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EVIDENCE OF SUPERDIFFUSIVE TRANSPORT OF ELECTRONS ACCELERATED AT INTERPLANETARY SHOCKS

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ABSTRACT

We study the transport properties of energetic electrons accelerated at corotating interaction regions in the solar wind considering the possibility of anomalous diffusion. It is shown that the particle time decay has a power-law behavior when a non-Gaussian propagator, appropriate for superdiffusive transport, is assumed for particles accelerated at a propagating shock. Looking at shock events detected by the *Ulysses* spacecraft at 5 AU, we found that 42–290 keV electron time profiles are well fitted by a power law corresponding to superdiffusive transport, i.e., $\langle \Delta x^2(t)\rangle \propto t^\alpha$, with $\alpha = 1.02$–1.38. This implies that particle propagation in the heliosphere can be intermediate between normal diffusion and ballistic motion.

Subject headings: acceleration of particles — diffusion — interplanetary medium — shock waves — turbulence

1. INTRODUCTION

Energetic particles are frequently observed in space, in particular during violent solar events (solar flares, coronal mass ejection–driven shocks) and in some regions of the interplanetary space where the fast solar wind ($V_{\rm SW} \simeq 750$ km s^{-1}), coming from solar coronal holes, encounters the slow wind ($V_{\rm SW} \simeq 400$ km s^{-1}), giving rise to two collisionless shocks moving in opposite directions—the reverse one sunward, and the forward one antisunward. Such structures, named corotating interaction regions (CIRs) (Gosling, Hundhausen, & Bame 1976), include a compressive region between the two shocks. Particles accelerated by the CIR shocks were observed, among others, by the *Ulysses* spacecraft, before and after the Jupiter encounter in 1992 February. Understanding how particles spread out in space is important for assessing the propagation of particles from the Sun to the Earth, for cosmic-ray acceleration and transport, and even for evaluating the influence of extragalactic cosmic rays on the fossil diversity on Earth (Medvedev & Melott 2007). However, transport properties in the interplanetary medium, due to the interaction of particles with magnetic turbulence, are poorly understood (Reames 1999; McKibben 2005; Stone et al. 2005). For instance, analysis of solar energetic particles (SEPs), observed both in space by *Wind* (Dröge 2003) and at ground level by polar neutron monitors (Ruffolo et al. 2006), indicates diffusive transport, for both protons and electrons, with rather large values of the parallel mean free path; on the other hand, comparing SEP data from *IMP-8* and *Ulysses*, we see that the transport parallel to the interplanetary magnetic field appears to be ballistic (or scatter-free) up to $\simeq$1 AU, while it could be diffusive up to about 3.2 AU (Reames 1999; Zhang et al. 2003; Lin 2005). Normal (Brownian-like) diffusion has long been considered (Jokipii 1966), also in connection with diffusive shock acceleration (DSA) (Bell 1978; Blandford & Ostriker 1978; Fisk & Lee 1980; Lee 1983), which is a first-order Fermi acceleration mechanism in which particles are accelerated during shock encounters. In this context, particles can reach very high energies if they can diffuse back to the shock owing to magnetic irregularities (e.g., Lee 2005). In addition, DSA can explain the observed cosmic-ray power-law spectra (Blandford & Eichler 1987; Hillas 2005). On the other hand, anomalous transport regimes were observed in fluid and plasma experiments (Solomon, Weeks, & Swinney 1993; Ratynskaia et al. 2006), so a key question is whether an energetic particle in space can propagate in an anomalous way, characterized by $\langle \Delta x^2(t)\rangle \propto t^\alpha$, both slower ($\alpha < 1$) and faster ($\alpha > 1$) with respect to Gaussian diffusion ($\alpha = 1$) (Zaslavsky et al. 1989; Perri et al. 2007). The faster regime, $\alpha > 1$, called superdiffusion, is also characterized by Lèvy random walks, which correspond to strong correlations in space and time (multisteps memory), and by long displacements, whose probability can be described by a non-Gaussian statistics (Klafter, Blumen, & Shlesinger 1987; Zaslavsky et al. 1989; Metzler & Klafter 2000; Zaslavsky 2002). Test particle numerical simulations have shown that different transport regimes, normal and anomalous, can be obtained for both perpendicular and parallel transport, depending on the features of magnetic turbulence (Giacalone & Jokipii 1999; Qin, Matthaeus, & Bieber 2002; Zimbardo, Pommois, & Veltri 2006; Pommois, Zimbardo, & Veltri 2007). Here we report the first direct observational evidence in space physics of particle superdiffusion parallel to the magnetic field.

2. PROPAGATOR FORMALISM

The analytical description of anomalous transport can be done by using the propagator $P(x, x', t, t')$ (Zumofen & Klafter 1993; Metzler & Klafter 2000; del-Castillo-Negrete, Carreras, & Lynch 2004; Webb et al. 2006), which is the probability of finding a particle, injected at x' and t', at position x at time t. At a sufficient distance from the Sun, the solar wind can be assumed to be statistically homogeneous, so the propagator will depend only on $x - x'$ and $t - t'$. In the case of normal diffusion, $\langle \Delta x^2(t)\rangle = 2Dt$, the propagator has a Gaussian form (Metzler & Klafter 2000; Webb et al. 2006) and the particle flux corresponds to an exponential decay, $J = K \exp[V_{\rm sh}|x|/D]$, where $V_{\rm sh}$ is the shock velocity, $|x|$ is the distance upstream from the shock, and D is the parallel diffusion coefficient (Fisk & Lee 1980; Lee 1983). Conversely, in the case of superdiffusive transport, the propagator can be obtained in the framework of continuous-time random walks (Zumofen & Klafter 1993; Klafter, Shlesinger, & Zumofen 1996; Metzler & Klafter 2000). The propagator is derived in the Fourier-Laplace space, and an explicit inversion is possible for limiting cases: for $x - x' \gg k_\mu^{1/2}(t - t')^{1/\mu - 1}$, where k_μ is an anomalous diffusion constant

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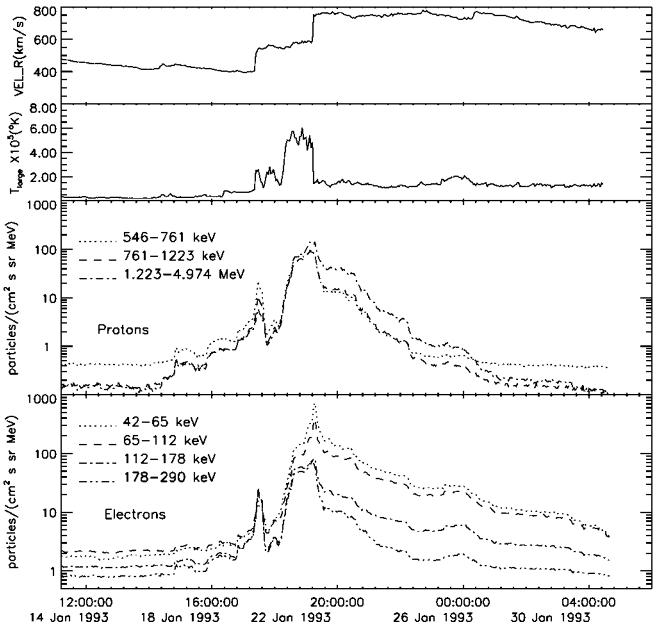

FIG. 1.—Plasma and energetic particle profiles for the *Ulysses* shock crossing of 1993 January 22. From the top to the bottom, the panels show 1 hr averages for the plasma radial velocity and the plasma temperature from SWOOPS (PI D. McComas); proton fluxes measured by HI-SCALE LEFS 60 (PI L. Lanzerotti) in the energy channels 546–761 keV, 761–1223 keV, and 1.233–4.974 MeV; and electron fluxes measured by HI-SCALE LEFS 60 in the energy channels 42–65 keV, 65–112 keV, 112–178 keV, and 178–290 keV. Note that each tick mark corresponds to 10 hr.

whose physical dimensions are $[l^2/t^{2/(\mu-1)}]$, the propagator has a power-law behavior given by

$$P(x-x',\ t-t') = b\,\frac{t-t'}{(x-x')^{\mu}}, \qquad (1)$$

where b and μ are constants, while it goes to zero for $x - x' > v(t-t')$, with v the particle velocity (Zumofen & Klafter 1993). For $2 < \mu < 3$ superdiffusion is obtained for large t, $\langle \Delta x^2(t)\rangle \propto t^{\alpha}$ with $\alpha = 4 - \mu$, while for $3 < \mu < 4$ transport is diffusive, even if the statistics is non-Gaussian (Klafter, Blumen, & Shlesinger 1987; Zumofen & Klafter 1993; Klafter, Shlesinger, & Zumofen 1996). We assume the shock to be planar and consider a simple one-dimensional geometry; in other words, each quantity depends only on the x-direction, that is, perpendicular to the shock front. The energetic particle fluxes, measured by a spacecraft at (x, t), are the superposition of particles accelerated at the shock moving according to $x' = V_{sh}t'$, with V_{sh} the upstream shock speed in the solar wind rest frame. We consider that the observer is at $x = 0$ upstream of the shock and is magnetically connected to the shock, which is coming from $x = -\infty$, so $t < 0$ for the relevant time interval. Accordingly, the particle omnidirectional distribution function $f(x, E, t)$ at the observer will be

$$f(x,\ E,\ t) = \int P(x-x',\ t-t') f_{sh}(x',\ E,\ t')\,dx'dt', \qquad (2)$$

with $f_{sh}(x', E, t') = f_0(E)\delta(x' - V_{sh}t')$, where $f_0(E)$ represents the distribution function of particles of energy E emitted at the shock. If we determine the energetic particle profile by using equation (1), i.e., at large distance from the source, we have

$$f(x,\ E,\ t) = f_0(E)b\int_{t_0}^{t} \frac{t-t'}{(x-x')^{\mu}}\,\delta(x' - V_{sh}t')\,dx'dt', \qquad (3)$$

where t_0 is the shock start time. Assuming that $2 < \mu < 3$, which corresponds to superdiffusion, and $t_0 = -\infty$, i.e., a shock starting very far away, we obtain

$$f(x,\ E,\ t) \simeq \frac{f_0(E)b}{\mu^2 - 3\mu + 2}\,\frac{(x - V_{sh}t)^{2-\mu}}{V_{sh}^2}. \qquad (4)$$

For $x = 0$ we have $f(0, E, t) \simeq 1/(-t)^{\mu-2}$, that is, a power-law profile with slope $\gamma = \mu - 2$. This slope does not change if the observer moves in the solar wind as $x = V_{sw}t$. Clearly, a power-law decay of the particle distribution function with small slope ($\gamma < 1$) implies superdiffusive transport.

3. ELECTRON DATA ANALYSIS AND DISCUSSION

The heliocentric distance of *Ulysses*, after the Jupiter encounter, was more than 5 AU, and the assumption of planar shock can be considered a reasonable first approximation. From 1992 July, *Ulysses* moved into the fast solar wind from a newly developed coronal hole, which gave rise to a long series of forward-reverse shock pairs associated with CIRs (Bame et al. 1993; Balogh et al. 1995). We concentrate on the period from mid-1992 (latitude S13°) to late 1993 (latitude S41°), because of the low influence of transient events (Desai et al. 1997), due to the decline in solar activity. We considered hourly averages both for electron and for proton fluxes obtained from the CDAWeb service of the National Space Science Data Center.[2]

A first event is shown in Figure 1, where the energetic particle fluxes are reported for the CIR of 1993 January 19–22. *Ulysses* was at a heliocentric distance of 5.01 AU and at a latitude of S25°. In this case the particle profile is particularly broad after the reverse shock at 02:57 of January 22. We can see that the electron fluxes vary by slightly more than 1 order of magnitude. In the semilog plots, power-law profiles are evidenced by lines that have upward concavity. Several bumps with timescales of 20–30 hr are seen in these time profiles upstream of the reverse shock, as well as in the following events, at all energy channels and also in proton profiles. As shown by Neugebauer et al. (2006) these irregularities in the energetic particle profiles are due to the low-frequency magnetic turbulence, normally present in the solar wind, which causes changes in the magnetic connection between the shock and the spacecraft. In Horbury et al. (1996) the breakpoint frequency of magnetic fluctuations as a function of heliocentric distance and latitude was analyzed. The reported breakpoint frequencies at 4–5 AU correspond to correlation times in the range of 10–30 hr (Horbury et al. 1996), which matches the characteristic temporal changes in the energetic particle profiles. Similar bumps were found by Dröge (2003) in the *Wind* electron profiles; he indicates local effects in the magnetic field structure as a possible cause of the bumps.

To better appreciate the power-law scaling of the energetic particle fluxes, we plot them in log-log axes, considering the logarithm of the observation time upstream of the shock minus the time of the shock crossing t_{sh}, i.e., $\log(|t - t_{sh}|)$. Figure 2

[2] See http://cdaweb.gsfc.nasa.gov.

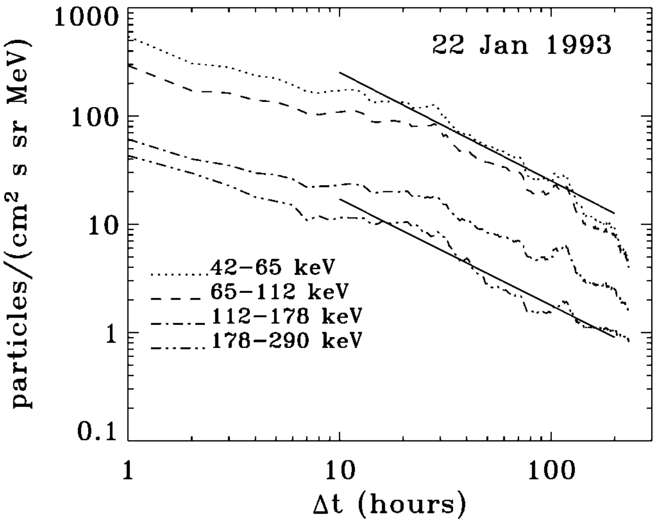

FIG. 2.—Electron fluxes upstream of the reverse shock of 1993 January 22 in log-log scale. Fits of the electron time profiles yield a power-law index of $\gamma \simeq 0.81$–0.98 (see Table 1), which implies superdiffusion with $\langle \Delta x^2(t) \rangle \simeq t^{1.02}$–$t^{1.19}$. Energy channels are indicated.

TABLE 1

ELECTRON TIME PROFILE FIT PARAMETERS

Date	Energy (keV)	γ	χ^2_{pl}	χ^2_e
1993 January 22 ……	42–65	1.0017 ± 0.0002	1.42	1.76
	65–112	0.92 ± 0.02	0.90	1.21
	112–178	0.81 ± 0.03	0.17	0.33
	178–290	0.98 ± 0.05	0.11	0.33
1993 May 10 ………	42–65	0.71 ± 0.08	0.10	0.03
	65–112	0.62 ± 0.07	0.03	0.11
	112–178	0.69 ± 0.08	0.03	0.15
	178–290	0.85 ± 0.08	0.07	0.18

reports the electron fluxes for the considered energy channels, with the solid lines representing the corresponding power-law fits. For clarity, not all of the fitting lines have been plotted. Denoting the energetic particle flux by J, which is proportional to the omnidirectional distribution function $f(x, E, t)$, we assume for the fit $J = A(\Delta t)^{-\gamma}$, with $\Delta t = |t - t_{sh}|$. The results of the fit, compared to those corresponding to an exponential decay obtained for normal diffusion, are reported in Table 1. Fits have been made in the tails of the electron distribution functions where equation (1) is valid. In addition, considering that the analyzed data are counting, we set the errors for the y-axis values to $\sqrt{y}$ (Poissonian statistics). It can be seen that for the power-law fit, values of the reduced χ^2, χ^2_{pl}, are much less than those of χ^2_e for the exponential decay, for most cases. Note that the power-law behavior is obtained over more than one decade in particle flux and over almost 200 hr in time, so the variations due to the turbulence do not appreciably influence the fit. For this event, values of $\gamma = 0.81$–0.98 imply $\mu = \gamma + 2 = 2.81$–2.98, that is, superdiffusion with $\langle \Delta x^2(t) \rangle \simeq t^{4-\mu} = t^{1.02}$–$t^{1.19}$.

Electron time profiles in log-log scale for a second event are shown in Figure 3. This event is associated to the CIR of 1993 May 10. *Ulysses* was at a heliocentric distance of 4.73 AU and a latitude of S30°. In this case only the reverse shock at 19:17 of May 10 was observed. We have found that the time profiles of energetic electrons observed by *Ulysses*, with energies between 42 and 290 keV, correspond to power laws, with slopes $\gamma \simeq 0.62$–0.85, implying superdiffusive transport with $\langle \Delta x^2(t) \rangle \simeq t^{1.15}$–$t^{1.38}$. These results show that the propagation of energetic particles in the turbulent environment of the solar wind is intermediate between diffusive and ballistic (or scatter free; Lin 2005; Pommois, Zimbardo, & Veltri 2007).

We made the same analysis for the proton time profiles and we found that their propagation corresponds mostly to normal diffusion, even if, in certain events, the decay of the particle flux in the tails of the distribution exhibits a power-law shape with an exponent $\gamma > 1$. The different transport properties of electrons and protons can be understood in terms of the Larmor radii and the consequent different interaction with magnetic turbulence. At 5 AU, assuming a magnetic field $B = 2$ nT, Larmor radii of the electrons at the analyzed energy channels range from $\rho_e = 80$ km to 360 km, while those of protons range from $\rho_p = 56{,}000$ km to 125,000 km. Magnetic turbulence in the solar wind exhibits a Kolmogorov-like spectrum, $\delta B^2(k) \simeq k^{-5/3}$ down to the dissipation scale λ_{diss}, which is usually assumed to be of the order of the thermal proton gyroradius ($\simeq 200$ km), below which the spectrum becomes much steeper. Introducing the resonant wavenumber $k_e = 1/\rho_e$ ($k_p = 1/\rho_p$) for electrons (protons), we can estimate $\delta B^2(k_e)/\delta B^2(k_p) \simeq (k_p/k_e)^{5/3} \simeq (\rho_e/\rho_p)^{5/3} \simeq 10^{-4}$; if the electron gyroradius is smaller than the thermal proton gyroradius, an even lower fluctuation level is sampled. The rate of pitch-angle diffusion depends both on the magnetic turbulence level and on the particle rigidity: electrons see weaker levels of turbulence; however, they can be scattered more easily owing to their low rigidity. Our results of superdiffusive transport can be understood considering that electrons can resonate in the dissipation range of turbulence where the spectrum has a fast decay, and this leads to a further decrease in pitch-angle diffusion. On the other hand, it has been shown by numerical simulations (Pommois, Zimbardo, & Veltri 2007) that parallel superdiffusive transport is possible either if the turbulence level is low or if the gyroradius is small, so establishing the dominant physical process that causes superdiffusion requires further investigations.

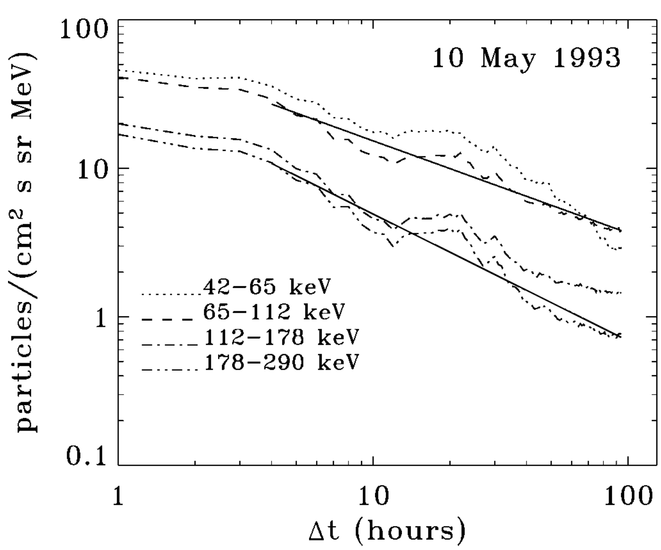

FIG. 3.—Electron fluxes upstream of the reverse shock of 1993 May 10, in log-log axes. For clarity, in this plot the electron fluxes of different channels have been displaced by factors equal to $\sqrt{10}$. A fit of the electron time profiles yields a power-law index of $\gamma \simeq 0.62$–0.85 (see Table 1), which also gives $\mu = 2.62$–2.85 and superdiffusion with $\langle \Delta x^2(t) \rangle \simeq t^{1.15}$–$t^{1.38}$.

In this Letter, by analyzing *Ulysses* electron data at 5 AU, we have shown that the energetic electron fluxes are well fitted by a power-law decay with a slope $\gamma \simeq 0.62$–0.98 over a period of 100–200 hr rather than by an exponential decay; the power-law profile is related to a mean square displacement growing faster than linearly in time, i.e., to a superdiffusive regime. Since superdiffusion allows a faster escape of particles from the shock region, the efficiency of the diffusive shock acceleration should be decreased. We argue that, at least for electrons, shock acceleration should be reformulated either in terms of non-Gaussian probability distributions or in terms of fractional Fokker-Planck equations (Metzler & Klafter 2000; Zaslavsky 2002). Indeed, in this case the concept of parallel mean free path (Ellison, Jones, & Baring 1999; Lee 2005), which is used to asses the DSA efficiency, is not useful owing to a divergence of the second-order moment of the jump length distribution (Klafter, Blumen, & Shlesinger 1987; Zaslavsky et al. 1989; Klafter, Shlesinger, & Zumofen 1996). Our results emphasize the importance of studying alternative acceleration mechanisms without pitch-angle scattering (Kuramitsu & Krasnoselskikh 2005; Jokipii & Giacalone 2007). The possibility of superdiffusion represents a new tool for understanding the propagation properties of SEPs and of energetic particles accelerated via the interaction with compressive zones in the heliosphere. In addition, it is useful in space weather forecasts because energetic particles can be used as a proxy for the arrival of strong solar disturbances on the geospace environment.

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REFERENCES

Balogh, A., et al. 1995, Space Sci. Rev., 72, 171
Bame, S. J., et al. 1993, Geophys. Res. Lett., 20, 2323
Bell, A. R. 1978, MNRAS, 182, 147
Blandford, R. D., & Eichler, D. 1987, Phys. Rep., 154, 1
Blandford, R. D., & Ostriker, J. P. 1978, ApJ, 221, L29
del-Castillo-Negrete, D., Carreras, B. A., & Lynch, V. E. 2004, Phys. Plasmas, 11, 3854
Desai, M. I., et al. 1997, Proc. 25 Int. Cosmic Ray Conf. (Potchefstroom), 365
Dröge, W. 2003, ApJ, 589, 1027
Ellison, D. C., Jones, F. C., & Baring, M. G. 1999, ApJ, 512, 403
Fisk, L. A., & Lee, M. A. 1980, ApJ, 237, 620
Giacalone, J., & Jokipii, J. R. 1999, ApJ, 520, 204
Gosling, J. T., Hundhausen, A. J., & Bame, S. J. 1976, J. Geophys. Res., 81, 2111
Hillas, A. M. 2005, J. Phys. G, 31, R95
Horbury, T. S., et al. 1996, A&A, 316, 333
Jokipii, J. R. 1966, ApJ, 146, 480
Jokipii, J. R., & Giacalone, J. 2007, ApJ, 660, 336
Klafter, J., Blumen, A., & Shlesinger, M. F. 1987, Phys. Rev. A, 35, 3081
Klafter, J., Shlesinger, M. F., & Zumofen, G. 1996, Phys. Today, 49, 33
Kuramitsu, Y., & Krasnoselskikh, V. 2005, Phys. Rev. Lett., 94, 031102
Lee, M. A. 1983, J. Geophys. Res., 88, 6109
———. 2005, ApJS, 158, 38
Lin, R. P. 2005, Adv. Space Res., 35, 1857
McKibben, R. B. 2005, Adv. Space Res., 35, 518
Medvedev, M. V., & Melott, A. L. 2007, ApJ, 664, 879
Metzler, R., & Klafter, J. 2000, Phys. Rep., 339, 1
Neugebauer, M., et al. 2006, J. Geophys. Res., 111, A12107
Perri, S., et al. 2007, Europhys. Lett., 78, 40003
Pommois, P., Zimbardo, G., & Veltri, P. 2007, Phys. Plasmas, 14, 012311
Qin, G., Matthaeus, W. H., & Bieber, J. W. 2002, Geophys. Res. Lett., 29, 1048
Ratynskaia, S., et al. 2006, Phys. Rev. Lett., 96, 105010
Reames, D. V. 1999, Space Sci. Rev., 90, 413
Ruffolo, D. 2006, ApJ, 639, 1186
Solomon, T. H., Weeks, E. R., & Swinney, H. L. 1993, Phys. Rev. Lett., 71, 3975
Stone, E. C., et al. 2005, Science, 309, 2017
Webb, G. M., et al. 2006, ApJ, 651, 211
Zaslavsky, G. M. 2002, Phys. Rep., 371, 461
Zaslavsky, G. M., et al. 1989, Soviet Phys.—JETP, 68, 995
Zhang, M., et al. 2003, J. Geophys. Res., 108, 1154
Zimbardo, G., Pommois, P., & Veltri, P. 2006, ApJ, 639, L91
Zumofen, G., & Klafter, J. 1993, Phys. Rev. E, 47, 851