

VELOCITY SPECTRUM FOR H I AT HIGH LATITUDES

A. CHEPURNOV¹, A. LAZARIAN¹, S. STANIMIROVIĆ¹, CARL HEILES², AND J. E. G. PEEK²

¹ Department of Astronomy, University of Wisconsin-Madison, 475 North Charter Street, Madison, WI 53706, USA

² Department of Astronomy, University of California, Berkeley, CA 94720, USA

Received 2009 September 24; accepted 2010 February 10; published 2010 April 20

ABSTRACT

In this paper, we present the results of the statistical analysis of high-latitude H I turbulence in the Milky Way. We have observed H I in the 21 cm line, obtained with the Arecibo³ L-Band Feed Array receiver at the Arecibo radio telescope. For recovering velocity statistics, we have used the velocity coordinate spectrum (VCS) technique. In our analysis, we have used direct fitting of the VCS model, as its asymptotic regimes are questionable for Arecibo’s resolution, given the restrictions from thermal smoothing of the turbulent line. We have obtained a velocity spectral index of 3.87 ± 0.11 , an injection scale of 140 ± 80 pc, and an H I cold phase temperature of 52 ± 11 K. The spectral index is steeper than the Kolmogorov index and can be interpreted as being due to shock-dominated turbulence.

Key words: ISM: lines and bands – methods: data analysis – techniques: spectroscopic – turbulence

Online-only material: color figure

1. INTRODUCTION

Galactic interstellar medium (ISM) is turbulent over a very wide range of spatial scales (Deshpande et al. 2000; Dickey et al. 2001; Elmegreen & Scalo 2004). This turbulence is a crucial parameter for understanding many astrophysical processes such as star formation, heat transfer, existence and evolution of ISM phases, cloud structure and dynamics, and cloud formation and destruction (see McKee & Ostriker 2007; Lazarian et al. 2009).

Vivid signatures of interstellar turbulence include the “big power law” of the electron density fluctuations (Armstrong et al. 1995), fractal structure of molecular clouds (Elmegreen & Falgarone 1996; Stutzki et al. 1998), and intensity fluctuations in channel maps (see Crovisier & Dickey 1983; Green 1993; Stanimirović et al. 1999; Deshpande et al. 2000; Elmegreen et al. 2001). For turbulence studies, the velocity fluctuations reflected by the channel maps are of evident importance.

One of the main approaches to characterizing ISM turbulence is based on using statistical descriptors. Many statistical tools for analyzing spectral line data cubes have been attempted: the principal component analysis (Heyer & Schloerb 1997), the spectral correlation function (Rosolowsky et al. 1999), and the velocity centroids method (Esquivel & Lazarian 2005a; Ossenkopf et al. 2006; Esquivel et al. 2007). Wavelets, in particular Δ -variance, have been shown to be very useful in studying inhomogeneous data (see Ossenkopf et al. 2008a, 2008b). These tools can be employed to investigate intensity fluctuations in spectral line data cubes that carry information on velocity turbulence. Nevertheless, the direct relation between the underlying velocity statistics and the measures available with the techniques above is far from being straightforward (see a discussion in Lazarian 2009).

The most direct and straightforward way of dealing with velocity fluctuations is to analyze the statistics of the Doppler-shifted spectral lines, most fully represented by the statistics of the position–position–velocity (PPV) data cubes. However, relating the fluctuations of intensity in the PPV domain to the underlying three-dimensional velocity and density statistics is a

problem that has been only recently addressed (see Lazarian & Pogosyan 2000; Lazarian 2009). Lazarian & Pogosyan (2000) developed the velocity-channel analysis (VCA) technique which connected observed intensity fluctuations with the underlying density and velocity fluctuations by manipulating velocity resolution of PPV data cubes.

In this paper, we apply a new statistical technique, the velocity coordinate spectrum (VCS) proposed initially in Lazarian & Pogosyan (2000) and elaborated in Lazarian & Pogosyan (2006) and Chepurnov & Lazarian (2006) (hereafter LP06 and CL06, respectively), to high-latitude H I observations obtained with the Arecibo radio telescope as a part of the all-sky survey undertaken by GALFA-H I. GALFA-H I is a consortium for Galactic studies with the Arecibo L-band Feed Array (ALFA). The survey specifications and strategy are described in Stanimirović et al. (2006), and data reduction methods are described in Peek & Heiles (2008). GALFA-H I data sets have a high spatial and velocity dynamic range and offer a good opportunity for testing the VCS technique. Earlier this technique was tested using synthetic observations (Chepurnov & Lazarian 2009).

This paper is organized as follows. In Section 2, we describe briefly the H I data used in this paper. In Section 3, we review the VCS technique and address several important issues that need to be considered before applying VCS to the H I data. In Section 4, we apply the VCS technique to the H I data. The discussion of our results, advantages and limitations of VCS are provided in Section 5. Applicability of asymptotic studies to our data is discussed in the Appendix.

2. H I DATA

We observed a high-latitude Galactic region, 16×7 square degrees large and centered on $\alpha = 2^{\text{h}}15^{\text{m}}$, $\delta = +9^{\circ}30'$, in 2005 May and June using Arecibo’s ALFA receiver and the GALSPECT spectrometer. ALFA is a seven-element focal plane array primarily designed for 21 cm observations. GALSPECT is a special-purpose spectrometer for Galactic science with ALFA. GALSPECT has a spectral resolution of 0.18 km s^{-1} and a fixed bandwidth of 1380 km s^{-1} . Each of the seven beams of ALFA has a 3.35 arcmin FWHM beamwidth with a beam ellipticity of 0.2 . The region contains much low-velocity, high-latitude

³ The Arecibo Observatory is part of the National Astronomy and Ionosphere Center, which is operated by Cornell University under a cooperative agreement with the National Science Foundation.

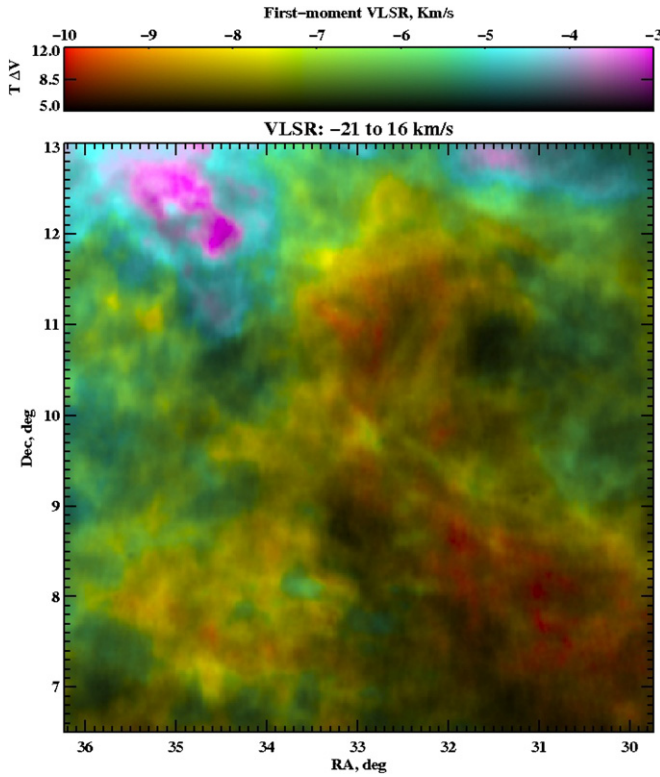


Figure 1. Intensity–velocity map of the analyzed region. In this image, color indicates the first-moment velocity, and brightness indicates the integrated line intensity over the VLSR range -21 to 16 km s^{-1} .

Galactic H I, as well as a sub-complex of very high velocity clouds whose analysis is detailed in Peek et al. (2007). The region was observed in a “basket-weave” or meridian-nodding mode, interlacing scans from day to day.

Data were reduced using the standard GALFA-H I reduction strategy (Peek et al. 2007), without the first sidelobe correction. The final data cube was at the end scaled to the equivalent region in the Leiden–Dwingeloo survey (LDS; Hartmann & Burton 1997) for a single, overall gain calibration.

ALFA’s pixels are known to have asymmetric first sidelobes, as well as significant stray radiation, i.e., unmapped, distant sidelobes. These can contaminate the data slightly—between 50% and 70% of the flux is in the main beam, 10% to 20% is in the first sidelobe, and 20% to 30% is in stray radiation, depending upon which ALFA pixel is measured.⁴ Unpublished work by Carl Heiles & Tom Troland (2003) leads us to believe that the stray radiation does not come from large angular distances from the main beam. This information is corroborated by the fact that the LDS spectra are quite consistent with our observed spectra, once scaled—if much of the flux came from sidelobes more distant than $36'$, the spectra would look significantly dissimilar.

In what follows, we use an approximately homogeneous 6.5×6.5 region, shown in Figure 1. The H I profiles throughout the analyzed region are shown in Figure 2. The average H I spectra derived from four image quadrants are relatively similar, suggesting that we can treat the whole region as being relatively homogeneous.

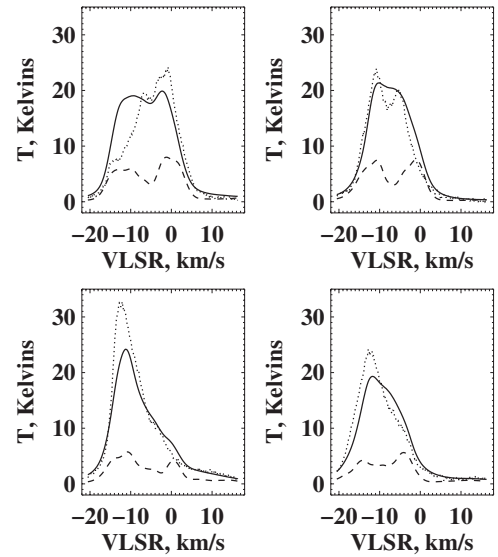


Figure 2. H I line profiles within the analyzed region. Spectra for the four quadrants of the image in Figure 1. The spatial arrangement of the panels is the same as for the image quadrants. For each panel, the solid profile is the average of all spectra in the quadrant; the dashed profile is the rms temperature over the quadrant; and the dotted profile is the spectrum of the center of the quadrant.

3. THE VCS TECHNIQUE

3.1. Basic Introduction

The VCS technique is based on calculating the one-dimensional power spectrum of intensity fluctuations along the velocity axis, $P_1(k_v)$. This spectrum varies with the angular resolution of the telescope, as is illustrated in Figure 3. By investigating how P_1 changes when changing from high to low angular resolution, we can estimate the power spectrum of velocity fluctuations and ISM parameters such as temperature and Mach number.

3.2. Theoretical Considerations

We provide a brief overview the main analytical results of LP00, LP06, and CL06 which are relevant to this paper.⁵ Before going into derivations, we first state our main assumptions.

We assume that the observed H I intensity fluctuations arise from turbulence which can be characterized by two power spectra: the power spectrum of velocity⁶ $P_v \sim k^{-\alpha_v}$ and the power spectrum of emissivity (proportional to density in the case of H I observations), $P_\varepsilon \sim k^{-\alpha_\varepsilon}$. Here k is the wavevector in the ordinary three-dimensional space (i.e., $k \sim 1/l$, where l is a spatial scale). Power spectra P_v and P_ε determine the energy distribution of turbulent motions and density fluctuations in space. Both P_v and P_ε contribute to the distribution of intensity fluctuations in the PPV space. Power spectra of velocity and density are Fourier transforms of the correlation functions of velocity and density, respectively. Those, however, are not

⁵ Note that we do not deal here with the VCS studies of turbulent volumes, where self-absorption is important, or with saturated absorption lines (Lazarian & Pogosyan 2008).

⁶ Much unfortunate confusion in the literature stems from the fact that the spectral indexes of turbulence may differ by a factor of 2 depending on whether the integration over two k directions is performed or not. For instance, the frequently quoted number for the Kolmogorov spectral index is $\alpha_v = 5/3$, which is obtained after the aforementioned integration. We deal with power spectra that are not integrated over $k^2 dk$; thus, the Kolmogorov spectrum index in this work is $\alpha_v = 11/3$. This definition of the spectral index is consistent with our previous papers.

⁴ See Heiles (2004), <http://www2.naic.edu/alfa/memos/general/>

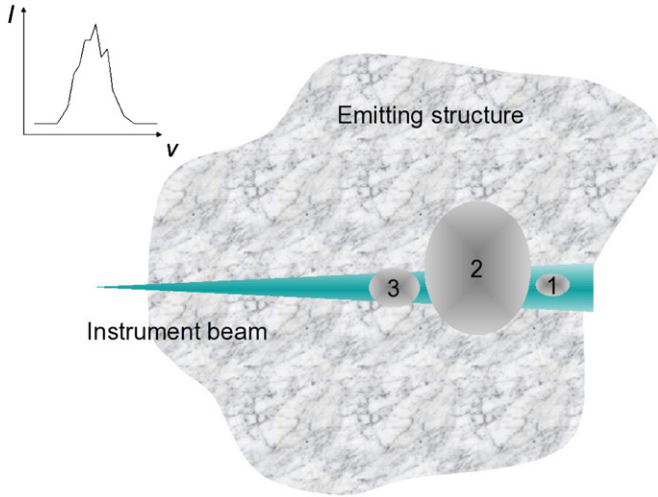


Figure 3. VCS technique: effects of resolution. The fluctuations along the velocity coordinate are analyzed. Eddies within the telescope size beam, e.g., eddy 1, are in a low-resolution mode. Eddies with a size exceeding the one of the beam, e.g., eddy 2, are in a high-resolution mode. The conical geometry of the emitting volume within the beam affects our analysis too.

(A color version of this figure is available in the online journal.)

directly available from observations. Thus, the approach first adopted in LP00 was to study proper PPV statistics and to relate them to the underlying structure function of velocity and correlation function of emissivity.

We start with the expression for a spectral line signal, or an H I intensity measured at velocity v_0 and at a given beam position $\hat{\mathbf{e}}$:

$$S(\hat{\mathbf{e}}, v_0) = \int w(\hat{\mathbf{e}}, \mathbf{r}) d\mathbf{r} \varepsilon(\mathbf{r}) f(v_r(\mathbf{r}) + v_r^{\text{reg}}(\mathbf{r}) - v_0), \quad (1)$$

where ε is the normalized emissivity, v_r^{reg} is the line-of-sight component of the regular velocity (e.g., the velocity arising from the galactic velocity shift), v_r is the line-of-sight component of the random turbulent velocity, and f denotes the convolution between the spectrometer channel sensitivity function⁷ and the Maxwellian distribution of velocities of gas particles, defined by temperature of an emitting medium. The window function w is defined as follows:

$$w(\hat{\mathbf{e}}, \mathbf{r}) \equiv \frac{1}{r^2} w_b(\hat{\mathbf{e}}, \hat{\mathbf{r}}) w_\varepsilon(\mathbf{r}), \quad (2)$$

where w_b is an instrument beam, pointed at the direction $\hat{\mathbf{e}}$, which depends on angular coordinates $\hat{\mathbf{r}}$, while w_ε is a window function defining the extent of the observed object.

The Fourier transform of a spectral line⁸ can be expressed as

$$\begin{aligned} \tilde{S}(\hat{\mathbf{e}}, k_v) &\equiv \frac{1}{2\pi} \int_{-\infty}^{\infty} S(v_0) e^{-ik_v v_0} dv_0 \\ &= \tilde{f}(k_v) \int w(\hat{\mathbf{e}}, \mathbf{r}) d\mathbf{r} \\ &\quad \cdot \varepsilon(\mathbf{r}) \exp(-ik_v(v_r(\mathbf{r}) + v_r^{\text{reg}}(\mathbf{r}))). \end{aligned} \quad (3)$$

Here, $\tilde{S}(\hat{\mathbf{e}}, k_v)$ is a function of the direction of observation determined by the vector $\hat{\mathbf{e}}$ and can easily be calculated from an observed PPV data cube. k_v is the wavevector in the velocity space and $k_v \sim 1/v$.

If we correlate $\tilde{S}$ taken in two directions, pointed by $\hat{\mathbf{e}}_1$ and $\hat{\mathbf{e}}_2$, we get the following measure, which can be used as a starting point for the mathematical formulation of the VCS technique, as well as the VCA technique:

$$\begin{aligned} K(\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, k_v) &\equiv \langle \tilde{S}(\hat{\mathbf{e}}_1, k_v) \tilde{S}^*(\hat{\mathbf{e}}_2, k_v) \rangle \\ &= \tilde{f}^2(k_v) \int w(\hat{\mathbf{e}}_1, \mathbf{r}) d\mathbf{r} \int w(\hat{\mathbf{e}}_2, \mathbf{r}') d\mathbf{r}' \\ &\quad \cdot \langle \varepsilon(\mathbf{r}) \varepsilon(\mathbf{r}') \rangle \langle \exp(-ik_v(v_r(\mathbf{r}) - v_r'(\mathbf{r}')) \rangle \\ &\quad \cdot \exp(-ik_v(v_r^{\text{reg}}(\mathbf{r}) - v_r'^{\text{reg}}(\mathbf{r}'))), \end{aligned} \quad (4)$$

where $\langle \dots \rangle$ denotes averaging⁹ and where we assumed that gas velocity and emissivity are uncorrelated. The latter is not a necessary condition as LP00 showed that important regimes of the statistical study can be recovered even if the two quantities are correlated to a maximal degree. In addition, studies of synthetic maps obtained using three-dimensional magnetohydrodynamic simulations that exhibit velocity and density correlations (see Lazarian et al. 2001) demonstrate that this assumption does not significantly affect the final result.

The first averaging in Equation (4) gives us the emissivity correlation function $C_\varepsilon(\mathbf{r} - \mathbf{r}')$, which for H I translates into the correlation function of overdensity, i.e., $\langle \rho \rangle^2 + C_{\Delta\rho}$, where $C_{\Delta\rho}$ is a correlation function of density fluctuations. An average of the exponent can be performed under the assumption that the velocity statistics are Gaussian¹⁰ (see LP00):

$$\begin{aligned} &\langle \exp(-ik_v(v_r(\mathbf{r}) - v_r'(\mathbf{r}')) \rangle \\ &= \exp\left(-\frac{k_v^2}{2} \langle (v_r(\mathbf{r}) - v_r'(\mathbf{r}'))^2 \rangle\right). \end{aligned} \quad (5)$$

To proceed, we assume that the beam separation and the beam width are both small enough that we can neglect the difference between v_r and v_z (we consider the z -axis to be a bisector of the angle between beams). We also assume that $v_z^{\text{reg}}(\mathbf{r})$ depends only on z and admits a linear approximation,

$$v_z^{\text{reg}}(z) = b(z - z_0) + v_{z,0}^{\text{reg}}, \quad (6)$$

where b characterizes a regular velocity shear. The case in which velocity shear b arises from Galaxy rotation is discussed in detail in LP00.

If we introduce a velocity structure tensor projection,

$$D_{vz}(\mathbf{r} - \mathbf{r}') \equiv \langle (v_z(\mathbf{r}) - v_z(\mathbf{r}'))^2 \rangle, \quad (7)$$

then Equation (4) can be rewritten as

$$\begin{aligned} K(\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, k_v) &= \tilde{f}^2(k_v) \\ &\quad \cdot \int w(\hat{\mathbf{e}}_1, \mathbf{r}) d\mathbf{r} \int w(\hat{\mathbf{e}}_2, \mathbf{r}') d\mathbf{r}' C_\varepsilon(\mathbf{r} - \mathbf{r}') \\ &\quad \cdot \exp\left(-\frac{k_v^2}{2} D_{vz}(\mathbf{r} - \mathbf{r}') - ik_v b(z - z')\right). \end{aligned} \quad (8)$$

⁷ i.e., amplitude-frequency characteristic of a spectrometer channel normalized to its integral value with frequency in velocity units.

⁸ Variable k_v plays here the role of k_z in LP00, being, however, different in dimension ($k_z = bk_v$, see Equation (6)). We use it here to avoid complications when $b = 0$.

⁹ Formally, this is an ensemble averaging, which is a mathematically rigorous concept, while in our case of galactic H I study, spatial averaging is applicable. This can be done by averaging over pairs of $\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2$ while keeping the distance between the $\hat{\mathbf{e}}_1$ and $\hat{\mathbf{e}}_2$ constants.

¹⁰ We assume that the velocity field has a Gaussian probability distribution function. The latter is fulfilled to high accuracy in both experimental (Monin & Yaglom 1975) and numerical (Biskamp 2003) data.

Further transformations lead to

$$K(\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, k_v) = \tilde{f}^2(k_v) \int g(\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, \mathbf{r}) d\mathbf{r} \cdot C_\varepsilon(\mathbf{r}) \exp\left(-\frac{k_v^2}{2} D_{vz}(\mathbf{r}) - ik_v b z\right), \quad (9)$$

where geometric factor g is given by

$$g(\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, \mathbf{r}) \equiv \int w(\hat{\mathbf{e}}_1, \mathbf{r}') w(\hat{\mathbf{e}}_2, \mathbf{r}' + \mathbf{r}) d\mathbf{r}', \quad (10)$$

and $\tilde{f}$ is a Fourier transform of the effective channel sensitivity function f .

The measure K given by Equation (9) depends both on the velocity wavevector k_v and on the angular distance between the vectors $\hat{\mathbf{e}}_1$ and $\hat{\mathbf{e}}_2$. If we integrate over k_v within an interval defined by the required channel width, we arrive at the formalism of the VCA technique. In the opposite limit, if we take coincident vectors $\hat{\mathbf{e}}_1$ and $\hat{\mathbf{e}}_2$ to obtain a one-dimensional spectrum P_1 , we arrive at the starting measure for the VCS technique:

$$P_1(k_v) \equiv K(\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, k_v)|_{\hat{\mathbf{e}}_1=\hat{\mathbf{e}}_2} = \tilde{f}^2(k_v) \int g(\mathbf{r}) d\mathbf{r} \cdot C_\varepsilon(\mathbf{r}) \exp\left(-\frac{k_v^2}{2} D_{vz}(\mathbf{r}) - ik_v b z\right). \quad (11)$$

where $g(\mathbf{r}) \equiv g(\hat{\mathbf{e}}, \hat{\mathbf{e}}, \mathbf{r})$ and $\hat{\mathbf{e}}$ is a beam direction.

As was shown in LP00 and CL06, Equation (11) has two asymptotic spectral regimes which depend on beamwidth. For the “high-resolution mode” (k_v is less than unity over rms velocity on the beam scale) the slope of P_1 is $2/(\alpha_v - 3)$, and, in the “low-resolution mode” it is $6/(\alpha_v - 3)$. We have assumed here a steep density spectrum¹¹ (i.e., when $\alpha_\varepsilon > 3$). The applicability of asymptotics depends on many factors and usually requires direct calculation of P_1 . In our analysis, we did direct calculations of Equation (11) as the asymptotic regime assumption is questionable for the analyzed PPV cube (see the Appendix for more details).

We further include the possibility that the observer is located inside or close to the emitting structure (i.e., lines of sight are converging as illustrated in Figure 3). This affects the geometric factor g , defined by Equations (10) and (2). CL06 showed that for a Gaussian beam with the radius θ_0 , the correspondent $g(\mathbf{r})$ for converging lines of sight can be reduced to

$$g(\mathbf{r}) = \frac{1}{\pi \theta_0^2} \int_0^\infty \frac{w_\varepsilon(z') w_\varepsilon(z' + |z|)}{z'^2 + (z' + |z|)^2} \cdot \exp\left(-\frac{R^2}{\theta_0^2(z'^2 + (z' + |z|)^2)}\right) dz', \quad (12)$$

where $\mathbf{R} \equiv (x, y)$. If we set w_ε as follows:

$$w_\varepsilon(z) = \begin{cases} 1, & z \in [z_0, z_1] \\ 0, & z \notin [z_0, z_1], \end{cases} \quad (13)$$

¹¹ Whether or not the last claim is true can be established with the analysis of column density maps. As we neglect the effects of self-absorption, the column densities can be obtained via v -integration of PPV cubes. Naturally, in the column density maps the spectrum is affected only by density and its slope is α_ε . The situation is a bit more complicated when the density is shallow ($\alpha_\varepsilon < 3$), which is the case in high Mach number turbulence (see Beresnyak et al. 2005) and the density combines with velocity to affect the P_1 slope. For a more detailed analysis, see LP00. To measure the slope of the density spectrum, the column density image can be used (see Section 4.1).

where z_0 and z_1 are the inner and outer borders of an emitting layer in a given direction, we have the following expression for $g(\mathbf{r})$:

$$g(\mathbf{r}) \approx \frac{-1}{2\sqrt{\pi}\theta_0 R z} \cdot \frac{\arctan(1 + \frac{2z_0}{z}) + \arctan(1 - \frac{2z_1}{z})}{(2z_0^2 + pz^2)^{-\frac{1}{2}} - (2z_1^2 - pz^2)^{-\frac{1}{2}}} \cdot \left(\operatorname{erf}\left(\frac{R}{\theta_0\sqrt{2z_0^2 + pz^2}}\right) - \operatorname{erf}\left(\frac{R}{\theta_0\sqrt{2z_1^2 - pz^2}}\right) \right), \quad (14)$$

where

$$p = \frac{z_1 + z_0}{z_1 - z_0}. \quad (15)$$

We discuss in Section 4.2 our selection of z_0 and z_1 limits.

We now need to express D_{vz} through the velocity spectrum. If we assume that $\mathbf{v}(\mathbf{r})$ is solenoidal¹² with a power-law power spectrum having cutoff at large scales, the velocity power spectrum can be written as follows (see, for example, Lesieur 1991):

$$F_{ij}(\mathbf{k}) = \frac{V_0^2}{k^{\alpha_v}} e^{-\frac{k_0^2}{k^2}} \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right), \quad (16)$$

where V_0^2 is the velocity power spectrum amplitude, α_v is the velocity spectral index, $k_0 = 2\pi/L_v$ is the cutoff wavevector bound with the injection scale L_v , and i and j are the component indices. Then D_{vz} can be represented as

$$D_{vz}(\mathbf{r}) = 2 \int d\mathbf{k} (1 - e^{i\mathbf{k}\mathbf{r}}) \hat{z}_i \hat{z}_j F_{ij}(\mathbf{k}). \quad (17)$$

(Summation over repeating indices is assumed here.)

4. DATA ANALYSIS

4.1. Model of P_1

We first calculate the two-dimensional spatial power spectrum of the H I column density image as this directly provides us with α_ε . This spectrum is shown in Figure 4 and has a power-law slope of ~ 3 (the significance of this slope is discussed in Section 5). This simplifies our analysis as the density (emissivity) correlation function (C_ε) has in this case weak (logarithmic) dependence on $\mathbf{r}$ and can be factored out of the integrand in Equation (11). This results in further expression for P_1 ,

$$P_{1,\text{mod}}(k_v) = \tilde{f}^2(k_v) P_0 \int g(\mathbf{r}) d\mathbf{r} \exp\left(-\frac{k_v^2}{2} D_{vz}(\mathbf{r})\right) + N_0, \quad (18)$$

$$\tilde{f}(k_v) = \frac{1}{2\pi} \exp\left(-\frac{kT k_v^2}{2m_p}\right), \quad (19)$$

where P_0 is the spectrum amplitude and N_0 is a constant, which depends on the detector noise and the resolution.

Therefore, by fitting the predicted $P_{1,\text{mod}}$ curve to the observational data we can determine the following parameters: P_0 , H I temperature (embedded in $\tilde{f}$), and velocity parameters α_v , L_v , and v_{turb} (all embedded in D_z). N_0 is estimated directly from the observational data.

¹² Numerical simulations show that most of the energy resides in solenoidal motions even for compressible driving.

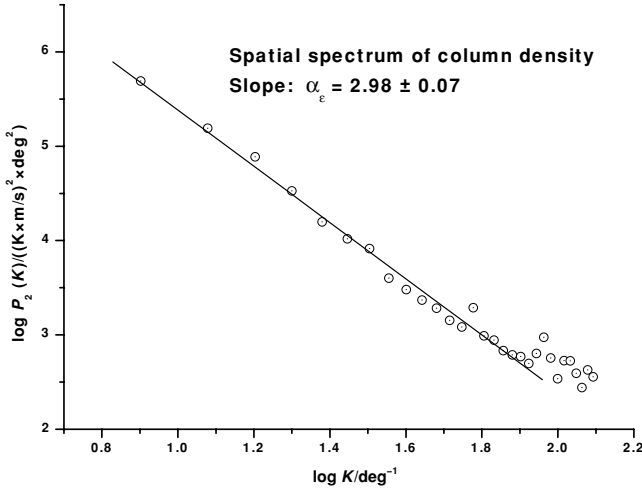


Figure 4. Spatial power spectrum of the H I column density image. The estimated spectral index of 3 results in a weak dependence of the three-dimensional density correlation function on radius vector.

We note that we omitted the influence of the instrumental channel function in Equation (19) and the regular velocity shear in Equation (18). The instrumental channel half-width is 94 m s^{-1} , which is much less than the thermal velocity of the cold phase, 670 m s^{-1} , and therefore not significant. In the direction of our observations, the regular velocity shear is $b = 2.34 \text{ m s}^{-1} \text{ pc}^{-1}$ which is much less than the shear resulting from turbulence at our largest scale, $v_{\text{turb}}/L_v = 40 \text{ m s}^{-1} \text{ pc}^{-1}$, and therefore negligible.

4.2. VCS Application

We calculate the velocity power spectrum P_1 for three different resolutions. First, we smooth the original PPV cube to resolutions of $0''.125$, $0''.250$, and $0''.500$. For each resolution element, we calculate P_1 using spatial averaging over the entire region. The resulting spectra are shown in Figure 5 with three different types of data points.

Next, we fit the observed velocity power spectra with the predicted curve. As $\alpha_\epsilon = 3$ we can use Equation (18) to fit the observed P_1 spectra with much simplified $P_{1,\text{mod}}$.

Two important issues should be noted regarding Equation (18).

1. The warm phase of the ISM is heavily suppressed by an exponential factor in Equation (19) and its impact is negligible regardless of its abundance. Therefore, our analysis is biased to the cold neutral medium.
2. The geometric factor $g(\mathbf{r})$ was taken in the form of Equation (14). The inner border of the emitting layer z_0 was determined by the Local Bubble, while the outer border z_1 was calculated from the layer height, which we assumed to be 200 pc . We assumed that the cold phase is distributed evenly enough¹³ between z_0 and z_1 , with $z_0 = 83 \text{ pc}$ and $z_1 = 270 \text{ pc}$, based on data from Lallement et al. (2003).

¹³ This term needs some justification. The thermal microphysics of cold neutral medium heating and cooling mechanisms dictate that it cannot exist below a minimum pressure $\frac{P_{\text{min}}}{k} \sim 1600 \text{ cm}^{-3} \text{ K}$ (Wolfire et al. 2003). With a typical column density $N(\text{HI}) \sim 5 \times 10^{20} \text{ cm}^{-2}$ and temperature $\sim 50 \text{ K}$, the total length along the line of sight cannot exceed $\sim 5 \text{ pc}$. We assume that this thickness is statistically uniformly fragmented between z_0 and z_1 .

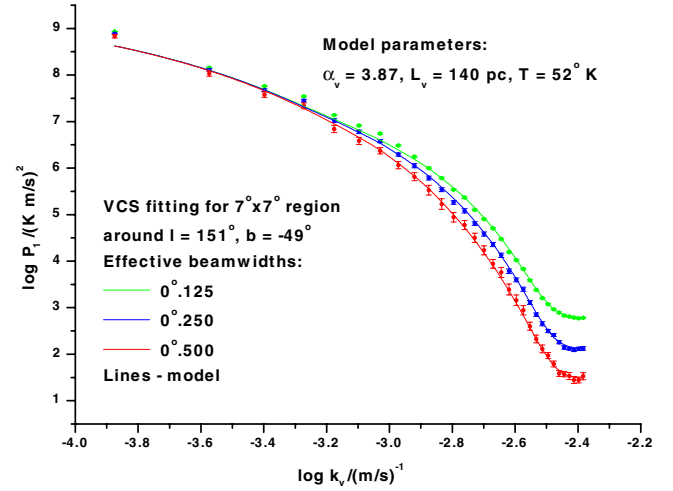


Figure 5. Fitting of the model VCS spectra to P_1 data, obtained from the GALFA-H I data cube for different effective resolutions. The leftmost points, most likely affected by the warm phase (their abscissa corresponds to the kinetic temperature of 6800 K), are excluded from the fitting.

The fitting is performed using Mathematica's function FindMinimum, which implements the algorithm of steepest descent. The resulting fits are very good, as shown in Figure 5. This process yields an estimate of the following parameters: the velocity spectral index $\alpha_v = 3.87 \pm 0.11$, the turbulent injection scale $L_v = 140 \pm 80 \text{ pc}$, the VCS amplitude P_0 , the gas temperature $T_{\text{cold}} = 52 \pm 11 \text{ K}$ (as discussed, this is biased toward the cold medium), and the (cold phase) gas Mach number $M_{\text{cold}} = 7.7^{+1.0}_{-0.7}$.

It is remarkable that most of the low- k_v part of the calculated spectra is in agreement with the rest of our dynamical range. We can guess that the reason is that the correspondent k_v 's are mostly not affected by the warm phase. We needed to exclude only the first point, which corresponds to $T = 6800 \text{ K}$ and is above the estimation.¹⁴

Despite its negligible impact on P_1 , the warm phase can dominate in a spectral line as a whole¹⁵ and in this case we can estimate its parameters too. If we know the characteristic velocity of the mean line profile v_{total} , we can calculate the warm phase thermal velocity as $\sqrt{v_{\text{total}}^2 - v_{\text{turb}}^2}$, if we assume that v_{turb} is the same for the both phases. Then we can calculate temperature and Mach number for the warm phase too. We get the warm phase temperature $T_{\text{warm}} = 4200^{+1000}_{-2500} \text{ K}$ and the warm phase Mach number $M_{\text{warm}} = 0.9^{+0.5}_{-0.1}$.

To calculate the statistical error of a fitting parameter, we deviate it from its optimum and let the other parameters compensate for the corresponding deviation of our target function. This gives us some other model curves, deviating from the optimal curves as well. We interpret the mean squared deviation between these two sets of model curves, normalized by variances of corresponding measured P_1 values, as squared normalized deviation of the parameter itself. As we know its absolute deviation, we determine its variance. By taking as a reference point the fitted model instead of the data, we separate the effect of deviation from the influence of possible systematic error and statistical

¹⁴ When calculating this temperature we assume that the thermal velocity projection can be represented $v_z = 1/k_v$.

¹⁵ Zeroth and first harmonics of the high-resolution P_1 , most likely affected by the warm phase, contain about 90% of the total energy of P_1 .

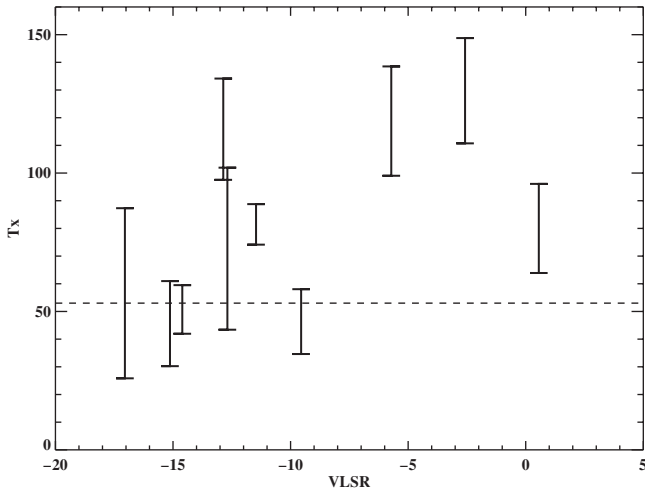


Figure 6. This figure exhibits the derived H I spin temperature T_x vs. $VLSR$ for six NVSS continuum sources that lie in the analyzed region; four sources have two measurable velocity components, providing a total of 10 samples shown. The dashed line is the temperature derived from our statistical analysis; see Table 1 for the corresponding estimation data.

scattering of data. This procedure guarantees uniqueness of the obtained solution too.

4.3. Verification of the Fitting Procedure

Numerical verification of the VCS technique was presented in Chepurinov & Lazarian (2009). However, the consistency of the fitting procedure needs to be checked as well. To do this, we can change the effective temperature of the PPV cube by convolving it over the velocity axis with a Maxwellian distribution. The temperature in the new cube is the sum of the original temperature (which we estimated to be 52 K) and that of the Maxwellian distribution. Temperature is the only fitting parameter we can easily change.

We have convolved our data with the 104 K Maxwellian distribution. Our fitting procedure gave $T = 154$ K, $\alpha_v = 3.87$, $L_v = 137$ pc, and $v_{\text{turb}} = 5.14$ km s⁻¹ (the original parameters were $T = 52$ K, $\alpha_v = 3.87$, $L_v = 138$ pc, and $v_{\text{turb}} = 5.15$ km s⁻¹). The expected temperature is $52 + 104 = 156$ K, very near to the value the procedure retrieved. Therefore, we see a good agreement between the estimate from our fitting procedure and the theoretically expected value when we increased the temperature by a factor of 3. All other parameters remained very near to their original values, which demonstrates good stability of the solution.

4.4. Verification of the Derived Temperature

Figure 6 exhibits the derived H I spin temperature T_x versus $VLSR$ for six NVSS continuum sources that lie in the analyzed region; four sources have two measurable velocity components, providing a total of 10 samples shown. The sources all have 1.4 GHz flux densities exceeding 0.75 Jy and constitute a complete set of sources in this region that yield reliably detected 21 cm absorption line profiles.

Each datum consists of two by-eye estimates: one a lower limit and the other an upper limit. The upper limit assumes that all the emission at the peak of the absorption line (the “expected emission temperature” $T_{B,\text{exp}}$) arises from gas at a uniform spin temperature. This situation is generally unrealistic, however, because unrelated gas along the line of sight contributes to

$T_{B,\text{exp}}$; hence, this assumption provides us with the upper limit. The lower limit arises by decomposing the emission profile into Gaussian components and assigning to $T_{B,\text{exp}}$ only the emission associated with the component that represents the absorbing gas. This implicitly assumes that the unrelated emission from warmer gas is not absorbed by the cold gas, so the warm gas lies in front of the cold absorbing gas; because some warm gas might lie behind, this assumption provides us with the lower limit. The limits in Figure 6 are approximate because they were determined by eye, not by least-squares fitting of Gaussian components, but the errors in these by-eye estimates are considerably smaller than the ranges of temperature shown.

The data in Figure 6 refer to cold gas. Typically, a significant fraction of the gas is warm and produces no detectable 21 cm line absorption. This warm gas is not represented in Figure 6. The limit of detectability depends on the flux of the continuum background source, and if this region had stronger sources we would have been able to plot data with warmer temperatures than Figure 6.

The bias toward cold gas that is introduced by these measurement considerations—which are observationally based—is similar in spirit to the bias inherent in our statistical analysis, which is also more sensitive to cold gas than warm. Thus, a comparison of the data in Figure 6 with our statistically derived spin temperature of 52 K is meaningful. In Figure 6, half of the data are consistent with the theoretically derived 52 K; the other half are warmer. Given the similarity between the observational and theoretical biases, we regard this agreement as satisfactory.

5. DISCUSSION AND SUMMARY

5.1. Retrieved Parameters

Applying the VCS technique to the $6.5^\circ \times 6.5^\circ$ site centered at $l = 151^\circ$, $b = -49^\circ$ we have determined the following H I parameters.

1. Velocity spectral index $\alpha_v = 3.87 \pm 0.11$.
2. Density spectral index $\alpha_\varepsilon = 3.0 \pm 0.1$.
3. Injection scale $L_v = 140 \pm 80$ pc.
4. Cold phase temperature $T_{\text{cold}} = 52 \pm 11$ K.
5. Cold phase Mach number $M_{\text{cold}} = 7.7^{+1.0}_{-0.7}$.

In addition, under the assumption that the warm medium dominates the line at low- k_v ’s, we get the warm phase temperature $T_{\text{warm}} = 4200^{+1000}_{-2500}$ K and the warm phase Mach number $M_{\text{warm}} = 0.9^{+0.5}_{-0.1}$.

The derived T_{cold} is very similar to what is typically assumed and measured for the cold neutral medium (CNM). For example, Heiles & Troland (2003) used an H I absorption/emission survey of 79 sources and found that the CNM spin temperature histogram peaks at about 40 K, while its median, weighted by column density, is 70 K. It is also in satisfactory agreement with our measurements of spin temperature within the analyzed region (see Section 4.4). The inferred T_{warm} is in agreement with the typical temperature of the warm neutral medium (WNM) of ~ 5000 (Wolfire et al. 2003). Similarly, T_{warm} in the range 500–5000 K was estimated observationally by Heiles & Troland (2003) to occupy 48% of the WNM and corresponds to the thermally unstable warm gas. Heiles & Troland (2003) also found that $M_{\text{cold}} \approx 3$ for the Milky Way, but has a large scatter and ranges from ~ 1 to ~ 7 . Our estimated M_{cold} is at the boundary of the Heiles & Troland (2003) range.

Under the assumption that the cold gas has a uniform distribution from 83 pc to 270 pc along the line of sight, we

estimated the injection scale of turbulence $L_v = 140 \pm 80$ pc. This is in agreement with the expected value of 100 pc associated with supernova explosions (compare to Haverkorn et al. 2008).

It is interesting that our velocity and density spectral indices are different from indices derived for the Ursa Major high-latitude cloud, $\alpha_v = \alpha_\epsilon = 3.6 \pm 0.2$ by Miville-Deschênes et al. (2003). These authors used a very different approach, velocity centroids, and the assumption that density and velocity fields are Gaussian. While in the case of Ursa Major field velocity and density indices are similar, in our case they are very different: $\alpha_v \approx 3.9$ and $\alpha_\epsilon \approx 3$. In addition, studies of centroids (Lazarian & Esquivel 2003; Esquivel & Lazarian 2005b; Ossenkopf et al. 2006; Esquivel et al. 2007) showed that ordinary centroids used in Miville-Deschênes et al. (2003) may represent only velocity statistics in subsonic turbulence. This is unlikely for most of the H I, which has an admixture of cold and warm gases.

Generally, the estimated parameters are reasonable and this suggests that the VCS technique could be used to estimate gas temperature and turbulent properties directly from H I emission profiles, instead of obtaining H I absorption spectra. However, we worked here with only a single region and further testing with observational data is essential to check VCS's reliability.

5.2. Advantages and Limitations of Our Approach

While the analysis of fluctuations in channel maps has been a relatively standard technique, the analysis of the fluctuations within PPV cubes along the velocity direction is quite a new approach. Naturally, we faced many new problems in this situation to which we have to present our solutions.

The most fundamental problem that we face dealing with the VCS technique is the necessarily limited inertial interval. We can demonstrate this assuming that the turbulence is Kolmogorov. In this case $v \sim l^{1/3}$ and an inertial range of 10^3 in terms of l translates into just one decade of inertial range in the velocity domain. While the actual astrophysical turbulence spans into many decades (see Armstrong et al. 1995), the measurements of the turbulent velocity fluctuations become very challenging because of the thermal broadening of lines. The latter depends on the mass of the species and is most prominent for atomic hydrogen. This was the reason that we found asymptotic studies not useful and adopted the fitting procedure described in this paper. We expect modifications of this procedure will be used in the future with other species.

However, in the no-asymptotic case we needed to fit several parameters. The pros and cons of the fitting procedure are interrelated. In general, it may be considered safer to measure just the index of the power slope, as is prescribed in the VCA technique (see Stanimirović & Lazarian 2001) rather than to fit several parameters simultaneously, including the injection scale and the gas temperature. However, the Doppler-shifted lines contain all this information and a successful fitting procedure can provide more than just the velocity spectral slope in question. In fact, we are developing a similar fitting procedure for the VCA technique (A. Chepurinov & A. Lazarian 2010, in preparation), which shows advantages compared to the usual employment of the channel map data.

An additional advantage of the fitting procedure is that some parameters of the model (e.g., temperature, injection scale, turbulent velocity) can be independently studied and tested. At the same time, in the situations when these parameters are not available by other means, the fitting procedure can provide estimates of them as discussed in LP06. Our verification procedure in Section 4.3 is encouraging in this respect.

The fact that the information for the VCS is taken from the fluctuations along the V-direction of the PPV cubes allows us to study spatially localized regions of turbulence and to map the distribution of turbulence within the studied turbulent volume, which is another advantage of VCS. In this respect, VCS can be successfully used in conjunction with other statistical tools which allow us to get insight into turbulence (see Burkhart et al. 2010).

For our analysis, we have chosen high-latitude Galactic H I. This gives advantages both through high resolution, which allows studies of different resolution limits of the VCS technique, and through allowing us to worry less about the effects of confusion that arises when H I in the plane of the Milky Way is studied. However, the downside of this is the necessity to deal with a more complex observational geometry, where fluctuations of different physical sizes are seen at the same angular scale (i.e., perspective). We have formalized the influence of observational geometry by introducing a geometric term, which contributes to the integrand in the expression for P_1 .

The column density map shows a spectral index of 3 with very good confidence. On the one hand, this allows us to omit the density factor in calculations of P_1 , if we take this value as a true estimation of the density spectral index. On the other hand, the VCA analysis for the absorbing medium predicts exactly this behavior for the optically thick case for any density spectral index (see LP04 for details).

At the same time, the temperature measured by the telescope is low enough to formally preclude the case of self-absorption. Nevertheless, it may still happen that H I is not self-absorbing in terms of total emission but is self-absorbing in terms of fluctuations of PPV statistics. For instance, it is well established (see LP00, LP06) that the PPV fluctuations are dominated by cold gas, while the contribution of the fluctuations in warm gas is exponentially suppressed. Thus, if the cold gas, which for the high latitudes sampled by our observations constitutes a small fraction of the total mass, still dominates the PPV fluctuations, we may have the situation that we observe. In this case, the density factor is undetermined. We can assume that the density spectrum is steep: in this case the density term can be factored out too (for asymptotic studies). However, the possibility of shallow density, i.e., $\alpha_\epsilon < 3$, exists. In this case, the spectral index of velocity is not $\alpha_v \approx 3.9$, but $\alpha_v = 3 + 0.9(\alpha_\epsilon - 2)$. The bracket in this case is less than 1, which means that the spectral index of velocity is $\alpha_v < 3.9$.

The Kolmogorov index corresponds to $\alpha_v \approx 3.7$, but turbulence with this index is known to produce the spectral index of density $\alpha_\epsilon \approx 3.7$ (Cho & Lazarian 2003). Supersonic turbulence is required to get $\alpha_\epsilon < 3.7$ (Beresnyak et al. 2005). Such turbulence corresponds to $\alpha_v > 3.7$. To satisfy these requirements one should have $2.77 < \alpha_\epsilon < 3$, which provides constraints on the density spectrum in the vicinity of $\alpha_\epsilon = 3$ that we assumed.

On the basis of the fact discussed above, we assumed that the fluctuations we measure are due to the velocity fluctuations only and correspond to the spectral index $\alpha_v \approx 3.9$, which is steeper than the Kolmogorov index. As our analysis is exponentially biased toward the cold gas, the spectrum measured is mostly of turbulence in the cold gas. In the situation when turbulence in the cold gas is a part of a large-scale cascade, as it is generally assumed, the turbulence in the cold gas is supersonic and the formation of the shock-type velocity spectrum observed is not surprising at all.

While our paper was at the referee stage, we learned about the new paper by Padoan et al. (2009). The paper also uses

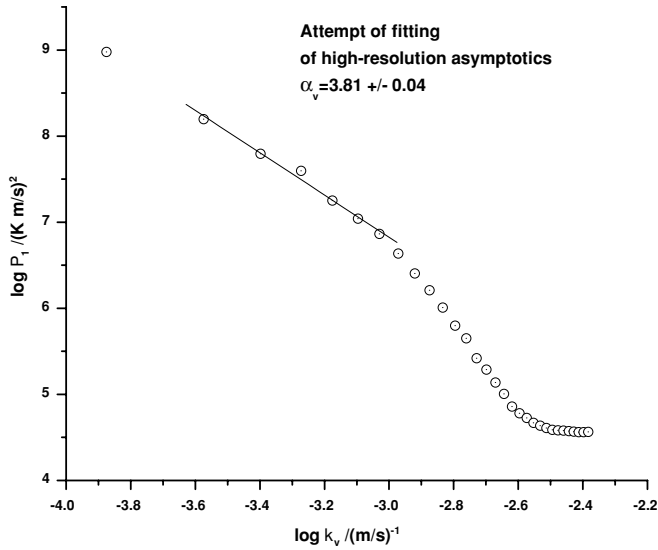


Figure 7. Attempt to fit high-resolution asymptotics. The derived velocity spectral index $\alpha_v = 3.81 \pm 0.04$ deviates by a systematic error from the spectral index obtained by the model fitting ($\alpha_v = 3.87 \pm 0.11$). The obtained value shows some systematic error and is actually a lower limit for α_v (see Figure 8, left panel).

the VCS technique, but it does not provide the fit to the data as we do in this paper. Instead, Padoan et al. (2009) compare the VCS power slope to the asymptotic predictions in Lazarian & Pogosyan (2000). We feel that this way of obtaining the turbulence power spectrum is subject to larger errors compared to the technique we use in this paper. Indeed, the dynamical range of the VCS fluctuations is rather restricted which limits the applicability of fitting the asymptotic slope.

However, the spectral index value, calculated in assumption of the asymptotic regime, can be used as a lower limit for α_v , if the emissivity term is negligible. Making a stronger statement needs an a posteriori check by direct calculation of P_1 . For instance, performing the same procedure as in Padoan et al. (2009), we would get the spectral index of turbulence of 3.81 ± 0.04 , which deviates by a systematic error from the numbers

Table 1
Spin Temperature Estimation Data for Figure 6

l	b	VLSR (km s ⁻¹)	$T_{\min}$ (K)	$T_{\max}$ (K)
149.684	-47.4998	-9.55142	34.6271	58.0417
151.554	-49.6997	-12.6884	43.4076	101.944
151.554	-49.6997	-5.71733	99.0173	138.530
153.901	-51.2807	-17.0453	25.8466	87.3100
153.901	-51.2807	-12.8627	97.5539	134.139
155.258	-45.8268	-14.6054	41.9442	59.5051
155.258	-45.8268	-2.58035	110.725	148.773
157.784	-48.1993	-11.4685	74.1393	88.7734
157.784	-48.1993	0.556630	63.8954	96.0905
159.721	-49.1208	-15.1283	30.2368	60.9685

Notes. In cases of more than one line for the same position, there are two recognizable velocity components.

3.87 ± 0.11 we obtain using our approach (see the Appendix for details).

5.3. Summary

We have applied the new VCS technique to the Arecibo high-latitude data and obtained the spectrum of velocity, which is steeper than the Kolmogorov one. The steeper turbulent velocity spectrum indicates the importance of shocks in the media, which are expected to make the spectrum of density shallower than the Kolmogorov density. This is the effect that we register studying the turbulent density. Our application of the VCS technique uses the model fitting procedure, which allows us to evaluate the injection scale of the turbulence, the temperature of the cold media, turbulent velocity, and Mach number. Assuming that the warm medium dominates the line at low k_v 's, we estimated the temperature and Mach number of the warm phase too. The obtained parameters for the region of the study are given in Section 5.1.

A.C. and A.L. acknowledge support from NSF grant AST0808118 and the Center for Magnetic Self-Organization in Laboratory and Astrophysical Plasmas. S.S. acknowledges support from NSF grant AST-0707679. C.H. acknowledges support from NSF grant AST-0709347.

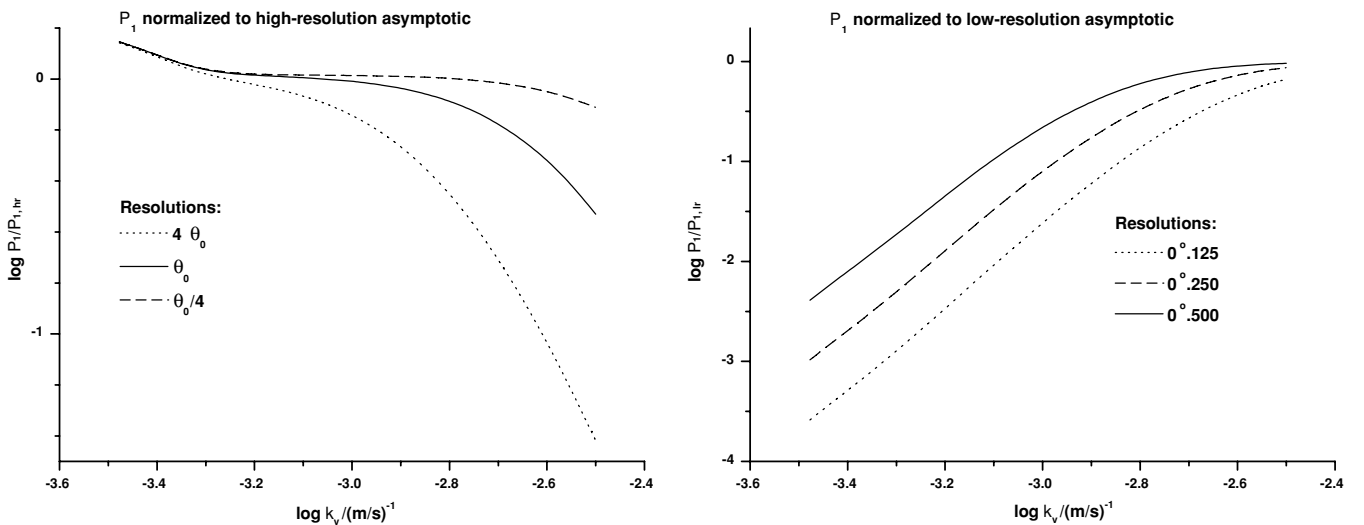


Figure 8. Left: P_1 normalized to the high-resolution asymptotic for the instrumental resolution (solid line) and resolutions four times lower (dotted line) and four times higher (dashed line). The assumption of the high-resolution asymptotic regime is applicable for the latter case only; for other cases there is some systematic error, more significant for the lower resolution. Right: P_1 normalized to the low-resolution asymptotic for resolutions 0.125 (dotted line), 0.250 (dashed line), and 0.500 (solid line). The assumption of the low-resolution asymptotic regime is not applicable.

APPENDIX

APPLICABILITY OF ASYMPTOTICS

LP00 and LP09 presented their final results in terms of asymptotics of P_1 for large k_v . While this is advantageous from a theoretical point of view, it presents some problems related to the analysis of observational data. Below we show that the use of asymptotics may require higher resolution and a larger dynamical range than is available from observations.

Let us check if the asymptotical approach is possible for our data. To do that, we can calculate the velocity spectral index, as if the asymptotics were applicable, and compare it to the value obtained by our fitting procedure. The correspondent linear regression is shown in Figure 7. We can see that there is some disagreement between the obtained spectral index $\alpha_v = 3.81 \pm 0.04$ and the value obtained by the model fitting ($\alpha_v = 3.87 \pm 0.11$), i.e., we can conclude that in our case the calculation assuming the high-resolution asymptotic regime produces a systematic error bigger than the statistical error of the asymptotic α_v .

As is clear from Figure 3, the notion of the high and lower resolution changes as the eddies of different sizes are studied. For the largest eddies and therefore for the small k_v we are in the limit of high resolution. However, this changes as we go to larger k_v . We can also compare the model P_1 with the predicted asymptotic for different resolutions; see Figure 8, left panel. We can see there that if the resolution were four times higher than that of our instrument, it would be possible to compute the velocity spectral index directly from the P_1 slope.

At the very beginning, it may not be obvious which we are in. What regime we are in may not be obvious from the very beginning. If we use the high-resolution asymptotics to find the velocity statistics, it is advisable to have an a posteriori check if the assumption of the high resolution is applicable. This complicates the analysis. But fitting of model P_1 , as is done in this paper, although harder to implement, is self-sufficient and more reliable. However, as can be seen in Figure 8, left panel, the asymptotic calculation can be used to get a lower limit for α_v , if we neglect the emissivity term.

What we mentioned above is related to the high-resolution asymptotic solution. What about low-resolution asymptotics obtained in LV00 and LV06? We can also check the low-resolution asymptotics (see Figure 8, right panel). We observe that such a regime is not present for the degraded resolutions we used for the model fitting. Theoretically it is possible to degrade resolution until the slope of P_1 saturates, and then calculate the velocity spectral index using the low-resolution asymptotics. However, in our case the high- k_v part of P_1 is affected by the temperature term, and this approach becomes unreliable for our data. In addition, P_1 gets noisier if we degrade the resolution.

All in all, the asymptotic formulae may not be straightforward to use with real observational data. Therefore, we advocate the numerical procedure presented in this paper as a more reliable way of handling our data set. This does not mean that the asymptotical analytical solutions are not useful. First of all, they provide a proper insight into qualitative properties of the PPV data cubes. In addition, if one uses a spectral line of a heavier species and has enough statistics and dynamical range for calculation of the low-resolution P_1 , the asymptotical

approach is applicable and self-sufficient. In general, we do not want to oppose the asymptotic solutions to the formulae that we use. One should remember that the asymptotic solutions were obtained by evaluating for high k_v the integral expressions employed in this work.

REFERENCES

- Armstrong, J. W., Rickett, B. J., & Spangler, S. R. 1995, *ApJ*, **443**, 209
 Beresnyak, A., Lazarian, A., & Cho, J. 2005, *ApJ*, **624**, L93
 Biskamp, D. 2003, *Magnetohydrodynamic Turbulence* (Cambridge: Cambridge Univ. Press)
 Burkhart, B., Stanimirović, S., Lazarian, A., & Kowal, G. 2010, *ApJ*, **708**, 1204
 Chepurinov, A., & Lazarian, A. 2006, arXiv:astro-ph/0611465v3
 Cho, J., & Lazarian, A. 2003, *MNRAS*, **345**, 325
 Chepurinov, A., & Lazarian, A. 2009, *ApJ*, **693**, 1074
 Crovisier, J., & Dickey, J. M. 1983, *A&A*, **122**, 282
 Deshpande, A. A., Dwarakanath, K. S., & Goss, W. M. 2000, *ApJ*, **543**, 227
 Dickey, J. M., McClure-Griffiths, N. M., Stanimirović, S., Gaensler, B. M., & Green, A. J. 2001, *ApJ*, **561**, 264
 Elmegreen, B. G., & Falgarone, E. 1996, *ApJ*, **471**, 816
 Elmegreen, B. G., Kim, S., & Staveley-Smith, L. 2001, *ApJ*, **548**, 749
 Elmegreen, B., & Scalo, J. 2004, *ARA&A*, **42**, 211
 Esquivel, A., & Lazarian, A. 2005a, *ApJ*, **784**, 320
 Esquivel, A., & Lazarian, A. 2005b, *ApJ*, **631**, 320
 Esquivel, A., Lazarian, A., Horibe, S., Cho, J., Ossenkopf, V., & Stutzki, J. 2007, *MNRAS*, **381**, 1733
 Green, D. A. 1993, *MNRAS*, **262**, 327
 Hartmann, D., & Burton, W. B. 1997, *Atlas of Galactic Neutral Hydrogen* (Cambridge: Cambridge Univ. Press)
 Haverkorn, M., Brown, J. C., Gaensler, B. M., & McClure-Griffiths, N. M. 2008, *ApJ*, **680**, 362
 Heiles, C. 2004, <http://www2.naic.edu/alfa/memos/general/>
 Heiles, C., & Troland, T. H. 2003, *ApJ*, **586**, 1067
 Heyer, M. H., & Schloerb, F. P. 1997, *ApJ*, **475**, 173
 Lallement, R., Welsh, B. Y., Vergely, J. L., Crifo, F., & Sfeir, D. 2003, *A&A*, **411**, 447
 Lazarian, A. 2009, *Space Sci. Rev.*, **143**, 357
 Lazarian, A., Beresnyak, A., Yan, H., Opher, M., & Liu, Y. 2009, *Space Sci. Rev.*, **143**, 387
 Lazarian, A., & Esquivel, A. 2003, *ApJ*, **592**, L37
 Lazarian, A., & Pogosyan, D. 2000, *ApJ*, **537**, 720
 Lazarian, A., & Pogosyan, D. 2006, *ApJ*, **652**, 1348
 Lazarian, A., & Pogosyan, D. 2008, *ApJ*, **686**, 350
 Lazarian, A., Pogosyan, D., Vazquez-Semadeni, E., & Pichardo, B. 2001, *ApJ*, **555**, 130
 Lesieur, M. 1991, *Turbulence in Fluids* (Dordrecht: Kluwer)
 McKee, C. F., & Ostriker, E. C. 2007, *ARA&A*, **45**, 565
 Miville-Deschênes, M., Levrier, F., & Falgarone, E. 2003, *A&A*, **593**, 831
 Monin, A. S., & Yaglom, A. M. 1975, *Statistical Fluid Mechanics: Mechanics of Turbulence* (Cambridge, MA: MIT Press)
 Ossenkopf, V., Esquivel, A., Lazarian, A., & Stutzki, J. 2006, *A&A*, **452**, 223
 Ossenkopf, V., Krips, M., & Stutzki, J. 2008a, *A&A*, **485**, 917
 Ossenkopf, V., Krips, M., & Stutzki, J. 2008b, *A&A*, **485**, 719
 Padoan, P., Juvela, M., Kritsuk, A., & Norman, M. L. 2009, *ApJ*, **707**, L153
 Peek, J. E. G., & Heiles, C. 2008, arXiv:0810.1283
 Peek, J. E. G., Putman, M. E., McKee, C. F., Heiles, C., & Stanimirović, S. 2007, *ApJ*, **656**, 907
 Rosolowsky, E. W., Goodman, A. A., Wilner, D. J., & Williams, J. P. 1999, *ApJ*, **524**, 887
 Stanimirović, S., & Lazarian, A. 2001, *ApJ*, **551**, L53
 Stanimirović, S., Staveley-Smith, L., Sault, R. J., Dickey, J. M., & Snowlen, S. L. 1999, in *IAU Symp. 190, New Views of the Magellanic Clouds*, ed. Y. H. Chu, et al. (San Francisco, CA: ASP), **103**
 Stanimirović, S., et al. 2006, *ApJ*, **653**, 1210
 Stutzki, J., Bensch, F., Heithausen, A., Ossenkopf, V., & Zielinsky, M. 1998, *A&A*, **336**, 697
 Wolfire, M. G., McKee, C. F., Hollenbach, D., & Tielens, A. G. G. M. 2003, *ApJ*, **587**, 278