of the eigenstates of $\vec{s}\cdot\vec{b}(t)$.

We obtain the equation for $|\psi'\rangle$ by differentiating Eq. (13) with respect to time, making use of the relation $U\vec{s}\cdot\vec{b}(t)U^{-1} = \vec{s}\cdot\vec{b}(0)$:

$$\frac{\partial|\psi'\rangle}{\partial t} = -\frac{i}{\hbar}\vec{s}\cdot\vec{\omega}\,U|\psi\rangle - \frac{i}{\hbar}U H|\psi\rangle$$

$$= -\frac{i}{\hbar}\vec{s}\cdot\vec{\omega}|\psi'\rangle + \frac{i}{\hbar}\Omega_p\,\vec{s}\cdot\vec{b}|\psi'\rangle \tag{14}$$

where $\Omega_p(t) = ge\,B(t)/2m_p c$ is the local precession frequency. We can now write

$$\vec{\omega} = \omega_z\vec{b} + \tfrac{1}{2}\,\omega_-(\vec{x} + i\vec{y}) + \tfrac{1}{2}\,\omega_+(\vec{x} - i\vec{y})$$

$$\vec{s} = s_z\vec{b} + \tfrac{1}{2}\,s_-(\vec{x} + i\vec{y}) + \tfrac{1}{2}\,s_+(\vec{x} - i\vec{y}) \tag{15}$$

where $\omega_\pm \equiv \omega_x \pm i\omega_y$, $s_\pm = s_x \pm is_y$. Equation (14) becomes

$$\left[\frac{\partial}{\partial t} - \frac{is_z}{\hbar}(\Omega_p - \omega_z)\right] |\psi'\rangle = -\frac{i}{2\hbar}(s_+\omega_- + s_-\omega_+)|\psi'\rangle \tag{16}$$

We now write

$$|\psi'\rangle = \sum c_m(t)\,|\psi_m'\rangle \tag{17}$$

where $|\psi_m'\rangle$ are the eigenstates of $\vec{s}\cdot\vec{z}(t)$ for eigenvalues $m\hbar$. Take as initial conditions $c_m{}'(0) = \delta_{m,n}$ and assume for $m \neq m$, $c_m{}' \ll c_m$. The non-zero matrix elements of $s_\pm$ are:

$$\langle m\,|s_+|\,m-1\rangle = \langle m-1\,|\,s_-|\,m\rangle = [(s+m)(s-m+1)]^{1/2}\hbar \tag{18}$$

Equation (16) now reduces to

$$\left[\frac{\partial}{\partial t} - i(m+1)(\Omega_p - \omega_z)\right] c_{m\pm 1}$$

$$= -\tfrac{1}{2}\,\omega_\mp\,[(s \pm m + 1)(s \mp m)]^{1/2}\,c_m \tag{19}$$

$$\left[\frac{\partial}{\partial t} - im(\Omega_p - \omega_z)\right] c_m = 0 \tag{20}$$

Next we define the new independent variable ϕ by

$$\phi = \int_0^t (\Omega_p(t') - \omega_z(t'))\,dt' \tag{21}$$

Equation (20) yields $c_m = \exp(im\phi)$, and integration of Eq. (19) yields

$$c_{m\pm 1} = -\frac{i}{2}[(s \pm m + 1)(s \mp m)]^{1/2}\exp[(m \pm 1)\phi]$$

$$\times \int_0^\phi \frac{d\phi'\,\omega_\mp(\phi')}{\Omega_p - \omega_z}\exp(\mp i\phi') \tag{22}$$

Let us apply Eq. (22) to the motion of a triton initially polarized in the $m_3 = 1/2$ state. If we set $\omega_z = 0$, Eq. (22) yields

$$c_{-1/2} = \frac{\exp(-i\phi)}{2}\int_0^\phi \exp(i\phi')\,\frac{\omega_+(\phi')}{\Omega_p(\phi')}\,d\phi' \tag{23}$$

Note:

$$d\omega/d\phi = (d\omega_-/dx)\,(dx/dt)\,(dt/d\phi)$$

$$= \vec{v}\cdot\nabla\omega_-/\Omega_p \approx (\rho_3/9H)\,\omega_-$$

where ρ_3 is the triton gyroradius and H is the scale size of variation of ω_-. Thus, if the scale of variation of the field is large compared to ρ_3, the rate of variation of $\omega_-(\phi)$ is slow compared to the factor $\exp(i\phi)$, and the net accumulation of $c_{-1/2}$ over a long time is exponentially small. (It is true that the gyromotion produces a variation of ω_- with ϕ, but the oscillation in ω_- is very small.) Equation (23) applies to atoms as well as ions. If an atom is polarized some distance from the plasma, it can be moved to the plasma through an inhomogeneous field without losing any polarization, provided the direction of the field does not rotate at a rate comparable to its precession frequency. Equations (21) and (23) demonstrate that any loss of polarization is exponential in the ratio of these rates. If the magnetic field is weaker than the critical field B_c (B_c = 300 G for T and 100 G for D), the electron and proton are coupled together and the electron is strongly coupled to the field, while the nucleus is coupled to the electron polarization, so Ω_p in Eq. (23) is replaced by the hyperfine frequency. For $B \ll B_c$ the nucleus is directly coupled to the magnetic field.

Inspection of Eq. (22) or Eq. (23) shows under what conditions we might expect the polarization to change. If B abruptly changes direction, as in a collision, Eq. (23) will yield a change in $c_{-1/2}$. Because of the factor $\exp(i\phi')$ we expect such changes to be random in phase so that only the square of such changes accumulates. Similarly, if a wave with a magnetic perturbation encounters the particle, it can make a resonant change if $\omega_-(\phi)$ is such as to have a unit harmonic. Such changes are considered in the next section.

4.2. Depolarization by collisions

The typical energy change of ions during collisions is of the order of kT, the thermal energy of the ions. For $T \approx 10$ keV, this energy is about eleven orders of magnitude larger than the energy necessary to flip a nuclear spin in a magnetic field of 50 kG. Intuitively, it is surprising that the nuclear spin orientation could survive collisions which are so much more energetic than its orientation energy. However, during the collision the actual torque exerted on the spin of the ion is very small. This, together with the short duration of the collision, leads to the conclusion that the change in spin polarization of the ion is very small. To estimate the amount of depolarization by collisions, consider a polarized nucleus with magnetic moment $\mu = ge\, \hbar/2m_p$ and a singly charged particle passing by with velocity v and impact parameter b. The magnetic field at the nucleus is $\cong ev/b^2c$ and the time is $2b/c$, so the time integrated torque on the nucleus is $2b/v \cdot (ev/b^2c) ge\, \hbar/2m_pc$. Equating this to the change in angular moment of the nucleus, which is $I\hbar$ times the change in direction of the spin $\delta\theta$, we obtain

$$\delta\theta \approx \left(\frac{ge^2}{m_p^2}\right)\frac{1}{bI} = g\frac{r_p}{bI} \tag{24}$$

where $r_p = e^2/m_pc^2$ is the classical proton radius. Notice that the change in direction is independent of the velocity of the particle or its mass, so $\delta\theta$ is the same for passing electrons and nuclei. $(\delta\theta)^2$ must be summed over all impacts to obtain an effective cross-section. This sum leads to the result

$$\sigma \approx \frac{2\pi}{I} g^2 \pi r_p^2 \ln\left(\frac{b_{max}}{b_{min}}\right) \tag{25}$$

This result, although of the correct order of magnitude, overestimates σ by about a factor of six because of angular factors. At $T = 10$ keV and $n = 10^{14}$ cm^{-3}, the logarithm is about 20 and the correct value of the cross-section is $\sigma \approx 255\ \pi r_p^2 \approx 1.8 \times 10^{-29}$ cm^2, which is a satisfactorily small cross-section. This result was given in Ref. [1].

For each nucleus and its m-state, a variety of depolarization types of collisions exist. For tritium there is depolarization by spin–orbit coupling or spin–spin coupling with electrons, deuterons or other tritons. We assume that the electrons are depolarized and the tritons are polarized in the same state as the test triton. The depolarizing deuteron may be

TABLE I. DEPOLARIZATION OF TRITIUM IN THE $m_3 = 1/2$ STATE

Spin–spin	
By unpolarized electrons	$19.6\ \pi r_p^{2\,*}$
By tritons, $m_3 = 1/2$	$12.9\ \pi r_p^{2\,*}$
By deuterons, $m_2 = 1$	$1.0\ \pi r_p^{2\,*}$
$m_2 = 0$	$0.83\ \pi r_p^{2\,*}$
$m_2 = -1$	$0.5\ \pi r_p^{2\,*}$
Spin–orbit	
By electrons	$255\ \pi r_p^2$
By ions	$255\ \pi r_p^2$

TABLE II. DEPOLARIZATION OF DEUTERIUM IN THE $m_2 = 1$ STATE

Spin–orbit	
By electrons	$10.8\ \pi r_p^2$
By ions	$10.8\ \pi r_p^2$
Spin–spin	
By unpolarized electrons	$0.8\ \pi r_p^{2\,*}$
By tritons, $m_3 = 1.2$	$0.84\ \pi r_p^{2\,*}$
$m_3 = -1/2$	$0.33\ \pi r_p^{2\,*}$
By deuterons, $m_2 = 1$	$0.04\ \pi r_p^{2\,*}$
Quadrupole	
By electrons	$6.6 \times 10^{-5}\ \pi r_p^{2\,*}$
By ions	$48\ \pi r_p^{2\,*}$

TABLE III. DEPOLARIZATION OF DEUTERIUM IN THE m = 0 STATE

Spin–orbit	
By electrons	$21.6\ \pi r_p^2$
By ions	$21.6\ \pi r_p^2$
Spin–spin	
By unpolarized electrons	$1.6\ \pi r_p^{2\,*}$
By tritons, $m_3 = 1/2$	$1.2\ \pi r_p^{2\,*}$
$m_3 = -1/2$	$1.2\ \pi r_p^{2\,*}$
By deuterons, $m_2 = 0$	$0.07\ \pi r_p^{2\,*}$
Quadrupole	
By electrons	$0.0004\ \pi r_p^{2\,*}$
By ions	$32.2\ \pi r_p^{2\,*}$

polarized in a different m-state. For a test deuteron there is a similar variety of cross-sections for depolarization by tritons, electrons or other deuterons. In addition, the deuteron can be depolarized by interaction of its electric quadrupole moment with fluctuating electric fields.

All of these cross-sections are considerably smaller than the cross-section for spin–orbit depolarization of tritons. For general use, these cross-sections are given in Tables I–III. For unduly complex calculations of these cross-sections, upper bounds are given (indicated by asterisks). The details for all these calculations are given in Ref. [29]. The calculation of the largest cross-section, the spin–orbit cross-section, is given in Appendix A of this paper.

The sum of all cross-sections for the depolarization of tritons by electrons is less than 2×10^{-29} cm^2. This leads to a depolarization rate

$$n\sigma \bar{v} \approx 8 \times 10^{-6}\ \mathrm{s}^{-1} \sqrt{T_4}\, n_{14} \tag{26}$$

where T_4 is the temperature in units of 10^4 eV and n_{14} is the density in units of 10^{14} cm^{-3}. The depolarization for nuclei is smaller by the square root of the mass ratio, and the depolarization for deuterons is smaller by a factor of 25. Thus, any depolarization by collisions can be ignored.

For electrons, however, this is different. The spin-orbit decay for electrons is obtained by inserting 1836 for g, so that the decay is faster by a factor of the order of 3×10^6 and the electrons depolarize in approximately 30 ms. Thus, we expect electrons to be unpolarized.

4.3. Combined effects of field inhomogeneities and collisions

In our discussion of the motion of an ion in an inhomogeneous field $\vec{B}$, we temporarily deferred the discussion of the ion gyromotion. If the magnetic field has a shear, the ion in its gyromotion will see a small change in the direction of the magnetic field, of the order of $\delta\theta = \rho_i\, |d\vec{b}/dx|$, where $\vec{b}$ is the unit vector in the direction $\vec{B}_0$ and ρ_i is the gyroradius of the ion. A simple application of Eq. (12) shows that such a gyromotion oscillation leads to an oscillation in $\omega_\pm$ of the order of $\Omega\delta\theta$ at the gyrofrequency rate Ω_i. This oscillation is non-resonant with the precession frequency Ω_p, so when Eq. (22) is applied it can be seen that an oscillation in the amplitude $c_{m\pm1}$ of the order of $1/2\ [(s_\pm\, m + 1)(s_\pm\, m)]^{1/2}\ \delta\theta \cdot \Omega_i/(\Omega_i - \Omega_{pr})$ is produced. For deuterons in the state m = 1, c_0 will oscillate by $(1/2)\sqrt{2}\ \delta\theta/(1-0.88) = 5\delta\theta$. However, a sudden collision could dephase the change in c_0 so that many collisions could gradually change c_0. Taking the sum over many collisions leads to depolarization. This is the depolarization process of Lodder [22] mentioned in Section 2.

For a quantitative evaluation of this depolarization rate, which we may term gyro-collisional depolarization, a kinetic calculation is necessary. Such a calculation is performed in Ref. [29]. In the present paper we content ourselves with a calculation based on the Langevin equations of motion

$$\frac{d\vec{r}}{dt} = \vec{v} \tag{27}$$

$$\frac{d\vec{v}}{dt} + \nu\,\vec{v} = \frac{e}{mc}\,\vec{v} \times \vec{B}_0 + \vec{A}(t) \tag{28}$$

We take a model for the sheared field:

$$\vec{b} = \vec{z}\left[1 - \left(\frac{x}{L}\right)^2\right]^{1/2} + \vec{y}\,\frac{x}{L} \tag{29}$$

where L is the scale length for shear. Then, applying the formalism of the beginning of this section and assuming $x \ll L$, we obtain from Eq. (12)

$$\vec{\omega} = \vec{b} \times \frac{d\vec{b}}{dt} = \frac{\vec{z} \times v_x \vec{y}}{L} = -\frac{v_x}{L}\,\vec{x} \tag{30}$$

where we have set $\omega_z = 0$.

Let us consider a deuteron which is originally in the m = 1 state; then Eq. (19) yields

$$\frac{dc_0}{dt} = -\tfrac{1}{2}\,\omega_+\, c_1 = -\frac{i\omega_x}{2}\, c_1 = \frac{iv_x}{2L}\, c_1 \tag{31}$$

From Eq. (21), $c_1 = \exp(i\Omega_p t)$, so that

$$c_0 = \frac{i}{2L}\int_0^t \exp(i\Omega_p t')\, v_x(t')\, dt' \tag{32}$$

The ensemble-averaged rate of secular increase in the polarization state m = 0 is then given by

$$\frac{|c_0^2|}{t} = \frac{2}{4L^2} \operatorname{Re} \int_0^\infty d\tau \exp(i\Omega_p \tau) \langle v_x(0)\, v_x(\tau)\rangle \quad (33)$$

The solution of Eq. (28) for $v_x(t)$ is

$$v_x(t) = \int_0^t A(t')\, e^{-\nu(t-t')} \cos(\Omega_2(t-t'))\, dt' \quad (34)$$

so that

$$\langle v_x(t)\, v_x(0)\rangle = \frac{v_\perp^2}{2} e^{-\nu t} \cos(\Omega_2 t) \quad (35)$$

We have taken the ensemble mean of A to be zero and used the fact that $\langle V_x^2\rangle = v^2/2$. Inserting Eq. (35) into Eq. (33) and taking $\nu \ll \Omega_p$ we have

$$\frac{|c_0|^2}{t} = \nu \frac{v_\perp^2}{8L^2\Omega_2}\left(\frac{\Omega_2}{\Omega_p - \Omega_2}\right)^2 = \nu \frac{v_\perp^2}{8L^2\Omega_2^2}\left(\frac{\Omega_2}{\Omega_p - \Omega_2}\right)^2 \quad (36)$$

Let us compare this result with Lodder's result. There are some differences. First, Lodder treats the deuteron by the method appropriate to a spin-1/2 particle, following the direction of the spin vector $\vec{p}$. As shown in Ref. [29] and in Section 5, a spin-1 particle such as a deuteron can be treated by the same formulation if all spins are initially in the m = 1 state. Then for small scattering of the $\vec{p}$ vectors, one has $|c_0^2| \approx \theta^2/2$. In these terms, Lodder's result can be written as

$$\frac{|c_0|^2}{t} = \frac{\nu_\theta}{2}\frac{\rho^2}{L^2}\left(\frac{\Omega_2}{\Omega_2 - \Omega_p}\right)^2 \quad (37)$$

where ν_θ is the pitch angle scattering rate; ν in Eqs (28) and (36) is equal to $\nu_\theta/2$. Lodder's ν_θ should be reduced by a factor of 1/2 because only scattering in the pitch angle direction counts. Another reduction by a factor of 1/2 is necessary because only the displacement of the guiding centre in the direction of the shear counts; finally, a further reduction by a factor of 1/2 should be made because it is the root mean square of the angle ϕ of the $\vec{p}$ vector that counts during the process. This brings the result of Lodder in agreement with Eq. (36).

Evaluating Eq. (36) numerically for a particle with total energy 3/2T gives

$$\frac{dP}{dt} = -0.169 \frac{n_{14}}{L_2^2 B_4^2 T_4^{1/2}} \quad (38)$$

where $P = |c_1|^2 - |c_0|^2$ is the usual definition of polarization, n_{14} is the density in units of 10^{14} cm^{-3}, L_2 is the shear length in metres, B_4 is the magnetic field in units of 10^4 G and T_4 is the temperature in units of 10 keV.

L^{-2} should be averaged over a volume. If q(0) = 1 on axis, q(a) = 2 and 1/q is linear, $\langle 1/L^2\rangle = 1/12R^2$; q is the safety factor and R is the major radius. For a reactor with $B_4 = 5$, $T_4 = 1$, $n_{14} = 3$ and $R_2 = 3$, we obtain $dP/dt = -1.8 \times 10^{-4}$ s^{-1}.

4.4. Depolarization by waves

From Eq. (22) it can be seen that the polarization state of the nucleus is most susceptible to variations in the direction of the magnetic field $\vec{b}$ which the nucleus sees as resonant with its natural precession frequency (in the local magnetic field). If the magnetic field is uniform, with variations due to small amplitude waves, $\vec{B}_0 + \delta\vec{B}$, there will be a contribution to polarization if the Doppler shifted frequency of the wave as seen by the ion is equal to the precession frequency. For a nucleus spiralling about the magnetic field B_0 there will also be a contribution to depolarization if the wave frequency ω satisfies the condition

$$\omega - k_\parallel v_\parallel + n\Omega_i = \Omega_p \quad (39)$$

where $k_\parallel$ is the component of the wave vector along B and n is any integer. This is because, as shown in Eq. (22), $\omega_\pm$ has frequency components $\omega \pm n\Omega$ which are due to the gyromotion of the particle (the higher harmonics n are generally weaker than the fundamental harmonic n = 0 if $k_\perp \rho \ll 1$).

Before writing down the general expression for depolarization by waves, let us estimate its magnitude. To be specific, let us consider a moving triton in the m = 1/2 state; $\vec{B}_0 + \delta\vec{B}$ is the field which the triton sees in its moving frame. Then from Eqs (11) and (12) we have

$$\frac{dc_{-1/2}}{dt} - \frac{i\Omega_p}{2} c_{-1/2} = \frac{i\Omega_p}{2}\frac{\delta B_x - i\delta B_y}{B_0} c_{1/2}$$
$$= \frac{i\Omega_p}{2}\left(\frac{\delta B_x - i\delta B_y}{B_0}\right) \exp\left(-\frac{i\Omega_p}{2} t\right) \quad (40)$$

$$c_{-1/2} = \exp(i\Omega_p t) \int_0^t \frac{i\Omega_p}{2}$$

$$\times \left(\frac{\delta B_x - i\delta B_y}{B_p}\right) \exp(i\Omega_p t')\, dt' \tag{41}$$

Several conclusions can be drawn from this equation. First, it is only the left circular part of the wave which causes a change in $c_{1/2}$, as would be expected from the fact that the triton precesses in the left-hand direction about $\vec{B}_0$. Second, only waves with harmonics near Ω_p (or those which differ from Ω_p by an integral multiple of Ω_3) affect polarization. Third, the amount of depolarization produced by such a wave in a time t is proportional to the amount of precession that would be produced by $\delta B_\perp$ alone in the absence of B_0. Using these facts, we can estimate the amount of depolarization produced by left circularly polarized waves with mean-square amplitudes $\delta B_\perp$ in the frequency range $1/2\ \Omega_p - 2\Omega_p$. Such waves are incoherent over a time Ω_p^{-1} and produce a change $\delta c_{-1/2} \approx 1/2(\delta B_\perp/B_0)\,\Omega_p t_c$ during the decorrelation time $t_c \approx \Omega_p^{-1}$. A simple estimate then shows that

$$\frac{(\delta c_{-1/2})^2}{t} \approx \frac{1}{8} \left\langle \left(\frac{\delta B_\perp}{B}\right)^2 \right\rangle \Omega_p \tag{42}$$

We have inserted another factor of 1/2 to take care of some cancellation in the integral of Eq. (41). Estimate (42) can be extended to deuterons by insertion of the proper matrix element.

As an example of the importance of wave depolarization, let us consider left circularly polarized waves with amplitude $\delta B \approx 1$ G and $B_0 = 5 \times 10^4$ G. Since $g_3 = 5.94$ for a triton,

$$\Omega_p = geB/2m_p c \approx 1.5 \times 10^9\ s^{-1}$$

and Eq. (42) yields

$$\frac{(\delta c_{-1/2})^2}{t} = 0.075\ s^{-1} \tag{43}$$

Thus, even waves of extremely small amplitude can lead to significant depolarization over times of interest. It is easy to show that a thermal level of waves produces negligible depolarization. However, if the waves were unstable in the range of the triton precession frequency, their amplitude would probably exceed $\delta B = 1$ G and polarization could not be maintained. A similar calculation for deuterons at rest and in the m = 1 state shows that for left circularly polarized waves in the range of the deuteron frequency, δc^2 is smaller by $8(g_2/g_3)^2 = 0.17$, so $(\delta c_0)^2/t = 0.013\ s^{-1}$. This is somewhat slower, so δB has to be of the order of 10 G to produce significant depolarization. This is still a small amplitude. Therefore, almost any wave energy near the deuteron cyclotron frequency at a non-thermal level would produce depolarization. On the other hand, in the absence of external heating by such waves, the presence of ordinary cyclotron damping should make the existence of unstable waves unlikely.

In order to obtain more quantitative results, consider a general spectrum of waves in a uniform magnetic field and plasma. Let the intensity of the magnetic field be given by

$$\langle \delta\vec{B}\,\delta\vec{B} \rangle = \sum_j \int d^3k\, \vec{I}^j(\vec{k}) \tag{44}$$

where j indicates the particular mode at wavenumber k.

In Ref. [29] it is shown that the rate of depolarization of a nuclear species i is

$$\frac{(\Delta c_{-1/2})^2}{t} = \alpha_i \sum_{n,j} \pi\Omega_p^2 \int d\vec{k}_\perp\, d\vec{k}_\parallel\, J_n^2\left(\frac{k_\perp v_\perp}{\Omega_c}\right)$$

$$\times [\vec{x} - i\vec{y}] \cdot \vec{I}^j \cdot [\vec{x} + i\vec{y}] \cdot \delta\,[\omega_{\vec{k}} - k_\parallel v_\parallel - n\Omega_i - \Omega_{pj}] \tag{45}$$

where $\alpha_i = 1$ for tritons. For deuterons in the m = 1 state, $\alpha_i = 2$, and $c_{-1/2}$ should be replaced by c_0. For deuterons in the m = 0 state, $\alpha_i = 4$, and $(\Delta c_{-1/2})^2$ should be replaced by $(\Delta c_1)^2 + (\Delta c_{-1})^2$. The operation $(\vec{x} - i\vec{y}) \cdot \vec{I}^j \cdot (\vec{x} + i\vec{y})$ selects waves of the proper polarization.

Equation (45) applies only to a uniform background B. If the wavelength of the wave is comparable to the size of the tokamak, dephasing of the precessing particle from resonance with the wave may occur because the precession rate varies. In this case the relevant bandwidth for the resonance is $(d\Omega_p/dt)^{1/2}$, with the time derivative taken following the particle.

If this bandwidth is wider than the spectrum of waves in resonance with the wave, then the depolarization rate must be recalculated. However, simply replacing Ω_p in Eq. (42) by $\Omega_p^2/(d\Omega_p/dt)^{1/2}$ should give a reasonable estimate for the depolarization rate. This remark is particularly relevant in the case of depolarization by coherent waves of a narrow bandwidth, as are used in radiofrequency heating, or for the trapped modes discussed by Coppi et al. [24–27].

5. KINETIC THEORY OF SPIN POLARIZATION

For a more complete treatment of the physics of spin polarized plasma, it is necessary to introduce a distribution function for the ions which includes their spin state as a variable in addition to their position $\vec{r}$ and their velocity $\vec{v}$. This method, which has been presented in Ref. [29], is briefly summarized below.

It is appropriate to treat $\vec{r}$ and $\vec{v}$ classically, but the spin must be treated quantum mechanically. However, the treatment of spin variables is difficult and it is fortunate that the spin state of a particle can be mapped onto a vector $\vec{p}$ whose equation of motion is simple.

First, consider a spin-1/2 particle such as a triton. Its spin state is represented by a two-component complex spinor

$$\psi = \begin{pmatrix}\alpha\\ \beta\end{pmatrix} \tag{46}$$

where $(\alpha)^2$ is the probability for m = + 1/2 and $(\beta)^2$ is the probability for m = −1/2. The Schrödinger equation for ψ is

$$i\hbar\frac{d\psi}{dt} = -2\mu\,\vec{B}\cdot\vec{S}\cdot\begin{pmatrix}\alpha\\ \beta\end{pmatrix} \tag{47}$$

where μ is the magnetic moment of the ion, $\vec{S} = 1/2\vec{\sigma}$ is the spin matrix vector, and σ indicates the Pauli matrices

$$\sigma_x = \begin{pmatrix}0&1\\1&0\end{pmatrix},\quad \sigma_y = \begin{pmatrix}0&-i\\i&0\end{pmatrix},\quad \sigma_z = \begin{pmatrix}1&0\\0&-2\end{pmatrix} \tag{48}$$

Now, let $\vec{p}$ be the vector

$$\vec{p} = (\alpha^*\beta^*)\,\vec{\sigma}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} \tag{49}$$

The time evolution of α and β, Eq. (47), induces a time dependence on $\vec{p}$ which is simply

$$\frac{d\vec{p}}{dt} = \Omega_p(\vec{b}\times\vec{p}) \tag{50}$$

where $\Omega_p = \mu B/\hbar$ is the precession frequency. The transformation relations between $\vec{p}$ and α, β are

$$\begin{aligned} 1 &= |\alpha|^2 + |\beta|^2 \\ p_z &= |\alpha|^2 - |\beta|^2 \\ p_x &= \alpha^*\beta + \alpha\beta^* \\ p_y &= -i(\alpha^*\beta - \alpha\beta^*) \end{aligned} \tag{51}$$

and

$$\alpha = [\tfrac{1}{2}(1+p_z)]^{1/2}\exp(i\phi), \qquad \beta = \frac{(p_x + ip_y)\exp(i\phi)}{[2(1+p_z)]^{1/2}} \tag{52}$$

From Eqs (51) we see that $p^2 = 1$. Thus $\vec{p}$ determines α and β up to the irrelevant phase factor $\exp(i\phi)$. The probabilities for m = ±1/2 are $(1 \pm p_z)/2$. (The number of free variables in ψ is 4 for the two complex numbers, minus 1 for the phase factor and minus 1 for the normalization, which is the number of free variables for a three-dimensional unit vector $\vec{p}$.)

For spin-1 particles, such as the deuteron, there are more degrees of freedom, and two polarization vectors, $\vec{p1}$ and $\vec{p2}$, are needed. However, there are two cases which are of particular interest; in these cases the two vectors can be taken as equal. In the first case the deuterons are completely polarized in the m = 1 state. In this case the spin of the deuteron can be represented by a single $\vec{p}$ which satisfies Eq. (50), with Ω_p the precession frequency of the deuteron. The probabilities for the m-states are:

$$\begin{aligned} &m = 1: && (1+p_z)/2 && m = 1 \text{ initially} \\ &m = 0: && [(1-p_z^2)/2]^{1/2} \\ &m = -1: && (1-p_z)/2 \end{aligned} \tag{53}$$

Thus, the $\vec{p}$ vector is initially in the +z direction.

In the second case the deuterons are completely polarized in the m = 0 state. The deuteron is again represented by a single $\vec{p}$ which satisfies Eq. (53), but the probabilities for the m-states are:

$$
\begin{aligned}
m = 1: &\quad [(1 - p_z^2)/2]^{1/2} \\
m = 0: &\quad p_z \qquad m = 0 \text{ initially} \\
m = -1: &\quad [(1 - p_z^2)/2]^{1/2}
\end{aligned}
\tag{54}
$$

Note that $\vec{p}$ is again initially in the +z direction, although the deuteron is polarized perpendicular to $\vec{z}$. In spite of this, $\vec{p}$ represents correctly the probabilities of the different spin states as they evolve in time.

In all cases, Eqs (50), (53) and (54) give correct answers quantum-mechanically, although they are of a classical form. It is striking that they are identical with the equation for a spiralling particle in a constant magnetic field. Thus they possess an adiabatic invariant p_z. This confirms the validity of the remarks at the end of Section 4.1.

Let us define a distribution function $F(t, \vec{r}, \vec{v}, \vec{p})$ for spin-1/2 particles where $Fd^3\vec{r}d^3\vec{v}d^2\vec{p}$ represents the number of particles in $d^3 r d^3 v$, with spin vector $\vec{p}$ in d^2p. The kinetic equation for this distribution function is

$$
\frac{\partial F}{\partial F} + v \cdot \nabla \cdot F + \frac{q}{m}\left(\vec{E} + \frac{\vec{v} \times \vec{B}}{c}\right) \cdot \nabla_v F + \Omega_p (\vec{b} \times \vec{p}) \cdot \nabla_p F = 0 \tag{55}
$$

where q is the charge and m is the mass of the particle. If we multiply this equation by $\vec{p}$ and integrate it over the sphere $p^2 = 1$, we obtain

$$
\frac{\partial P}{\partial t} + v \cdot \nabla P + \frac{q}{m}\left(\vec{E} + \frac{\vec{v} \times \vec{B}}{c}\right) \cdot \nabla_v P + \Omega_p P \times \vec{b} = 0 \tag{56}
$$

where

$$
\vec{P}(t, \vec{r}, \vec{v}) = \int d^2 p F(t, \vec{r}, \vec{v}, \vec{p}) \tag{57}
$$

is the mean polarization in $d^3\vec{r}d^3\vec{v}$. The average probability that a particle is in the m = 1/2 state is $1/2(1 + (P_z/f))$, where f is the normal distribution function. Thus we have four distribution functions in phase space $\vec{r}$, v, namely f and the three components of $\vec{P}$. We also have four Vlasov-like equations.

For spin-1 particles the identical formalism holds in the two special cases. However, the first case gives the average probabilities for the m = 1 and m = −1 states, but it does not give the probabilities for the m = 0 state. Similarly, the second case gives only the probability for the m = 0 state.

The 'generalized' kinetic equations (55) and (56) are very useful in describing polarized plasmas. In Ref. [29] they are used to solve a number of problems in a more complete fashion than is done in this paper.

6. SUMMARY AND CONCLUSIONS

The concept of a polarized plasma generalizes that of an ordinary plasma by explicitly taking into account the nuclear spin states. If these states can be ordered, the plasma could be regarded as a polarized plasma. A polarized plasma has a variety of advantages for fusion. For example, ignition can be more easily achieved, at values of the plasma beta lower by a factor of as much as 1.5. These advantages may be critical to making fusion economical, especially if fusion is marginally economical without polarization. This of course supposes that polarization can be fully exploited.

There are two conditions that must be met for polarization to be feasible. First, it must be possible to develop large sources of polarized fuel for a fusion device. Four possible methods of producing polarized fuel are described in Section 2. There are good reasons to believe that each of these methods could be developed so that a large enough source of polarized fuel to drive a fusion device can be obtained. At present, however, each of these methods presents some difficulties which have to be overcome.

The second condition concerns the question of the survivability of the polarized state of a plasma under fusion conditions. There is no direct experimental evidence of the extent to which a polarized plasma will stay polarized. It is clear that interactions of the plasma with the surrounding walls represent the greatest danger to polarization. If a proper wall material cannot be found, then polarization must be abandoned as a viable process. Amorphous graphite may be a satisfactory wall material. However, only a direct experimental test under fusion conditions can determine whether graphite is an acceptable material or whether another acceptable material exists.

In addition to surviving interactions with the wall, the plasma polarization must also survive internal depolarization mechanisms, such as that discussed by

Lodder [22] and the depolarization of stray waves resonant with the precession frequency suggested by Coppi et al. [24–27]. Again a direct test is needed to prove that these processes and perhaps others are not serious. From theoretical calculations it appears that with a proper choice of geometries and magnetic fields these depolarization processes can be avoided.

In summary, at present it is uncertain whether or not polarization can be used to reduce the costs of fusion. If it can, the benefits to be obtained should be substantial enough to justify its development.

Appendix A

DEPOLARIZATION OF TRITONS BY ELECTRONS (Spin–orbit cross-section)

We calculate the cross-section σ by the Born approximation. This is legitimate because the cross-sections of various partial waves in a Coulomb field are near to those of free waves. From Eq. (126.7) of Ref. [32] we have:

$$d\sigma = \frac{m_e^2}{4\pi^2\hbar^4}\left|\int U \exp(-i\vec{q}\cdot\vec{r})\, dV\right|^2 do$$

$$= \frac{m_e^2}{4\pi^2\hbar^4}\left|\int \left\langle -\tfrac{1}{2};\ \exp(-i\vec{k}'\cdot\vec{r})\right| \frac{g_3 e^2}{2m_p c^2 m_e}\frac{\vec{s}\cdot\vec{\ell}}{r^3}\right|$$

$$\times \tfrac{1}{2}\exp(i\vec{k}\cdot\vec{r})\Big\rangle\, dV\Big|^2 do \qquad \text{(A-1)}$$

where $|1/2; \exp(i\vec{k}\cdot\vec{r})|$ is the state with $m_3 = 1/2$ and with the electron in a plane wave state, $d\sigma$ is the differential cross-section for the electron scattered into the solid angle do and $m_s = -1/2$ and $\vec{q} = \vec{k}' - \vec{k}$. Now,

$$\vec{\ell} = \hbar(\vec{r}\times\vec{k})$$

and

$$\langle 1/2|\vec{s}|1/2\rangle = (\hbar/2)(\vec{x}\times i\vec{y})$$

so

$$d\sigma = \frac{g_3^2 r_p^2}{16\pi^2}\left|\frac{\vec{k}\times(\vec{x}+i\vec{y})}{2}\cdot\int\frac{\vec{r}\exp(-i\vec{q}\cdot\vec{r})}{r^3}\,dV\right|^2 do \qquad \text{(A-2)}$$

It is easy to show that

$$\int\frac{\vec{r}\exp(-i\vec{q}\cdot\vec{r})}{r^3}\,dV = \frac{4\pi\vec{q}}{q^2} \qquad \text{(A-3)}$$

$$d\sigma = \frac{g_3^2 r_p^2}{4}\left|\frac{\vec{k}\times(\vec{x}+i\vec{y})\cdot\vec{q}}{q^2}\right|^2 do \qquad \text{(A-4)}$$

Averaging Eq. (A-4) over all orientations for the collision we find that the bracket averages to $2/3|\vec{k}\times\vec{q}/q^2|^2 = 2/3\,(\sin^2\theta/4(\sin^2\theta/2))^2$. Thus with $do = 2\pi\sin\theta\, d\theta$, we have for the total cross-section

$$\sigma = \frac{2}{3}\frac{g_3^2 r_p^2}{4}\frac{2\pi}{16}\int_0^\pi\frac{\sin^3\theta\, d\theta}{\sin^4\theta/2} \qquad \text{(A-5)}$$

The integral diverges for small θ corresponding to the usual divergence at large impact parameters. We cut it off at $\theta_{min} = 2e^2/mv^2$ $b_{max} = b_0\omega_p/c$, where $b_0 = e^2/m_e v^2$, and obtain

$$\sigma = \frac{\pi}{3} g_3^2 r_p^2 \log\left(\frac{c}{\sqrt{e}\, b_0\,\omega_p}\right) \qquad \text{(A-6)}$$

Substituting $g_3 = 5.94$, $mv^2/2 = 10^4$ eV, $n = 10^{14}$ cm^{-3}, we obtain the results given in Section 4.2 and in Table I.

ACKNOWLEDGEMENTS

Many people have been of great help to the authors, indicating sources of information or participating in some of the calculations. In particular, the authors are very grateful to H.P. Furth, M. Goldhaber, W. Happer, M.N. Rosenbluth, B. Coppi and W. Stodiek for continual encouragement and help. The authors are also grateful to E. Courant, J. Dawson, R. Dicke, W. Haberli, D. Kerst, A. Krisch, R.V. Pound and T. Tombrello for particularly valuable suggestions.

Useful discussions of the experimental feasibility of spin polarization with J. Cecchi, R. Knize, W. Happer, F. Calaprice, A. MacDonald, J. Strachan and W. Heidbrink are also acknowledged.

This work was supported by the United States Department of Energy, under Contract No. DE-ACO2-76-CHO-3073. R.M.K. is particularly grateful for the hospitality of the Institute of Plasma Physics, Nagoya, and of the Heliotron Laboratory in Kyoto.

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(Manuscript received 13 June 1984
Final manuscript received 30 June 1986)