Journal of Physics E: Scientific Instruments

Temperature sensor characteristics and measurement system design

To cite this article: J P Bentley 1984 *J. Phys. E: Sci. Instrum.* **17** 430

View the article online for updates and enhancements.

You may also like

- Simplified calibration process and uncertainty assessment for sampling large numbers of single-use thermistors for upper-air temperature measurement
Inseok Yang, Sunghun Kim, Young Hee Lee et al.
- A water calorimeter for on-site absorbed dose to water calibrations in ^{60}Co and MV-photon beams including MRI incorporated treatment equipment
Leon de Prez, Jacco de Pooter, Bartel Jansen et al.
- A simple experiment to determine the characteristics of an NTC thermistor for low-temperature measurement applications
A Mawire

This content was downloaded from IP address 18.116.67.212 on 17/05/2024 at 02:12

J. Phys. E: Sci. Instrum., Vol. 17, 1984. Printed in Great Britain

Temperature sensor characteristics and measurement system design

J P Bentley
Department of Electrical, Instrumentation and Control Engineering, Teesside Polytechnic, Middlesbrough, Cleveland TS1 3BA, UK

Abstract. This article firstly describes the characteristics of three types of sensors currently in wide use: platinum resistance thermometers, thermistors and thermocouples. It then explains how the transfer characteristics of each sensor can be established using readily available temperature standards. The article then shows how these characteristics can be used to design signal conditioning circuits which convert the sensor output into a convenient voltage signal. Bridge circuits are necessary for the resistance thermometer and thermistor. It is possible to design a bridge with a nonlinear transfer characteristic which almost compensates for that of the thermistor. The design of an automatic reference junction circuit for a thermocouple is discussed. The article concludes by describing how a microcomputer can be incorporated into these systems as a signal processing element to improve accuracy. In the case of the thermocouple, the computer solves a quartic equation to arrive at an improved estimate of temperature.

1. Introduction

Thermocouples, platinum resistance thermometers and thermistors are the three most commonly used contacting temperature sensors. Thermocouples are the most popular industrial temperature sensors and, by correct choice of thermocouple materials and protective sheath, it is possible to measure temperature over a wide range for many different fluids and processes. A copper against constantan (type T) thermocouple, for example, can be used to measure temperatures down to -100 °C and is resistant to both oxidising and reducing atmospheres. A platinum/rhodium against platinum (type R) thermocouple can be used up to about 1400 °C provided it is protected from metallic vapours using a protective sheath of refractory material. Unless each thermocouple is individually calibrated the overall accuracy, i.e. system measurement error, cannot be reduced much below $\pm 1\%$ of span. For applications requiring greater accuracy platinum resistance thermometers are used. Here accuracies of the order of $\pm 0.2\%$ of span are possible with a well designed system. Platinum resistance thermometers can be used to measure temperatures down to -200 °C but the upper limit to their range is only 500 °C. Thermistors have high sensitivity particularly between -50 and $+100$ °C; this makes them especially useful for detecting small changes in ambient or near ambient temperatures encountered in environmental, meteorological, medical and domestic applications. They cannot however be used above 300 °C; in order to achieve an acceptable degree of accuracy thermistors must be individually calibrated and compensation made for their extreme nonlinearity.

This paper firstly discusses the characteristics of these three important temperature sensors, and how their main transfer characteristics can be measured using readily available temperature standards, i.e. how they can be calibrated. It then shows how to use these characteristics in the design of signal conditioning circuits which convert the sensor output to a convenient voltage signal. Details of practical resistance thermometer and thermocouple installations are also included. The paper concludes by describing how a microcomputer can be incorporated into these systems as a signal-processing element to improve overall accuracy.

2. Platinum resistance thermometers

2.1. Characteristics

The resistance of most metals increases reasonably linearly with temperature in the range -100 to $+500$ °C. The general relationship between the resistance $R_T \Omega$ of a metal and temperature T °C is a power series of the form:

$$R_T = R_0(1 + \alpha T + \beta T^2 + \gamma T^3 + \ldots) \qquad (1)$$

where R_0 is the resistance at 0 °C and $\alpha, \beta, \gamma \ldots$ are temperature coefficients of resistance. Although relatively expensive, platinum is usually chosen for industrial resistance thermometers; cheaper metals, notably nickel and copper, are used for less demanding applications. Platinum is preferred because it is chemically inert, has linear and repeatable resistance–temperature characteristics, and can be used over a fairly wide temperature range (-200 to $+500$ °C) in many types of environments. It can be refined to a high degree of purity thus ensuring that statistical variations in resistance between elements, at the same temperature, (tolerance values) are small. This means that it is only necessary to calibrate individual elements in applications requiring the highest accuracy.

Table 1 gives resistance values at different temperatures for elements with a resistance of 100 Ω at 0 °C conforming to existing British (BS1904) and German (DIN43760) standards. Examination of this data shows that the nonlinearity is small; the maximum nonlinearity as a percentage of the total resistance change between 0 and 200 °C being 0.76%. Figure 1 shows the tolerance limits for a 100 Ω element laid down by British and German standards.

The amount of electrical power produced in the element should be limited in order to avoid self-heating effects; in a typical element 10 mW of power causes a temperature rise of 0.3 °C so that for high-accuracy applications, the maximum current should be limited to about 5 mA. One type of probe is constructed using the partially supported arrangement shown in

0022-3735/84/060430 + 10 $02.25 © 1984 The Institute of Physics

Table 1. Resistance–temperature values for 100 Ω platinum element.

BS 1904		DIN 43760 and proposed new British Standard				
0	°C	0	2	4	6	8
100.00	0	100.00	100.78	101.56	102.34	103.12
103.90	10	103.90	104.68	105.46	106.24	107.02
107.79	20	107.79	108.57	109.35	110.12	110.90
111.67	30	111.67	112.45	113.22	113.99	114.77
115.54	40	115.54	116.30	117.08	117.85	118.62
119.40	50	119.40	120.16	120.93	121.70	122.47
123.24	60	123.24	124.01	124.77	125.54	126.31
127.07	70	127.07	127.84	128.60	129.37	130.13
130.89	80	130.89	131.66	132.42	133.18	133.94
134.70	90	134.70	135.46	136.22	136.98	137.74
138.50	100	138.50	139.26	140.02	140.77	141.53
142.29	110	142.29	143.04	143.80	144.55	145.31
146.06	120	146.06	146.81	147.57	148.32	149.07
149.82	130	149.82	150.57	151.33	152.08	152.83
153.57	140	153.58	154.32	155.07	155.82	156.57
157.31	150	157.31	158.06	158.81	159.55	160.30
161.04	160	161.04	161.79	162.53	163.27	164.02
164.76	170	164.76	165.50	166.24	166.98	167.72
168.46	180	168.46	169.20	169.94	170.68	171.42
172.16	190	172.16	172.90	173.63	174.37	175.10
175.83	200	175.84	176.57	177.31	178.04	178.78
179.83	210	179.51	180.24	180.97	181.71	182.44
183.16	220	183.17	183.90	184.63	185.36	186.09
186.82	230	186.82	187.54	188.27	189.00	189.72
190.45	240	190.45	191.18	191.90	192.63	193.35

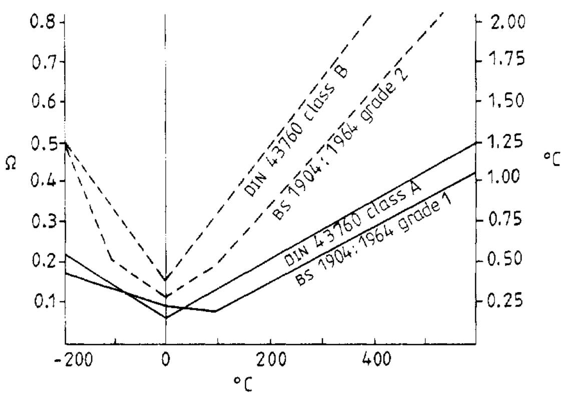


Figure 1. Tolerance limits for 100 Ω platinum element.

figure 2(*a*) (Kent Industrial Measurements 1981). Here fine platinum wire is wound into a very small spiral and is inserted into axial holes in a high-purity alumina insulator, the spiral being unsupported between the insulators. A small quantity of glass adhesive is introduced into the holes and the unit is fired, thus securely fixing a part of each turn onto the alumina. Figure 2(*a*) also shows the element housed in a protective stainless steel sheath. The time constant of the element and sheath, i.e. the time taken for a 63% response to a step change in temperature, is approximately 80 seconds in water. Platinum elements using a thin film rather than fine wire are becoming increasingly popular. One method of construction is to deposit specially formulated platinum ink on an alumina substrate; the film is laser trimmed to give a resistance of 100 ± 0.1 Ω at 0 °C and covered by a ceramic coating (RS Components 1983). The film element is mounted in a stainless steel protective sheath of length 150 mm and diameter 4 mm (figure 2(*b*)). The last 25 mm of the sheath (containing the sensor) is flattened to a rectangle of width 6 mm and thickness 1.6 mm to give improved response time. The time constant is typically 2 seconds in water. Information on protective sheaths for thermocouples and their dynamic response is given in § 4.4.

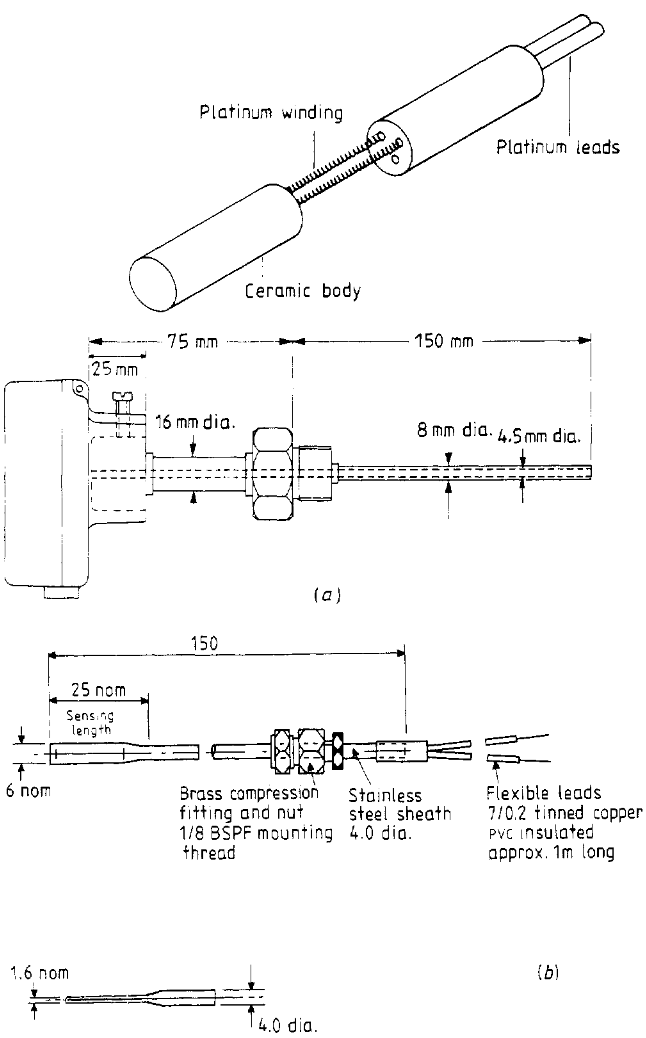


Figure 2. (*a*), Element with 'partially supported' construction and a typical sheath; (*b*), thin film element in sheath.

2.2. Calibration

The ultimate standard for temperature measurement is the International Practical Temperature Scale of 1968 (IPTS-68). The scale is based on eleven primary fixed points to which values have been assigned which conform as closely as possible with thermodynamic temperatures. The fixed points are the triple points, freezing points and boiling points of pure substances (National Physical Laboratory 1982, Barber 1969). Examples are the triple point of water (273.16 K, 0.01 °C), the boiling point of water (steam point 100 °C) and the freezing point of zinc (419.58 °C). An alternative to the boiling point of water is the freezing point of tin (231.97 °C); this is useful in the calibration of a temperature sensor which is to be used above 200 °C.

The scale specifies that a standard platinum resistance thermometer, calibrated at various fixed points, be used as an interpolating instrument below 630.74 °C and a standard platinum against 10% rhodium/platinum thermocouple to interpolate above 630.74 °C. IPTS-68 also specifies 23 secondary fixed points which include the boiling point of nitrogen (77.35 K, −195.80 °C) and the ice point (273.15 K, 0 °C).

The resistance–temperature relationship for a platinum resistance element has only a small amount of nonlinearity and is well described by the quadratic equation

$$R_T = R_0(1 + \alpha T + \beta T^2). \tag{2}$$

This means that in order to find R_0, α and β the resistance should be measured at three fixed points. For applications not requiring the highest accuracy in the range 0 to 200 °C, the ice point, steam point and tin point are the most convenient. Substituting the resistance values of table 1 into equation (2) yields the following set of equations:

$$\begin{aligned} &\text{at } 0\ ^\circ\text{C} && 100.00 = R_0 \\ &\text{at } 100\ ^\circ\text{C} && 138.50 = R_0[1 + \alpha(100) + \beta(100)^2] \\ &\text{at } 232\ ^\circ\text{C} && 187.54 = R_0[1 + \alpha(232) + \beta(232)^2]. \end{aligned} \tag{3}$$

These can be solved to give $R_0 = 100.00\ \Omega$, $\alpha = 3.9092 \times 10^{-3}\ ^\circ\text{C}^{-1}$, $\beta = -5.917 \times 10^{-7}\ ^\circ\text{C}^{-2}$. In practice because of statistical variations within a batch of elements, the resistance values actually measured for a given element will be slightly different from those used in equation (3) giving slightly different values of R_0, α and β.

As stated in § 2.1 calibration of resistance thermometers need only be performed in applications requiring the highest accuracy. From figure 1 we see that the tolerance limits for a 100 Ω element conforming to BS1904:1964 Grade 1 are small, for example within ±0.1 Ω between 0 and 150 °C. Thus calibration is only necessary if the final measurement system is to have an accuracy of better than ±0.1%. The accuracy of the calibration should be substantially better than ±0.1%, i.e. ±0.01% to ±0.02%. In order to achieve this level of accuracy special equipment and precautions are necessary. Practical details of the construction of boiling and freezing point equipment are given in the British Calibration Service guidance publication on the calibration of thermometers (Barber 1969).

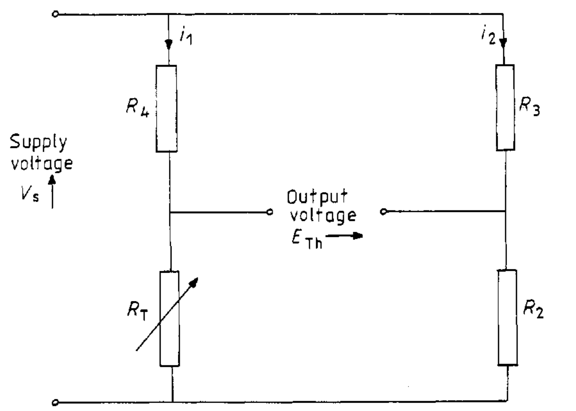


Figure 3. Basic deflection bridge.

2.3. Design of signal conditioning circuit

Figure 3 shows the basic deflection bridge circuit necessary to convert a resistance output into a convenient voltage signal. It can be shown (Bentley 1983) that the bridge may be represented by a Thevenin equivalent circuit consisting of a voltage source E_{Th} and a series resistance R_{Th} where:

$$E_{Th} = V_s\left(\frac{R_T}{R_T + R_4} - \frac{R_2}{R_2 + R_3}\right) \tag{4a}$$

$$R_{Th} = \frac{R_T R_4}{R_T + R_4} + \frac{R_2 R_3}{R_2 + R_3}. \tag{4b}$$

From 4(a) we see that the relationship between bridge output voltage E_{Th} and sensor resistance R_T is in general nonlinear. Since the platinum element itself has very little nonlinearity, a linear bridge is preferable: this can be achieved by making the ratio R_4/R_T (and hence R_3/R_2) large. Consider a specific example where we require E_{Th} to have a range of 0 to 100 mV corresponding to a temperature range T of 0 to 100 °C, E_{Th} being approximately proportional to T. From 4(a) we have

$$E_{Th} = V_s\left(\frac{1}{1 + \dfrac{R_4}{R_T}} - \frac{1}{1 + \dfrac{R_3}{R_2}}\right). \tag{5}$$

At 0 °C, $R_T = R_0$ and $E_{Th} = 0$.

i.e.
$$0 = V_s\left(\frac{1}{1 + \dfrac{R_4}{R_0}} - \frac{1}{1 + \dfrac{R_3}{R_2}}\right)$$

giving

$$R_4 = R_0\frac{R_3}{R_2} \quad \text{i.e. } R_4 = 100\frac{R_3}{R_2}. \tag{6}$$

Substituting for R_4 in equation (5) gives

$$E_{Th} = V_s\left(\frac{1}{1 + \dfrac{R_0}{R_T}\cdot\dfrac{R_3}{R_2}} - \frac{1}{1 + \dfrac{R_3}{R_2}}\right). \tag{7}$$

If we make (R_3/R_2) large compared with 1, then E_{Th} is approximately given by

$$\begin{aligned} E_{Th} &= V_s\left(\frac{R_2}{R_3}\right)\left(\frac{R_T}{R_0} - 1\right) \\ &= V_s\left(\frac{R_2}{R_3}\right)(\alpha T + \beta T^2). \end{aligned} \tag{8}$$

Neglecting the βT^2 term we have $E_{Th} \approx V_s(R_2/R_3)\alpha T$ i.e. an

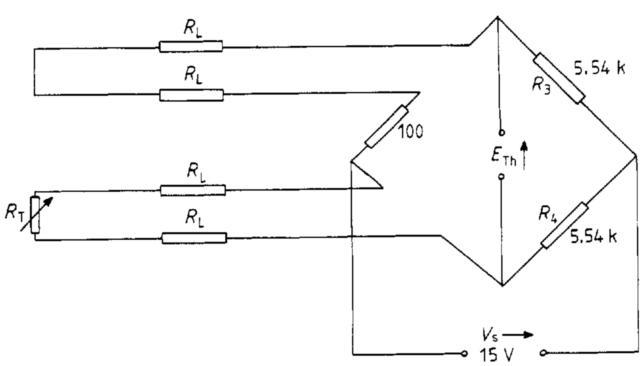


Figure 4. Designed platinum resistance thermometer bridge circuit with four-wire connections.

approximately linear relationship between E_{Th} and T. However, equation (7) should be used for the design calculation; at $T=100$ °C, $R_T=138.50\ \Omega$ and $E_{Th}=0.1$ V, assuming $V_s=15$ V we have:

$$0.1=15\left(\frac{1}{1+\dfrac{100}{138.50}\dfrac{R_3}{R_2}}-\frac{1}{1+\dfrac{R_3}{R_2}}\right). \tag{9}$$

Solving the resulting quadratic equation and taking the positive root gives $(R_3/R_2)=55.4$. Choosing $R_2=100\ \Omega$ gives $R_3=5.54$ kΩ; the individual values of R_3 and R_3 are not critical (except where temperature compensation of the leads is required) it is their ratio which is crucial to the design. From equation (6) we require $R_4=5.54$ kΩ in order to balance the bridge at 0 °C. The maximum current through the platinum resistance element

$$i_1^{\max}=\frac{15}{5540+100}=2.66 \text{ mA}$$

and the maximum power $=(2.66\times10^{-3})^2\times100=0.7$ mW; this low power dissipation causes negligible self-heating effects. For accurate work it is essential to be able to adjust R_4 to exactly balance the bridge at the ice point and to adjust the supply voltage V_s to give exactly 0.1 V output voltage at the steam point.

This simple circuit has one important defect. In an industrial situation the resistance sensor will be situated on the plant, the bridge circuit in a control room, typically 100 m away. The connection is made using leads with a typical resistance of 0.026 $\Omega\ \mathrm{m}^{-1}$ i.e. each lead has a resistance R_L of 2.6 Ω. Changes in plant ambient temperature will significantly affect lead resistance R_L and bridge output voltage E_{Th}. Thus if the total lead resistance $2R_L$ is 5.2 Ω at 20 °C, a fall in plant ambient temperature to 10 °C will cause R_L to fall to 5.0 Ω. This fall in lead resistance of 0.2 Ω causes the bridge output voltage to fall by 0.5 mV. This is equivalent to an error of 0.5 °C in measured temperature which is unacceptable in a system specifically chosen for its high accuracy. This problem is overcome using a connecting cable with three or four cores rather than just two. The additional wires are incorporated in the other arm of the bridge so that any changes in R_L tend to cancel out with negligible effect on E_{Th}; figure 4 shows a modified bridge with 'four wire' connections (note that it is now necessary to make $R_2 \simeq R_T$ at the working temperature). Some installations avoid this problem by locating the bridge circuit in a field-mounted transmitter which is close to the resistance thermometer, so that no long leads are required. The transmitter gives a voltage or current output signal exactly proportional to the resistance of the element.

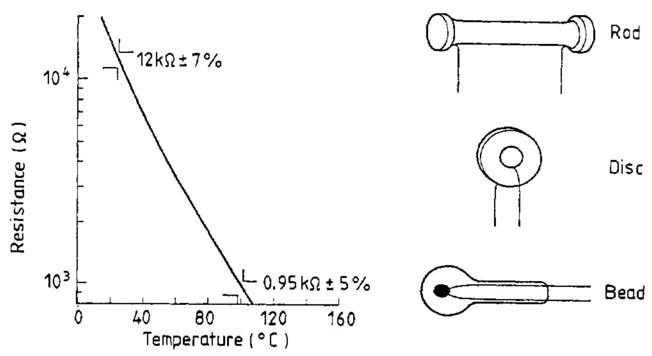


Figure 5. Thermistor characteristics and construction.

3. Thermistors

3.1. Characteristics

Thermistors are semiconductor materials whose resistance changes with temperature. The most commonly used type is prepared by mixing oxides of the iron group of transition metal elements such as chromium, manganese, iron, cobalt and nickel. The resistance of this type of thermistor decreases with temperature, so that they are referred to as negative temperature coefficient (NTC) thermistors. The relationship between resistance and temperature is highly nonlinear (figure 5) and is described by the equation:

$$R_\theta=A\exp(\beta/\theta) \tag{10}$$

where R_θ is the resistance at temperature θ K and A and β are constants for the thermistor. Thermistors are usually in the form of beads, rods or discs (figure 5): bead thermistors are enclosed in a glass envelope. A typical NTC thermistor (Mullard 1974, figure 5) has a resistance of 12 kΩ at 25 °C (298 K) falling to 0.95 kΩ at 100 °C (373 K) and $\beta=3750$ K. The manufacturer's tolerance limits on the above figures are ±7%, i.e. ±840 Ω, at 25 °C, and ±5%, i.e. ±48 Ω, at 100 °C; these limits are far wider than for platinum elements. Thus in order to make accurate measurements with thermistors they must be calibrated individually and compensation or correction made for their extremely nonlinear characteristics. Thermistors are limited to temperature ranges between −80 and 300 °C. Again the amount of electrical power produced in the thermistor should be limited to that which can be dissipated to its surrounding. In a typical thermistor the self-heating effect is 1 °C rise for every 7 mW of power in still air whereas it is reduced to 1 °C rise for 19 mW if a heat sink is provided. Thermistors made from silicon with a positive temperature coefficient (PTC) are also available; the resistance of a typical element increases from 500 Ω at −55 °C to 1900 Ω at 125 °C (Texas Instruments 1980).

3.2. Calibration

From equation (10) we see that the resistance–temperature characteristics of a thermistor are defined by two constants A and β. Thus we need only to measure the resistance of the element at two fixed points; the most convenient are the ice point $\theta=273.15$ K and the steam point $\theta=373.15$ K. Resistance values for a typical thermistor are $R_{273}=8979\ \Omega$ and $R_{373}=365.5\ \Omega$, giving the equations:

$$\begin{aligned}8979&=A\exp(\beta/273)\\365.5&=A\exp(\beta/373).\end{aligned} \tag{11}$$

Solving these gives $\beta=3260$ K and $A=0.0585\ \Omega$.

3.3. Design of signal conditioning circuit

As stated above there is a nonlinear relation between resistance R_θ and temperature θ (equation (10)) for a thermistor. The relation between deflection bridge output voltage E_{Th} and sensor resistance R_θ is also nonlinear (equation 4(a)). The nature of these nonlinearities is such that it is possible to design a deflection bridge so that the bridge nonlinearity partially compensates for the thermistor nonlinearity over a limited temperature range (figure 6). The overall relation between E_{Th} and θ is then approximately linear.

Suppose we require a bridge output voltage in the range 0 to 2.55 V (the usual input range for an 8-bit analogue-to-digital

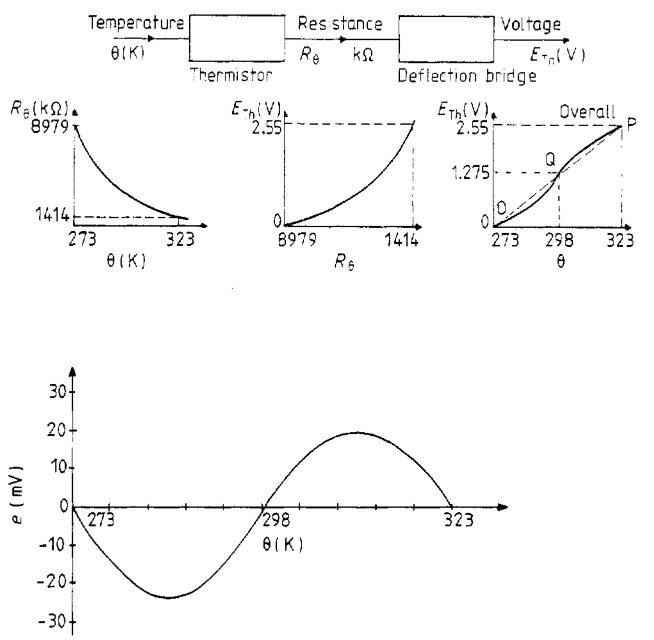


Figure 6. Thermistor-bridge linearisation and nonlinear error function.

converter), approximately proportional to input temperature in the range 273 to 323 K. Almost minimum nonlinearity is obtained if we design the bridge so that $E_{Th} = 1.275$ V (midpoint of voltage range) at $\theta = 298$ K (midpoint of temperature range). The bridge is then designed by solving the following three equations for V_s, R_4 and the ratio R_3/R_2:

$$0 = V_s\left(\frac{1}{1+\frac{R_4}{R_{273}}} - \frac{1}{1+\frac{R_3}{R_2}}\right) \text{(point O)} \qquad (12a)$$

$$1.275 = V_s\left(\frac{1}{1+\frac{R_4}{R_{298}}} - \frac{1}{1+\frac{R_3}{R_2}}\right) \text{(point Q)} \qquad (12b)$$

$$2.550 = V_s\left(\frac{1}{1+\frac{R_4}{R_{323}}} - \frac{1}{1+\frac{R_3}{R_2}}\right) \text{(point P).} \qquad (12c)$$

The values R_{298}, R_{323} are found from the thermistor equation $R_\theta = 0.0585 \exp(3260/\theta)$ to be $R_{298} = 3297\ \Omega$ and $R_{323} = 1414\ \Omega$. From equation (12*a*) we have

$$R_4 = R_{273}(R_3/R_2) = 8979(R_3/R_2). \qquad (13)$$

Substituting equation (13) in (12*b*) and (12*c*) yields two equations involving V_s and (R_3/R_2); dividing these gives a single equation in (R_3/R_2) which can be solved to give $(R_3/R_2) = 0.259$. Choosing $R_2 = 1000\ \Omega$ gives $R_3 = 259\ \Omega$ (as with the resistance thermometer bridge individual values of R_3 and R_3 are not critical); from (13) we have $R_4 = 2326\ \Omega$ and substituting for R_4, (R_3/R_2) in (12*c*) gives $V_s = 6.13$ V. The maximum current through the thermistor

$$i_1^{\text{MAX}} = \frac{6.13}{2326 + 1414} = 1.64 \text{ mA}$$

and the maximum power $= (1.64 \times 10^{-3})^2\ 1414 = 3.8$ mW. The self-heating effect is therefore only around 0.5 °C in still air and considerably less in liquids.

For accurate work, it is essential to be able to adjust R_4 to exactly balance the bridge at the ice point and to adjust supply voltage V_s to give exactly 2.55 V output voltage at 50 °C. In order to measure 50 °C accurately, a platinum resistance thermometer should be used as an interpolating instrument. This is calibrated using ice, steam and tin fixed points as explained in § 2.2. The resistance–temperature relationship for the platinum standard is then accurately known and given by:

$$R_T = 100(1 + 3.9092 \times 10^{-3}T - 5.917 \times 10^{-7}T^2) \qquad (14)$$

From (14) $R_T = 119.4\ \Omega$ at $T = 50$ °C; this means that V_s should be adjusted until $E_{Th} = 2.55$ V when the resistance of the platinum standard is 119.4 Ω. The platinum resistance standard can then be used to measure the overall voltage–temperature relation; typically the maximum nonlinearity is ± 25 mV i.e. $\pm 1\%$ of full scale deflection. Figure 6 also shows the remaining nonlinear error function $e(\theta)$; this is the difference between the actual and ideal voltage–temperature relations. It is approximately a sinusoid of amplitude 25 mV i.e. 0.5 °C.

4. Thermocouples

4.1. *Characteristics*

Thermocouples are the most commonly used industrial temperature sensor. If two different metals A and B are joined together there is a difference in electrical potential across the junction. This contact potential depends on the metals A and B and the temperature T °C of the junction, and is given by a power series of the form:

$$E_T^{AB} = a_1T + a_2T^2 + a_3T^3 + a_4T^4 + \ldots. \qquad (15)$$

A thermocouple is a closed circuit consisting of two junctions (figure 7) at different temperatures T_1 and T_2 °C. If a high-impedance voltmeter is introduced into the circuit so that current flow is negligible, then the measured EMF is, to a close approximation, the difference of the contact potentials i.e.:

$$E_{T_1,T_2}^{AB} = E_{T_1}^{AB} - E_{T_2}^{AB}$$
$$= a_1(T_1 - T_2) + a_2(T_1^2 - T_2^2) + a_3(T_1^3 - T_2^3) + a_4(T_1^4 - T_2^4) + \ldots. \qquad (16)$$

Thus the measured EMF depends on the temperatures T_1, T_2 of both junctions. In the following discussion T_1 will be the temperature to be measured, i.e. the temperature of the measurement junction and T_2 will be the temperature of the reference junction. In order to infer T_1 accurately from the measured EMF, the reference junction temperature T_2 must be known.

Figure 7 also shows a typical industrial thermocouple installation; a chromel against alumel (type K) thermocouple is measuring the temperature of steam in a pipe. The thermocouple is extended to the control room using extension or compensation leads made of chromel and alumel. The reference junction is therefore located in the control room; here the temperature T_2 is nominally 20 °C but variations of ± 5 °C are quite possible. An obvious solution is to enclose the reference junction in a temperature controlled enclosure, e.g. a refrigerator at 0 °C. A better alternative is based on the law of intermediate temperatures; this states that for a given pair of metals:

$$E_{T_1,T_3} = E_{T_1,T_2} + E_{T_2,T_3} \qquad (17)$$

where T_2 is the intermediate temperature. If $T_3 = 0$ °C, then:

$$E_{T_1,0} = E_{T_1,T_2} + E_{T_2,0}. \qquad (18)$$

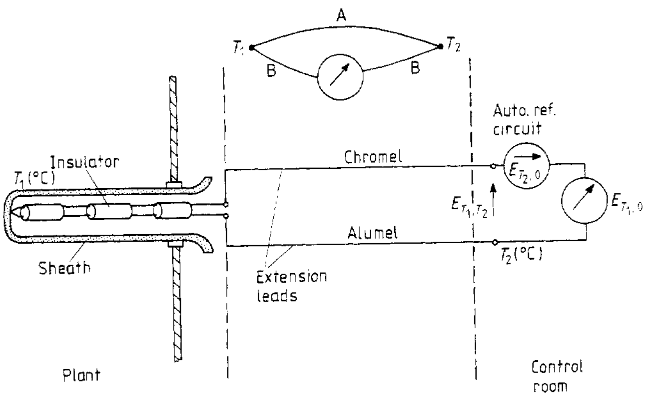


Figure 7. Thermocouple principle and typical industrial installation.

Thus if we introduce a second source of EMF $E_{T_2,0}$ into the circuit, in series with thermocouple EMF E_{T_1,T_2}, then the resultant voltage is $E_{T_1,T_2} + E_{T_2,0}$ which is equal to $E_{T_1,0}$. Thus a voltage detector or amplifier measures the EMF relative to an apparent fixed reference temperature of 0 °C, even though the actual reference temperature is varying about a mean of 20 °C. The circuit which provides $E_{T_2,0}$ is referred to as an automatic reference junction circuit, its design is discussed in § 4.3.

This is still the most popular type of installation in hazardous areas i.e. regions of a plant where an electrical spark could ignite a flammable gas mixture. However, in nonhazardous areas (and with more recently developed transmitters, in hazardous areas as well) it is possible to locate the reference junction circuit and amplifier in a field-mounted transmitter which is close to the thermocouple. This gives a current or voltage output signal proportional to $E_{T_1,0}$ which is transmitted to the control room using normal cable.

Table 2 (Bentley 1983) summarises the measurement range, EMF values and tolerances of four thermocouples in common industrial use. The magnitude of thermocouple nonlinearity can be estimated from the EMF values given. For example a copper against constantan thermocouple, used between 0 and 400 °C, has an EMF $E_{T_1,0}$ of 0 μV at 0 °C, 20 869 μV at 400 °C and 9286 μV at 200 °C. A straight line relation between 0 and 400 °C would give 10 435 μV at 200 °C; the nonlinearity at 200 °C is therefore −1149 μV or −5.5 per cent of FSD. Typical tolerances are of the order of ±1 per cent, i.e. about 10 times greater than for platinum resistance thermometers. For base metal thermocouples (types J, T, K), the extension or compensation leads are made of the same metals as the thermocouple itself. For rare metal thermocouples (e.g. type R), the metals in the extension leads are copper and copper–nickel which have similar thermoelectric properties to platinum and platinum–rhodium but are far cheaper.

Table 2. Characteristics of commonly used thermocouples.

Type	Temperature ranges (°C)	EMF values (μV)	Tolerances	Extension leads	Characteristics
Iron v. constantan Type J	16 SWG 0–500 10 SWG 0–600	$E_{100,0} = 5268$ $E_{200,0} = 10777$ $E_{300,0} = 16325$ $E_{500,0} = 27388$	0 to 300 °C ± 3 °C above 300 °C ± 1%	as for thermocouple	Oxidising and reducing atmospheres have little effect; should be protected from moisture, oxygen and sulphur-bearing gases
Copper v. constantan Type T	21 SWG − 100–+400	$E_{-100,0} = -3378$ $E_{+100,0} = 4277$ $E_{200,0} = 9286$ $E_{400,0} = 20869$	0 to 100 °C ± 1 °C above 100 °C ± 1%	as for thermocouple	Recommended for low and sub-zero temperatures Resists oxidising and reducing atmospheres up to approx. 350 °C Requires protection from acid fumes
Nickel–chromium v. nickel–aluminium Chromel v. alumel Type K	16 SWG 0–950 10 SWG 0–1150	$E_{100,0} = 4095$ $E_{250,0} = 10151$ $E_{500,0} = 20640$ $E_{1000,0} = 41269$	0 to 400 °C ±3 °C above 400 °C ± 0.75%	as for thermocouple	Recommended for oxidising and neutral conditions Rapidly contaminated in sulphurous atmospheres Not suitable for reducing atmospheres
Platinum v. platinum −13% rhodium Type R	0–1400 (depending on sheath)	$E_{300,0} = 2400$ $E_{600,0} = 5582$ $E_{900,0} = 9203$ $E_{1200,0} = 13224$	0 to 1100 °C ± 1 °C 1100 to 1400 °C ± 2 °C	Copper Copper/nickel	Rapidly poisoned by reducing atmospheres. Particularly susceptible to many metal vapours, therefore important that nonmetal sheaths be used

4.2. Calibration

As an example let us consider the calibration of a copper–constantan (type T) thermocouple. As mentioned above this is considerably nonlinear and experience has shown that a quartic equation is necessary to represent the EMF temperature variation in the range −200 to +200 °C with sufficient accuracy. Terminating equation (16) at the fourth-order term gives

$$E_{T_1,T_2} = a_1(T_1 - T_2) + a_2(T_1^2 - T_2^2) + a_3(T_1^3 - T_2^3) + a_4(T_1^4 - T_2^4) \quad (19)$$

and we see that four fixed points are necessary to find the constants a_1, a_2, a_3, a_4. The most convenient are the nitrogen point (−196 °C), ice point (0 °C), steam point (100 °C) and the tin point (232 °C). Typical results obtained with a Cambridge slide-wire potentiometer fitted with a Weston standard cell are:

$$E_{-196,22} = -6412\ \mu\text{V},\ E_{0,22} = -868\ \mu\text{V},\ E_{100,22} = 3404\ \mu\text{V},\ E_{232,22} = 10\ 160\ \mu\text{V}.$$

The reference junction temperature was measured using the interpolating platinum resistance thermometer standard described in §§ 2.2 and 3.3. From (19) the resulting set of

Table 3. Choice of protective sheath material for thermocouples.
Metal sheaths

Material	OD	Max. length	Max. working temperature (°C)	Properties and uses
Mild steel	11 mm 16 mm 25 mm 35 mm	 6000 mm 6000 mm 6000 mm	550 °C	Used in applications where oxidising conditions are not too severe, or slightly reducing atmospheres. Oil baths, water, steam where rusting would not be a problem.
Stainless steel	8 mm 16 mm 25 mm	3000 mm 6000 mm 6000 mm	800 °C	Used instead of mild steel if rusting is a problem. Food, drink and general industrial use.
Calorised mild steel	16 mm 25 mm	1500 mm 1500 mm	800 °C	For use in sulphurous, oxidising and reducing atmospheres. Tin and inert salt baths, but not molten lead or aluminium.
Inconel	16 mm 27 mm	6000 mm 6000 mm	1100 °C	Very resistant to oxidisation. Widely used in heat treatment and in sulphur-free atmospheres, cyanide and neutral salt baths.
Chrome iron (27%)	16 mm 25 mm	900 mm 1500 mm	1150 °C	For use in high-temperature sulphurous and reducing atmospheres. Also suitable for use in zinc-based alloys.
Cast nichrome	32 mm	1500 mm	1050 °C	Resistant to oxidation, providing the atmosphere is sulphur-free.

Refractory sheaths

Material	OD	Max. length	Max. working temperature (°C)	Porous or impervious	Properties and uses
Mullite	8 mm 17 mm 30 mm	900 mm 1400 mm 1400 mm	1600 °C	Impervious	High resistance to flux attack. Relatively inert in sulphur and carbon atmospheres. Good resistance to thermal shock. Recommended protection for rare metal elements in severe conditions. Used for inner and outer sheaths.
Silicon carbide	30 mm	1400 mm	1400 °C	Porous	Very resistant to thermal shock but not for use in highly oxidising atmospheres. Used as an outer sheath with an impervious inner sheath under severe conditions and where rapid temperature changes occur.
Recrystallised alumina	9 mm	450 mm	1750 °C	Impervious	The most impervious of all refractory materials. Expensive and normally restricted to smaller sizes of sheath.

equations for polynomial coefficients is:

$$\begin{aligned}
-6412 &= (-196-22)a_1 + (196^2-22^2)a_2 \\
&\quad + (-196^3-22^3)a_3 + (196^4-22^4)a_4 \\
-868 &= (0-22)a_1 + (0^2-22^2)a_2 + (0^3-22^3)a_3 \\
&\quad + (0^4-22^4)a_4 \\
3404 &= (100-22)a_1 + (100^2-22^2)a_2 + (100^3-22^3)a_3 \\
&\quad + (100^4-22^4)a_4 \\
10\,160 &= (232-22)a_1 + (232^2-22^2)a_2 + (232^3-22^3)a_3 \\
&\quad + (232^4-22^4)a_4
\end{aligned} \tag{20}$$

Equations (20) can be solved by partial pivoting and gaussian elimination to give

$$a_1 = 38.4699,\ a_2 = 0.04542,\ a_3 = -3.0617 \times 10^{-5},\ a_4 = 1.396 \times 10^{-8}.$$

The use of a fifth fixed point, e.g. the boiling point of carbon tetrachloride (77 °C), provides an independent check on the accuracy of these coefficients.

4.3. *Design of signal conditioning circuit*

We now discuss the design of the automatic reference junction circuit introduced in § 4.1. This provides an EMF source equal to $E_{T_2,0}$ i.e. that of a thermocouple with junction temperatures T_2 and 0 °C, where T_2 is the control room temperature. From equation (19) we have

$$E_{T_2,0} = a_1 T_2 + a_2 T_2^2 + a_3 T_2^3 + a_4 T_2^4 \tag{21}$$

but since $T_2 \approx 20$ °C we can ignore terms beyond the first and simplify the equation to $E_{T_2,0} \approx a_1 T_2$; this means we require a circuit giving a voltage output signal proportional to T_2. This can be obtained with a metal resistance thermometer incorporated into a deflection bridge circuit with a large value of R_3/R_3 (§ 2.3). Thus we require the bridge output voltage E_{Th} to equal $E_{T_2,0}$; using linear approximations to equations (8) and (21) we have

$$V_s \frac{R_2}{R_3} \alpha T_2 = a_1 T_2, \text{ i.e. } V_s \frac{R_2}{R_3} = \frac{a_1}{\alpha}. \tag{22}$$

Using typical figures, $V_s = 5$ V, $a_1 = 38.5 \times 10^{-6}$ V °C^{-1}, $\alpha = 6.8 \times 10^{-3}$ °C^{-1} (nickel), gives $R_3/R_2 = 880$. Individual values can then be assigned to R_3 and R_2. R_4 is chosen so that the bridge is balanced at 0 °C since obviously no reference junction compensation is required when $T_2 = 0$ °C.

4.4. *Protection of thermocouples*

In order to protect the thermocouple element from contamination, oxidation and corrosion, an outer protective covering or sheath is normally fitted. Table 3 (Kent Industrial Measurements 1981) lists sheath materials in common use and gives their characteristics and applications. Metal sheaths are suitable for temperatures up to 1150 °C depending on type. Refractory sheaths are used to protect rare metal thermocouples, e.g. platinum against platinum–rhodium (type R), up to temperatures of 1750 °C. Figure 8 (Kent Industrial Measurements 1981) shows the construction and dimensions of typical thermocouple and sheath assemblies. The sheaths shown are made from either mild steel, Iconel or chrome iron, have insertion lengths between 300 and 1500 mm and are used to protect thermocouples of the nickel–chromium against nickel–aluminium type (type K).

The time constant τ of a bare thermocouple is given by

$$\tau = \frac{MC}{UA}$$

where M = mass of thermocouple in kg, C = specific heat in J kg^{-1} °C^{-1}, U = heat transfer coefficient between thermocouple and process fluid in W m^{-2} °C^{-1} and A = effective heat transfer area in m^2.

Heat transfer coefficient U depends critically on the properties and velocity of the process fluid: for a fast moving liquid U is typically 625 but only about 25 for a slow moving gas stream. The corresponding bare thermocouple time constants are typically 0.1 s in the liquid and 2.5 s in the gas. A typical sheath has a mass which is far greater than that of the thermocouple so that the dynamic response of the thermocouple–sheath assembly is far slower than that of the thermocouple itself. The time constant can be minimised by ensuring good heat transfer between thermocouple and sheath. The tip of the thermocouple must touch the sheath, at the very least, and thermal contact can be further improved by either filling the sheath with oil, mercury or powdered metal or using a crimped metal sleeve. Figure 9 compares the step response of a bare thermocouple with that of the thermocouple enclosed in a stainless steel sheath of length 350 mm, outside diameter 20 mm, inside diameter 10 mm. In both cases the element was quickly transferred from air at 20 °C to boiling water. The time constant of the bare thermocouple was approximately 0.1 s, whereas the time constant of the thermocouple–sheath assembly was approximately 10 s.

In applications where high speed of response is important but a thick protective sheath is not necessary, then mineral-insulated thermocouples can be used. These consist of a thin-walled metal sheath (figure 10, Kent Industrial Measurements 1981), typically between 1.5 and 6.0 mm in diameter, and much smaller mass than the assemblies shown in figure 8. The thermocouple wires are inserted and the sheath is filled with highly compacted magnesium oxide which is an electrical insulator but a good heat conductor. There are two main types: insulated hot junction or bonded hot junction. In the insulated hot junction type the wires are welded together but insulated from the sheath so that the insulation resistance can be checked before and after installation. The time constant of this type (step test between air at 20 °C and water at 90 °C) varies between 0.24 s for a diameter of 1.5 mm to 3.90 s for a diameter of 6.0 mm. In the bonded hot junction type both the wires and sheath are welded together; this type has an even shorter time constant.

5. Use of microcomputers to improve system accuracy

A microcomputer can be incorporated into each of the three temperature measurement systems discussed in order to improve overall system accuracy. The platinum resistance bridge, thermistor bridge and thermocouple can be interfaced to a microcomputer using a instrumentation amplifier and an analogue-to-digital converter. The principles of analogue-to-digital conversion and converters have been reviewed in an earlier article in the *Instrument Science and Technology* series (Owens 1982). The operation of analogue-to-digital conversion introduces quantisation errors, the magnitude of the error depending on the number of binary digits used. An 8-bit converter has a quantisation error of up to $\pm\frac{1}{2}$ LSB, i.e. $\pm 100/2 \times 255 = \pm 0.196\%$, whereas the maximum error for a 12-bit converter is $\pm 100/2 \times 4095 = \pm 0.012\%$. The number of digits used should be such that the quantisation error is small compared with the errors to be corrected by the microcomputer.

A platinum resistance thermometer bridge designed and set

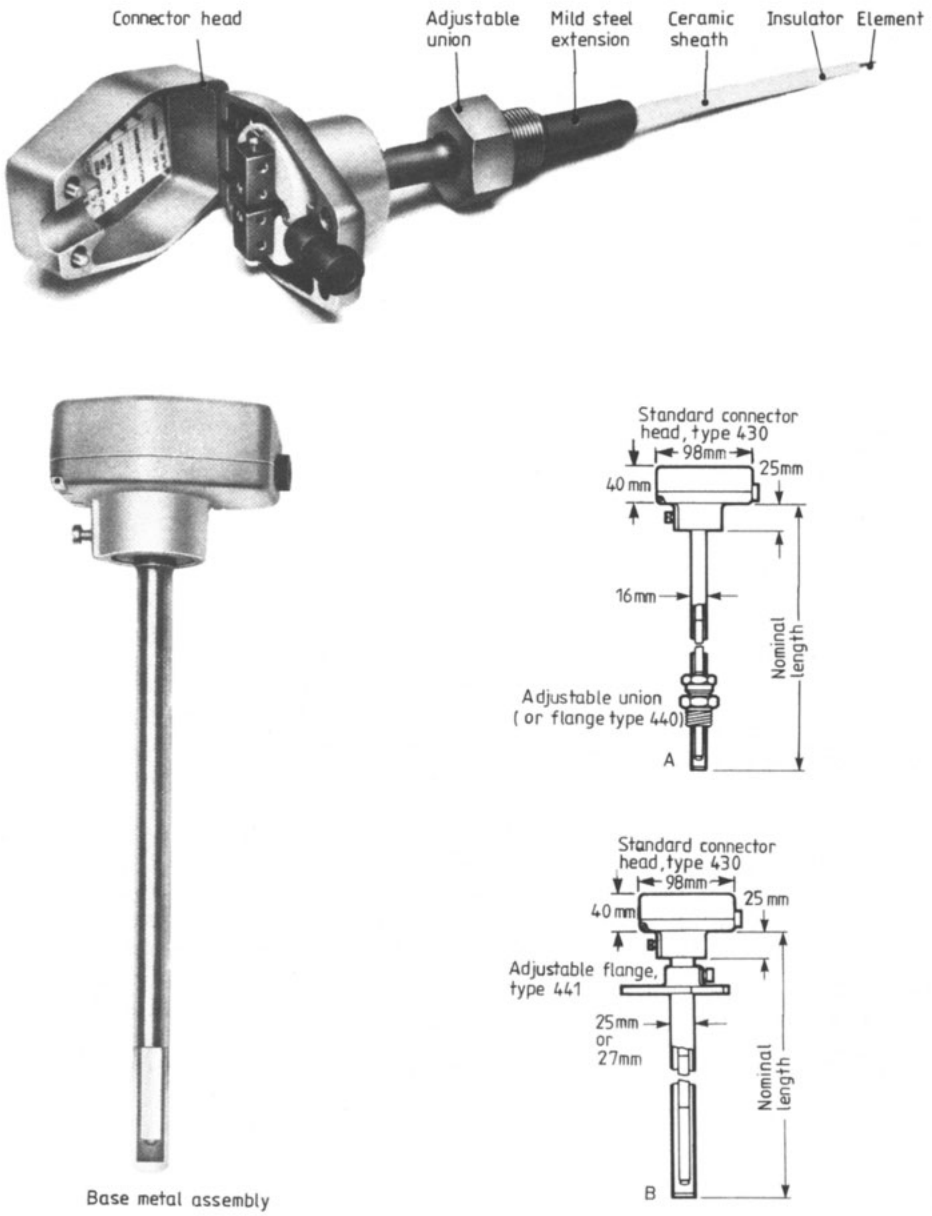


Figure 8. Typical thermocouple and sheath assembly.

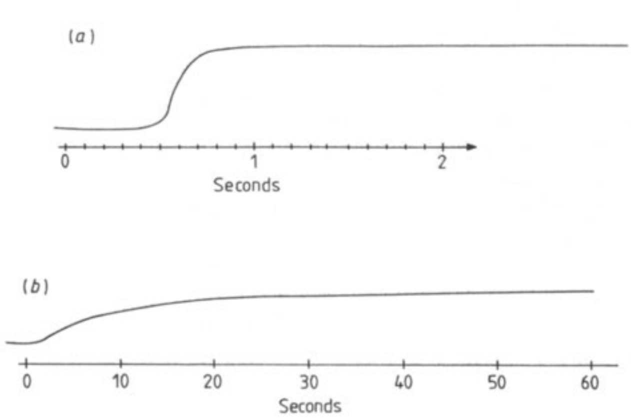


Figure 9. Step responses for (*a*), bare thermocouple and (*b*), thermocouple in sheath.

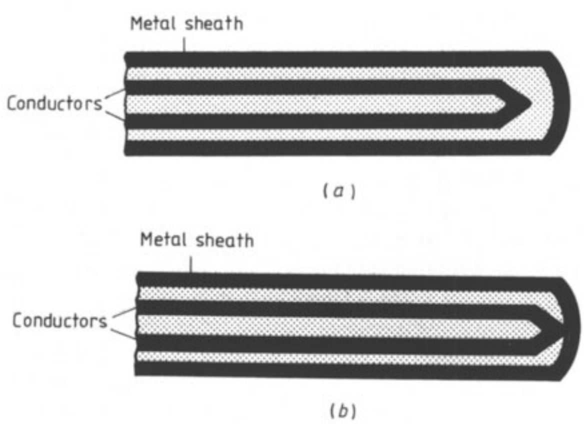


Figure 10. Construction of mineral-insulated thermocouples: (*a*), insulated hot junction; (*b*), bonded hot junction.

up as described in § 2.3 will give an output voltage range of 0 to 100 mV linearly related to temperature in the range 0 to 100 °C.

Thus the measured temperature is simply equal to the bridge voltage in millivolts. However, for high accuracy applications, the microcomputer finds T, by solving equations (7) and (14); in (7) E_{Th} is measured and V_s, R_0, R_3/R_2 are known so that R_T can be calculated; substituting for R_T in (14) gives a quadratic equation which can be solved for T.

The thermistor bridge circuit of § 3.3 gives an output voltage range E_{Th} of 0 to 2.55 V approximately linearly related to an input temperature range θ of 273 to 323 K, the maximum nonlinearity being about $\pm 1\%$ of FSD. The computer makes an initial estimate of θ using

$$\theta_{\text{est}} = 273 + (50/2.55)E_{\text{Th}}. \tag{23}$$

This estimate can be refined by adding the nonlinear correction function shown in figure 6; this is approximately a sinusoid of amplitude 0.5 K so that the corrected temperature is

$$\theta_{\text{corr}} = \theta_{\text{est}} + 0.5 \sin 2\pi\left(\frac{\theta - 273}{50}\right). \tag{24}$$

However, to realise the full potential of this correction a 12-bit rather than an 8-bit analogue-to-digital converter should be used.

If the copper–constantan thermocouple described in § 4.1 is used with the automatic reference junction circuit of § 4.3 then the computer effectively measures $E_{T_1,0}$, the thermocouple EMF relative to a fixed reference temperature of 0 °C. From equation (19) we have

$$E_{T_1,0} = a_1T_1 + a_2T_1^2 + a_3T_1^3 + a_4T_1^4 \tag{25}$$

where a_1, a_2, a_3, a_4 are known (§ 4.2). Thus measured junction temperature T_1 can be found by solving the quartic:

$$f(T_1) = 0 \tag{26}$$

where $f(T_1) = a_4T_1^4 + a_3T_1^3 + a_2T_1^2 + a_1T_1 - E_{T_1,0}$. The computer solves this iteratively using the Newton–Raphson method. A first guess of

$$T_{1,1} = \frac{E_{T_1,0}}{a_1} \tag{27}$$

is then used to calculate a second improved guess:

$$T_{1,2} = T_{1,1} - \frac{f(T_{1,1})}{f'(T_{1,1})} \tag{28}$$

where

$$f'(T_1) = \frac{\mathrm{d}}{\mathrm{d}T_1} f(T_1).$$

The process is repeated using

$$T_{1,r+1} = T_{1,r} - \frac{f(T_{1,r})}{f'(T_{1,r})} \tag{29}$$

until the magnitude of the difference between successive guesses is less than or equal to 0.1 °C i.e. until

$$|T_{1,n+1} - T_{1,n}| \leqslant 0.1. \tag{30}$$

The number of iterations needed to converge is usually three or four and never more than eight.

6. Conclusion

All three temperature sensors discussed in this paper possess characteristics such as nonlinearity and statistical tolerance variations which, potentially, could limit the accuracy of measurement systems based on them. However, if high overall system accuracy is required, it can be achieved by putting effort into the calibration of individual sensors, the design of compensating signal conditioning circuits and the development of microcomputer software which solves the nonlinear equations associated with the sensors.

Acknowledgment

Microcomputer software for the fitting of thermocouple polynomials to fixed point data and the numerical solution of quartic equations was developed by Mr C J Oliver as part of his final-year project work in this Department.

References

Barber C R 1969 *The Calibration of Thermometers* (London: British Calibration Service Guidance Publication)

Bentley J P 1983 *Principles of Measurement Systems* (London: Longman)

Kent Industrial Measurements 1981 *Specification Sheets for Resistance Thermometer and Thermocouple Assemblies*

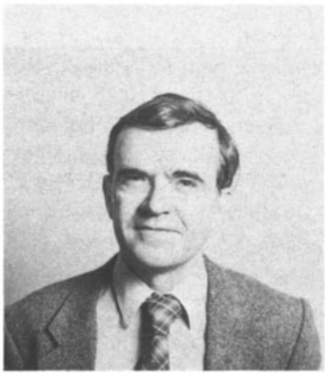
Mullard Limited 1974 *Technical Information on Two-point NTC Thermistors for Measurement and Control*

National Physical Laboratory 1982 *Booklet on Measurement Services – Temperature*

Owens A R 1982 Digital signal conditioning and conversion *J. Phys. E: Sci. Instrum.* **15** 789–806

RS Components 1983 *Data Sheet R/3914 on platinum resistance temperature detectors*

Texas Instruments 1980 *Technical Information on TSP102, TL170 Series Semiconductor Sensors.*



John P Bentley is currently Senior Lecturer in Instrumentation in the Department of Electrical, Instrumentation and Control Engineering at Teesside Polytechnic. He has published several papers covering a broad range of instrumentation topics. His book *Principles of Measurement Systems*, intended mainly as an instrumentation textbook for undergraduates, was published in September 1983. His current research interests include the development of vortex flowmeters for gas flows in large pipes and the dynamic modelling of pressure transducers.