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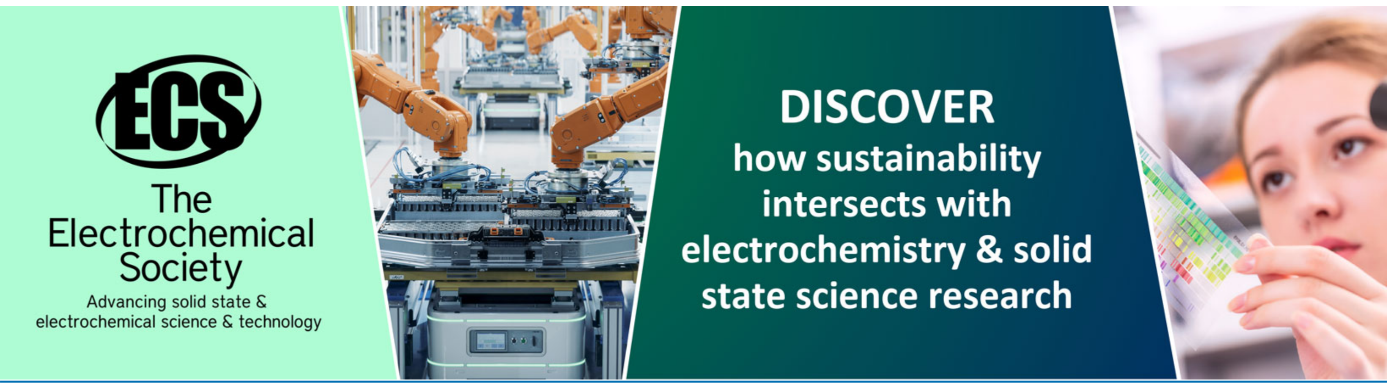

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Detailed balance limit of the efficiency of tandem solar cells

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Abstract. The fundamental (detailed balance) limit of the performance of a tandem structure is presented. The model takes into account the fact that a particular cell is not only illuminated by part of the solar irradiance but also by the electroluminescence of other cells of the set. Whereas, under 1 sun irradiance, a single solar cell only converts 30% of the solar energy, a tandem structure of two cells can convert 42%, a tandem structure of three cells can convert 49%, etc. Under the highest possible light concentration, these efficiencies are 40% (one cell), 55% (two cells), 63% (three cells), etc. The model also allows us to predict the ideal efficiency of a stack with an infinite number of solar cells. Such a tandem system can convert 68% of the unconcentrated sunlight, and 86% of the concentrated sunlight.

1. Introduction

Shockley and Queisser (1961) proposed the detailed balance limit of a single p–n junction solar cell. In this ideal diode, only radiative recombination occurs between the conduction band and the valence band. Nonradiative processes (i.e. Auger effects) are absent in this limit case. The efficiency η was calculated as a function of the bandgap E_g. A maximum of 30% was found for an energy gap of about 1 eV.

On the other hand, various authors (Alvi *et al* 1976, Loferski 1976, Masden and Backus 1978, Moon *et al* 1978, Bennett and Olsen 1978, Gokcen and Loferski 1978) studied recently tandem structures, under both natural and concentrated sunlight. These authors used various empirical data for the solar spectrum and for the diode material constants.

It is of interest to know the basic limit of the efficiency of tandem solar cells. This limiting efficiency might not be dependent on any semiconductor property, except the various bandgaps. Therefore we have applied the detailed balance principle to tandem structures.

Figure 1 shows a simple example of a tandem structure of two homojunction solar cells ($E_{g1} \geqslant E_{g2}$). In each cell not only a part of the solar spectrum is absorbed, but also a part of the electroluminescent spectrum emitted by the other cell. This makes the calculations rather laborious. By enhancing the value of the load R_1 for example, the working point (I_1, V_1) of the first cell is moved. But as the emission spectrum of this cell is proportional to $\exp(qV_1/kT)$, the light incident on the second cell is enhanced and its I–V characteristic is shifted, so that the working point (I_2, V_2) also changes.

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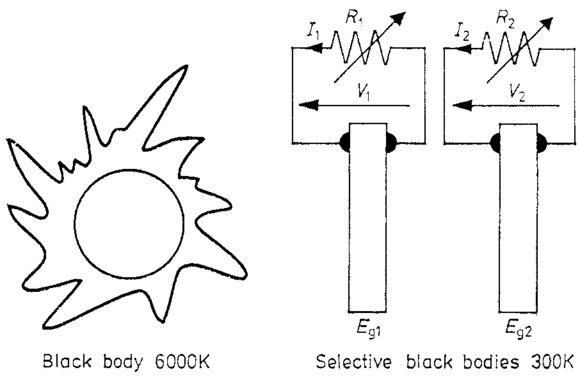

Figure 1. Configuration of a tandem structure consisting of two solar cells ($E_{g1} \geqslant E_{g2}$).

2. Mathematical treatment

In the tandem structure, all n solar cells are assumed to have the same temperature T_c. The I–V characteristic of the ith cell is given by

$$I_i(V_i) = I_{0i} \left[\exp\left(qV_i/kT_c\right) - 1\right] - I_{1i}. \tag{1}$$

The reverse saturation current I_{0i} is determined by the radiative recombination between free holes and electrons (Shockley and Queisser 1961):

$$I_{0i} = qF_{0i} = 2qA \int_{E_{gi}/h}^{\infty} N(\nu, T_c)\, d\nu \tag{2}$$

where A is the surface of the solar cell

$$N(\nu, T) = \frac{2\pi}{c^2} \frac{\nu^2}{\exp(h\nu/kT) - 1} \tag{3}$$

is the Planck black-body radiation flux, and E_{gi} is the bandgap of the ith solar cell.

The light-generated current is given by

$$I_{1i} = q(F_{si} - F_{0i}) \tag{4}$$

where F_{si} is the photon flux incident to the ith cell. The first cell is illuminated by the sun, and by the light emitted by the second cell:

$$F_{s1} = f_\omega A \int_{E_{g1}/h}^{\infty} N(\nu, T_s)\, d\nu + A \exp\left(\frac{qV_2}{kT_c}\right) \int_{E_{g1}/h}^{\infty} N(\nu, T_c)\, d\nu \tag{5a}$$

where f_ω is the geometrical factor taking into account that the sun is seen only under a limited solid angle (Shockley and Queisser 1961) and where T_s denotes the black-body temperature of the sun (assumed 6000 K). An arbitrary cell is illuminated by part of the solar spectrum, by the light emitted by the preceding cell and by the light emitted by the following cell. If the various cells are arranged so that the bandgaps form a decreasing series ($E_{g1} \geqslant E_{g2} \geqslant \ldots \geqslant E_{gn}$), we have (for $i = 2, 3, \ldots, n-1$):

$$F_{si} = f_\omega A \int_{E_{gi}/h}^{E_{g(i-1)}/h} N(\nu, T_s)\, d\nu + A \exp\left(\frac{qV_{i-1}}{kT_c}\right) \int_{E_{g(i-1)}/h}^{\infty} N(\nu, T_c)\, d\nu$$
$$+ A \exp\left(\frac{qV_{i+1}}{kT_c}\right) \int_{E_{gi}/h}^{\infty} N(\nu, T_c)\, d\nu. \tag{5b}$$

The last solar cell of the stack is of course not illuminated by a following cell:

$$F_{sn}=f_\omega A\int_{E_{gn}/h}^{E_{g(n-1)}/h} N(\nu, T_s)\,d\nu + A\exp\left(\frac{qV_{n-1}}{kT_c}\right)\int_{E_{g(n-1)}/h}^{\infty} N(\nu, T_c)\,d\nu. \tag{5c}$$

The total electrical power

$$P=-\sum_{i=1}^{n} V_iI_i$$

generated by the system is maximised by $\partial P/\partial V_i=0$. Putting $x_i=qV_i/kT_c$, this gives rise to the following set of equations:

$$(1+x_1)\exp x_1=(F_{s1}/F_{01})+\tfrac{1}{2}\,x_2\exp x_1 \tag{6a}$$

$$(1+x_i)\exp x_i=\frac{F_{si}}{F_{0i}}+\tfrac{1}{2}\,x_{i-1}\exp x_i\,\frac{\int_{E_{g(i-1)}/h}^{\infty} N(\nu, T_c)\,d\nu}{\int_{E_{gi}/h}^{\infty} N(\nu, T_c)\,d\nu}+\tfrac{1}{2}\,x_{i+1}\exp x_i \tag{6b}$$

$$(i=2, 3, \ldots, n-1)$$

$$(1+x_n)\exp x_n=\frac{F_{sn}}{F_{0n}}+\tfrac{1}{2}\,x_{n-1}\exp x_n\,\frac{\int_{E_{g(n-1)}/h}^{\infty} N(\nu, T_c)\,d\nu}{\int_{E_{gn}/h}^{\infty} N(\nu, T_c)\,d\nu}. \tag{6c}$$

The solution $x_1, x_2, \ldots, x_n$ of this set determines the working points (V_1, I_1), (V_2, I_2), $\ldots, (V_n, I_n)$ at which the cells have to be operated in order to extract a maximum of power from the complete tandem stack. We will call these points the maximum power points. It is interesting to note that the largest rectangles $I_i \times V_i$ in the respective characteristics $I_i(V_i)$ are determined by the voltages $V_i=(kT_c/q)y_i$, where $y_1, y_2, \ldots, y_n$ is the solution of the equation set

$$(1+y_i)\exp y_i=F_{si}/F_{0i} \qquad (i=1, 2, \ldots, n). \tag{7}$$

So we can conclude that the maximum power points do *not* coincide with the maximum rectangle points (more precisely, all x_i are somewhat larger than the corresponding y_i).

This is in contrast with the single solar cell, where the maximum power point and the maximum rectangle point are one and the same point on the characteristic.

Once the set equations for x_i are solved, the maximum power P_m is given by

$$P_m=kT_cx_1\exp x_1(x_1-\tfrac{1}{2}\,x_2)\,F_{01}+\sum_{i=2}^{n-1} kT_cx_i\exp x_i \times[(x_i-\tfrac{1}{2}\,x_{i+1})F_{0i}-\tfrac{1}{2}\,x_{i-1}F_{0(i-1)}]+kT_cx_n\exp x_n[x_nF_{0n}-\tfrac{1}{2}x_{n-1}F_{0(n-1)}]. \tag{8}$$

3. Results

The aforementioned calculations have been performed for a device temperature T_c of 300 K. Unconcentrated sunlight ($f_\omega=2{\cdot}18\times10^{-5}$) as well as concentrated sunlight have been examined. The case $f_\omega=1$ (this means a concentration ratio of 45 900) was studied particularly, because it is assumed to be the highest concentration physically possible: the solar cell stack 'sees' the sun in all directions of the hemisphere (solid angle $=2\,\pi$).

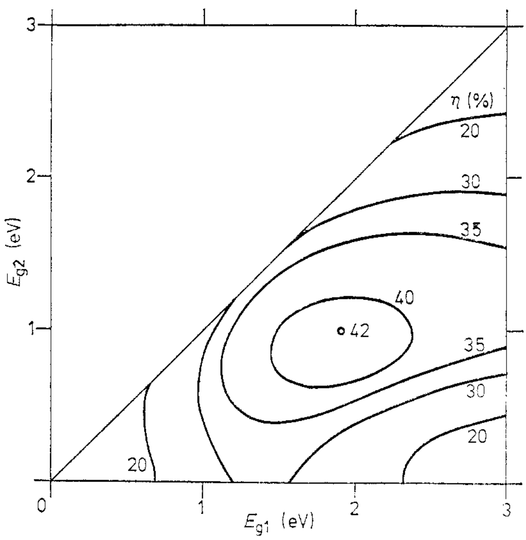

Figure 2. Limit efficiency η (%) of a two-cell tandem as a function of the two semiconductor bandgaps E_{g1} and E_{g2}.

Figure 2 shows the calculated limit efficiency $\eta(E_{g1}, E_{g2})$ for the case of unconcentrated sunlight. A maximum of 42·3% occurs for $E_{g1} = 1{\cdot}9$ eV and $E_{g2} = 1{\cdot}0$ eV.

This efficiency is a lot higher than the detailed balance efficiency of a single solar cell. A maximum of 30·4% occurs there for $E_g = 1{\cdot}3$ eV. The higher conversion efficiency of the tandem structure is of course due to a better match to the solar spectrum. The mutual profit by the different cells of each others electroluminescence is a second (although much smaller) advantage of the tandem system. If one calculates the efficiency of the tandem $E_{g1} = 1{\cdot}9$ eV and $E_{g2} = 1{\cdot}0$ eV without taking into account the mutual irradiance, one

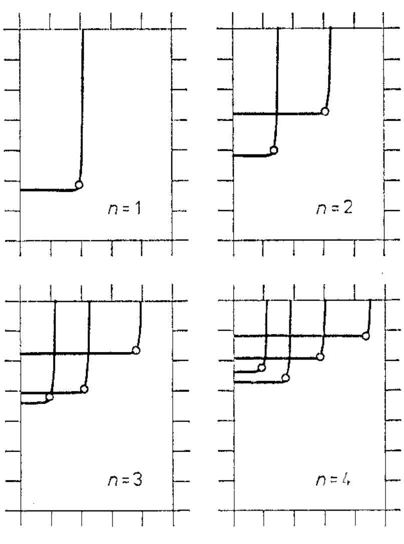

Figure 3. *I–V* characteristics of the optimal tandem stack of *n* cells, with indication of the *n* maximum power points. Horizontal scale 500 mV/div., vertical scale 100 A m^{-2}/div.

finds an efficiency of 42·2%. So, we can conclude that the electroluminescent spectra contribute only about 0·1%.

Calculations for tandem structures with n stacked cells give the optimal set of bandgaps $E_{g1}, E_{g2}, \ldots, E_{gn}$. Table 1 gives the results if concentration is absent. Figure 3 shows the corresponding I–V characteristics and working points. Table 2 gives the results for the concentration ratio 45 900.

Table 1. The optimal set of bandgaps E_{gi} for tandem structures with n stacked cells in unconcentrated sunlight.

n	η (%)	E_{g1} (eV)	E_{g2} (eV)	E_{g3} (eV)	E_{g4} (eV)
1	30	1·3	—	—	—
2	42	1·9	1·0	—	—
3	49	2·3	1·4	0·8	—
4	53	2·6	1·8	1·2	0·8

Table 2. The optimal set of bandgaps E_{gi} for tandem structures with n stacked cells in sunlight concentrated in the ratio 45 900:1.

n	η (%)	E_{g1} (eV)	E_{g2} (eV)	E_{g3} (eV)	E_{g4} (eV)
1	40	1·1	—	—	—
2	55	1·7	0·8	—	—
3	63	2·1	1·2	0·6	—
4	68	2·5	1·6	1·0	0·5

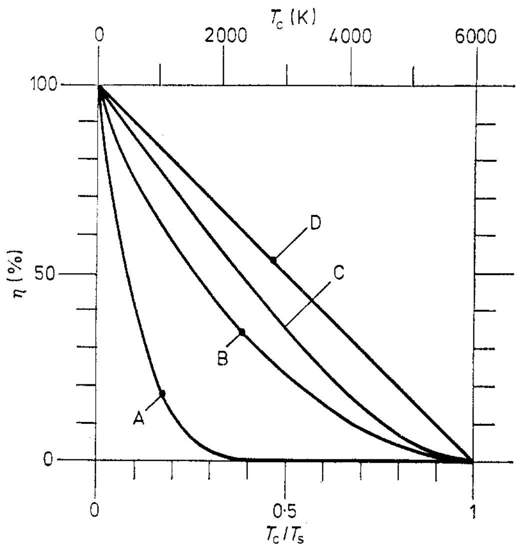

Figure 4. Efficiency η of an infinite tandem structure as a function of the cell temperature T_c for: A 1 sun illumination ($f_\omega = 2{\cdot}18 \times 10^{-5}$); B 45 900 suns illumination ($f_\omega = 1$). As a comparison are shown: C the maximum efficiency proposed by Landsberg; D the Carnot efficiency.

4. An infinite number of cells

As we have seen, the mutual irradiance of the cells by the others' electroluminescent spectra plays only a secondary role, if the number of cells (n) in the stack is small. The effect, however, plays a more fundamental role if one increases considerably the number n.

Indeed, if n increases significantly, the bandgap increments $E_{g(i-1)}-E_{gi}$ become smaller, and thus also the photon flux from the sun on the ith cell:

$$f_\omega A \int_{E_{gi}/h}^{E_{g(i-1)}/h} N(\nu, T_s)\,d\nu \tag{9}$$

i.e. the first term in equation (5*b*), whereas the reverse saturation current I_{0i} given by (2) stays constant.

When $E_{g(i-1)}-E_{gi}$ becomes very small (for unconcentrated sunlight, this means smaller than about 50 meV) the directly absorbed sunlight (9) alone becomes weak, and the voltage factor and fill factor of the ith cell would be affected if this direct sunlight were the only incident radiation. Taking the two other spectra into account in equation (5*b*), the light-generated current determined by equations (4) and (5) stays much larger than the reverse saturation current determined by equation (2), even if the bandgap increment $E_{g(i-1)}-E_{gi}$ tends to zero. And, indeed, one can make sure that the optimum set of bandgaps, for n becoming infinite, is a continuously decreasing E_g.

By making the following substitutions:

$$x_i \to x$$
$$x_{i+1} \to x+dx$$
$$x_{i-1} \to x-dx$$
$$E_{gi} \to E_g$$
$$E_{g(i-1)} \to E_g-dE_g$$

formula (6*b*) becomes

$$(1+x)\exp x = f_\omega \frac{N(E_g/h, T_s)}{N(E_g/h, T_c)} \tag{10}$$

or

$$(1+x)\exp x = f_\omega \frac{\exp(E_g/kT_c)-1}{\exp(E_g/kT_s)-1}. \tag{11}$$

The electrical power generated is then given by

$$P_m = A\frac{kT_c}{h}\int_{E_{gn}}^{\infty} x^2 \exp x\, N(E_g/h, T_c)\,dE_g \tag{12}$$

where the integral's lower limit E_{gn} is the last (and thus the smallest) bandgap of the stack. It has to be chosen appropriately to avoid a series of solar cells with negative x (and thus negative open circuit voltage). It should be chosen so that the corresponding x from equation (11) equals zero. This gives $E_{gn}=0{\cdot}25$ eV and $E_{gn}=0$ for concentration ratio 1 and 45 900 respectively. Observing this precaution, one can easily calculate P_m from equation (11) and (12). One finds finally an efficiency of 68·2% for 1 sun illumination intensity and 86·8% for 45 900 suns intensity.

A discussion of the $I(V)$ characteristics in this limiting case of a continuously decreasing E_g is given in the appendix.

It is interesting to compare the above found efficiencies with the numbers published by Parrott (1979): 64% for 1 sun intensity and 88% for extreme concentration. These latter theoretical efficiencies were also calculated for a stack of an infinite number of solar cells, but with a much more complicated configuration: the 'edge-illuminated multigap solar cell'.

As the conversion efficiency of the infinite tandem cell does not depend on any material constant (not on bandgap values as is the case for a finite tandem), the only parameters that can still vary are the cell temperature T_c and the geometrical factor f_ω. Figure 4 shows the efficiency as a function of the temperature for the two important values of f_ω. It is interesting to compare these curves with the Carnot efficiency $1-(T_c/T_s)$ and with the efficiency $1-\frac{4}{3}(T_c/T_s)+\frac{1}{3}(T_c/T_s)^4$ proposed by Landsberg and Mallinson (1976) and Landsberg (1977) for the upper limit for solar energy conversion.

5. Conclusions

We have calculated the limit efficiency of a tandem solar cell system, consisting of n homojunction cells in optical series, for $n=1$, 2, 3, 4 and ∞. We found conversion efficiencies which put an upper limit to the various semi-empirical efficiencies published earlier by other authors.

The highest efficiency found amounts to 86·8% and is realised by a tandem of an infinite number of cells, with a smoothly varying series of bandgaps (from 0 to ∞), illuminated by heavily concentrated sunlight.

Appendix

Some additional remarks might be useful for the infinite stack of semiconductors with smoothly varying bandgap. From equations (2) and (5*b*) we find that the parameters of the $I(V)$ characteristic

$$I(x)=qF_0 \exp x-qF_s \tag{A1}$$

have the following values:

$$F_0=2A\int_{E_g/h}^{\infty} N(E_g/h, T_c)\,d\nu \tag{A2}$$

and

$$F_s=2A \exp x\int_{E_g/h}^{\infty} N(E_g/h, T_c)\,d\nu-A \exp x\,N(E_g/h, T_c)(dE_g/h)$$
$$+f_\omega A\,N(E_g/h, T_s)(dE_g/h). \tag{A3}$$

It is clear that these relations are only valid if x varies also smoothly within the cell stack, i.e. if $x(E_g)$ is a continuous function. This is only realised if the loads R vary continuously. Under these conditions formula (A1) can be rewritten

$$I(x)=qF_0{}^* \exp x-qF_s{}^* \tag{A4}$$

where $F_0{}^*$ and $F_s{}^*$ are now both independent of $x(E_g)$, i.e. independent of the load function $R(E_g)$:

$$F_0{}^*=(A/h)N(E_g/h, T_c)\,dE_g \tag{A5}$$

and

$$F_s^* = (A/h) f_\omega N(E_g/h, T_s)\, dE_g. \tag{A6}$$

We see that the maximum power condition, expressed by equation (11), is identical to the following equation:

$$(1+x) \exp x = F_s^*/F_0^*. \tag{A7}$$

Thus in this case of an infinite number of cells, the maximum power points are the maximum rectangle points of the characteristics (A4). We repeat that passing through such an $I(V)$ characteristic is not realised if the load of only one individual cell is varied, but only if also the loads of neighbouring cells are varied, in such a way that the load remains a continuous function of the bandgap.

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