

HALL EFFECT CONTROLLED GAS DYNAMICS IN PROTOPLANETARY DISKS. II. FULL 3D SIMULATIONS TOWARD THE OUTER DISK

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ABSTRACT

We perform three-dimensional stratified shearing-box magnetohydrodynamic (MHD) simulations on the gas dynamics of protoplanetary disks with a net vertical magnetic flux of B_{z0} . All three nonideal MHD effects, Ohmic resistivity, the Hall effect, and ambipolar diffusion, are included in a self-consistent manner based on equilibrium chemistry. We focus on regions toward outer disk radii, from 5 to 60 AU, where Ohmic resistivity tends to become negligible, ambipolar diffusion dominates over an extended region across the disk height, and the Hall effect largely controls the dynamics near the disk midplane. We find that at around $R = 5$ AU the system launches a laminar or weakly turbulent magnetocentrifugal wind when the net vertical field B_{z0} is not too weak. Moreover, the wind is able to achieve and maintain a configuration with reflection symmetry at the disk midplane. The case with anti-aligned field polarity ($\mathbf{\Omega} \cdot \mathbf{B}_{z0} < 0$) is more susceptible to the magnetorotational instability (MRI) when B_{z0} decreases, leading to an outflow oscillating in radial directions and very inefficient angular momentum transport. At the outer disk around and beyond $R = 30$ AU, the system shows vigorous MRI turbulence in the surface layer due to far-UV ionization, which efficiently drives disk accretion. The Hall effect affects the stability of the midplane region to the MRI, leading to strong/weak Maxwell stress for aligned/anti-aligned field polarities. Nevertheless, the midplane region is only very weakly turbulent in both cases. Overall, the basic picture is analogous to the conventional layered accretion scenario applied to the outer disk. In addition, we find that the vertical magnetic flux is strongly concentrated into thin, azimuthally extended shells in most of our simulations beyond 15 AU, leading to enhanced radial density variations known as zonal flows. Theoretical implications and observational consequences are briefly discussed.

Key words: accretion, accretion disks – instabilities – magnetohydrodynamics (MHD) – methods: numerical – protoplanetary disks – turbulence

1. INTRODUCTION

Gas dynamics in protoplanetary disks (PPDs) is largely controlled by nonideal magnetohydrodynamic (MHD) effects, which include Ohmic resistivity, the Hall effect, and ambipolar diffusion (AD), because of the weak level of ionization. The three effects coexist in PPDs, with Ohmic resistivity dominating dense regions (the midplane region of the inner disk) and AD dominating tenuous regions (the inner disk surface and the outer disk), and the Hall-dominated region lies in between. While Ohmic resistivity and AD have been studied extensively in the literature, the role of the Hall effect remains poorly understood. This paper is the continuation of our exploration on the role of the Hall effect in PPDs, following Bai (2014, hereafter Paper I), where an extensive summary of the literature and background information were provided in great detail.

One of the major new elements introduced by the Hall effect is that the gas dynamics depends on the polarity of the external poloidal magnetic field ($\mathbf{B}_0$) threading the disk. Such an external field is expected to be present in PPDs as inherited from the star-formation process (see McKee & Ostriker 2007 and Crutcher 2012 for an extensive review) and is also required to explain the observed accretion rate in PPDs (Bai & Stone 2013b; Bai 2013; Simon et al. 2013a, 2013b). Observationally, the large-scale magnetic fields have been found to thread star-forming cores (Chapman et al. 2013; Hull et al. 2014). It is conceivable that the large-scale fields with $\mathbf{B}_0 \cdot \mathbf{\Omega} > 0$ and $\mathbf{B}_0 \cdot \mathbf{\Omega} < 0$

are equally possible, where $\mathbf{\Omega}$ is along the disk rotation axis. In PPDs, one would expect different physical consequences for different field polarities in regions where the Hall effect is important ($\lesssim 50$ –60 AU).

In Paper I, we focused on the inner region of PPDs ($R \lesssim 15$ AU), where the midplane region is dominated by Ohmic resistivity and the Hall effect, and the disk upper layer is dominated by AD. Without including the Hall effect, it has been found that the magnetorotational instability (MRI; Balbus & Hawley 1991) is completely suppressed in the inner disk, leading to a laminar flow, and the disk launches a magnetocentrifugal wind (Bai & Stone 2013b; Bai 2013). When the Hall effect is included (Paper I), the basic picture of a laminar wind still holds, but the radial range where a laminar wind solution can be found depends on magnetic polarity: for $\mathbf{B}_0 \cdot \mathbf{\Omega} > 0$, the range of a stable wind solution is expected to extend to $R \sim 10$ –15 AU, but for $\mathbf{B}_0 \cdot \mathbf{\Omega} < 0$, the stable region is reduced to only up to ~ 3 –5 AU. In addition, when $\mathbf{B}_0 \cdot \mathbf{\Omega} > 0$, the horizontal magnetic field is strongly amplified as a result of the Hall-shear instability, where shear amplification of the radial field into the toroidal field is fed by the conversion of the toroidal field back to the radial field because of the Hall effect (see also Kunz 2008; Lesur et al. 2014). With opposite polarity, the Hall effect acts destructively with shear and strongly reduces the horizontal magnetic field.

The works in Paper I mainly employ quasi-one-dimensional (1D) simulations to construct laminar wind solutions for the inner disk. In this paper, we shift toward the outer disk and consider regions where the MRI is expected to set in (beyond

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$\sim 3\text{--}10$ AU), up to the radius where the Hall effect has a significant influence (~ 60 AU). We conduct full three-dimensional (3D) simulations to accommodate turbulent fluctuations and potentially large-scale variations. In this range of disk radii, the midplane region is largely dominated by both the Hall effect and AD, and AD is progressively more dominant toward the disk surface layer. Without including the Hall effect, it was found that the MRI is able to operate in the AD-dominated midplane, although the level of turbulence is strongly reduced because of AD (Bai 2013; Simon et al. 2013a). In addition, as far-UV (FUV) ionization penetrates deeper (geometrically) toward the outer disk, MRI can operate much more effectively in the much-better-ionized surface FUV layer to drive disk accretion (Perez-Becker & Chiang 2011; Simon et al. 2013a). Including the Hall effect, we expect a modification of the gas dynamics in the disk midplane region that is dependent on the polarity of the large-scale magnetic field.

We begin by studying the properties of the MRI in the presence of both the Hall effect and AD using unstratified shearing-box simulations and discuss its relevance in PPDs in Section 2. In Section 3 we describe the numerical setup for our full 3D stratified simulations of PPDs and run parameters. We then present simulation results at two focusing radii, 30 AU (Section 4) and 5 AU (Section 5), emphasizing the role played by the Hall effect. We briefly discuss simulations at other disk radii (15 and 60 AU) in Section 6, which help map out the dependence of PPD gas dynamics on disk radii. In Section 7, we summarize the main results and discuss observational consequences, caveats, and future directions.

2. MRI WITH HALL EFFECT AND AMBIPOLAR DIFFUSION

We begin by considering how the Hall effect affects the general properties of the MRI in the outer region of PPDs. We conduct unstratified shearing-box simulations that include both the Hall term and AD. These generalize previous MRI simulations with a single nonideal MHD effect (Bai & Stone 2011; Kunz & Lesur 2013) and are more appropriate for the midplane regions of outer PPDs. The results will serve to guide more realistic simulations for the rest of this paper.

All simulations in this paper are performed using the ATHENA MHD code (Stone et al. 2008), with the relevant nonideal MHD terms implemented in our earlier works (Bai & Stone 2011, Paper I). Standard shearing-box equations and the definitions of nonideal MHD effects can be found in Section 2.2 of Paper I. In brief, dynamical equations are written in Cartesian coordinates in the corotating frame with a local disk patch with angular velocity $\Omega \mathbf{e}_z$. As a convention, (x, y, z) represent radial, azimuthal, and vertical coordinates, respectively. For the unstratified simulations considered in this section, the vertical gravity and Ohmic resistivity terms are dropped. An isothermal equation of state $P = \rho c_s^2$ is adopted with c_s being the sound speed. In code units, we have $\rho_0 = c_s = \Omega = 1$, where ρ_0 is the initial gas density (or midplane density for stratified simulations in the following sections). The unit for the magnetic field is chosen such that magnetic permeability $\mu = 1$. Nonideal MHD diffusion coefficients are characterized by dimensionless numbers (to be introduced in this section), based on which one can convert physical units to code units.

In the following, we first discuss the relative importance of the Hall effect and AD in the relevant regions of PPDs. We then discuss the MRI linear dispersion relation in the presence of both the Hall and AD terms. Finally, we proceed to

nonlinear unstratified shearing-box simulations. Our coverage of parameter space is by no means complete, but they are chosen to be relevant to the regions of PPDs that we study in the remainder of the paper (midplane regions up to ~ 60 AU).

2.1. Relative Importance of the Hall Effect and Ambipolar Diffusion in PPDs

The Hall effect is characterized by a physical length scale l_H . In the absence of charged grains, it reads as (Kunz & Lesur 2013)

$$l_H \equiv \frac{v_A}{\omega_H} = \left(\frac{\rho}{\rho_i} \right) \left(\frac{v_A}{\omega_i} \right), \quad (1)$$

where $v_A = B/\sqrt{4\pi\rho}$ is the Alfvén velocity, ω_i is the ion cyclotron frequency, $\omega_H = (\rho_i/\rho)\omega_i$ is the Hall frequency, and ρ_i and ρ are the mass densities of the ions and the bulk of the gas, respectively, with $\rho_i \ll \rho$ for weakly ionized gas. Note that both v_A and ω_i are proportional to the magnetic field strength, so l_H is field-strength independent and is determined solely by the ionization fraction. In disks, it is natural to normalize l_H by the disk scale height $H \equiv c_s/\Omega$. The associated Hall diffusivity η_H can be expressed as

$$\eta_H = v_A l_H. \quad (2)$$

At a fixed ionization fraction ($\propto \rho_i/\rho$), we have $\eta_H \propto B/\rho$.

AD is characterized by the frequency with which neutrals collide with ions $\gamma_i \rho_i$, where γ_i is the coefficient of momentum transfer for ion-neutral collisions. In the disk, it is natural to normalize $\gamma_i \rho_i$ to the disk orbital frequency, by defining

$$Am \equiv \frac{\gamma_i \rho_i}{\Omega}, \quad (3)$$

which is the Elsasser number for AD. Generally, AD plays an important role in the gas dynamics when $Am \lesssim 10$ (Bai & Stone 2011). The associated ambipolar diffusivity is given by

$$\eta_A = v_A^2 / \gamma_i \rho_i. \quad (4)$$

At a fixed ionization fraction, we have $\eta_A \propto (B/\rho)^2$.

The above definitions apply when electrons and ions are the main charged species, where the physics can be described most easily. Generalizations to include charged grains can be found in, e.g., Wardle (2007) and Bai (2011a), which are used in our vertically stratified simulations in subsequent sections.

Both η_H and η_A are inversely proportional to the ionization fraction, while different scalings of η_H and η_A with gas density indicate that AD becomes progressively more important toward low-density regions (e.g., the outer disk). To quantify this, we take the product of the two dimensionless numbers l_H/H and Am , which is independent of the ionization fraction:

$$Am \cdot \left(\frac{l_H}{H} \right) = \frac{\gamma_i \rho}{\omega_i} \left(\frac{v_A}{c_s} \right) \propto \frac{\sqrt{\rho}}{c_s}. \quad (5)$$

When adopting the minimum-mass solar nebula disk model (MMSN; Weidenschilling 1977; Hayashi 1981), we have at disk midplane $\rho_0 \propto R^{-11/4}$, $c_s \propto R^{-1/4}$ and hence $Am \cdot (l_H/H) \propto R^{-9/8}$. More specifically, we find²

$$Am \cdot \left(\frac{l_H}{H} \right) \approx 0.64 \left(\frac{R}{10 \text{ AU}} \right)^{-9/8}. \quad (6)$$

² The factors γ_i and ω_i depend on the mass of the ions. However, for the ion mass $m_i \gg m_H$, the dependence diminishes. The value computed here assumes the gas mean molecular weight $\mu = 2.33m_H$, following the formulas in Bai (2011a).

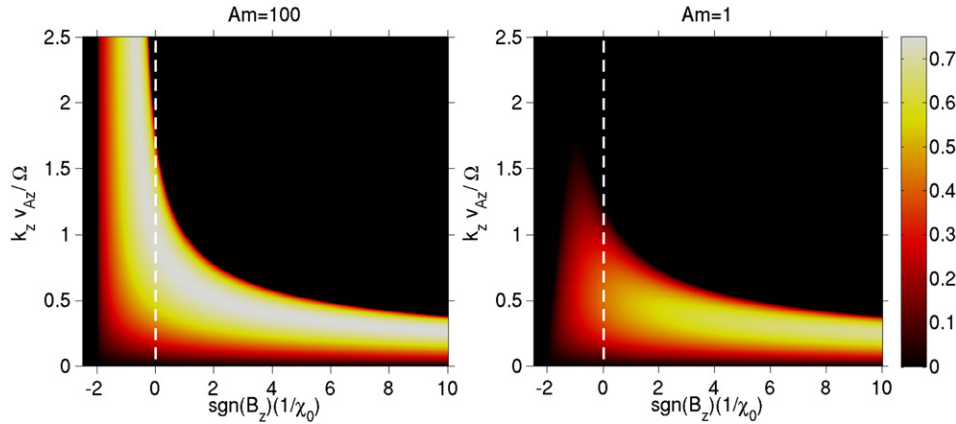


Figure 1. Linear growth rate of the MRI in the presence of the Hall effect and AD, in the case of a pure vertical background magnetic field and for modes with pure vertical wavenumbers $k_z = k$. The growth rate is drawn as a function of normalized wavenumber kv_A/Ω and $\text{sgn}(B_z)(1/\chi_0)$, with the two panels showing the results for fixed $Am = 100$ (ideal MHD) and $Am = 1$ (strong AD). Note that no unstable mode exists for $\text{sgn}(B_z)(1/\chi_0) \leq -2$.

In the outer region of PPDs, the value of Am is found to be of order unity for a wide range of disk radii (Bai 2011a, 2011b), and this formula provides a very useful relation in estimating the importance of AD and the Hall effect in PPDs. If we consider the Hall effect to be important when $l_H/H \gtrsim 0.1$, then the influence of the Hall effect extends to ~ 50 – 60 AU.

For the MRI, the importance of the Hall effect and AD is characterized by their respective Elsasser numbers, defined as $v_A^2/\eta\Omega$, with η being η_H or η_A . With the AD Elsasser number introduced in Equation (3), the Hall Elsasser number can be written as (also see in Paper I)

$$\chi \equiv \frac{\omega_H}{\Omega}. \quad (7)$$

Note that χ depends on field strength ($\propto B$) and also

$$\frac{l_H}{H} = \frac{1}{\chi} \frac{v_A}{c_s} = \frac{1}{\chi} \sqrt{\frac{2}{\beta}} = \frac{X}{\sqrt{2\beta}}, \quad (8)$$

where the plasma $\beta = 8\pi P/B^2$ is the ratio of gas to magnetic pressure, and $X \equiv 2/\chi$ is another commonly adopted dimensionless quantity in the literature (Sano & Stone 2002a, 2002b). The nonideal MHD term with an Elsasser number of order unity or less indicates that the term is physically significant.

A comparison between χ and Am reveals the relative importance between the Hall effect and AD, where a larger value indicates less importance. Using Equation (6), we find

$$\frac{Am}{\chi} \approx 4.5 \sqrt{\frac{\beta}{100}} \left(\frac{R}{10 \text{ AU}} \right)^{-9/8}. \quad (9)$$

Again, we see that Am and χ are likely of the same order for a wide range of disk radii given the expected magnetic field strength of $\beta \lesssim 100$ (saturated β) in the outer disk.

In our definition, ω_H , l_H , and χ are all positive. On the other hand, the Hall term also depends on the polarity of the magnetic field relative to Ω . To distinguish the two cases, we always state explicitly in this paper the polarity of the background magnetic field $B_{z0} > 0$ or $B_{z0} < 0$ for fields aligned and anti-aligned with Ω .

2.2. Linear Properties

The linear dispersion relations of the MRI for general axisymmetric perturbations in the Hall and AD regimes have been

derived separately in Balbus & Terquem (2001) and in Kunz & Balbus (2004) and Desch (2004). The authors considered a general background field configuration $\mathbf{B}_0 = B_{z0}\mathbf{e}_z + B_{\phi0}\mathbf{e}_\phi$ and general axisymmetric perturbations of the form $\exp(i\mathbf{k} \cdot \mathbf{x} + \sigma t)$ with $\mathbf{k} = k_x\mathbf{e}_x + k_z\mathbf{e}_z$. The main results reveal that for linear MRI modes, the Hall term is coupled only to the vertical magnetic field, and the AD term is also coupled to the toroidal field. As a result, the presence of a background toroidal field has little effect on the Hall MRI, but it facilitates the MRI to operate in the AD-dominated regime with $Am \lesssim 1$. A joint dispersion relation including all nonideal MHD terms was given by Pandey & Wardle (2012). They showed that while contributions from the Hall and AD terms are independent, the joint effect is that regimes stable to pure Hall MRI can be rendered unstable because of AD, a situation which again requires a net toroidal field and strong AD ($Am \lesssim 1$).

Exploring the full parameter space of the MRI in the presence of the Hall and AD effects with different field orientations using nonlinear simulations is beyond the scope of this work. Here, we restrict ourselves to a pure vertical background field with either $B_{z0} > 0$ or $B_{z0} < 0$. This choice makes the dispersion relation much simpler, and the most unstable mode has a pure vertical wavenumber of $k_z = k$. For these modes, AD behaves the same way as Ohmic resistivity by replacing η_A with η_O (in the linear regime). This case also covers the most essential MRI physics relevant to PPDs: the Hall term is not directly coupled to the toroidal field, and for AD, the background toroidal field does not strongly affect the level of MRI turbulence for $Am \gtrsim 1$ (Bai & Stone 2011).

In reference to previous works (e.g., Wardle 1999), we show in Figure 1 the MRI growth rate for pure vertical modes $k = k_z$ as a function of dimensionless wavenumber kv_{A0}/Ω and $1/\chi_0$, where subscript 0 represents χ and v_A determined from the background field. Similarly, we use β_0 to denote the plasma β for the background field. The magnetic polarity is reflected from $\text{sgn}(B_{z0})$. We consider two cases with $Am = 1$ and $Am = 100$.

For $Am = 100$ (very weak AD), the dispersion relation is well described by pure Hall MRI. For $B_{z0} > 0$, the most unstable mode always has the maximum growth rate of $0.75\Omega^{-1}$, and the most unstable wavelength λ_m shifts progressively to larger scales with $\lambda_m \propto \chi_0^{-1/2}$ as the Hall term strengthens ($\chi_0 \rightarrow 0$). Normalizing to disk scale height, we find

$$\frac{\lambda_m}{H} \approx 4\pi \sqrt{\frac{l_H}{3H}} \left(\frac{2}{\beta_0} \right)^{1/4} \approx 0.5 \sqrt{\frac{3l_H}{H}} \left(\frac{10^4}{\beta_0} \right)^{1/4}. \quad (10)$$

Table 1
List of Unstratified Simulation Runs with $B_{z0} > 0$

Run	Res.	Am	l_H	β_0	χ_0	E_k	E_M	α_{Rey}	α_{Max}	α	α_{mag}
Q3A1B4-R24	24	1	0.3	10^4	0.047	4.6(−2)	2.4(−3)	3.0(−4)	5.0(−4)	8.0(−4)	0.21
Q3A1B4-R48	48	1	0.3	10^4	0.047	3.1(−2)	3.6(−3)	3.8(−4)	7.8(−4)	1.2(−3)	0.22
Q3A1B5-R24	24	1	0.3	10^5	0.015	1.4(−2)	3.7(−3)	4.3(−4)	8.8(−4)	1.3(−3)	0.24
Q3A1B5-R48	48	1	0.3	10^5	0.015	1.4(−2)	4.2(−3)	5.1(−4)	9.8(−4)	1.5(−3)	0.23
Q1A1B4-R24	24	1	0.1	10^4	0.14	1.6(−2)	2.6(−3)	5.9(−4)	6.1(−4)	1.2(−3)	0.24
Q1A1B4-R48	48	1	0.1	10^4	0.14	1.7(−2)	4.9(−3)	9.3(−4)	1.2(−3)	2.1(−3)	0.25
Q1A1B5-R24	24	1	0.1	10^5	0.045	1.0(−2)	1.7(−3)	3.8(−4)	2.3(−4)	6.1(−4)	0.14
Q1A1B5-R48	48	1	0.1	10^5	0.045	9.8(−3)	1.1(−3)	3.9(−4)	2.3(−4)	6.1(−4)	0.20

Notes. Values written in $a(-b)$ denote $a \times 10^{-b}$, l_H is normalized to H , and E_k and E_M are normalized to the midplane gas pressure $\rho_0 c_s^2$. See Section 2.3 for details.

For $B_{z0} < 0$, unstable modes exist only when $(1/\chi_0) < 2$, and the unstable wavenumber can extend virtually to infinity when $(1/\chi_0) > 1/2$.

For $Am = 1$, we see that small-scale modes are strongly suppressed. For $B_{z0} < 0$, the most unstable modes have wave numbers of $k_m v_A / \Omega \sim 0.5$. In the absence of the Hall effect ($1/\chi_0 = 0$), λ_m is increased by a factor of ~ 2 because of AD. For $B_{z0} > 0$ and toward a stronger Hall term ($1/\chi_0 \gtrsim 5$), λ_m is less affected by AD because it is shifted to larger scales, and the maximum growth rate is only slightly reduced.

2.3. Unstratified Shearing-box Simulations

Our unstratified shearing-box simulations mainly serve for calibrating and interpreting stratified simulation results. Therefore, we do not aim at a thorough parameter study but mainly focus on parameter regimes relevant to real PPDs. In this regard, we consider the following set of parameters.

1. The Hall length $l_H = 0.1H$ or $0.3H$.
2. Net vertical field strength, with $\beta_0 = 10^4$ and 10^5 .
3. Magnetic field polarity, $B_{z0} > 0$ or $B_{z0} < 0$.
4. The value of $Am = 1$, occasionally 10 and 100.

Our simulations use a fixed box size of $4H \times 4H \times 2H$ in (x, y, z) dimensions. Note that our simulation box height is $2H$ rather than H typically used in unstratified shearing-box simulations, providing the potential to accommodate larger spatial structures while not being unrealistically tall for real disks. Our unstratified simulations can be performed with relatively high spatial resolution, 48 cells per H in the x - z plane (24 in the y dimension). We cannot afford the same resolution for our stratified runs in Sections 3–5. Therefore, we also conduct simulations with half the resolution to justify the use of lower resolution in our stratified simulations.

We have chosen the value of $Am = 1$ that is appropriate for the midplane region of the outer disk. From Equation (6), the Hall length of $l_H \sim 0.1$ to $0.3H$ applies to the range of $R \sim 20$ to 50 AU. Given $\beta_0 = 10^4$ and 10^5 , the corresponding value of χ_0 ranges from 0.015 to 0.14.

For $B_{z0} < 0$, and for this range of χ_0 , there is no linearly unstable MRI mode. However, this does not necessarily relate to the nonlinear sustainability, given the relatively small value of l_H . Therefore, we first run simulations in the ideal MHD limit to time $t = 60 \Omega^{-1}$, then we turn on nonideal MHD terms and evolve further to time $t = 300 \Omega^{-1}$. In Figure 2, we show the time evolution of three runs with $Am = 1, 10$, and 100 at fixed $l_H = 0.1$ and $\beta_0 = 10^4$. We see that for $Am = 100$,

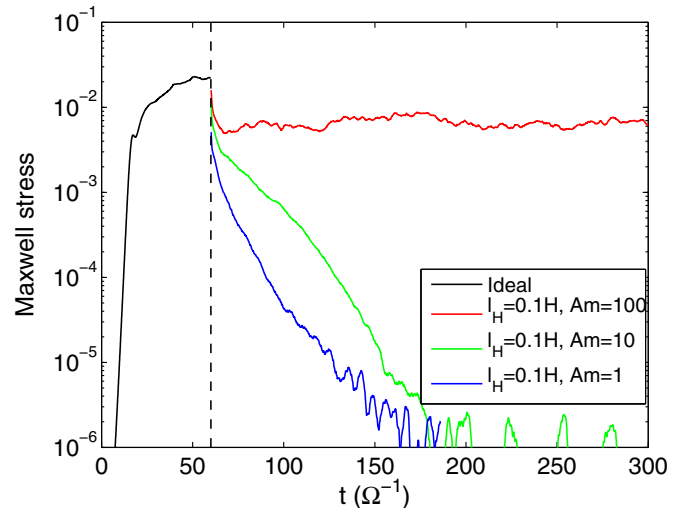


Figure 2. Nonlinear sustainability of the MRI turbulence in the case of $B_{z0} < 0$. The run is initialized under ideal MHD with $\beta_0 = 10^4$ until $t = 60 \Omega^{-1}$ before the Hall (with $l_H = 0.1H$) and AD terms are turned on. Without linearly unstable MRI modes, turbulence is sustained for $Am = 100$ but decays for $Am = 10$ and 1 .

the MRI turbulence can be sustained but at a lower level, while for $Am = 10$ and 1 , turbulence is suppressed. We have also tested with other values of β_0 and l_H , and we find that as long as $Am = 1$, no sustained MRI turbulence is possible. This implies that under this configuration the midplane region of the outer disk is likely the exact analog of the conventional “dead zone.”

For $B_{z0} > 0$, the background field configuration is unstable to the MRI. We provide the list of runs and diagnostic quantities in Table 1. The runs are named in the form $Qx Ay Bz - R w$, where $x = 10 l_H / H$, $y = Am$, $z = \log_{10}(\beta_0)$, and w is the numerical resolution (24 or 48 per H). In all cases, we have fixed the value of $Am = 1$. We find that vigorous turbulence is quickly developed for all runs. Many of these runs show secular effects in their evolution (to be discussed later), so we run these simulations for a very long time to $t = 1440 \Omega^{-1}$ and extract turbulence statistics by performing time and volume averages after $t = 1120 \Omega^{-1}$ (denoted by the overline). Major diagnostic quantities include kinetic energy density $E_k = \overline{\rho v^2}/2$, magnetic energy density $E_M = \overline{B^2}/2$, Maxwell stress $\alpha^{\text{Max}} \equiv -\overline{B_x B_y}$, and Reynolds stress $\alpha^{\text{Rey}} \equiv \overline{\rho v_x v_y}$ (the normalization factor $\rho_0 c_s^2$ is omitted because it is one in code units). The total Shakura–Sunyaev α is $\alpha^{\text{Max}} + \alpha^{\text{Rey}}$. Another useful diagnostic

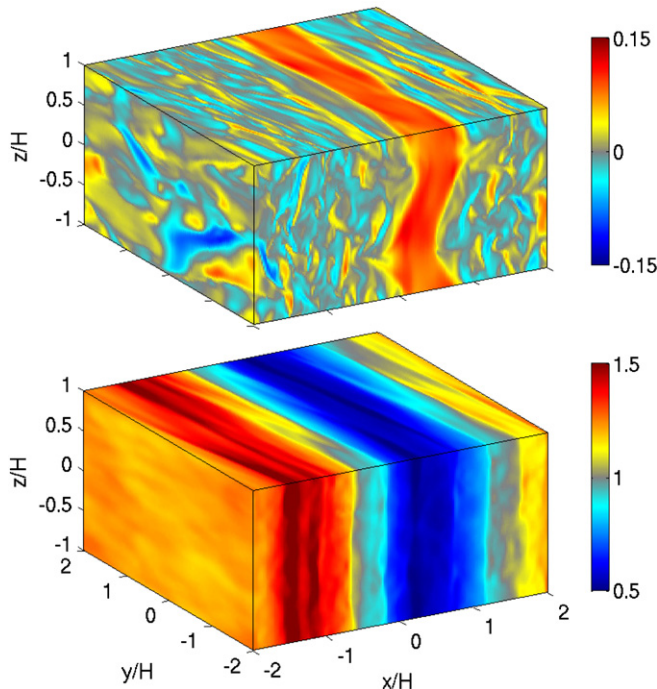


Figure 3. Snapshot from the end of our unstratified run Q3A1b4-R48 with $Am = 1$, $l_H = 0.3H$, and $B_{z0} > 0$. The top panel shows the vertical magnetic field B_{z0} , and the bottom panel shows the gas density ρ .

is $\alpha^{\text{mag}} \equiv \alpha^{\text{Max}}/E_M$ (e.g., Hawley et al. 2011; Sorathia et al. 2012), which is considered a useful indicator for numerical convergence.

First, we find that for a relatively large $l_H = 0.3H$ and a relatively strong field $\beta_0 = 10^4$ a strong zonal field is gradually built up on relatively long timescales ($\sim 10^3 \Omega^{-1}$). This results from the concentration of the vertical magnetic flux pertaining to the Hall effect, as studied in detail in Kunz & Lesur (2013). In Figure 3, we show the final snapshot of our run Q3A1B4-R48 at time $t = 1440 \Omega^{-1}$, which clearly shows the zonal field structure. On the other hand, we find that the zonal field coexists with vigorous turbulence and gives an α value of $\sim 10^{-3}$. The presence of vigorous turbulence, rather than remaining in the low-transport state, is largely due to relatively strong AD with $Am = 1$: additional magnetic dissipation acts against the buildup of magnetic flux (Kunz & Lesur 2013). We do not observe prominent zonal field structures in runs with smaller l_H and weaker magnetic fields.

In the meantime, we find that in essentially all of our unstratified simulations, the density variation also shows significant zonal structure, leading to strong zonal flows to balance the pressure gradient of the zonal density variation (Johansen et al. 2009). Such density variation is not captured in Kunz & Lesur (2013) because of their usage of incompressible code. The density structure from our run Q3A1B4-R48 is shown in the bottom panel of Figure 3, where the density variation reaches $\sim 50\%$. As a result, the kinetic energy displayed in Table 1 is largely dominated by the kinetic energy associated with the zonal flow ($v_y \sim 0.2\text{--}0.3c_s$). Other runs develop weaker zonal density variations and weaker zonal flows as well. These structures also develop over long timescales of $\sim 10^3 \Omega^{-1}$ and show secular variations. A full discussion on such zonal flows is beyond the scope of this paper, but phenomenologically we observe from our unstratified simulations that a stronger zonal

flow is launched when l_H is larger and when the background field is stronger.³

In all of our simulations, sustained MRI turbulence at the level of $\alpha \sim 10^{-3}$ is obtained. A stronger background vertical field leads to stronger turbulence, and a larger l_H also leads to stronger turbulence until the zonal field configuration is developed, where the turbulence level is reduced. We caution that for the parameters considered here, the most unstable MRI mode is not well resolved. For the best-resolved case (run Q3A1B4-R48), we find from Equation (10) that the most unstable wavelength amounts to about 13 cells. We do not expect our simulations to show unambiguous convergence on the value of α (and in fact the value of α is also affected by the development of the zonal flows, which show long timescale variations). Nevertheless, by looking at the value of α_{mag} , we find that low- and high-resolution simulations give consistent values in all cases except for run Q1A1B5. Moreover, by inspecting snapshots in runs with different resolutions, we find that their evolutionary behaviors are qualitatively similar in all cases. This gives us confidence that the 24 cells per H adopted in our stratified runs is adequate to capture the essential properties of the MRI in the Hall-AD regime.

In sum, our unstratified simulations of the MRI in the presence of both the Hall effect and AD indicate that under conditions appropriate for the outer region of PPDs ($Am \sim 1$), the MRI cannot be self-sustained in the midplane if $B_{z0} < 0$, while for $B_{z0} > 0$, the self-sustained turbulence exists at the level of $\alpha \sim 10^{-3}$. When $B_{z0} > 0$, we observe zonal fields develop when the Hall term and background field are relatively strong, and we find that zonal flows develop in all cases.

3. SETUP OF 3D STRATIFIED SIMULATIONS

We perform a series of 3D stratified shearing-box simulations where all nonideal MHD effects are included self-consistently. The setup of the simulations closely follows Paper I (with the formulation given in Sections 2.1–2.2 and the methodology given in Section 3.1). We briefly summarize the simulation setup below with minor updates.

We consider an MMSN disk. While submillimeter continuum observations suggest that disk surface-density profiles are typically shallower than MMSN scalings, the inferred disk surface densities at a few tens of AU are approximately consistent with MMSN values (e.g., Andrews et al. 2009). In addition, by exploring a wide range of disk radii from 5 AU to 60 AU, we cover all of the essential physical effects as the midplane regions transition from being Hall dominated to AD dominated. Different disk surface-density profiles may shift the transition radius but must exhibit the same physical effects.

At a given radius R , we produce a diffusivity table based on equilibrium chemistry using the chemical reaction network developed in our earlier works (Bai & Goodman 2009; Bai 2011a) and the latest version of the UMIST database (McElroy et al. 2013). Dust grains of $0.1 \mu\text{m}$ size and an abundance of 10^{-4} are assumed as in our previous works, where the total surface area is consistent with typical grain-coagulation calculations (e.g., Birnstiel et al. 2011). Cosmic rays, X-rays, and radioactive decay are included as standard ionization sources. We further include an effective treatment of the FUV ionization. It is calibrated with the models of Walsh et al. (2010, 2012),

³ We note that MRI alone also generically leads to zonal flows and magnetic flux concentration, as discussed in detail in Bai & Stone (2014), but they are overwhelmed by the strong Hall effect in these *unstratified* Hall MRI simulations. Stratified simulations do yield different results, to be discussed in Sections 4.4.

Table 2
List of Stratified Simulation Runs

Run	R (AU)	Hall?	B_{z0}	β_0	Box Size (H)	T (Ω^{-1})	α^{Max}	α^{Rey}	δv_z	$\dot{M}_{\text{out}}$	$ T_{z\phi}^{\text{Max}} $	Section
R5b5H+	5	Yes	+	10^5	$4 \times 8 \times 12$	360	6.5(−3)	1.6(−5)	1.0(−2)	3.0(−4)	3.1(−4)	5
R5b5H−	5	Yes	−	10^5	$4 \times 8 \times 12$	360	4.2(−4)	3.4(−5)	5.0(−3)	1.1(−4)	1.64(−4)	5
R5b4H−	5	Yes	−	10^4	$4 \times 8 \times 12$	360	1.3(−3)	4.6(−4)	1.4(−3)	3.2(−4)	6.7(−4)	5
R15b5H+	15	Yes	+	10^5	$4 \times 8 \times 12$	720	2.3(−3)	1.3(−4)	5.2(−3)	2.9(−4)	2.5(−4)	6.1
R15b5H−	15	Yes	−	10^5	$4 \times 8 \times 12$	720	7.2(−4)	2.8(−5)	2.4(−3)	2.3(−4)	2.4(−4)	6.1
R15b4H+	15	Yes	+	10^4	$4 \times 8 \times 12$	720	2.3(−3)	3.1(−4)	8.2(−3)	6.1(−4)	8.8(−4)	6.1
R15b4H−	15	Yes	−	10^4	$4 \times 8 \times 12$	720	3.0(−3)	1.7(−4)	8.6(−3)	8.0(−4)	1.1(−3)	6.1
R30b5H+	30	Yes	+	10^5	$6 \times 12 \times 12$	960	1.9(−3)	3.9(−4)	2.0(−2)	2.2(−4)	2.1(−4)	4
R30b5H0	30	No	+	10^5	$6 \times 12 \times 12$	960	1.5(−3)	2.9(−4)	1.5(−2)	2.3(−4)	2.2(−4)	4
R30b5H−	30	Yes	−	10^5	$6 \times 12 \times 12$	960	1.4(−3)	2.2(−4)	1.3(−2)	2.3(−4)	2.2(−4)	4
R30b4H+	30	Yes	+	10^4	$6 \times 12 \times 12$	960	6.1(−3)	4.4(−4)	2.0(−2)	1.5(−3)	1.7(−3)	4
R30b4H0	30	No	+	10^4	$6 \times 12 \times 12$	960	4.8(−3)	5.4(−4)	2.4(−2)	1.1(−3)	1.4(−3)	4
R30b4H−	30	Yes	−	10^4	$6 \times 12 \times 12$	960	5.0(−3)	6.5(−4)	2.4(−2)	1.2(−3)	1.4(−3)	4
R60b5H+	60	Yes	+	10^5	$4 \times 8 \times 12$	720	2.9(−3)	5.7(−4)	2.5(−2)	2.4(−4)	2.2(−4)	6.2
R60b5H−	60	Yes	−	10^5	$4 \times 8 \times 12$	720	2.6(−3)	5.0(−4)	2.1(−2)	2.4(−4)	2.1(−4)	6.2
R60b4H+	60	Yes	+	10^4	$4 \times 8 \times 12$	720	9.3(−3)	4.4(−4)	8.9(−3)	2.0(−3)	1.9(−3)	6.2
R60b4H−	60	Yes	−	10^4	$4 \times 8 \times 12$	720	7.3(−3)	4.3(−4)	1.1(−2)	1.8(−3)	2.0(−3)	6.2

Notes. α^{Max} and α^{Rey} are computed within $z = \pm 4.5H$, $T_{z\phi}^{\text{Max}}$ is evaluated at $z = \pm 4.5H$, and δv_z is the turbulent vertical velocity within $z = \pm 2H$. See Section 4 for details.

recalculated based on our X-ray ionization parameters with X-ray luminosity $L_X = 10^{30} \text{ erg s}^{-1}$ and temperature $T_X = 5 \text{ keV}$. The FUV substantially increases ionization at the disk surface, and the gas essentially behaves in the ideal MHD regime in the FUV ionization layer. The diffusivities have the form $\eta_O \propto B^0$, $\eta_H \propto B$, and $\eta_A \propto B^2$, which are applicable given the small grain abundance.

Unlike in Paper I, simulations in this work are full 3D because we expect the MRI turbulence to develop. All of our simulations have a vertical domain extending from $z = -6H$ to $6H$ using a resolution of 24 cells per H in x and z and half the resolution in y . A density floor of $5 \times 10^{-6} \rho_0$ is applied for all simulations to avoid numerical difficulties in the strongly magnetized disk surface region (where $\rho_0 = 1$ is the midplane gas density in code units). For the simulations in Section 4 (at $R = 30 \text{ AU}$), we use a very extended horizontal box size of $6H \times 12H$ in (x, y) to better accommodate potentially large-scale structures. Note that for an MMSN disk at 30 AU , the disk aspect ratio $H/R \approx 0.078$, so the radial box size is $\sim 14 \text{ AU}$, which is about the maximum size where a shearing-sheet approximation can be considered as reasonable. A smaller horizontal domain size of $4H \times 8H$ is used for simulations in Sections 5–6 to reduce computational cost.

All simulations are started with all nonideal MHD terms turned on and are initialized with a uniform vertical magnetic field B_{z0} characterized by a midplane plasma β_0 , together with a sinusoidally varying (in x) vertical field B_{z1} to avoid strong initial channel flows (Bai & Stone 2013a). We choose B_{z1} to be $4B_{z0}$ with a relatively high frequency of radial variations to make simulations saturate more quickly:

$$B_z = B_{z0} + 4B_{z0} \sin(8\pi x/L_x). \quad (11)$$

Simulations are typically run to $t = 960 \Omega^{-1}$ or $720 \Omega^{-1}$.

We have slightly modified the vertical outflow boundary condition compared with Paper I. Here, the boundary condition assumes hydrostatic equilibrium in ρ , outflow in v_z , and zero gradient in B_z , v_x , and v_y (same as Paper I), and B_x and B_y are reduced proportionally with density in the ghost zones (different from Paper I, same as in Simon et al. 2013a). We do observe that the evolution of mean magnetic fields somewhat depends on the treatment of the outflow boundary condition, which reflects the limitation of the shearing box when using open boundaries in the presence of disk outflow. Some of its influences will be discussed in the main text. Nevertheless, the general flow properties do not sensitively depend on the choice of vertical boundary condition (Fromang et al. 2013).

We consider four disk radii: $R = 5 \text{ AU}$, 15 AU , 30 AU , and 60 AU . At each radius, we consider $\beta_0 = 10^4$ and 10^5 and for both magnetic polarities. We mainly focus on two disk radii: $R = 30 \text{ AU}$ (Section 4), where we further conduct Hall-free simulations for detailed comparison; and $R = 5 \text{ AU}$ (Section 5), where comparisons with quasi-1D simulations in Paper I will be made. All 3D simulations are listed in Table 2. Simulation runs are named as RxbH*, where x represents the disk radius in AU, $y = \log_{10} \beta_0$, and * can be 0, +, or − for simulations excluding the Hall term (0), with the Hall term and $B_{z0} > 0$, and with the Hall term and $B_{z0} < 0$.

4. SIMULATION RESULTS: 30 AU

We focus on $R = 30 \text{ AU}$ in this section. We choose this radius because we find that at this location the Hall effect around the disk midplane is about as equally important as AD. The disk is likely to develop more stable configurations at smaller disk radii (for $B_{z0} > 0$), as found in Paper I, and the Hall effect becomes less prominent toward larger radii. This location has

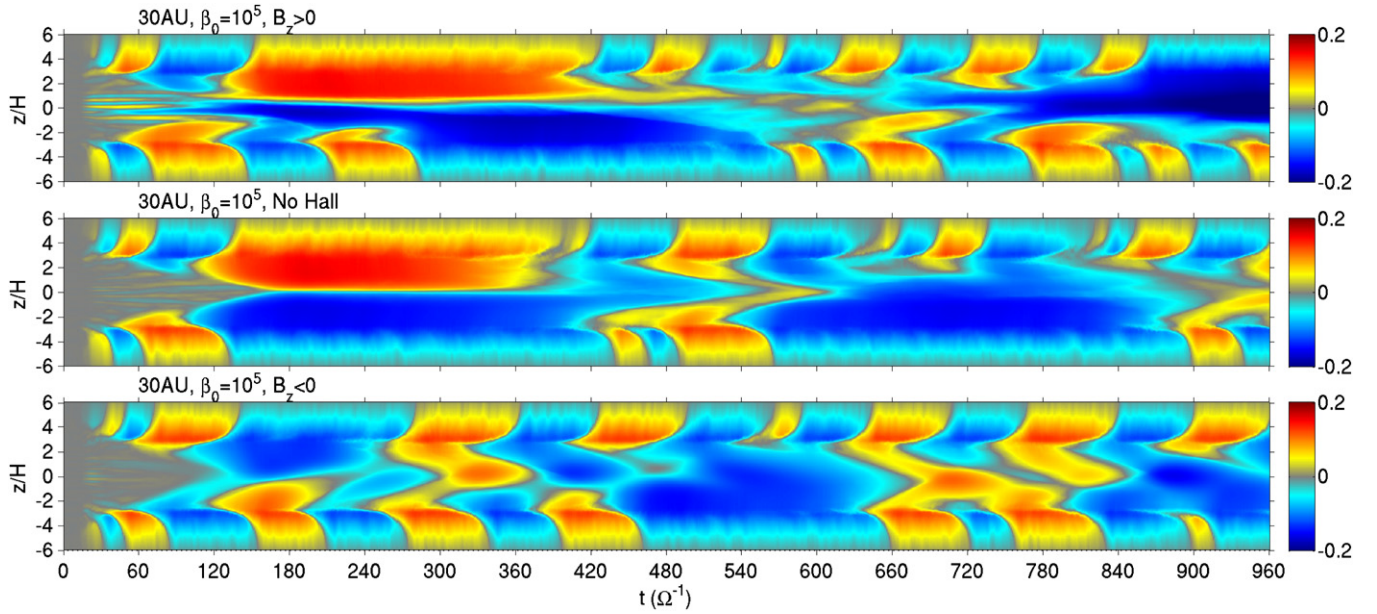


Figure 4. Time evolution of the vertical profile of horizontally averaged B_y in our runs at 30 AU with $\beta_0 = 10^5$. The top, middle, and bottom panels correspond to runs R30b5H+, R30b5H0, and R30b5H–, i.e., Hall turned on with $B_{z0} > 0$, Hall-free, and Hall turned on with $B_{z0} < 0$.

been explored in Simon et al. (2013a, 2013b), where only AD was taken into account with a fixed profile of $Am = 1$ near the midplane. Our new simulations self-consistently take into account the ionization-recombination chemistry, together with the Hall effect included for the first time.

We perform a total of six runs with $\beta_0 = 10^5$ and 10^4 . All runs lead to vigorous MRI turbulence in the surface layer, and in the presence of a net vertical magnetic field, they always launch outflows. Different aspects of these runs are discussed in the subsections below.

4.1. Evolution of a Large-scale Toroidal Field

The global evolution of the system is largely controlled by magnetic fields, so we first discuss the overall evolution of a large-scale toroidal field from our simulations as a standard diagnostic. Starting from runs with $\beta_0 = 10^5$, R30b5H+, R30b5H0, and R30b5H–, we show in Figure 4 the time evolution of the horizontally averaged B_y for all three runs. Because the initial conditions for these simulations are identical (except for a negative sign of B_{z0} for run R30b5H–), these runs initially proceed in a similar way. The Hall and AD terms become progressively more important as (midplane) magnetic fields become stronger, and the three runs then evolve differently. All three cases show a prominent level of dynamo activities emanating from the surface layer, where the sign of mean B_y alternates over time. The alternation behavior is quite irregular and to some extent similar to ideal MHD simulations with a modestly strong vertical magnetic flux ($\beta_0 \gtrsim 10^3$, Bai & Stone 2013a). It contrasts with the conventional MRI dynamo (zero net vertical magnetic field in ideal MHD) that exhibits very periodic cycles of about 10 orbits (e.g., Davis et al. 2010; Shi et al. 2010).

We next discuss simulations with $\beta_0 = 10^4$, with three runs R30b4H+, R30b4H0, and R30b4H–. Similar to the weaker field case, all three runs develop vigorous turbulence mainly in the surface layer because of FUV ionization (see next subsection). The time evolution of horizontally averaged B_y for the three runs is shown in Figure 5. We see that the MRI dynamo is suppressed

in all cases, and the mean toroidal field is predominantly one sign. This is generally a consequence of a stronger net vertical field, which is an analog of the ideal MHD case (Bai & Stone 2013a). While the system is turbulent, the toroidal field is always the dominant field component, and when the dynamo is suppressed, this field component is dominated by the mean field. However, by viewing individual simulation snapshots, localized patches possessing opposite signs for the toroidal field do exist in runs R30b4H0 and R30b4H–. In the latter case, the region with opposite B_y gradually grows and eventually leads to the reversal of the mean toroidal field in the disk (bottom panel of the figure). We have continued this run further and found that the mean B_y will reverse again after another $\sim 300 \Omega^{-1}$, and this cycle is likely to continue. Similarly, the positive B_y region started to dominate the upper half of the disk near the end of our run R30b4H0.

The secular evolution of the mean field discussed above exists in all of our simulations to a certain extent, and this is partly related to the limitations of the shearing-box approach: with an imposed net vertical field that presumably connects to infinity, the mean field in the disk should be in causal contact with the field beyond, but the causal connection is truncated with the prescribed outflow boundary condition. Because most activities in disks are magnetically driven, the secular evolution of the mean fields also makes the level of turbulence in disks time variable. For example, in run R30b4H–, the midplane region exhibits stronger turbulent activities around time $t = 480\text{--}600 \Omega^{-1}$ with turbulent velocity about a factor of three higher than some other periods. Therefore, readers should bear in mind the potential uncertainties that are due to such variabilities.

4.2. Stress Profiles and Level of Turbulence

Based on the time evolution of the mean field, we extract useful diagnostic quantities and average them in time from $t = 480 \Omega^{-1}$ onward for simulations with $\beta_0 = 10^5$, and from $t = 360 \Omega^{-1}$ onward for simulations with $\beta_0 = 10^4$. The mean vertical profiles of these quantities are shown in Figures 6 and 7.

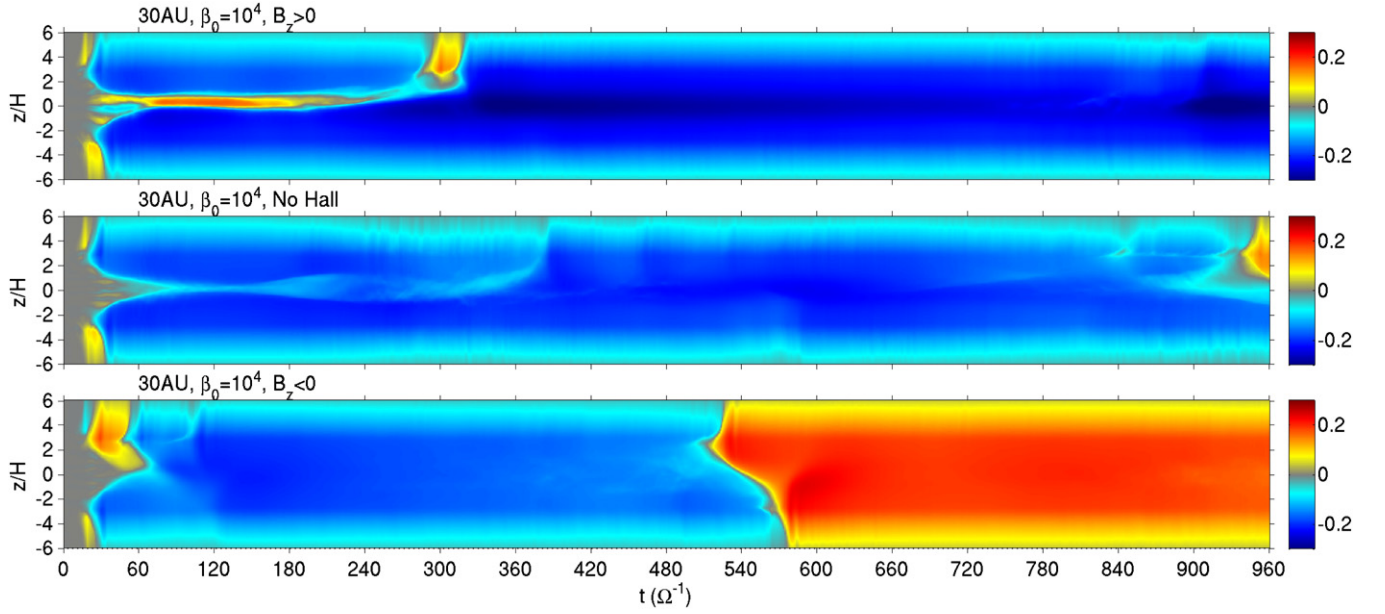


Figure 5. Same as Figure 4, but for runs at 30 AU with $\beta_0 = 10^4$.

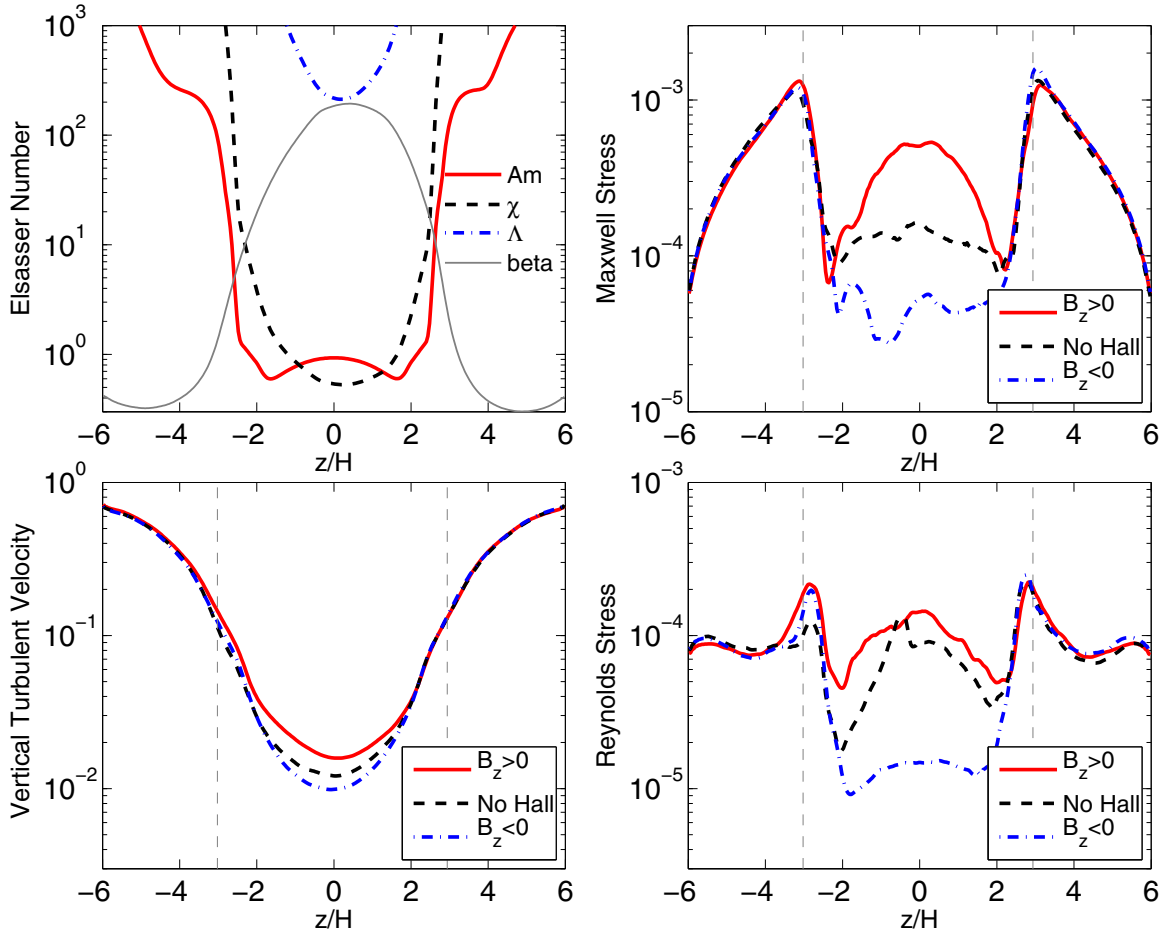


Figure 6. Vertical profiles of various horizontally averaged diagnostic quantities from our runs at 30 AU with $\beta_0 = 10^5$. Top left: the Ohmic (Λ), Hall (χ), and ambipolar (Am) Elsasser numbers in blue dash-dotted, black dashed, and red solid lines, together with plasma β in a thin gray line. The profile is extracted from the Hall-free run R30b5H0 (almost identical to the other two runs). Bottom left: vertical turbulent velocity. The remaining three panels show various profiles for all three runs of R30b5H+ (red solid), R30b5H0 (black dashed), and R30b5H- (blue dash-dotted). The gray vertical dashed lines mark the location where $Am = 100$ in run R30b5H0.

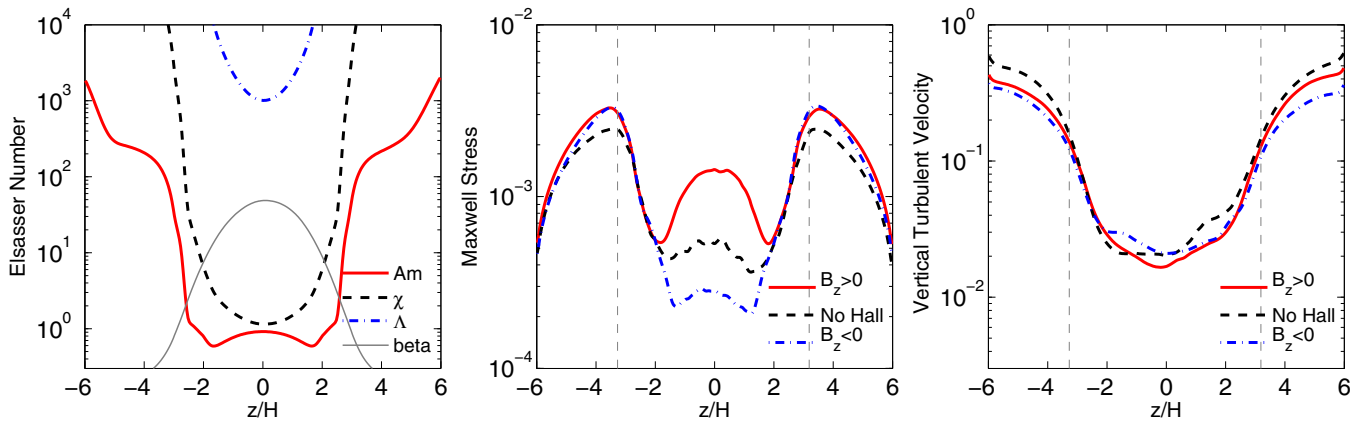


Figure 7. Same as Figure 6 without the bottom right panel, but for runs at 30 AU with $\beta_0 = 10^4$. The vertical dashed line labels the location where $Am = 100$ in run R30b4H0.

The relative importance of various nonideal MHD effects can be best viewed from the top left panel of Figure 6 and the left panel of Figure 7. They show the profiles of the Elsasser numbers (based on the Hall-free run in each case, but the runs with the Hall term generally give almost the same profiles). Clearly, Ohmic resistivity is completely negligible with $\Lambda \gg 100$ at all heights. With $\beta_0 = 10^5$, both the Hall effect and AD are important within $z \sim \pm 2\text{--}2.5H$ with χ and Am being around one, and the range of influence of AD extends higher from the midplane than the Hall effect. The Hall effect is less important relative to AD with stronger net flux $\beta_0 = 10^4$ because the resulting total field is stronger. Beyond $z \sim 2.5H$, FUV ionization catches up and all nonideal MHD effects are greatly reduced. Beyond $z = \pm 3H$, the gas behaves essentially in the ideal MHD regime with $Am > 100$.

Vigorous MRI turbulence takes place beyond about $z \sim \pm 2.5H$ thanks to FUV ionization. As a result, the profile of the Maxwell stress $T_{R\phi}^{\text{Max}} = -B_x B_y$ peaks at around $z = \pm 3H$, as shown in the top right panel of Figure 6 and the middle panel of Figure 7. Beyond $z \sim \pm 3H$, the Maxwell stress drops because disk density drops, and it enters the magnetically dominated corona (plasma $\beta < 1$). All three runs at a given β_0 show very similar properties in this region because the gas behaves in the ideal MHD regime. Runs with $\beta_0 = 10^4$ have a systematically higher Maxwell stress than the corresponding $\beta_0 = 10^5$ runs by a factor of three to four as a result of a stronger background field.

The midplane region is where three simulations at fixed β_0 are expected to differ because of the Hall effect. The most prominent difference lies in the Maxwell stress. The runs with $B_{z0} > 0$ give the highest stress that peaks at the midplane. This is related to the Hall-shear instability (Kunz 2008), which operates only when $B_{z0} > 0$ and is responsible for generating stronger horizontal magnetic fields and hence Maxwell stress in the inner disk (Lesur et al. 2014; Paper I). Here, the effect is much less prominent than in the inner disk studied in Paper I and Lesur et al. (2014) because the Hall effect is only modestly significant ($\chi \sim 1$). The runs with $B_{z0} < 0$ give the lowest midplane Maxwell stress, while the Maxwell stress from R30b5H0 (without the Hall term) lies in between. This is again consistent with the expectation from Paper I that the horizontal magnetic field tends to be reduced for negative B_{z0} .

As discussed in Section 2, for $B_{z0} > 0$, the midplane region is unstable to the MRI, and the level of MRI turbulence is expected to be stronger than in the Hall-free case. For $B_{z0} < 0$,

self-sustained MRI turbulence is not expected because of the Hall effect. To characterize the level of turbulence, we consider the vertical component of the rms velocity, which is shown in the bottom left panel of Figure 6 and the right panel of Figure 7 for the two sets of runs. They are computed based on the turbulent kinetic energy at each height.⁴ In the same way, we define δv_z to be the rms vertical velocity fluctuation within $z = \pm 2H$ for all of our runs, which is included in Table 2.

We see that the turbulent rms vertical velocity reaches $\sim 0.3\text{--}0.8c_s$ at the disk surface ($z \sim \pm 4H$) for all of these runs, while it is reduced by more than one order of magnitude to $\sim 0.01\text{--}0.03c_s$ around the disk midplane. For $\beta_0 = 10^5$, the run with $B_{z0} > 0$ gives a higher midplane turbulent velocity, the run with $B_{z0} < 0$ gives the lowest, and the Hall-free run lies in between, which is consistent with our expectation. Nevertheless, the difference is within a factor of two, so the role of the Hall effect in the midplane turbulent activities is only modest. While we caution that the level of turbulence in the $B_{z0} > 0$ case may be underestimated because of the lack of numerical resolution, the overall scenario is similar to the Hall-free case: turbulence in the midplane region is substantially reduced largely because of AD, consistent with the earlier stratified AD simulations of (Simon et al. 2013a, where the midplane region was termed an “ambipolar-damping” zone). In the case of $B_{z0} < 0$, because the MRI cannot be self-sustained at the disk midplane, the midplane turbulent motion is largely induced by the strong MRI turbulence from the disk surface layer. This is a direct analog of the *conventional* “Ohmic dead zone” of the inner disk (e.g., Fleming & Stone 2003; Turner & Sano 2008; Oishi & Mac Low 2009).⁵

For $\beta_0 = 10^4$, we find that the level of midplane turbulence in all three runs are very similar (modulo some secular variations not reflected in the time-averaged plots), despite the marked difference in Maxwell stress. We have checked that for $B_{z0} > 0$, the midplane Maxwell stress is dominated by contributions from the large-scale field ($-\overline{B_x B_y}$), while for $B_{z0} < 0$, the midplane Maxwell stress is almost entirely due to the turbulent field. The turbulent contributions of the midplane Maxwell

⁴ A turbulent velocity calculated in this way is partially weighted by density. Results without additional weighting are almost identical.

⁵ On the other hand, because Ohmic resistivity and AD scale with magnetic field strength very differently, increasing the net vertical field tends to make midplane regions more laminar in the outer disk because of AD (Simon et al. 2013a), rather than more turbulent in the conventional Ohmic dead-zone case (Gressel et al. 2012; Okuzumi & Hirose 2012).

stress from the two runs R30b4H+ and R30b4H− are in fact similar. We have also checked that for $\beta_0 = 10^5$ the midplane Maxwell stress is always dominated by turbulent stress. The low level of turbulence in run R30b4H+ may be considered to be a consequence of the strong mean toroidal field ($\overline{B_y}$), which dominates the magnetic field strength and tends to suppress turbulent motions (but see also Section 4.4).

Overall, based on the six simulations with different strengths and polarities of the net vertical field, it is clear that the Maxwell stress profile (and hence the radial transport of angular momentum) is layered. Moreover, it appears that $\delta v_z \approx 0.01\text{--}0.02c_s$ is a good proxy for the level of turbulence in the midplane region of the outer disks, with much stronger turbulence in the FUV ionization layer at the disk surface.

4.3. Angular Momentum Transport and Disk Outflow

Outflow is always launched in shearing-box simulations in the presence of a net vertical magnetic flux (e.g., Suzuki & Inutsuka 2009). Although this outflow may serve as a wind-launching mechanism, the kinematics of the outflow is not well characterized in shearing-box simulations because the rate of mass outflow does not converge with simulation box height (Fromang et al. 2013) and there are also symmetry issues (Bai & Stone 2013a). Therefore, we do not aim at fully characterizing the outflow properties but simply provide some basic diagnostics for reference. We calculate the rate of mass outflow leaving the simulation box $\dot{M}_{\text{out}}$. This is computed by time averaging the sum of vertical mass flux at the two vertical boundaries. We also calculate the $z\phi$ component of the Maxwell stress tensor $T_{z\phi}^{\text{Max}} = -B_z B_\phi$, which determines the rate of wind-driven angular momentum transport (if the outflow is eventually incorporated into a global magnetocentrifugal wind). If the flow is laminar, $T_{z\phi}$ can be conveniently evaluated at the base of the wind, where the toroidal velocity transitions from sub-Keplerian to super-Keplerian (Bai & Stone 2013b; Bai 2013). Because most of our simulation runs are highly turbulent at the disk surface, and there are ambiguities in defining the base of the wind (and whether the outflow can become a global wind at all; Bai & Stone 2013a), we simply provide a reference value of time-averaged $|T_{z\phi}^{\text{Max}}|$ evaluated at $z = \pm 4.5H$ in Table 2.

The value of the Shakura–Sunyaev α for a stratified disk can be written as

$$\alpha = \frac{\int T_{R\phi} dz}{c_s^2 \int \rho dz}, \quad (12)$$

where $T_{R\phi}$ has contributions from both the Maxwell stress ($-B_x B_y$) and Reynolds stress ($\rho v_x v_y$), leading to α^{Max} and α^{Rey} in Table 2. From the lower right panel of Figure 6, we see that the vertical profile of the Reynolds stress is generally a factor of several smaller than the Maxwell stress. Because of uncertainties in characterizing the outflow from shearing-box simulations, we truncate the vertical integral at $z = \pm 4.5H$ in the equation above. For the six runs, the values of α are found to be around $1.5\text{--}2 \times 10^{-3}$ for $\beta_0 = 10^5$ and $5\text{--}6 \times 10^{-3}$ for $\beta_0 = 10^4$. As discussed in the previous subsection, a stronger net vertical field yields larger stress. These α values are also comparable to results from Simon et al. (2013a) with FUV penetration depth 0.01 g cm^{-2} , but they have incorporated more realistic chemistry.

In a steady state, the total accretion rate driven by radial transport of angular momentum (given by α) and the putative wind-driven accretion (given by $T_{z\phi}$) can be approximately

written as (e.g., Bai 2013)

$$\begin{aligned} \dot{M} &\approx \frac{2\pi}{\Omega} \alpha c_s^2 \Sigma + \frac{8\pi}{\Omega} R |T_{z\phi}|, \\ \dot{M}_{-8} &\approx 0.82 \left(\frac{\alpha}{10^{-3}} \right) R_{\text{AU}}^{-1/2} + 4.1 \left(\frac{|T_{z\phi}|}{10^{-4} \rho c_s^2} \right) R_{\text{AU}}^{-3/4}, \end{aligned} \quad (13)$$

where R_{AU} is the radius measure in AU, and we have assumed an MMSN disk model in the second equation, with $\dot{M}_{-8}$ being the accretion rate measured in $10^{-8} M_\odot \text{ yr}^{-1}$.

Using the values from Table 2 with $R = 30$ AU, we find that based on radial transport alone, the resulting accretion rate is about $0.24\text{--}0.33 \times 10^{-8} M_\odot \text{ yr}^{-1}$ for the three runs with $\beta_0 = 10^5$ studied here, which is somewhat smaller than desired. If there were contributions from disk wind, the estimated wind-driven accretion rate is about $0.7 \times 10^{-8} M_\odot \text{ yr}^{-1}$. The sum of the two contributions just matches the desired rate of $10^{-8} M_\odot \text{ yr}^{-1}$. For $\beta_0 = 10^4$, the accretion rate resulting from radial angular momentum transport gives $\sim 0.72\text{--}0.91 \times 10^{-8} M_\odot \text{ yr}^{-1}$, with a potential contribution from the wind to give $\sim 5 \times 10^{-8} M_\odot \text{ yr}^{-1}$.

4.4. Zonal Field and Zonal Flow

For our 30 AU simulations, we find in using Equation (8) and from the Elsasser number plots in Figures 6 and 7 that $l_H \approx 0.2H$ around the disk midplane, which is about the threshold value to trigger the zonal field configuration in the unstratified case, as discussed in Kunz & Lesur (2013). In Section 2 we showed in Figure 3 that a strong zonal field and zonal flow are present in unstratified simulations when $\beta_0 = 10^4$ and $B_{z0} > 0$. To check whether our stratified simulations reveal similar behaviors, we show in Figure 8 the time evolution of mean gas density ρ and B_z for runs 30Aub4H± and 30Aub5H±, averaged in the y and z dimensions, within the disk region $-2H \leq z \leq 2H$.

We find that, strikingly, for all runs, the vertical magnetic flux is concentrated into thin (axisymmetric) shells, while in regions outside these shells, the net vertical flux is close to zero. In the meantime, there are very prominent radial density variations that are characteristic of strong zonal flow. There is a clearly secular evolution of the vertical magnetic flux distribution and zonal flows; this is also related to the secular behaviors discussed in Section 4.1. At first glance, these features appear to be consistent with those shown in Figure 3 from our unstratified simulations. However, there are distinct differences. In particular, both the $B_{z0} > 0$ and $B_{z0} < 0$ cases show such zonal fields, while from unstratified simulations a zonal field is expected only from the $B_{z0} > 0$ case. Also, the width of the zonal field is very small ($< 0.5H$), while from unstratified simulations the width is generally wider than H .

In fact, we find that the concentration of magnetic flux is a generic behavior in shearing-box simulations with a net vertical magnetic flux. Not only simulations with the Hall effect, but also our Hall-free simulations at 30 AU, together with many simulations at other disk radii, show this behavior to some level. We also find that the concentration is less prominent when the net vertical field is weaker, as one compares the top and bottom panels in Figure 8. Accompanying the magnetic flux concentration is the strong zonal flow, the density of which varies across the domain up to $\sim 30\%$. An enhanced zonal flow in the presence of a net vertical magnetic flux was reported in Simon & Armitage (2014) based on stratified shearing-box simulations in the AD-dominated outer disk. Such zonal flows also exist in

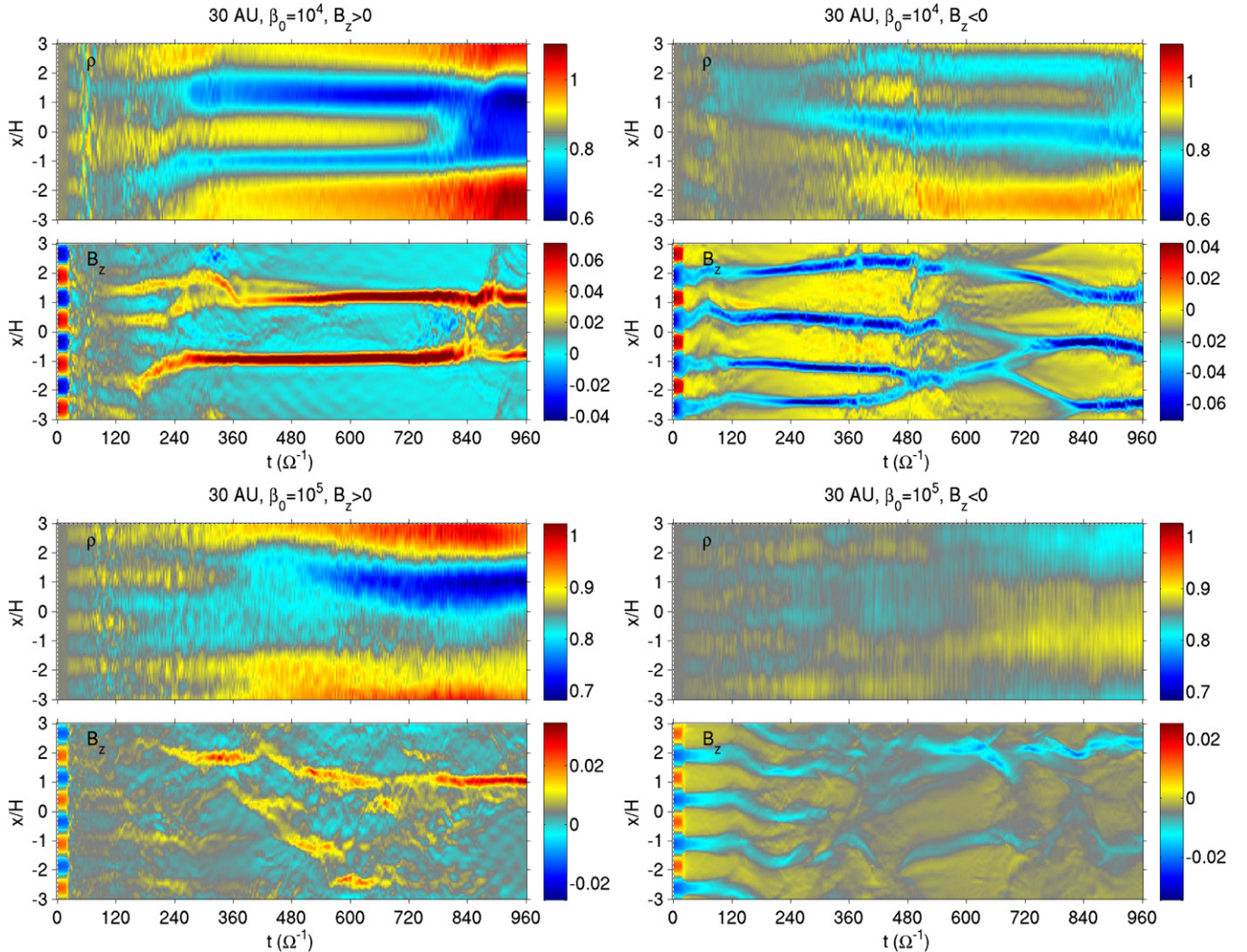


Figure 8. Time evolution of the radial profiles of mean gas density ρ (upper panels) and mean vertical magnetic field B_z (lower panels) averaged over the y - z plane within $z = \pm 2H$ in our runs R30b4H+ (upper left), R30b4H- (upper right), R30b5H+ (lower left), and R30b5H- (lower right). The color scales are centered in their mean values (in code units).

our earlier simulations including both Ohmic resistivity and AD closer in (at 10–20 AU; Bai 2013), and we have verified that, in general, there is only one single “wavelength” of the density/pressure variation across the radial domain, regardless of the radial domain size (Bai 2013, unpublished).

In our follow-up study (Bai & Stone 2014), we show that in the presence of a net vertical magnetic field, the magnetic flux concentration is a natural consequence of the MRI itself. Regions with a strong magnetic flux yield a larger Maxwell stress, which drives enhanced zonal flows with flux-concentrated regions corresponding to density minima. AD further enhances the concentration to yield sharp vertical flux profiles. From Figure 8, we see that the location where magnetic flux concentrates significantly correlates with the density minimum, especially in runs R30b4H+ and R30b5H+. The correlation is less pronounced in runs with $B_{z0} < 0$, which reflects the fact that MRI is not operating in the midplane. Moreover, because the surface layer is fully MRI turbulent, the magnetic flux distribution is also likely affected by the surface MRI in the ideal MHD regime.

In sum, the zonal field and zonal flow observed in our stratified simulations are not due to the Hall effect as reported in unstratified simulations, but are correlated phenomena generic

to the MRI with a net vertical magnetic flux. Although the saturation of the zonal flow is artificially affected by the simulation box size, its association with the magnetic flux concentration is very likely a physical phenomenon. Our local simulations here serve as a first study of the PPD gas dynamics including all nonideal MHD effects, and more works, especially global simulations, are needed to quantify their properties.

5. SIMULATIONS AT 5 AU

Our second focused location is at the relatively small radius of $R = 5$ AU, which complements our studies in Paper I.⁶ Using quasi-1D simulations, we found in Paper I that for $B_{z0} > 0$ the inner disk launches a laminar magnetocentrifugal wind and very efficiently drives disk accretion. In constructing the wind solutions, we enforced reflection symmetry about the disk midplane so that the wind solution has the desired symmetry properties to match a physical magnetocentrifugal wind (i.e., the horizontal component of the magnetic field must flip across the disk). It remains to be demonstrated that this wind

⁶ To better compare with the results in Paper I, we run the simulations at 5 AU with the same vertical outflow boundary condition as in Paper I instead of the modified version in the rest of the simulations.

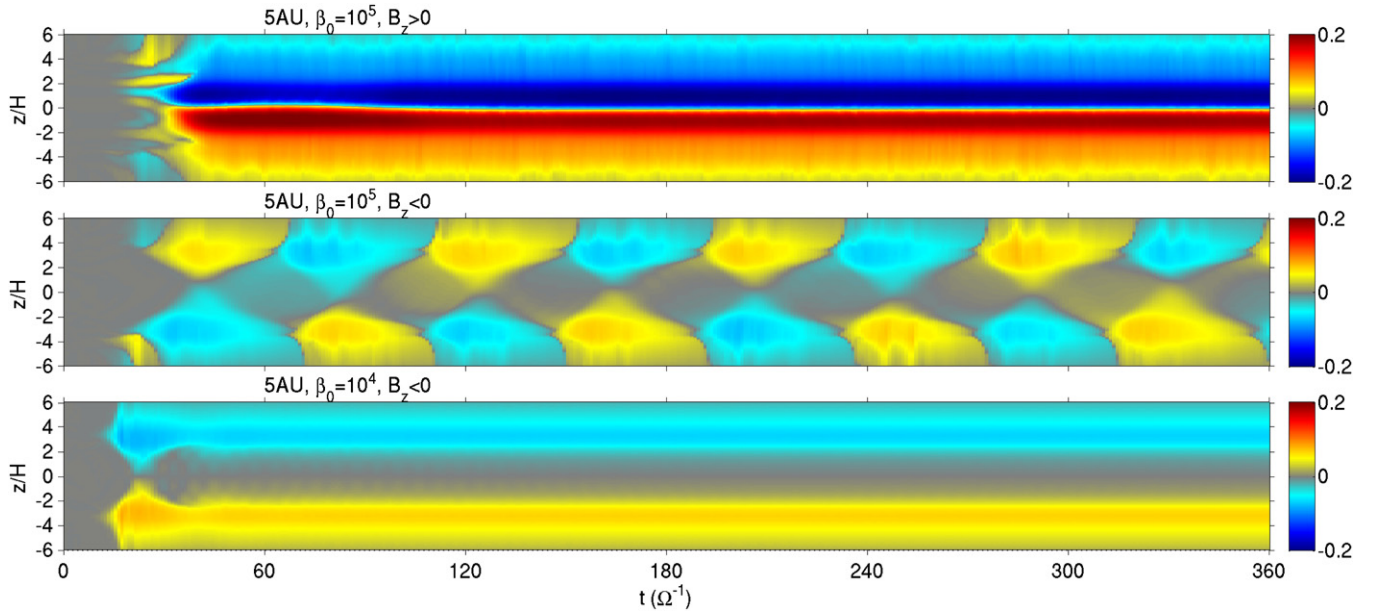


Figure 9. Time evolution of the vertical profile of horizontally averaged B_y in our runs at 5 AU. The top, middle, and bottom panels correspond to runs R5b5H+, R5b5H−, and R5b4H−.

configuration is stable in 3D without enforcing the symmetry. Another important result from [Paper I](#) is that for $B_{z0} < 0$, we did not find any stable wind configuration for the typically expected level of vertical magnetic field strength at this location ($\beta_0 = 10^5$ – 10^6) because MRI sets in in a very narrow range of disk height. It remains to be demonstrated how the disk behaves under this situation.

We have performed three runs. For $B_{z0} > 0$ we consider $\beta_0 = 10^5$ (run R5b5H+), and for $B_{z0} < 0$ we consider $\beta_0 = 10^5$ and 10^4 (runs R5b5H− and R5b4H−). From [Paper I](#), we expect largely laminar configurations to be developed for runs R5b5H+ and R5b4H−, launching a magnetocentrifugal wind; the MRI should set in for run R5b5H−. In Figure 9, we again show the space-time plot of the horizontally averaged B_y for the three runs. Given the highly regular patterns seen in this figure, it suffices to run these simulations just to $t = 360 \Omega^{-1}$ and then perform a time average from $t = 180 \Omega^{-1}$ onward.

5.1. Simulation with $B_{z0} > 0$

For run R5b5H+, we see from the top panel of Figure 9 that the system is able to achieve a largely laminar state as desired. More interestingly, the toroidal field changes sign almost exactly at the disk midplane, automatically maintaining the reflection symmetry (more specifically, even- z symmetry; see Figure 9 of [Bai & Stone 2013b](#)). Achieving this field geometry is essential for physically launching a magnetocentrifugal wind. Our full 3D simulation therefore supports the procedure adopted in [Paper I](#) where the reflection symmetry across the midplane was enforced.

Checking the time-averaged vertical profiles of the main diagnostic quantities, we find that the result is almost identical with Figure 9 of [Paper I](#) (with a slight difference because our box extends to $z = 6H$ rather than $8H$). For this solution, the horizontal magnetic field near the midplane is strongly amplified by the Hall shear instability, and the flip of this horizontal field creates a strong current density at the midplane. This contrasts with the study by [Bai \(2013\)](#), where without the Hall term, the strong current layer was found to be located offset from the

midplane at $z_{SC} \approx 1.3H$ in this particular case (see his Table 2 for run S-R5-b5). It appears that with the Hall term included, the horizontal magnetic field tends to flip right across the midplane rather than from the upper layers.

In Figure 10, we further show the vertical profiles of time-averaged Maxwell stress and vertical turbulent velocities. For run R5b5H+, the Maxwell stress profiles peak close to the disk midplane at a rather high level close to $10^{-2} \rho_0 c_s^2$ as a result of the Hall shear instability. The dip at the midplane is due to the flip of the horizontal field, all in agreement with the results in [Paper I](#). For the turbulent velocity profiles, however, we find that an appreciable level of turbulence is present in this run. The turbulent velocity is again on the order of $0.01c_s$ around the midplane and increases toward the surface layer at a level very similar to that in the outer disk studied in the previous section. Since we expect the system to be stable to the MRI, the turbulence mainly originates from elsewhere: at the midplane, we find that the strong current layer tends to exhibit small amplitude corrugation from time to time, resembling the tearing modes in reconnecting with the current sheet. Such corrugating motion is likely the source of most random velocities that propagate toward disk surface layers and become amplified because of the rapid density drop.

In sum, for $B_{z0} > 0$, our 3D simulation with the full box well reproduces the quasi-1D simulations with enforced reflection symmetry in [Paper I](#): we expect that accretion is mainly driven by the magnetocentrifugal wind, with a significant contribution from the radial transport of angular momentum via the large-scale Maxwell stress and magnetic braking (see Table 2 of [Paper I](#)). The wind-driven accretion flow mostly proceeds in the strong current layer where the toroidal magnetic field flips ([Bai & Stone 2013b](#)), and here it takes place exactly at disk midplane. Our 3D simulation further reveals the presence of turbulence, which largely originates from the midplane region where the relatively strong large-scale horizontal magnetic fields flip. The level of turbulence is similar to that in the outer disk. We also comment that because the system is stable to the MRI, magnetic flux concentration into thin shells is not observed in this simulation.

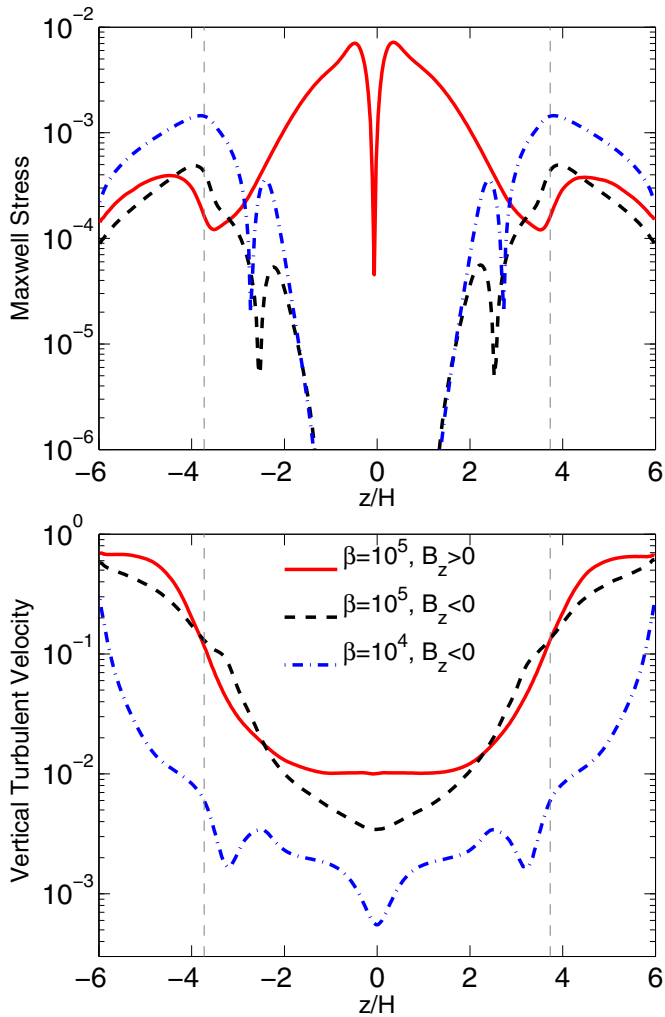


Figure 10. Vertical profiles of Maxwell stress (top) and vertical turbulent velocity (bottom) for all three runs at 5 AU, as marked in the legend. The vertical dashed lines label the location where $Am = 100$ in run R5b5H–.

5.2. Simulations with $B_{z0} < 0$

For run R5b5H–, the system is expected to be unstable to the MRI in a narrow range of disk height at about $|z| \sim 2\text{--}3H$. This can roughly be identified from the left panel of Figure 9 in Paper I, where the Hall Elsasser number χ passes one at around $z = 2.5H$, and the plasma β there is still much larger than unity (based on the Hall-free run in dashed lines). A detailed explanation of the onset of the instability is given in Section 5.2 of Paper I, but, in brief, it is related to the fact that for $B_{z0} < 0$ the Hall term makes the most unstable MRI wavelength shift to a shorter wavelength when the Elsasser number χ_0 is of order unity, allowing the unstable modes (that are otherwise too long) to fit into the disk. Using full 3D simulations, we see from the middle panel of Figure 9 that the large-scale toroidal magnetic field flips in a highly periodic manner, and the origin of the periodic flips directly connects to the unstable region. Interestingly, the toroidal fields in the upper and lower halves always have opposite signs, and the midplane horizontal field is very weak (and goes through zero). We have also found that the overall mean field evolution can be almost exactly reproduced from our quasi-1D simulation of Paper I. An outflow is launched whose mass outflow rate is smaller than but the same order of magnitude as the rate from our run R5b5H+ (see Table 2).

Therefore, at a given time, the magnetic field configuration can be considered physical for a magnetocentrifugal wind. However, because the toroidal (and hence radial) field constantly changes sign, the wind keeps oscillating between the radial inward and outward directions. This is inconsistent with global wind geometry and reflects the limitation of the local shearing-box framework (Bai & Stone 2013a). While periodic field flips are likely physical phenomena that are inherent with the onset of the MRI, global simulations are necessary to determine the fate of the outflow.

The onset of the MRI also leads to some level of turbulence, as seen from the bottom panel of Figure 10. Away from the region where the MRI operates, turbulent motion largely results from a passive response to the MRI activities, and the midplane has the weakest level of turbulent motion. Despite different origins, the level of turbulence is comparable to run R5b5H+, especially at the surface.

The fact that the mean toroidal field periodically changes sign makes it ambiguous in estimating the role of disk wind in transporting angular momentum (the net wind-driven accretion rate would be zero considering the periodic flips). Here we set it aside and look at the radial transport of angular momentum from the Maxwell stress, as shown in the top panel of Figure 10. We see that the Maxwell stress peaks at about $|z| \sim 4H$, but at a relatively low level. We estimate the total α to be only about 4.5×10^{-4} , corresponding to an accretion rate of $\sim 1.6 \times 10^{-9} M_{\odot} \text{ yr}^{-1}$ using Equation (13). This is about an order of magnitude smaller than the expected level of $10^{-8} M_{\odot} \text{ yr}^{-1}$.

We further performed run R5b4H– with a stronger net vertical field $\beta_0 = 10^4$. Based on Paper I, we expect the system to be stable to the MRI and to develop a laminar magnetocentrifugal wind. This is again confirmed using full 3D simulations, with the general wind properties almost identical to the one obtained in Paper I. In particular, our full 3D run automatically obeys the reflection symmetry across the midplane, confirming that solutions with enforced symmetry in Paper I are generally physical. Note that the toroidal field is close to zero near the midplane as a result of the Hall term. The level of random motion in our run R5b4H– is systematically weaker than all other runs, confirming its intrinsically laminar nature. One can obtain from Table 2 the Maxwell stress and the wind stress to derive the accretion rate resulting from the radial transport and wind, or directly look at Table 2 of Paper I for more accurate estimates. We see that the radial transport is completely negligible compared with the wind-driven accretion rate. The latter gives $\sim 10^{-7} M_{\odot} \text{ yr}^{-1}$, which is an order of magnitude more than sufficient.

In sum, it appears that for $B_{z0} < 0$, while results from our shearing-box simulations are likely robust, they also raise puzzling issues regarding the mechanism to transport angular momentum. For a relatively weak net vertical field ($\beta_0 \sim 10^5$), MRI sets in, leading to a periodically oscillating outflow. Based on shearing-box simulations, we are unable to tell whether the outflow drives angular momentum transport, but radial transport of angular momentum by Maxwell stress appears to be too inefficient. For a relatively strong net vertical field ($\beta_0 \sim 10^4$), the system unambiguously launches the magnetocentrifugal wind with a physical wind geometry, but it drives a more rapid accretion than typically observed. At this point, it is unclear how the system can achieve an accretion rate at the desired level of $\sim 10^{-8} M_{\odot} \text{ yr}^{-1}$, an issue that can only be clarified from global simulations.

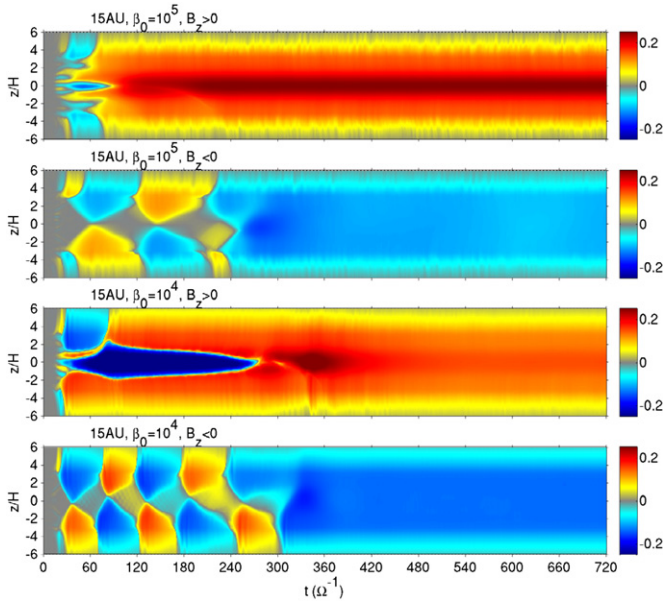


Figure 11. Time evolution of the vertical profile of horizontally averaged B_y in our runs at 15 AU. Shown from top to bottom are runs R15b5H+, R15b5H-, R15b4H+, and R15b4H-.

6. SIMULATIONS AT OTHER DISK RADII

In this section, we perform simulations at two other locations, 15 AU and 60 AU, from which we can study the radial dependence of PPD gas dynamics and the role played by the Hall effect. At each location, we consider $\beta_0 = 10^4$ and 10^5 and different magnetic polarities. All nonideal MHD terms are included.

6.1. Results from 15 AU

At 15 AU, our quasi-1D simulations suggest a laminar configuration for $B_{z0} > 0$ with $\beta_0 = 10^4$, while MRI should set in otherwise. In Figure 11, we show the space-time plot of the horizontally averaged toroidal field. In Figure 12, we further show the time-averaged profiles of Maxwell stress and vertical turbulent velocity for all four runs, where the time averages are taken from time $t = 420 \Omega^{-1}$ onward. We see that for all four runs the system eventually settles into a state where the large-scale toroidal field remains one sign across the entire disk, so the symmetry of the outflow would be undesirable for a global wind. Nevertheless, we again set aside the issue of symmetry and focus on other properties.

For $B_{z0} > 0$, and comparing runs R15b5H+ with R15b4H+, it is counterintuitive to notice from both figures that a stronger mean toroidal magnetic field is generated when the net vertical field is weaker (R15b5H+), leading to stronger Maxwell stress around disk midplane. A look at all of the simulation data reveals that for run R15b4H+ almost the entire vertical magnetic flux is concentrated in a single thin shell, while the rest of the radial zones have a net vertical flux that is effectively zero. As a result, magnetic field amplification by the Hall shear instability is suppressed for the bulk of the disk. A strong zonal flow is also formed with a high density contrast of 30%, and the shell of magnetic flux coincides with the density minimum. The highly nonuniform distribution of magnetic flux also makes the flow properties in this run deviate from the wind solution in Paper I (see his Table 2). On the other hand, for run R15b5H+, the magnetic flux distribution is much more uniform, leading to effective growth of the horizontal magnetic field that is due to

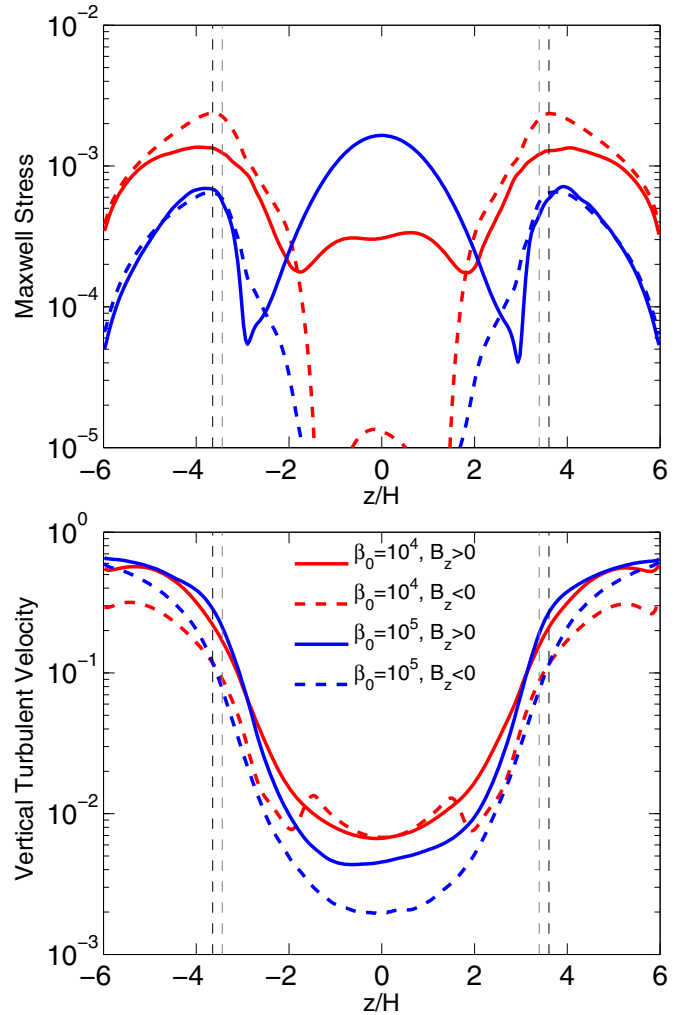


Figure 12. Vertical profiles of Maxwell stress (top) and vertical turbulent velocity (bottom) for all four runs at 15 AU, as marked in the legend. The vertical dashed lines mark the location where $Am = 100$ in run R15b4H- (dark) and R15b5H- (light).

Hall shear instability, producing stronger Maxwell stress at the disk midplane. At this point, it is again unclear how realistic the level of magnetic flux concentration is, and the results shown here should be treated with caution.

For $B_{z0} < 0$, we see that for both runs with $\beta_0 = 10^4$ and 10^5 , the initial evolution of mean B_y closely resembles our run R5b5H- with quasi-periodic flips. This is again because the MRI sets in a thin layer where the Hall Elsasser number transitions through order unity. Later on, a field of one sign takes over and dominates the entire disk. There are also MRI activities in the FUV layer, though the level is weaker than in their 30 AU counterpart (e.g., seen from the peak Maxwell stress). To some extent, this location represents a transition between the 5 AU and 30 AU cases: in the former case, the MRI is triggered mainly in the Hall-dominated layer, and in the latter case, the MRI is active mainly in the FUV layer. As usual, the midplane horizontal magnetic field is suppressed due to the Hall effect, and most of the Maxwell stress originates from the FUV layer.

From the values of $\alpha_{\phi}^{\text{Max}}$ and $T_{z\phi}^{\text{Max}}$ listed in Table 2 and using Equation (13), we see that the net vertical magnetic flux has to be at least $\beta_0 = 10^4$ in order for the accretion rate to reach levels comparable to $10^{-8} M_{\odot} \text{ yr}^{-1}$. On the other hand, if a magnetocentrifugal wind is operating, the level of $T_{z\phi}^{\text{Max}}$ from

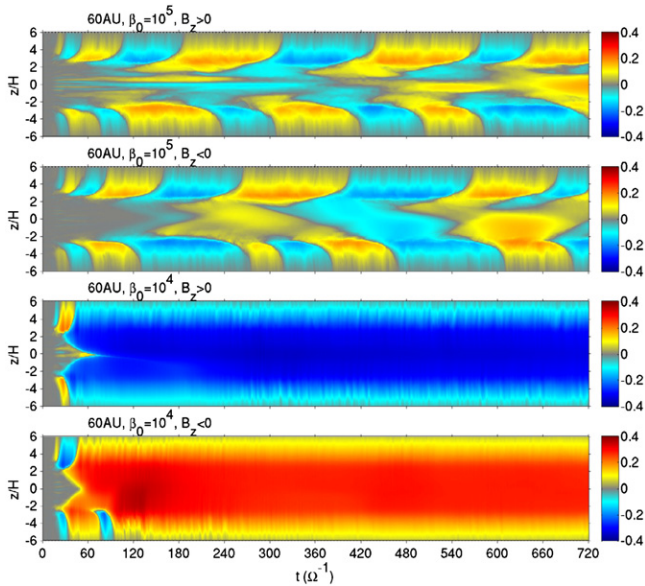


Figure 13. Time evolution of the vertical profile of horizontally averaged B_y in our runs at 15 AU. Shown from top to bottom are runs R60b5H+, R60b5H-, R60b4+, and R60b4H-.

a weak net vertical field with $\beta_0 = 10^5$ is sufficient to drive the accretion rate above the desired level. Overall, the turbulent velocity is smallest at midplane either because of weak MRI turbulence ($B_{z0} > 0$ with weak field) or induced random motion from MRI activities in the disk surface ($B_{z0} < 0$), similar to the 30 AU case.

6.2. Results at 60 AU

At 60 AU, the relative importance of the Hall effect is reduced by a factor of about two compared with the 30 AU case (see Equation (9)), and it is only marginally important at the disk midplane. AD is the dominant effect in most regions of the disk. Also, given the approximately constant penetration column density of the FUV ionization, it penetrates deeper geometrically (in terms of disk scale height) at the more tenuous outer disk. In Figure 13, we show the space-time plot of the horizontally averaged toroidal field. In Figure 14, we further show the time-averaged profiles of Maxwell stress and vertical turbulent velocity for all four runs, where the time averages are taken from time $t = 300 \Omega^{-1}$ onward. The general evolution of the systems are in many ways similar to our runs at 30 AU, where the MRI drives vigorous turbulence in the surface FUV layer, with the midplane region only weakly turbulent. Here we mainly focus on the differences and the overall trend toward larger disk radii.

At $\beta_0 = 10^5$, dynamo activities constantly flip the mean toroidal field similar to but apparently more regularly than in the 30 AU case for both magnetic polarities. For $\beta_0 = 10^4$, the dynamo is suppressed, and the entire disk is dominated by a mean toroidal field with a single sign. When $B_{z0} < 0$, we do not observe the mean field changing sign as for the 30 AU counterpart shown in Figure 5. In fact, the toroidal field in the entire disk has the same sign throughout the saturated state of the simulation, so we do not expect this sign flip to occur. We speculate that the flip we observed at 30 AU is associated with the relatively strong Hall effect at the disk midplane, but it is unlikely to occur toward the outer disk as the Hall effect becomes less dominant.

At 60 AU, the contrast in Maxwell stress between the $B_{z0} > 0$ and $B_{z0} < 0$ cases at disk midplane is still very evident.

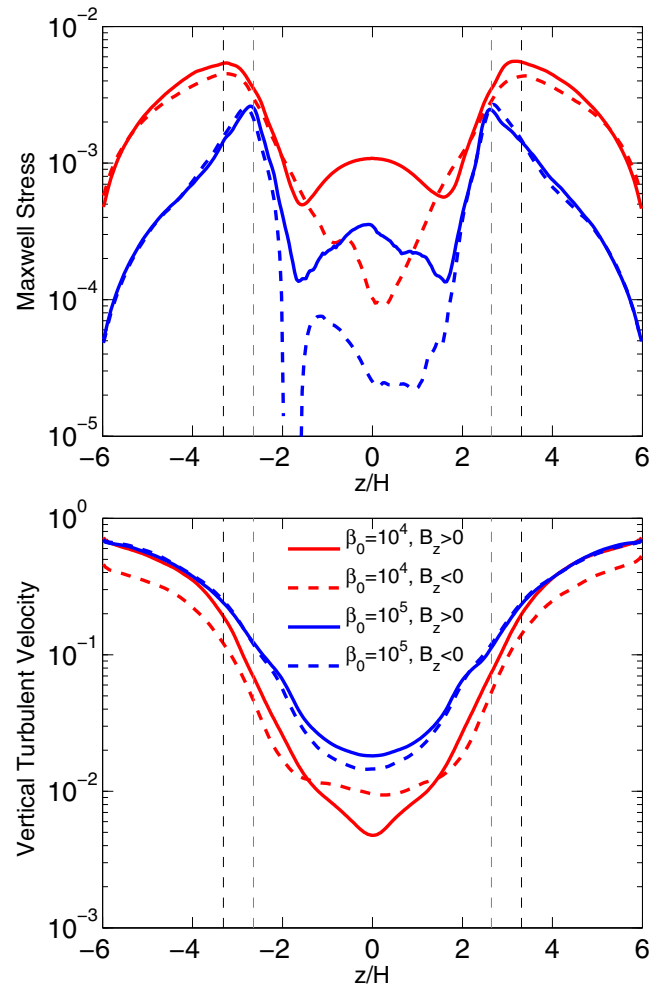


Figure 14. Vertical profiles of Maxwell stress (top) and vertical turbulent velocity (bottom) for all four runs at 60 AU, as marked in the legend. The vertical dashed lines mark the location where $Am = 100$ in run R60b4H- (dark) and R60b5H- (light).

The level of turbulence is found to be higher for runs with a weaker net vertical field $\beta_0 = 10^5$, which may be because in runs with $\beta_0 = 10^4$ the turbulent motion is limited by the relatively strong large-scale toroidal field, but it may also be due to the strong concentration of magnetic flux into thin shells where a large fraction of the simulation domain has effectively zero net vertical flux.

Deeper penetration of FUV ionization allows the MRI to be fully active over thicker surface layers. With a larger gas density in the active layer (in code units), the turbulent magnetic fields from the MRI become stronger (at fixed β_0) than their 30 AU counterparts, giving larger values of α^{Max} . Again, we find that for the Maxwell stress alone to drive the accretion rate of $\sim 10^{-8} M_{\odot} \text{ yr}^{-1}$, the net vertical flux needs to be $\beta_0 \sim 10^4$ or stronger. The magnetocentrifugal wind, if operating in the outer disk, would drive an accretion with rate $\sim 0.4\text{--}4 \times 10^{-8} M_{\odot} \text{ yr}^{-1}$ for $\beta_0 = 10^5\text{--}10^4$.

7. SUMMARY AND DISCUSSIONS

7.1. Summary

In this work, we have studied the gas dynamics of PPDs, focusing on regions toward the outer disk (from 5 to 60 AU), taking into account all nonideal MHD effects in a self-consistent manner. In these regions, the Hall effect generally dominates

near the disk midplane, AD plays an important role over a more extended region across the disk height, and the very surface layer behaves in the ideal MHD regime because of FUV ionization. In the presence of the Hall effect, the gas dynamics depends on the polarity of the net vertical magnetic field (B_{z0}) threading the disk relative to the rotation axis (along $\hat{z}$). Because the Hall effect becomes progressively less important relative to AD with increasing disk radius, we estimate based on the MMSN disk model that the Hall effect controlled polarity dependence extends to about 60 AU.

We first conducted unstratified MRI simulations that included both the Hall effect and AD. We find that at the conditions expected in the outer region of PPDs (the midplane plasma β_0 for the net vertical field being 10^4 – 10^5) the MRI leads to turbulence when $B_{z0} > 0$, but the MRI cannot be self-sustained when $B_{z0} < 0$. For $B_{z0} > 0$, the level of MRI turbulence is of the order $\alpha \sim 10^{-3}$ (with AD Elsasser number $Am = 1$). We confirm that the strong zonal field configuration of Kunz & Lesur (2013) can be achieved with a sufficiently strong Hall effect, and we find that in the meantime it leads to strong zonal flows. In addition, a numerical resolution of 24 cells per $H = c_s/\Omega$ is in general adequate to resolve the bulk properties of the MRI turbulence.

We then focus on self-consistent stratified MRI simulations at various disk radii from 5 AU to 60 AU, with the main results summarized as follows.

At a relatively small disk radius (~ 5 AU), and for $B_{z0} > 0$, we confirm and justify the results from Paper I that the system launches a strong magnetocentrifugal wind and is able to achieve a physical wind geometry, with the horizontal magnetic field flipped exactly at the disk midplane. Although Maxwell stress is enhanced because of the Hall shear instability, accretion is largely driven by the wind and proceeds primarily through the midplane strong current layer. In addition, the midplane region is weakly turbulent, which likely results from the flip of the relatively strong horizontal magnetic field. The turbulent motion gets amplified toward the disk surface as gas density drops.

For $B_{z0} < 0$, our full 3D simulations confirm the results from Paper I that the system is unstable to the MRI in a thin Hall-dominated layer when the net vertical field is relatively weak ($\beta_0 = 10^5$). This results in periodic flips of the large-scale horizontal magnetic field over time, accompanied by a radially oscillating disk outflow or wind. The fate of the outflow or wind and whether it drives accretion are uncertain based on shearing-box simulations. Radial transport of angular momentum by Maxwell stress is found to be too inefficient by one order of magnitude. A stable magnetocentrifugal wind with a physical wind geometry can be achieved with a stronger net vertical field ($\beta_0 = 10^4$), which very efficiently drives accretion with $\dot{M} \gtrsim 10^{-7} M_\odot \text{ yr}^{-1}$. It is uncertain whether and how the system can achieve the typically observed rate of $10^{-8} M_\odot \text{ yr}^{-1}$.

At a relatively large disk radius (~ 30 AU), we find that the Hall effect mainly affects the Maxwell stress at disk midplane, with $B_{z0} > 0$ ($B_{z0} < 0$) giving an enhanced (reduced) stress that is similar to that found at the inner disk (Paper I; Lesur et al. 2014). Nevertheless, the strongest Maxwell stress results from vigorous MRI turbulence in the surface layer because of FUV ionization (Perez-Becker & Chiang 2011; Simon et al. 2013a). While self-sustained MRI is expected at disk midplane when $B_{z0} > 0$ but not when $B_{z0} < 0$, the level of turbulence in all cases appears very similar, with a vertical turbulent velocity of the order $\delta v_z \sim 0.01$ – $0.03 c_s$. The turbulent motion in the latter case is largely induced from stronger turbulence in the surface

layer, analogous to the conventional ‘‘Ohmic dead zone’’ picture (e.g., Fleming & Stone 2003). Overall, the gas dynamics in the outer regions of PPDs shows a clear layered structure consisting of a highly turbulent surface FUV ionization layer and a weakly turbulent midplane region that is due to damping through a combination of AD, the Hall effect, and large-scale magnetic field structure.

We find that for a relatively weak net vertical field ($\beta_0 = 10^5$), the MRI dynamo leads to repeated flips of the large-scale toroidal field with very irregular cycles. Dynamo activities tend to be suppressed for stronger fields ($\beta_0 = 10^4$). Our simulations also show secular behavior in the evolution of the mean toroidal field, especially in simulations at 30 AU. This is to a certain extent related to the limitations of the shearing box because the net vertical magnetic flux ought to be connected to infinity but gets truncated by the vertical boundary condition without reaching all of the critical points (e.g., Fromang et al. 2013).

We also find that most of our simulations show a strong concentration of vertical magnetic flux into a thin axisymmetric shell at a certain radial location, and the rest of the regions have a net vertical flux that is close to zero. The concentration is generally stronger in simulations with a stronger net vertical field ($\beta_0 = 10^4$) and toward outer disk radii ($\gtrsim 15$ AU). Accompanying the magnetic flux concentration is an enhanced density variation (zonal flow) across the radial domain, with most flux concentrated in low-density regions. The concentration differs from the zonal field from unstratified Hall-MRI simulations (Kunz & Lesur 2013), but it is a generic property of the MRI turbulence in the presence of a net vertical magnetic flux, which is studied in detail in our followup paper (Bai & Stone 2014). Global simulations are required to determine the saturation amplitude of the concentration and transport of magnetic flux across the disk.

Although all of our simulations launch disk outflows, it is uncertain whether such outflows (at $\gtrsim 15$ AU) can be incorporated into a global magnetocentrifugal wind because of the MRI dynamo and symmetry issues (Bai & Stone 2013a), but if they do, the level of net vertical flux of $\beta_0 = 10^5$ and stronger is generally sufficient to drive accretion at the desired level of $10^{-8} M_\odot \text{ yr}^{-1}$. On the other hand, to purely rely on radial transport of angular momentum by Maxwell and Reynolds stresses, the level of the net vertical field must be $\beta_0 = 10^4$ or stronger, assuming an MMSN disk model. This level of field translates to physical field strength according to

$$B = 18.6 \beta^{-1/2} R_{\text{AU}}^{-13/8} \text{ G}. \quad (14)$$

For reference, we find for $\beta_0 = 10^4$ that $B_{z0} \sim 0.7$ mG at 30 AU.

7.2. Discussion

Combining the results from this paper and Paper I, we see that the Hall effect has a major influence on the disk dynamics in the inner region of PPDs ($\lesssim 15$ AU), where the wind properties, stability to the MRI, and the amplification or reduction of the midplane horizontal magnetic field show a strong dependence on the polarity of the net vertical magnetic field. The Hall effect also affects the stability to the MRI in the midplane region of the outer disk, although the level of midplane turbulence appears insensitive to the Hall effect. Overall, it is likely that wind-driven accretion dominates the inner disk, and accretion can be largely driven by the MRI through the surface FUV layer in the outer disk. This picture was already outlined in the discussion of Bai (2013), which incorporated numerical simulation results

without the Hall effect (Bai & Stone 2013b; Simon et al. 2013a). On the other hand, the detailed behavior in the inner disk region, as well as the transition from the largely laminar inner disk to the MRI turbulent outer disk, is expected to have a strong polarity dependence because of the Hall effect, as summarized in the previous subsection and also in Paper I and Lesur et al. (2014).

Several observational consequences are expected based on our current simulation results. First, the fact that the inner disk launches a magnetocentrifugal wind can be detectable through gas tracers. In fact, signatures of low-velocity disk outflow have been routinely inferred from blue-shifted emission line profiles, such as from CO, O I, and Ne II lines (e.g., Hartigan et al. 1995; Pascucci & Sterzik 2009; Pontoppidan et al. 2011; Herczeg et al. 2011; Sacco et al. 2012; Rigliaco et al. 2013). While conventionally interpreted as signatures of photoevaporation (e.g., Gorti et al. 2009; Owen et al. 2010), a magnetocentrifugal wind is likely to produce similar signatures because it possesses low velocities near the launching point before getting strongly accelerated and diluted. In reality, both mechanisms are likely to cooperate to launch the disk wind. We note that a purely photoevaporative wind is likely to be *angular-momentum* conserving because the radial driving force does not exert any torque on the outflow, while a strongly magnetized magnetocentrifugal wind is more likely to be *angular-velocity* conserving near the base of the wind where the gas is forced to move along supra-thermal magnetic fields anchored to the disk (e.g., Spruit 1996). Searching for distinguishable signatures between the two scenarios would be important for understanding the nature of the observed disk outflows.

Second, we expect the level of turbulence in the outer disk to be layered, where the level of turbulence is expected to be of the order $\delta v_z \sim 10^{-2} c_s$ at the midplane and increases to near sonic level toward the disk surface. An empirical constraint on the level of turbulence in the outer regions of PPDs has already been reported based on the turbulent line width of the CO (3–2) transition (Hughes et al. 2011). This line is optically thick and probes the disk surface layer with a line width constrained to be $\lesssim 10\%$ – 40% of sound speed, consistent with a fully turbulent surface layer. With superb sensitivity and resolution, ALMA is expected to constrain the variations in turbulence level at different disk heights using different line tracers. Together with theoretical efforts in modeling turbulent line width (e.g., Simon et al. 2011, 2013b), we expect ALMA data to provide direct evidence of a layered structure in the outer PPDs.

Third, the weakly turbulent outer disk with a toroidal-dominated field configuration may lead to grain alignment and dust polarization (Cho & Lazarian 2007). We have found that in the outer disk ($\gtrsim 30$ AU) the net vertical field needs to be $\beta_0 \sim 10^4$ or stronger for Maxwell stress to drive an accretion rate of $10^{-8} M_\odot \text{ yr}^{-1}$. For this level of net vertical field, we see that the MRI dynamo is suppressed, and the entire field is dominated by a large-scale toroidal magnetic field, whose strength at the disk midplane corresponds to a plasma $\beta \sim 10$ – 20 (e.g., see Figure 13). Using Equation (14), we find that the midplane toroidal field can be at least ~ 3 – 8 mG at 60–100 AU. Based on Equation (1) of Hughes et al. (2009), and using the MMSN disk model at midplane with grain size of 10–100 μm and dust aspect ratio $s = 3$, we find that the critical strength for grain alignment to occur is ~ 1 – 40 mG at 60–100 AU. Although there are large theoretical uncertainties, we see that the match is marginal, and the field strength in the outer disk can either be just enough to promote grain alignment, or a little too weak to align the grains. Several observational attempts to search for dust polarization

in Class II disks have failed (Hughes et al. 2009, 2013). Very recently, however, the successful detection of dust polarization toward younger sources has been reported, with an inferred field configuration resembling a large-scale toroidal field (Rao et al. 2014; Stephens et al. 2014). This might indicate that the disk magnetic field fades over time. Again, future dust polarization observations by ALMA will likely provide better constraints on the geometry, strength, and evolution of disk magnetic fields.

From this work, together with Paper I, we have explored the main parameter space in the gas dynamics of PPDs using local shearing-box simulations. There are other unexplored parameters and uncertainties, including the abundance and size distribution of grains, where small grains may reduce the importance of the Hall effect and AD and hence promote the MRI (Bai 2011a, 2011b). Also, the cosmic-ray ionization rate may be reduced and modulated by the stellar wind (Cleeves et al. 2013; it is less affected by disk wind), the X-ray luminosity can be highly variable because of stellar flares (Wolk et al. 2005; Ilgner & Nelson 2006), and FUV photons may be shielded by the dust in the disk wind from the inner disk (Panoglou et al. 2012; Bans & Königl 2012). It is likely that grain abundance and FUV ionization are more sensitive parameters (Bai & Stone 2013b; Simon et al. 2013a; Paper I), and X-ray ionization is less sensitive but also important (Bai 2011a; Paper I).

Probably the largest uncertainties in our work come from the use of a local shearing-box framework, and there are several outstanding issues related to net vertical magnetic flux. With a net vertical flux, it is well known that the properties of the disk outflow are not well characterized in shearing-box simulations, largely because the vertical gravitational potential is ever increasing in the local approximation (Fromang et al. 2013; Bai & Stone 2013b). Issues related to the symmetry and fate of the outflow are notorious (Bai & Stone 2013a, 2013b). Moreover, the evolution of a large-scale magnetic field can be affected by the vertical outflow boundary condition. In addition, magnetic flux concentration and zonal flows are again not well characterized in shearing-box simulations (Bai & Stone 2014). Global disk simulations with vertical stratification and net vertical magnetic flux have recently been carried out (Suzuki & Inutsuka 2014), yet many of these issues remain not quite addressed because of limited domain size in the θ dimension. In the future, it is crucial to perform global simulations with a sufficiently large vertical domain to accommodate the disk outflow or wind and the fine resolution in the disk to resolve the disk microphysics.

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