The Astrophysical Journal, 654: L37–L40, 2007 January 1

A PHYSICAL FRAMEWORK FOR GRAND UNIFICATION OF GALAXIES AND ACTIVE GALACTIC NUCLEI. I. ORIGIN OF THE BLACK HOLE MASS–BULGE VELOCITY DISPERSION RELATION

Renyue Cen[1]

Received 2006 October 6; accepted 2006 November 16; published 2006 December 20

ABSTRACT

It is shown that if gas accretion via a disk onto the central supermassive black hole is efficient only for a surface density $\Sigma \geq 10$ g cm^{-2}, the black hole mass–galactic bulge velocity dispersion relation (Tremaine et al.) is borne out and so may be the modest dispersion in that relation, in the context of hierarchical structure formation theory. The relation is not expected to evolve with redshift in this model.

Subject headings: accretion, accretion disks — black hole physics — galaxies: formation — stars: formation

1. INTRODUCTION

Both the observed correlation between the mass of the central supermassive black hole (SMBH) and the velocity dispersion of the host galactic bulge, and the small dispersion about that correlation (Tremaine et al. 2002), are intriguing and not well understood. Several interesting models have been offered to possibly provide an explanation for such a relation (Silk & Rees 1998; Ostriker 2000; Adams et al. 2001; Colgate et al. 2003). In this Letter we provide an alternative model and make the case that, if there is a critical surface density for accretion disks at $\Sigma \sim 10$ g cm^{-2}, then the exact observed relation as well as the modest dispersion in the relation can be obtained in the cold dark matter model.

2. SYNCHRONOUS GROWTH OF SUPERMASSIVE BLACK HOLE AND BULGE

During a significant merger event, gravitational torques drive a significant amount of gas toward the central region (Barnes & Hernquist 1991; Mihos & Hernquist 1996). What is the density run of the resulting gas disk?

Simulations have shown that $M(<j) \propto j$ for dark matter halos (Bullock et al. 2001; Cen et al. 2004) at the low-j end, where $M(<j)$ is the amount of matter with specific angular momentum smaller than j. It seems reasonable to expect that such a relation may be extended to gas in halos, since both dark matter and gas are subject to largely the same gravitational forces (van den Bosch et al. 2002). When the gas in the small j end is channeled to the central region during the merger and cools, with lower j gas self-adjusting to settle at smaller radii, it would produce a self-gravitating gas disk whose surface density runs as

$$\Sigma(r) = \Sigma_0 (r/r_0)^{-1}. \tag{1}$$

This occurs when a pre-existing gravitational potential well is small compared to that produced by the newly formed gas disk. On the other hand, if the newly formed gas disk only incrementally fortifies an existing gravitational potential well, the gas disk should also follow equation (1) in a steady state, since the existing gravitational potential well corresponds to a flat rotation curve. Thus, it seems that a flat rotation curve may be maintained in the galactic bulge through a sequence of largely random mergers. Indeed, observations indicate that the rotation curves in galactic bulges are nearly flat within a factor of 2 over a radial span of three to four decades (Sofue & Rubin 2001), except in the very inner region within the sphere of influence of the central SMBH. These considerations lead us to conclude that it may be assumed, with good accuracy, that the velocity dispersion σ within the bulge region is constant and the infalling gas from each merger event produces a gas disk of a surface density run governed by equation (1). The exact value of Σ_0 at a fixed r_0 will depend on the strength of the merger event. We note that both Σ_0 and r_0 cancel out in the final expression that we derive below.

Following a merger, when the disk gas within a physical radius r_0 forms into stars (except the very inner small disk which will be assumed to accrete mostly onto the SMBH and is a very small fraction of the overall mass), an amount of stellar mass equal to

$$\Delta M_* = 2\pi \Sigma_0 r_0^2 \tag{2}$$

will be added to the bulge within r_0.

Let us now propose that only the gas in the inner region where the surface density exceeds Σ_c at a radius $r_c(\Sigma_0, r_0)$ will be accreted onto the central SMBH. It can then be shown that the following mass,

$$\Delta M_{\rm BH} = \frac{(\Delta M_*)^2}{2\pi r_0^2 \Sigma_c}, \tag{3}$$

will be added to the SMBH due to gas accretion. To make things simple, we assert that $\Delta M_{\rm BH}$ in equation (3) holds, even if r_c is smaller than the Bondi radius, within which SMBH may start to dominate the gravitational potential well and the rotation velocity rises. We assume that in this case, instead, that there will be an initial, unsteady state of gas accretion onto the SMBH, until a steady state is reached that conforms to the rotation velocity profile. In a hierarchical cold dark matter structure formation model (Spergel et al. 2006) galaxies grow through mergers and acquisitions (Lacey & Cole 1993; Kauffmann et al. 1993). Equation (3) suggests that the central SMBH grow synchronously with the bulges of galaxies, albeit at a different rate: the growth of black holes is more heavily facilitated by large mergers. Integrating equation (3) yields

$$M_{\rm BH} = \frac{\sum (\Delta M_*)^2}{2\pi r_0^2 \Sigma_c} + C, \tag{4}$$

[1] Princeton University Observatory, Princeton University, Princeton, NJ; cen@astro.princeton.edu.

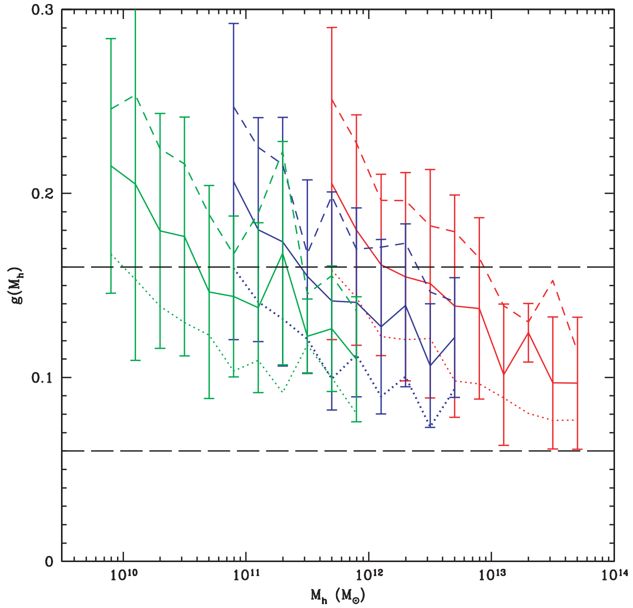

FIG. 1.—$g(M)$ as a function of halo mass M_h. We assume that each merger channels an amount of gas $\Delta M_* \propto M_2 R$ for $R \equiv M_1/M_2 \geq R_{\rm th}$ and zero otherwise, where M_1 and M_2 denote the mass of the small and large halos of the merging pair, respectively. Three simulations are run each with 512^3 particles, in a box of size (50, 25, 12.5) Mpc h^{-1} comoving, respectively, shown from right to left in red, blue, and green. The dotted, solid, and dashed sets of curves have $R_{\rm th} = (0.1, 0.2, 0.3)$, respectively. The 1 σ variance is displayed only for the case with $R_{\rm th} = 0.2$. The three simulations (and some additional ones, not shown here) are run to test resolution effects. It is clear that the upturn toward small mass for each curve is resolution effect; low-mass ratio merger events in small halos and hence $g(M_h)$ are underestimated due to the resolution effect. The two horizontal long-dashed lines approximately bracket the range of $g = 0.11 \pm 0.05$. The standard cold dark matter model with a cosmological constant using the parameters determined by WMAP3 (Spergel et al. 2006) is used.

where $\sum (\Delta M_*)^2$ denotes the sum of the squares of stellar mass increments in the bulge interior to r_0 from mergers over the entire growth history. Since the initial mass of the central black hole is zero or small, if there is a small seed black hole, the integration constant C is, for all practical purposes, zero. We rewrite equation (4) as

$$M_{\rm BH} = \frac{g(M_*)M_*^2}{2\pi r_0^2 \Sigma_c}, \qquad (5)$$

where we have defined

$$g(M_*) \equiv \frac{\sum (\Delta M_*)^2}{M_*^2}, \; M_* \equiv \sum \Delta M_*, \qquad (6)$$

where M_* is the stellar mass within r_0 at the redshift under consideration. Note that the left-hand side of equation (5) automatically includes contributions of pre-existing BHs in merging galaxies. Equation (5) may be cast into the following form:

$$M_{\rm BH} = \frac{2g(M_*)\sigma^4}{\pi G^2 \Sigma_c} = 1.3 \times 10^8 \left(\frac{\sigma}{200 \text{ km s}^{-1}}\right)^4 \times \left(\frac{\Sigma_c}{10 \text{ g cm}^{-2}}\right)^{-1} \left(\frac{g(M_*)}{0.11}\right) M_\odot, \qquad (7)$$

where σ is 1-d velocity dispersion of the bulge and G is the gravitational constant. Equation (7) would be in remarkably good agreement with the observed one, $M_{\rm BH} = (1.3 \pm 0.2) \times 10^8 (\sigma/200 \text{ km s}^{-1})^{4.02 \pm 0.32} M_\odot$ (Tremaine et al. 2002), if the two parameters, g and Σ_c, in the equation have the fiducial constant values given.

All cosmological uncertainties in $M_{\rm BH}$ have now been condensed into $g(M_*)$, which depends on the assembly history of a bulge, for a given M_*; i.e., $g(M_*)$ may not only depend on M_* but also have multiple values (i.e., a dispersion) at a fixed M_*. Since $g(M_*)$ is not easily computable, we instead make the assumption that the amount of gas driven toward the central region in a merger is proportional to the strength of a merger event times the host mass. A precise definition of the merger strength is, however, difficult to pin down. The mass ratio of the merger pair, $R = M_1/M_2$ (where M_1 and M_2 denote the mass of the small and large halo of the merging pair, respectively), impact parameter, orbital inclination, initial orbital energy, the sizes of the halos, a pre-existent bulge, etc., may all play a role to a varying extent (Mihos & Hernquist 1996). Instead, we simply use R as a proxy to characterize a merger strength. Since the neglected factors that may be involved, such as the orbital inclination and a pre-existing bulge, may largely behave like random variables in the overall growth history of a galaxy, it may be a good approximation to absorb all factors into R, which is known to be important from simulations in terms of driving gas inward, if not the most important. As shown below, the exact definition of the merger strength in terms of R do not appear to matter as far as g is concerned, hence justifying our simplified approach. We also assume that there is a threshold (lower bound), $R_{\rm th}$, in order to drive a significant amount of gas toward the central region. Again, it turns out that the results do not materially depend on $R_{\rm th}$. We would like to point out that a merger threshold may be operating, since bulgeless spiral galaxies do exist, indicating that the growth of a galaxy as a whole does not necessarily result in a bulge. In our picture, bulgeless spiral galaxies would have grown through mergers of strength below $R_{\rm th}$ and accretion.

We use the simulation-based merger-tree method of Monaco et al. (2002) called "Pinocchio" (PINpointing Orbit-Crossing Collapsed HIerarchical Objects) to compute the merger histories of halos in the standard cold dark matter model with a cosmological constant using the parameters determined by WMAP3 (Spergel et al. 2006): $\Omega = 0.27$, $\Lambda = 0.73$, $H_0 = 72$ km s^{-1} Mpc^{-1}, $n_s = 0.95$, $\sigma_8 = 0.77$. Three simulations are run, each with 512^3 particles, in a box of size (50, 25, 12.5) Mpc h^{-1} comoving, respectively. Each simulation outputs a list of halos at any specified redshift, where each halo is associated with a linked list (i.e., a merger tree) that allows one to trace back every merger event in its past with detailed information including the merger ratio R among others. Because $g(M_h)$ is expected to depend sensitively on mass resolution for low-mass halos and box size for large-mass halos, we present three simulations of different resolution and box sizes to illustrate such effects. Some additional simulations were also made to help understand these effects.

Figure 1 shows $g(M_h)$ as a function of halo mass M_h, where we assume that each merger channels an amount of gas $\Delta M_* \propto M_2 R$. In Figure 2 we use a different dependence of the amount of gas that is driven to the center on the merger mass ratio, $\Delta M_* \propto M_2 R^2$. We see that $g(M_h)$ depends weakly on M_h. The dependences of $g(M_h)$ on both R and $R_{\rm th}$ are also weak. Note that each curve from each simulation box tends to rise at the low halo mass end. We understand that this is a resolution effect, because low-mass ratio merger events in small

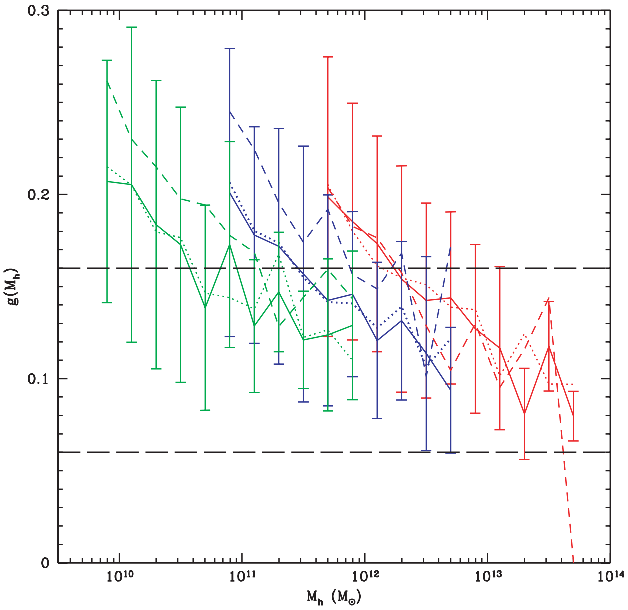

Fig. 2.—Similar to Fig. 1, except that we assume each merger channels an amount of gas $\Delta M_* \propto M_2 R^2$.

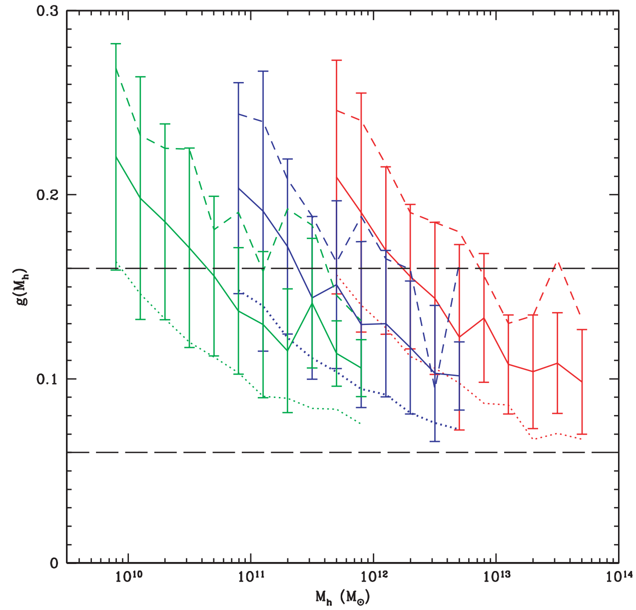

Fig. 3.—$g(M)$ as a function of halo mass M_h at $z = 0, 1, 2$ with dotted, solid, and dashed curves, respectively, all with $R_{\rm th} = 0.2$, with the assumption that $\Delta M_* \propto M_2 R$.

halos and hence $g(M_h)$ are underestimated due to resolution effect. Since the overall stellar mass increase in the bulge is linearly proportional to the stellar mass increase ΔM_* while the increase in SMBH mass depends quadratically on ΔM_*, g should artificially rise if small ΔM_* are not accounted for, hence the rise of g at low-mass end in Figure 1 (and Figs. 2 and 3). Our extensive testing verified this, which is easily seen in Figure 1 by comparing the results from three shown simulation boxes. Therefore, the rise of g at the low-mass end is an artifact.

The results found are consistent with g being constant, $g = 0.11 \pm 0.05$ (Figs. 1 and 2, *horizontal long-dashed lines*), bracketing the computed range of halo masses where merger-tree simulation results are deemed to be reliable and for $R_{\rm th} = 0.1$–0.3. The relatively small dispersion in g and its near constancy with respect to M_h may be reflective of temporal smoothing of the growth of a black hole/bulge by a sequence of largely "random" merger events of varying strengths and a relatively featureless cold dark matter power spectrum. In this sense of "randomness," our working assumption that the overall growth history of the bulge and SMBH in a halo, on average, may be approximated to be dependent only of R, may be justified, as long as other likely factors, such as impact parameter, orbital inclination, initial orbital energy, the sizes of the halos, a pre-existent bulge, etc., are random variables over the history.

Equation (7) also indicates that the dispersion in the mass of SMBH at a fixed bulge velocity dispersion may also be contributed by a dispersion in Σ_c, which may be of astrophysical origin. The fact that the observed dispersion in $M_{\rm BH}$-σ relation is no larger than ~0.3 dex (Tremaine et al. 2002) implies that the uncertainty Σ_c should be no larger than a factor of ~2. In other words, it seems that whatever physics dictates Σ_c operates in a precise fashion. We see that if $\Sigma_c \sim 10$ g cm^{-2}, the observed $M_{\rm BH}$-σ relation (Tremaine et al. 2002) is produced.

The derived result, equation (7), appears to represent a "fundamental" line, relating $M_{\rm BH}$ to a dynamical quantity of the bulge, in this case, the velocity dispersion σ. It might be that, if one were to express $M_{\rm BH}$ against, say, the total stellar mass in the bulge M_*(tot), the scatter would be significantly larger. This seems to be expected given the nonlinear dependence of $\Delta M_{\rm BH}$ on ΔM_* (eq. [3]).

Figure 2 shows $g(M_h)$ for $z = (0, 1, 2)$. We see that g does not evolve significantly in this redshift range. We do not show results at higher redshifts due to the merger-tree simulation particle mass resolution effect and significantly larger cosmic variances. But there is no indication that g will not be constant and will be significantly different at higher redshifts. Therefore, it is expected that the local observed BH mass–bulge velocity dispersion relation (Tremaine et al. 2002) is expected to hold at all redshifts. This provides a unique and critical test of the model.

3. DISCUSSION AND CONCLUSIONS

We have proposed a simple model for the growth of SMBHs based on the hierarchical formation of galaxies and a critical assumption that gas accretion onto an SMBH is only effective for surface densities in excess of $\Sigma \sim 10$ g cm^{-2}. We show that the exact observed relation between the mass of the SMBH and the velocity dispersion of the bulge (Tremaine et al. 2002) is obtained in this model (eq. [7]). In addition, the small dispersion in the relation may also be expected in this model, although it is less certain due to a lack of clear understanding of the physics with regard to the required critical surface density. It appears that this $M_{\rm BH}$-σ relation represents a fundamental line, relating $M_{\rm BH}$ to the dynamic state of the bulge (the velocity dispersion σ). A definitive additional prediction is that the $M_{\rm BH}$-σ relation does not evolve with redshift in this model, which provides a test.

On the other hand, we show that, while the overall growth of the SMBH is roughly synchronous with the growth of their host galactic bulges, it has a different rate and in general the increase in mass of the SMBH is not necessarily proportional to the increase in bulge mass. As a result, it is expected that the correlation between the mass of the SMBH and the overall bulge mass may display a much large scatter, consistent with observations (Magorrian et al. 1998; Ferrarese & Merritt 2000).

The physical origin for such a critical surface density in the standard accretion disk (Shakura & Sunyaev 1973) is, however, still unclear. A general requirement may be that gas accretion onto the central SMBH is dominant over star formation at $\Sigma \geq 10$ g cm^{-2} and the reverse is true at $\Sigma \leq 10$ g cm^{-2}. It is interesting to note that $\Sigma_c \sim 10$ corresponds to an optical depth of ∼10. Therefore, Σ_c may be directly or indirectly related to a transition from an optically thin to optically thick disk. The accretion disk model based on the large-scale vortices of the Rossby vortex instability may offer one solution (Colgate et al. 2003). The possible existence of an unstable disk at some large radii, which is expected to be prone to star formation, may offer another solution (Goodman 2003). Further investigations are warranted.

I thank Rashid Sunyaev for a stimulating conversation and an anonymous referee for constructive comments. I am grateful to Pierluigi Monaco for providing a merger tree code (Pinocchio) and useful correspondences regarding the usage of the code. This work is supported in part by grants NNG05GK10G and AST 05-07521.

REFERENCES

Adams, F. C., Graff, D. S., & Richstone, D. O. 2001, ApJ, 551, L31
Barnes, J., & Hernquist, L. 1991, ApJ, 370, L65
Bullock, J. S., Dekel, A., Kolatt, T. S., Kravtsov, A. V., Klypin, A. A., Porciani, C., & Primack, J. R. 2001, ApJ, 555, 240
Cen, R., Dong, F., Bode, P., & Ostriker, J. P. 2004, ApJ, submitted (astro-ph/0403352)
Colgate, S. A., Cen, R., Li, H., Currier, N., & Warren, M. S. 2003, ApJ, 598, L7
Ferrarese, L., & Merritt, D. 2000, ApJ, 539, L9
Goodman, J. 2003, MNRAS, 339, 937
Kauffmann, G., White, S. D. M., & Guiderdoni, B. 1993, MNRAS, 264, 211
Lacey, C., & Cole, S. 1993, MNRAS, 262, 627
Magorrian, J., et al. 1998, AJ, 115, 2285
Mihos, J. C., & Hernquist, L. 1996, ApJ, 464, 641
Monaco, P., Theuns, T., & Taffoni, G. 2002, MNRAS, 331, 587
Ostriker, J. P. 2000, Phys. Rev. Lett., 84, 5258
Shakura, N. I., & Sunyaev, R. A. 1973, A&A, 24, 337
Silk, J., & Rees, M. J. 1998, A&A, 331, L1
Sofue, Y., & Rubin, V. 2001, ARA&A, 39, 137
Spergel, D. N., et al. 2006, ApJ, submitted (astro-ph/0603449)
Tremaine, S., et al. 2002, ApJ, 574, 740
van den Bosch, F. C., et al. 2002, ApJ, 576, 21