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ELECTRON-POSITRON PAIR PRODUCTION IN THE ELECTROSPHERE OF QUARK STARS

T. HARKO AND K. S. CHENG
Department of Physics, University of Hong Kong, Pok Fu Lam Road, Hong Kong, Hong Kong SAR, China;
harko@hkucc.hku.hk; hrspksc@hkucc.hku.hk
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ABSTRACT

We study Schwinger pair creation of charged particles due to the inhomogeneous electric field created by the thin electron layer at the surface of quark stars (the electrosphere). As suggested earlier, due to the low photon emissivity of the quark-gluon plasma and of the electrosphere, electron-positron pair emission could be the main observational signature of quark stars. To obtain the electron-positron pair creation rate we use the tunneling approach. Explicit expressions for the fermion creation rate per unit time per unit volume are derived, which generalize the classical Schwinger result. The finite-size effects in pair production, due to the presence of a boundary (the surface of the quark star), are also considered in the framework of a simple approach. It is shown that the boundary effects induce large quantitative and qualitative deviations of the particle production rate from what one deduces with the Schwinger formula and its generalization for the electric field of the electrosphere. The electron-positron pair emissivity and flux of the electrosphere of quark stars due to pair creation is considered, and the magnitude of the boundary effects for these parameters is estimated. Due to the inhomogeneity of the electric field distribution in the electrosphere and to the presence of the boundary effects, at high temperatures ($T \geq T_{\rm cr} \approx 0.1$ MeV) we find a lower electron-positron flux than previously estimated. The numerical value of the critical temperature $T_{\rm cr}$ depends on the surface potential of the star. We briefly consider the effect of the magnetic field on the pair creation process and show that the magnetic field can drastically enhance the pair creation rate.

Subject heading: elementary particles

1. INTRODUCTION

The Schwinger mechanism of pair production (Schwinger 1951), first proposed to study the production of electron-positron pairs in a strong and uniform electric field, has been applied to many problems in contemporary physics. Strong electromagnetic fields lead to two physically important phenomena: pair production and vacuum polarization. A strong electric field makes the quantum electrodynamic (QED) vacuum unstable, and consequently it decays by emitting a significant number of boson or fermion pairs (Greiner et al. 1985).

For a spin $\frac{1}{2}$ particle, Schwinger's predicted production rate per unit time and volume w is given by (Schwinger 1951; Soffel et al. 1982)

$$w(E_0) = eE_0 \int \frac{d^2k_i}{(2\pi)^2} \sum_{n=1}^{\infty} \frac{1}{n} \exp\left[-\frac{\pi n\left(m_e^2 + k_i^2\right)}{|eE_0|}\right], \tag{1}$$

where m_e and e are the electron mass and charge, respectively, k_i is the transverse momentum, and E_0 is the (constant) electric field. In this paper we use units so that $\hbar = c = k_{\rm B} = 1$, where $k_{\rm B}$ is Boltzmann's constant. In these units, e is equal to $\alpha^{1/2}$ and 1 MeV $= 5.064 \times 10^{-3}$ fm^{-1}.

In most of the physical applications the proper time method introduced by Schwinger (1951) has been used to calculate the pair production rate. The real part of the effective action leads to vacuum polarization and the imaginary part to pair production. Although that method is conceptually well defined and technically rigorous, it is generally difficult to apply it to concrete physical problems such as inhomogeneous electromagnetic fields.

Schwinger's result was generalized to electric fields $E_3 = E_3(x_\pm)$, which depend on either light cone coordinates $x_\pm = x_3 \pm x_0$, but not on both, in Tomaras et al. (2000). The form of the result is exactly the same as in the original formula given by equation (1). The case of electric fields depending on both x_+ and x_-, $E_3 = E_3(x_+, x_-)$, was considered in Avan et al. (2003).

An alternative approach to pair creation was initiated by Casher et al. (1979, 1980), who rederived Schwinger's pair production rate by semiclassical tunneling calculations. The boson and fermion pair production rate by strong static uniform or inhomogeneous electric fields was derived, in terms of instanton tunneling through potential barriers in the space-dependent gauge, in Kim & Page (2002).

The Schwinger mechanism for particle production in a strong and uniform electric field for an infinite system was generalized to the case in which the strong field is confined between two plates separated by a finite distance by Wang & Wong (1988). The production rates, obtained by solving the Klein-Gordon and Dirac equations in a linear vector potential, can be expressed in an exact analytical form. The numerical evaluations of the production rates have shown large deviations from the Schwinger formula, thus indicating a large finite-size effect in particle production. The explicit finite-size corrections to the Schwinger formula for the rate of pair production for uniform electric fields confined to a bounded region were calculated, by using the Balian-Bloch expansion of the Green functions, by Martin & Vautherin (1988, 1989).

One important astrophysical situation in which electron-positron pair creation could play an extremely important role is the case of the electrosphere of quark stars (Usov 1998a, 1998b, 2001; Page & Usov 2002). Quark stars could be formed as a result of a hadron-quark phase transition at high densities and/or temperatures (Itoh 1970; Bodmer 1971; Witten 1984). If the hypothesis of the quark matter is true, then some neutron stars could actually be strange stars, built entirely of strange matter (Alcock et al. 1986; Haensel et al. 1986). For a review of strange star properties, see Cheng et al. (1998a).

Several mechanisms have been proposed for the formation of quark stars. Quark stars are expected to form during the collapse of the core of a massive star after the supernova explosion, as a result of a first- or second-order phase transition, resulting in deconfined quark matter (Dai et al. 1995). The proto–neutron star core, or the neutron star core, is a favorable environment for the conversion of ordinary matter into strange quark matter (Cheng & Dai 1998; Cheng et al. 1998b). Another possibility is that some neutron stars in low-mass X-ray binaries can accrete sufficient mass to undergo a phase transition to become strange stars (Cheng & Dai 1996). This mechanism has also been proposed as a source of radiation emission for cosmological γ-ray bursts (Cheng & Dai 1996). Some basic properties of strange stars such as mass, radius, collapse, and nucleation of quark matter in neutron star cores have been also studied (Glendenning et al. 1995, 1997; Cheng & Harko 2000; Harko & Cheng 2000, 2002; Harko et al. 2004).

The structure of a realistic strange star is very complicated, but its basic properties can be described as follows (Alcock et al. 1986). Beta-equilibrated strange quark star matter consists of an approximately equal mixture of up (u), down (d), and strange (s) quarks, with a slight deficit of the latter. The Fermi gas of $3A$ quarks constitutes a single color-singlet baryon with baryon number A. This structure of the quarks leads to a net positive charge inside the star. Since stars in their lowest energy state are supposed to be charge neutral, electrons must balance the net positive quark charge in strange matter stars. The electrons, being bounded to the quark matter by the electromagnetic interaction and not by the strong force, are able to move freely across the quark surface, but clearly they cannot move to infinity because of the electrostatic attraction of quarks. For hot stars the electron distribution could extend up to $\sim 10^3$ fm above the quark surface (Kettner et al. 1995; Cheng & Harko 2003; Usov et al. 2005). The electron distribution at the surface of the quark star is called electrosphere. The effect of the color-flavor locked phase of strange matter on the electric field in the electrosphere was discussed by Usov (2004).

Photon emissivity is the basic parameter for determining macroscopic properties of stellar-type objects. Alcock et al. (1986) have shown that because of very high plasma frequency near the strange matter edge, photon emissivity of strange matter is very low. For temperatures $T \ll E_p$, where $E_p \approx 23$ MeV is the characteristic transverse plasmon cutoff energy, the equilibrium photon emissivity of strange matter is negligibly small, compared to the blackbody one. The spectrum of equilibrium photons is very hard, with $\omega > 20$ MeV (Chmaj et al. 1991).

The bremsstrahlung emissivity of the quark matter and of the electrosphere have been estimated recently in Cheng & Harko (2003), Jaikumar et al. (2004), and Harko & Cheng (2005). By taking into account the effect of the interference of amplitudes of nearby interactions in a dense medium (the Landau-Pomeranchuk-Migdal effect) and the absorption of the radiation in the external electron layer, the emissivity of the quark matter could be 6 orders of magnitude lower than the equilibrium blackbody radiation (Cheng & Harko 2003). The presence of the electric field strongly influences the radiation spectrum emitted by the electrosphere and strongly modifies the radiation suppression pattern (Harko & Cheng 2005). All the radiation properties of the electrons in the electrosphere essentially depend on the value of the electric potential at the quark star surface.

The Coulomb barrier at the quark surface of a hot strange star may also be a powerful source of e^-e^+ pairs, which are created in the extremely strong electric field of the barrier. In the case of energy loss via the production of e^-e^+ pairs, the energy flux is given by $F_\pm = \epsilon_\pm \dot{n}$, with $\epsilon_\pm = m_e + T$ and

$$\dot{n} = \left(\frac{9T}{2\pi\epsilon_{\rm F}^2}\right)\sqrt{\frac{\alpha}{\pi}}\exp\left(-\frac{2m_e}{T}\right)n_e T^2 J(\xi)\Delta r_E, \tag{2}$$

where $\epsilon_{\rm F} = (\pi^2 n_e)^{1/3}$ is the Fermi energy of the electrons, $\xi = 2(\alpha/\pi)^{1/2}(\epsilon_{\rm F}/T)$,

$$J(\xi) = \frac{1}{3}\frac{\xi^3 \ln(1+2\xi^{-1})}{(1+0.074\xi)^3} + \frac{\pi^5}{6}\frac{\xi^4}{(13.9+\xi)^4}, \tag{3}$$

and Δr_E is the thickness of the region with the strong electric field (Usov 1998a, 2001).

At surface temperatures of around 10^{11} K, the luminosity of the outflowing plasma may be of the order of $\sim 10^{51}$ ergs s^{-1} (Usov 1998a, 1998b). Moreover, as shown by Page & Usov (2002), for about 1 day for normal quark matter and for up to 100 yr for superconducting quark matter, the thermal luminosity from the star surface, due to both photon emission and e^-e^+ pair production, may be orders of magnitude higher than the Eddington limit. Hence, electron-positron pair creation due to the electric field of the electrosphere could be one of the main observational signatures of quark stars.

It is the purpose of this paper to consider the Schwinger process of pair creation in the electrosphere of the quark stars, by systematically taking into account the main physical characteristics of the environment in which pair production takes place. The electric field in the electrosphere is not a constant, as assumed in the Schwinger model, but it is a rapidly decreasing function of the distance z from the quark star surface. Therefore, in order to realistically describe the pair production process one must consider electron-positron creation in an inhomogeneous electric field. To study the pair production process we adopt the tunneling approach, and the fermion production rate in the electric field of the electrosphere is derived. The production rate is a local quantity that depends on the distance to the quark star's surface and is a rapidly decreasing function of z. The emissivity and energy flux due to pair creation at the quark star surface are also considered. An important factor that can significantly reduce the pair production rate is the presence of a boundary (the quark star surface). For electric fields perpendicular to the boundary there is a significant reduction in the magnitude of the pair production rate, which is also associated with an important qualitative change in the process, which becomes a periodic function of the distance to the quark star surface.

This paper is organized as follows. In § 2 we review the basic properties of the electrosphere of quark stars. The electron-positron rate production in the electric field of the electrosphere is obtained in § 3. The boundary effects are considered in § 4. In § 5 we calculate the electron-positron pair flux of the electrosphere and compare our results with those obtained by Usov (1998a, 1998b, 2001). A brief summary of our results is given in § 6.

2. STRUCTURE OF THE ELECTROSPHERE OF STRANGE STARS

In the electrosphere, electrons are held to the strange quark matter (SQM) surface by an extremely strong electric field. The thickness of the electrosphere is much smaller than the stellar radius $R \simeq 10^6$ cm, and a plane-parallel approximation can be used to study its structure (Usov et al. 2005). In this approximation all values depend only on the coordinate z, where the z-axis is perpendicular to the SQM surface ($z = 0$) and directed outward. To find the distributions of electrons and electric fields in the vicinity of the SQM surface, we use a simple Thomas-Fermi model considered by Alcock et al. (1986) and take into account the finite-temperature effects as discussed by Kettner et al. (1995).

This model can be solved exactly, and all physical quantities of interest (chemical potential, electric field, etc.) can be expressed in an exact analytical form (Cheng & Harko 2003; Usov et al. 2005). The chemical equilibrium in the electrosphere implies that the electron chemical potential μ_e satisfies the condition $-V(\infty) + \mu_e(\infty) = -V + \mu_e$, where V is the electrostatic potential per unit charge and $V(\infty)$ and $\mu_e(\infty)$ are the values of the electrostatic potential and of the electron's chemical potential at infinity, respectively. Since far outside the star both $V(\infty)$ and $\mu_e(\infty)$ tend to zero, it follows that $\mu_e = V$ (Alcock et al. 1986).

The structure of the electrosphere and the corresponding radiation processes essentially depend on the value of V_q, the electric charge density inside the quark star. When the temperature of the quark star core drops below 10^9 K, the strange matter becomes superfluid. At this temperature quarks can form colored Cooper pairs near the Fermi surface and become superconducting. From the Bardeen-Cooper-Schrieffer (BCS) theory it follows that the critical temperature T_c at which the transition to the superconducting state takes place is $T_c = \Delta/1.76$, where Δ is the pairing gap energy (Blaschke et al. 2000). An early estimation of Δ gave $\Delta \sim 0.1$–1 MeV (Bailin & Love 1984), but some recent studies considering instanton-induced interactions between quarks estimated $\Delta \sim 100$ MeV (Alford et al. 1998).

Strange quark matter in the color-flavor locked (CFL) phase of quantum chromodynamics (QCD), which occurs for $\Delta \sim 100$ MeV, could be rigorously electrically neutral, despite the unequal quark masses, even in the presence of the electron chemical potential (Alford et al. 1998). Hence, for the CFL state of quark matter $V_q = 0$ and no electrons are present inside or outside the quark star (Lugones & Horvath 2003).

However, Page & Usov (2002) pointed out that for sufficiently large m_s the low-density regime is rather expected to be in the "two-color-flavor superconductor" phase, in which only u and d quarks of two colors are paired in single condensate, while those of the third color, and s quarks of all three colors, are unpaired. In this phase, some electrons are still present. In other words, electrons may be absent in the core of strange stars but present, at least, near the surface where the density is lowest. Nevertheless, the presence of the CFL effect can reduce the electron density at the surface, and hence it can also significantly reduce the intensity of the electric field and the electromagnetic emissivity of the electrons in the electrosphere. Therefore, in order to describe the radiation properties of the electrosphere we assume that $V_q \neq 0$.

The Poisson equation for the electrostatic potential $V(z, T)$ generated by the finite-temperature electron distribution reads (Alcock et al. 1986; Kettner et al. 1995)

$$\frac{d^2V}{dz^2} = \frac{4\alpha}{3\pi}\left[\left(V^3 - V_q^3\right) + \pi^2\left(V - V_q\right)T^2\right], \quad z \leq 0, \tag{4}$$

$$\frac{d^2V}{dz^2} = \frac{4\alpha}{3\pi}\left(V^3 + \pi^2T^2V\right), \quad z \geq 0, \tag{5}$$

where T is the temperature of the electron layer, which can be taken as a constant, since we assume the electrons are in thermodynamic equilibrium with the constant-temperature quark matter. In equations (4)–(5), z is the space coordinate measuring height above the quark surface, α is the fine-structure constant, and $V_q/3\pi^2$ is the quark charge density inside the quark matter. The boundary conditions for equations (4)–(5) are $V \to V_q$ as $z \to -\infty$ and $V \to 0$ for $z \to \infty$. In the case of the zero-temperature electron distribution at the boundary $z = 0$ we have the condition $V(0) = \frac{3}{4}V_q$ (Alcock et al. 1986).

The general solution of equation (5) is given by (Cheng & Harko 2003)

$$V(z, T) = \frac{\sqrt{2}\pi T}{\sinh\left[2\sqrt{\alpha\pi/3}T(z + z_0)\right]}, \tag{6}$$

where z_0 is a constant of integration. Its value can be obtained from the condition of the continuity of the potential across the star's surface, requiring $V_q(0, T) = V(0, T)$, where $V_q(z, T)$ is the value of the electrostatic potential in the region $z \leq 0$, described by equation (4). Therefore,

$$z_0 = \frac{1}{2}\sqrt{\frac{3}{\alpha\pi}}\frac{1}{T}\,\mathrm{arcsinh}\left[\frac{\sqrt{2}\pi T}{V_q(0, T)}\right]. \tag{7}$$

The number density distribution n_e of the electrons at the quark star surface can be obtained from $n_e(z, T) = V^3/3\pi^2 + VT^2/3$ (Kettner et al. 1995; Cheng & Harko 2003) and is given by

$$n_e(z, T) = \frac{\sqrt{2}\pi}{3}\frac{1 + \cosh^2\left[2\sqrt{\alpha\pi/3}T(z + z_0)\right]}{\sinh^3\left[2\sqrt{\alpha\pi/3}T(z + z_0)\right]}T^3. \tag{8}$$

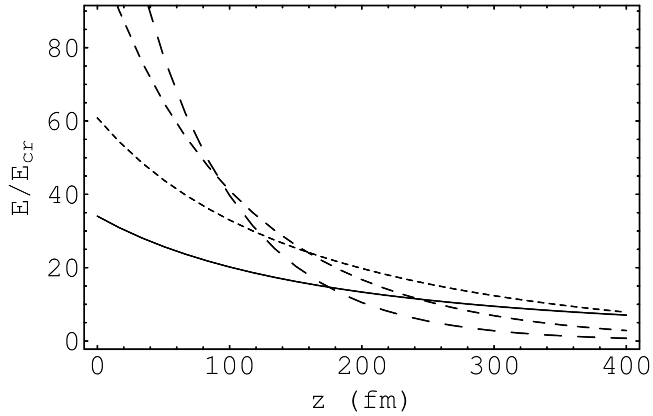

FIG. 1.—Ratio of the electric field of the electrosphere E and the critical electric field $E_{\rm cr} = m_e^2/e$ as a function of the distance z (in fermis) for different values of the temperature: $T = 0.01$ MeV (*solid curve*), $T = 5$ MeV (*dotted curve*), $T = 10$ MeV (*short-dashed curve*), and $T = 15$ MeV (*long-dashed curve*). In all cases $V_q(0, T) = 15$ MeV.

In the limit of zero temperature, $T \to 0$, we obtain $V(z) = a_0/(z + b)$, where $a_0 = (3\pi/2\alpha)^{1/2}$ and b is an integration constant that can be determined from the boundary condition $V(0) = \frac{3}{4}V_q$, which gives $b = (4a_0/3V_q)$. Therefore, in this case we find for the electron particle number distribution the expression $n_e(z) = (1/3\pi^2)a_0^3/(z + b)^3$.

In the absence of a crust of the quark star, the electron layer can extend to several thousand fermis outside the star's surface.

The strength of the electric field E outside the quark star surface is given by

$$E(z,T) = \sqrt{\frac{4}{3\pi}}V\sqrt{\frac{V^2}{2} + \pi^2 T^2} \tag{9}$$

and can be expressed as

$$E(z,T) = \sqrt{\frac{8\pi^3}{3}}\frac{\cosh\left[2\sqrt{\alpha\pi/3}T(z + z_0)\right]}{\sinh^2\left[2\sqrt{\alpha\pi/3}T(z + z_0)\right]}T^2. \tag{10}$$

The ratio of the electric field of the electrosphere of quark stars and of the critical electric field $E_{\rm cr} = m_e^2/e$ is represented, as a function of the distance from the quark star surface and for different values of the temperature, in Figure 1.

3. ELECTRON-POSITRON PAIR PRODUCTION RATES IN THE ELECTROSPHERE

Let us consider a quantized Dirac field coupled to a classical external electromagnetic field described by the potential A_μ. The probability P of remaining in the ground state; i.e., the probability of emitting no pairs, is given by $P = |\langle 0|S|0\rangle|^2$, where S is the S-matrix defined as $S = \hat{T}\exp(i\int L_I\, d^4x)$, $\hat{T}$ is the time-ordering operator, and L_I is the interaction Lagrangian (Soffel et al. 1982). The probability P can also be written as $P = \exp\left[-\int w(x)\, d^4x\right]$, where $w(x) = 2\,{\rm Im}\, L_{\rm eff}(x)$, where $L_{\rm eff}(x)$ is the one-loop effective Lagrangian density, which includes all orders in the external field but neglects self-interactions of the matter fields. The quantity $w(x)$ can be interpreted as the pair production rate per unit time and unit volume at the spacetime point $x = (x_0, x_1, x_2, x_3)$ (Greiner et al. 1985; Martin & Vautherin 1989).

An alternative description of the pair creation process can be obtained by assuming that the vacuum decays if there exists an ingoing antiparticle mode that is at the same time an outgoing particle mode. An in-vacuum is defined via the ingoing (anti)particle basis, ${}_{-}a_l\,|{\rm in, vac}\rangle = {}_{+}a_k\,|{\rm in, vac}\rangle = 0$, whereas the out-vacuum is defined by the outgoing states ${}^{-}a_l\,|{\rm out, vac}\rangle = {}^{+}a_k\,|{\rm out, vac}\rangle = 0$, where a are the particle creation and annihilation operators, respectively (Soffel et al. 1982). The number of the outgoing particles in the mode k created in the in-vacuum is $N_{kk'} \equiv \langle {\rm vac, in}|{}^{+}a_k^{+}\,{}^{+}a_{k'}|{\rm vac, in}\rangle = \sum_l |\beta_{kl}|^2\delta(k - k')$, where the coefficients β_{kl} are the elements of the single-particle S-matrix. Due to the validity of the energy and momentum conservation laws, one can generally choose $\beta_{kl} = \beta_k\delta_{kl}$, and thus the number of created particles in the mode k is simply given by $N_{kk'} = |\beta_k|^2\delta(k - k')$. The delta function can be reexpressed as a delta function over the frequencies of the associated modes, $\beta_{kl} = \mathcal{T}_k\delta(\omega_l - \omega_k)$. Then the rule $[\delta(\omega_l - \omega_k)]^2 \to (1/2\pi)\delta(\omega_l - \omega_k)\int dt$ can be used to calculate a continuous rate of creation of particle-antiparticle pairs (Soffel et al. 1982; Kim & Page 2002),

$$w = \frac{d}{dt}\langle N\rangle = \frac{1}{2\pi}\int |\mathcal{T}_k|^2\, d\omega. \tag{11}$$

The physical meaning of the particle creation process is the following: a wave packet of negative frequencies incident on an electric field oriented along the z-direction will be partly reflected by the electric field and partly transmitted to $z \to \infty$ as a wave with positive frequencies. This process is nothing other than tunneling (Soffel et al. 1982; Greiner et al. 1985; Wang & Wong 1988). However, in the following we restrict the tunneling probability to the transmission probability through the potential barrier but exclude any nonzero transmission probability above the potential barrier.

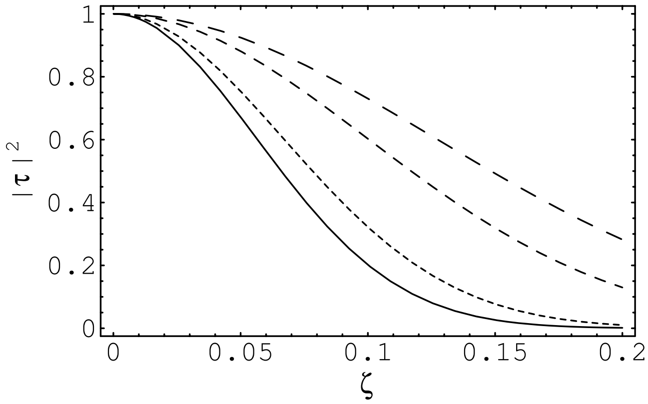

FIG. 2.—Transmission probability $|\mathcal{T}|^2 = \exp(-2\sigma)$ for electron-positron pair creation in the electrosphere of quark stars as a function of the parameter $\zeta = (m_e^2 + k_\perp^2)^{1/2}/\omega$ for different values of the temperature energy ratio: $2\pi^2T^2/\omega^2 = 0.1$ (*solid curve*), $2\pi^2T^2/\omega^2 = 1$ (*dotted curve*), $2\pi^2T^2/\omega^2 = 3$ (*short-dashed curve*), and $2\pi^2T^2/\omega^2 = 5$ (*long-dashed curve*).

In the Wentzel-Kramers-Brillouin (WKB) approximation the transmission probability $\mathcal{T}$ can be approximately given as $|\mathcal{T}|^2 = \exp\left(-2\int_{\rm barrier}|k_z|\,dz\right) = \exp(-2\sigma)$, where $\sigma = \int_{\rm barrier}|k_z|\,dz$ and k_z is the longitudinal momentum of the electron (Soffel et al. 1982; Wang & Wong 1988; Kim & Page 2002).

For a fixed frequency ω, the momentum k_z of an electron in the electrosphere of the quark stars is given by

$$k_z = \sqrt{m_e^2 + k_\perp^2 - [\omega - V(z)]^2}, \tag{12}$$

where $k_\perp^2 = k_x^2 + k_y^2$ (Soffel et al. 1982; Kim & Page 2002). The mean particle production number $\langle N\rangle$ is given by summing over all modes (Greiner et al. 1985),

$$\langle N\rangle = \int e^{-2\sigma}\frac{dt\,d\omega}{2\pi}\frac{dx\,dk_x}{2\pi}\frac{dy\,dk_y}{2\pi}. \tag{13}$$

In order to estimate the electron-positron pair production rate in the electrosphere, we first have to find the thickness of the classically forbidden zone or, equivalently, the limits of integration $z_\pm$ in the transmission probability. They can be obtained as solutions of the equation $k_z(z) = 0$ and are given by

$$z_\pm = \frac{1}{2}\sqrt{\frac{3}{\alpha\pi}}\frac{1}{T}\,\mathrm{arcsinh}\left(\frac{\sqrt{2}\pi T}{\omega \mp \sqrt{m_e^2 + k_\perp^2}}\right) - z_0. \tag{14}$$

Therefore, by taking into account the form of the electrostatic potential in the electrosphere we obtain for σ the expression

$$\sigma = \int_{z_-}^{z_+}\sqrt{m_e^2 + k_\perp^2 - \left\{\omega - \frac{\sqrt{2}\pi T}{\sinh\left[2\sqrt{\alpha\pi/3}\,T(z + z_0)\right]}\right\}^2}\,dz. \tag{15}$$

With the help of the transformation

$$\eta = \frac{1}{\sqrt{m_e^2 + k_\perp^2}}\left\{\omega - \frac{\sqrt{2}\pi T}{\sinh\left[2\sqrt{\alpha\pi/3}\,T(z + z_0)\right]}\right\},$$

we obtain the following integral representation for σ:

$$\sigma = \sqrt{\frac{3\pi}{2\alpha}}\frac{m_e^2 + k_\perp^2}{\omega^2}\int_{-1}^{1}\frac{\sqrt{1-\eta^2}\,d\eta}{\left[\left(\sqrt{m_e^2 + k_\perp^2}/\omega\right)\eta - 1\right]\sqrt{2\pi^2T^2/\omega^2 + \left[\left(\sqrt{m_e^2 + k_\perp^2}/\omega\right)\eta - 1\right]^2}}. \tag{16}$$

The variation of the transmission probability $|\mathcal{T}|^2 = \exp(-2\sigma)$ is represented, as a function of the parameter $\zeta = \left(m_e^2 + k_\perp^2\right)^2/\omega$ and for different values of the temperature and particle energy ratio $2\pi^2T^2/\omega^2$, in Figure 2. For large values of the particle energy,

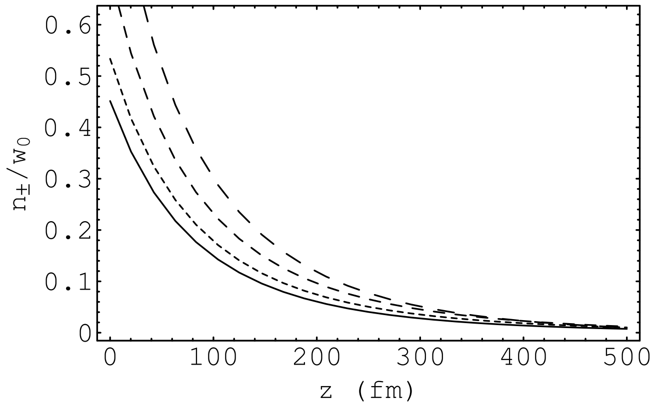

FIG. 3.—Ratio of the electron-positron pair production rate $n_{\pm}$ in the electrosphere and of the Schwinger rate w_0 for a constant electric field $E_0 = 50E_{\rm cr}$ as a function of the distance z (in fermis) for different values of the temperature: $T = 0.5$ MeV (*solid curve*), $T = 1.5$ MeV (*dotted curve*), $T = 2.5$ MeV (*short-dashed curve*), and $T = 3.5$ MeV (*long-dashed curve*). In all cases the surface electrostatic potential of the quark star is $V_q = 15$ MeV.

$\omega \gg (m_e^2 + k_\perp^2)^{1/2}$, $\zeta \to 0$, and the transmission probability is equal to 1, $|\mathcal{T}|^2 \to 1$. The transmission probability also increases with the temperature of the electrosphere.

In the process of particle production in the electrosphere of quark stars due to the tunneling from the negative energy state, a positive energy particle (an electron) is created, leaving a hole in the negative energy continuum, which can be taken to be an antiparticle (a positron) moving in a direction opposite the direction of the created particle. The created pair of particles is characterized by the energy (frequency) ω. For such a pair, one cannot, strictly speaking, specify a particular point as the location where the pair is produced (Wang & Wong 1988). One can only say that the pair of particles begins to emerge between the points z_- and z_+. Nevertheless, one can associate the point z, which is the solution of the equation $\omega = V(z)$, with the location in the vicinity of which a pair of particles is produced. With this approximate association, the energy of the produced particle is then identified by an approximate location, and the energy interval also can be approximately related to a spatial interval at which the pair of particles is produced via the relation $d\omega = (\partial V/\partial z)\, dz = eE(z, T)\, dz$.

Moreover, we introduce polar coordinates in the momentum space so that $k_x = k \cos\theta$ and $k_y = k \sin\theta$. Therefore, the electron-positron pair production rate $\dot{n}_{\pm}$, giving the number of electron-positron pairs created per unit time and per unit volume by the electric field of the electrosphere at the surface of quark stars, $\langle N\rangle/\Delta t \Delta x \Delta y \Delta z$, can be written as

$$\dot{n}_{\pm}(z, T) = \frac{s}{4\pi^2} eE(z, T) \int_0^\infty k \exp\left(-2\sqrt{\frac{3\pi}{2\alpha}} \frac{m_e^2 + k^2}{V^2(z, T)} \int_{-1}^{1} \frac{\sqrt{1-\eta^2}\left\{\left[\sqrt{m_e^2 + k^2}/V(z, T)\right]\eta - 1\right\}^{-1} d\eta}{\sqrt{2\pi^2 T^2/V^2(z, T) + \left\{\left[\sqrt{m_e^2 + k^2}/V(z, T)\right]\eta - 1\right\}^2}}\right) dk, \qquad (17)$$

where s describes the spin degrees of freedom of the produced particles ($s = 1$ for bosons and $s = 2$ for fermions).

Equation (17) has a general formal structure that is very similar to that of equation (1), describing the electron-positron pair production in a constant electric field. In both cases the pair production rate is proportional to the intensity of the electric field, multiplied by an integral over the transverse momentum of the particles. However, since in the electrosphere $E(z, T) \to 0$ for large z, the particle production rate also naturally tends to zero, and outside the electrosphere or in the regions with low electric field the particle production ceases. The variation of the pair production rate in the electrosphere, as a function of the distance z, is represented for different values of the temperature in Figure 3.

By introducing a new variable $\zeta = \left(m_e^2 + k^2\right)^{1/2}/V(z, T)$, the pair production rate can also be represented as

$$\dot{n}(z, T) = \frac{s}{4\pi^2} eE(z, T) V^2(z, T) \int_{m_e/V(z, T)}^\infty \zeta \exp\left\{-2\sqrt{\frac{3\pi}{2\alpha}}\zeta^2 \int_{-1}^{1} \frac{\sqrt{1-\eta^2}(\zeta\eta - 1)^{-1} d\eta}{\sqrt{\left[2\pi^2 T^2/V(z, T)^2\right] + (\zeta\eta - 1)^2}}\right\} d\zeta. \qquad (18)$$

In order to find an approximate representation of the pair production rate we power expand the integrand in equation (16). In the first order in η we obtain

$$\frac{\sqrt{1-\eta^2}}{\left[\left(\sqrt{m_e^2 + k_\perp^2}/\omega\right)\eta - 1\right]\sqrt{2\pi^2 T^2/\omega^2 + \left[\left(\sqrt{m_e^2 + k_\perp^2}/\omega\right)\eta - 1\right]^2}} \approx \frac{\omega}{\sqrt{2\pi^2 T^2 + \omega^2}} + \frac{\sqrt{m_e^2 + k_\perp^2}}{\sqrt{2\pi^2 T^2 + \omega^2}}\left(\frac{\omega^2}{2\pi^2 T^2 + \omega^2} + 1\right)\eta + O(\eta^2). \qquad (19)$$

Therefore, in the first order of approximation σ is given by

$$\sigma \approx \sqrt{\frac{3\pi}{\alpha}}\frac{m_e^2 + k_\perp^2}{\omega\sqrt{\omega^2/2 + \pi^2 T^2}}. \tag{20}$$

By fixing the electron energy so that $\omega = V(z, T)$ and taking into account the expression for the electric field in the electrosphere, given by equation (10), it follows that the electron-positron pair production rate can be written as

$$\begin{aligned}\dot{n}_\pm(z, T) &\approx \frac{2}{4\pi^2} eE(z, T) \int_0^\infty \exp\left(-2\sqrt{\frac{3\pi}{\alpha}}\frac{m_e^2 + k^2}{V\sqrt{V^2/2 + \pi^2 T^2}}\right) k\, dk \\ &\approx \frac{2}{4\pi^2} eE(z, T) \int_0^\infty \exp\left[-\pi\frac{m_e^2 + k^2}{eE(z, T)}\right] k\, dk \\ &\approx \frac{1}{4\pi^3} e^2 E^2(z, T) \exp\left[-\pi\frac{E_{\rm cr}}{E(z, T)}\right].\end{aligned} \tag{21}$$

This is exactly the leading term in the Schwinger formula for the pair creation in a constant electric field E_0 but with the constant electric field substituted with the inhomogeneous, z-dependent electric field in the electrosphere, $E_0 \to E(z, T)$. Therefore, in the first approximation, it is possible to study electron-positron pair production in arbitrary electric fields in the electrosphere by simply substituting the constant electric field in the Schwinger formula with the inhomogeneous electric field.

In the second order of approximation we obtain

$$\sigma \approx \sqrt{\frac{3\pi}{\alpha}}\frac{5}{6}\frac{m_e^2 + k^2}{\omega\sqrt{\omega^2/2 + \pi^2 T^2}} + \frac{1}{3}\sqrt{\frac{3\pi}{\alpha}}\frac{\left(m_e^2 + k^2\right)^2}{\omega\sqrt{\omega^2/2 + \pi^2 T^2}}\left[\frac{1}{\omega^2} + \frac{1}{4}\frac{1}{\omega^2/2 + \pi^2 T^2} + \frac{3}{8}\frac{\omega^2}{(\omega^2/2 + \pi^2 T^2)^2}\right], \tag{22}$$

giving

$$\dot{n}_\pm(z, T) \approx \frac{3}{8\pi^2} e^2 E^2(z, T) \exp\left[\frac{25\pi}{48\lambda(z, T)}\right][3\lambda(z, T)]^{-1/2}\{1 - {\rm erf}[\chi(z, T)]\}, \tag{23}$$

where $\lambda(z, T) = eE[1/V^2 + (1/3\pi)V^2/E^2 + (2/3\pi^2)V^6/E^4]$, $\chi = [\lambda(z, T)/3\pi]^{1/2} E_{\rm cr}/E + 5\sqrt{\pi}/4\sqrt{3}\lambda(z, T)$, and ${\rm erf}\,(x) = (2/\sqrt{\pi}) \times \int_0^x \exp(-u^2)\, du$.

4. BOUNDARY EFFECTS IN PAIR PRODUCTION IN THE ELECTROSPHERE

The electron-positron pair creation in the electrosphere takes place very close to the surface of the quark star, which represents the boundary of the system. Therefore, electron-positron pair creation is localized to the half-space $z \geq 0$, and the pair creation takes place in electric field localized to a bounded region in the space. Boundary effects in electron-positron pair creation by electric fields confined in a finite region of space have been previously investigated, with the general result that finite-size effects induce large deviations of the production rate from what one deduces from the Schwinger formula (Wang & Wong 1988; Martin & Vautherin 1988, 1989).

In analyzing the surface effects on pair production there are two very different situations, which lead to very different results. The surface effects essentially depend on the orientation of the electric field with respect to the boundary (Martin & Vautherin 1988, 1989). In the case of an electric field parallel to the boundary, the pair production rate per unit time and unit volume in the region $z \geq 0$ is $w(z) = \sum_{n=1}^\infty w_n[1 - \exp(-eEz^2/n\pi)]$, where $w_n = (e^2E^2/4\pi^3)\exp(-n\pi E_{\rm cr}/E)$ is the Schwinger production rate in a constant field with infinite extension. As one can see for $z = 0$ the boundary effects cancel the Schwinger volume term, and thus electron-positron pair creation cannot take place on a boundary parallel to the field (Martin & Vautherin 1988, 1989). Generally, there is a significant reduction of the Schwinger pair production rate near a surface that is parallel to the electric field. For an electric field of the order of $E = 5E_{\rm cr}$, the reduction in the production rate at a distance of $z = 197.5$ fm from the parallel boundary is, for the case of the dominant term with $n = 1$, $1 - \exp(-eEz^2/\pi) = 0.33$, while for $z = 987.362$ fm the reduction rate is 0.999. For the case of an electric field $E = 50E_{\rm cr}$, we have a reduction rate of 0.98 at $z = 197.5$ fm and 1 at $z = 987.362$ fm.

However, the case of the electric field parallel to the boundary is not relevant for the quark star electrosphere, in which the electric field, which is oriented outward, is perpendicular to the surface of the star (the boundary). In the case of the electric field perpendicular to the boundary one can find the correction terms by using a method based on approximating the Green functions in terms of classical paths. The first-order correction to the effective Lagrangian for particle production is (Martin & Vautherin 1989)

$$L_{\rm eff}^{(1)}(z) = -\frac{1}{8\pi}\int_0^\infty \frac{ds}{s^2}\left[eE\coth(eEs) - \frac{1}{s}\right]\exp\left[-im_e^2 s + ie\frac{E}{2}z^2\coth\left(\frac{eEs}{2}\right)\right]. \tag{24}$$

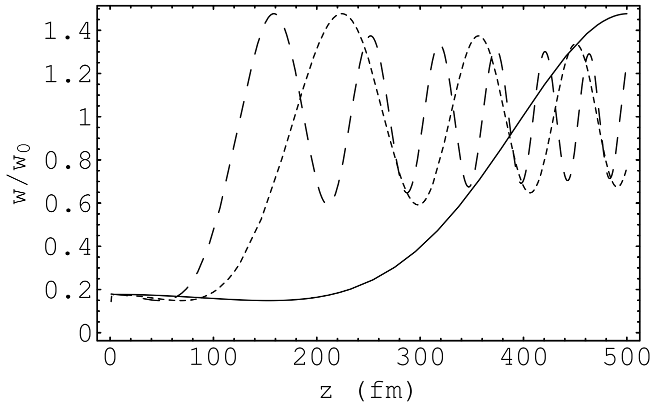

FIG. 4.—Ratio of the pair production rate $w = w_0 + w_1$ for a constant electric field, with the boundary effects included, and the Schwinger rate w_0 as a function of the distance z (in fermis) for different values of the electric field: $E = 5E_{\rm cr}$ (*solid curve*), $E = 25E_{\rm cr}$ (*dotted curve*), and $E = 50E_{\rm cr}$ MeV (*dashed curve*).

The first-order correction $w_1(z)$ to the pair production rate is given by $w_1(z) = 2\,{\rm Im}\,L_{\rm eff}^{(1)}(z)$, and for $m_e \approx 0$ can be written in the form (Martin & Vautherin 1989)

$$w_1(z) = -\frac{1}{4\pi^2}e^2E^2\int_1^\infty \Phi(s)\sin\left(\frac{eE}{2}z^2s\right)ds, \tag{25}$$

where

$$\Phi(s) = \frac{1}{(s^2-1)\{\ln[(s+1)/(s-1)]\}^2}\left\{s+\frac{1}{s}-\frac{2}{\ln[(s+1)/(s-1)]}\right\}. \tag{26}$$

The ratio of the total production rate $w = w_0 + w_1$ and of the Schwinger production rate $w_0 = (e^2E^2/4\pi^3)\exp(-\pi E_{\rm cr}/E)$ is represented, for different values of the electric field, in Figure 4.

For large z the function ${\rm Im}\,L_{\rm eff}^{(1)}(z)$ has the asymptotic behavior

$$ {\rm Im}\;L_{\rm eff}^{(1)}(z) \approx -\frac{1}{8\pi^2}e^2E^2\frac{\sin[(eE/2)z^2]}{\ln[(eE/2)z^2]} + O\left\{\frac{1}{\ln[(eE/2)z^2]}\right\}, \tag{27}$$

while for small z it is given by

$$ {\rm Im}\;L_{\rm eff}^{(1)}(z) \approx -\frac{e^2E^2}{48\pi} + \frac{eE^2}{8\pi^2}\left(\frac{eE}{2}z^2\right)c, \tag{28}$$

where $c = \frac{1}{3} - \int_1^\infty\left[s\Phi(s) - \frac{1}{3}\right]ds \approx -0.1$ (Martin & Vautherin 1989).

From these equations and from Figure 4 it follows that when the boundary is perpendicular to the field the pair production rate exhibits oscillations as a function of the distance to the boundary. No such oscillations occur when the electric field is parallel to the boundary. For a discussion of the physical origin of this phenomenon and its possible implications, see Martin & Vautherin (1989).

As we have shown in § 3, in the first approximation we can substitute the inhomogeneous electric field in the results derived for the constant-field case. We follow this approach in the study of the effect of the boundary (the quark star surface) on the pair production in the electrosphere. Hence, we assume that the boundary effect in the pair production rate is given by

$$w_1(z,T) \approx -\frac{1}{4\pi^2}e^2E^2(z,T)\int_1^\infty \Phi(s)\sin\left[\frac{eE(z,T)}{2}z^2s\right]ds, \tag{29}$$

which is a straightforward generalization of the constant-field case.

Therefore, the total rate production $\dot{n}_\pm^{(b)}(z,T) = \dot{n}_\pm(z,T) + w_1(z,T)$ of the electron-positron pairs in the electrosphere, with the boundary contribution included, is given by

$$\dot{n}_\pm^{(b)}(z,T) \approx \frac{s}{4\pi^2}eE(z,T)V^2(z,T)\int_{m_e/V(z,T)}^\infty \sigma\exp\left[-2\sqrt{\frac{3\pi}{2\alpha}}\sigma^2\int_{-1}^1\frac{\sqrt{1-\eta^2}(\sigma\eta-1)^{-1}\,d\eta}{\sqrt{2\pi^2T^2/V(z,T)^2+(\sigma\eta-1)^2}}\right]d\sigma$$
$$-\frac{1}{4\pi^2}e^2E^2(z,T)\int_1^\infty\Phi(s)\sin\left[\frac{eE(z,T)}{2}z^2s\right]ds. \tag{30}$$

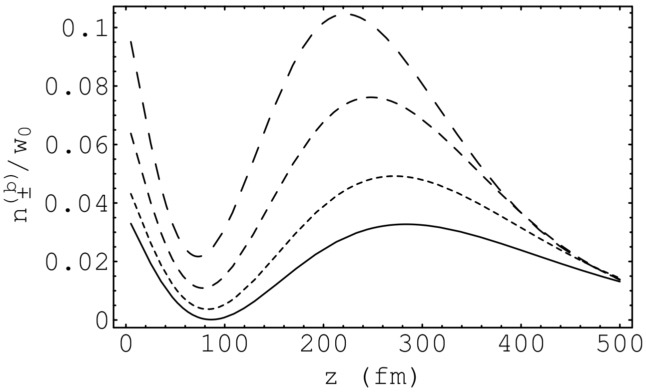

FIG. 5.—Ratio of the pair production rate $n_{\pm}^{(b)} = n_{\pm} + w_1$ in the electrosphere of the quark stars, with the boundary effects included, and the Schwinger rate of pair creation in a constant electric field w_0 as a function of the distance z (in fermis) for different values of the temperature: $T = 0.5$ MeV (*solid curve*), $T = 1.5$ MeV (*dotted curve*), $T = 2.5$ MeV (*short-dashed curve*), and $T = 3.5$ MeV (*long-dashed curve*). The value of the constant electric field used to calculate the Schwinger rate is $E_0 = 50E_{\rm cr}$. In all cases the surface electrostatic potential of the quark star is $V_q = 15$ MeV.

The ratio of the total pair production rate in the electrosphere, with the boundary effects included, and the Schwinger pair production rate in a constant electric field is represented, for different values of the temperature, in Figure 5. As expected, the boundary effects significantly reduce the creation of electron-positron pairs.

5. ELECTRON-POSITRON PAIR FLUX OF THE ELECTROSPHERE

For an electron in the electrosphere sitting near the Fermi surface the energy is given locally by $\epsilon = \mu_e(z, T) - V(z, T)$. Since from the boundary conditions it follows that $\mu_e(z, T) \to 0$ and $V(z, T) \to 0$ for $z \to \infty$, the energy of the electrons satisfies the condition $\epsilon = 0$. From this result it immediately follows that $\partial\epsilon/\partial z = 0$. Electron-positron pairs are created with an energy $\epsilon_{\pm} = m_e + T$, which is greater than the energy of the electrons in the electrosphere, $\epsilon_{\pm} > \epsilon$.

We define the electron-positron pair emissivity $Q_{\pm}$ of the electrosphere as the energy created per unit time and per unit volume by the electric field, given by the product of the number of pairs multiplied by the energy of each pair. The mathematical definition of the electron-positron pair emissivity in vacuum is

$$Q_{\pm}(z, T) = \epsilon_{\pm}\frac{\langle N\rangle}{\Delta t\Delta x\Delta y\Delta z} = \epsilon_{\pm}n_{\pm}(z, T) = (m_e + T)n_{\pm}(z, T). \tag{31}$$

The quantity $Q_{\pm}(z, T)$ is in general a local quantity, depending on the distance to the quark star surface and on the temperature in the electrosphere. The variation of the electron-positron emissivity is represented, for different values of the temperature and for a fixed quark star surface potential, in Figure 6. In the case of a constant electric field the electron-positron emissivity is $Q_{\pm}^{(0)}(T) = \epsilon_{\pm}w_0 = (m_e + T)(e^2/4\pi^3)E^2\exp(-\pi E_{\rm cr}/E)$. For an electric field $E = 50E_{\rm cr}$ we obtain $Q_{\pm}^{(0)}(T) = 1.28(m_e + T)$, which for $T = 3.5$ MeV gives $Q_{\pm}^{(0)}(T) = 5.12$ MeV^5.

The electron-positron pair flux of the electrosphere in vacuum is defined as

$$F_{\pm}^0(T) = \frac{1}{\pi}\int_0^{\infty} Q_{\pm}(z, T)\,dz = \frac{1}{\pi}\int_0^{\infty}\epsilon_{\pm}n_{\pm}(z, T)\,dz = \frac{(m_e + T)}{\pi}\int_0^{\infty} n_{\pm}(z, T)\,dz. \tag{32}$$

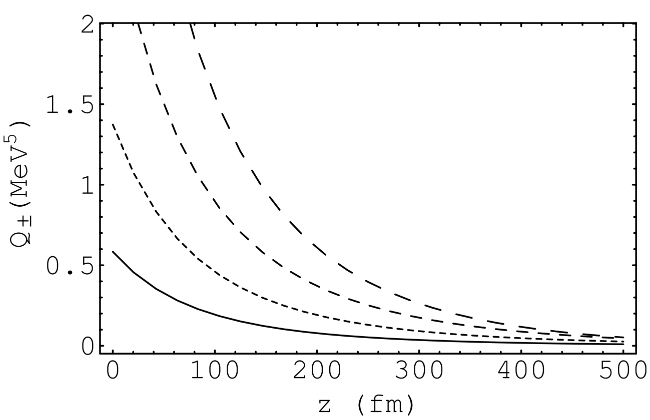

FIG. 6.—Electron-positron pair emissivity of the electrosphere of the quark stars as a function of the distance z (in fermis) for different values of the temperature: $T = 0.5$ MeV (*solid curve*), $T = 1.5$ MeV (*dotted curve*), $T = 2.5$ MeV (*short-dashed curve*), and $T = 3.5$ MeV (*long-dashed curve*). In all cases the surface electrostatic potential of the quark star is $V_q = 15$ MeV.

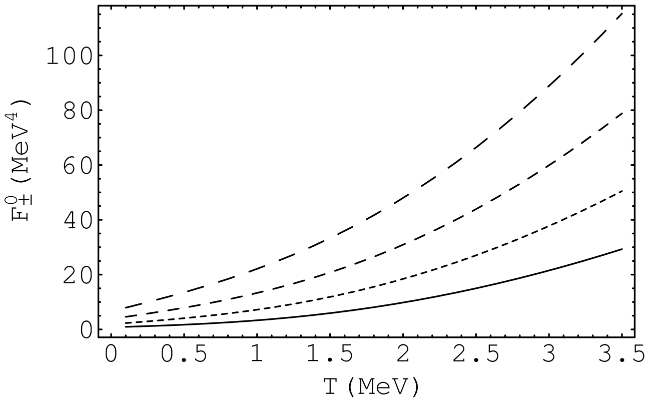

FIG. 7.—Vacuum electron-positron pair flux $F^0_\pm$ of the electrosphere of the quark stars as a function of the temperature T (in MeV) for different values of the surface electrostatic potential: $V_q = 8$ MeV (*solid curve*), $V_q = 10$ MeV (*dotted curve*), $V_q = 12$ MeV (*short-dashed curve*), and $V_q = 14$ MeV (*long-dashed curve*).

In the case of a constant electric field $E = 50E_{\rm cr}$ and assuming that the electrosphere extends up to $d = 500$ fm, we obtain $F^0_\pm(T) = (m_e + T)(e^2/4\pi^4)E^2 \exp(-\pi E_{\rm cr}/E)d \approx m_e + T$, which for $T = 3.5$ MeV gives $F^0_\pm(T) \approx 4$ MeV^4. The variation of the electron-positron flux is represented, as a function of the temperature and for different values of the quark star surface potential, in Figure 7.

The variation of the electron-positron emissivity in the presence of a boundary perpendicular to the electric field is represented, as a function of the distance z to the star's surface, in Figure 8. The boundary of the star strongly suppresses the pair production and induces an important qualitative change in the behavior of the emissivity Q. The electron-positron flux in the electrosphere, by taking into account the boundary effects, is represented in Figure 9. There is a significant effect in the electron-positron flux due to the suppression of pair creation by the quark star's surface.

The possibility of the thermal contribution to the pair creation process was discussed by Gies (1999). In the first loop approximation there is no thermal contribution to the imaginary part of the effective Lagrangian and therefore to the particle production rate. This means that the pair production rate in strong electric fields is basically independent of temperature, and the temperature dependence of this effect in the electrosphere of the quark stars is indirect and is due to the temperature dependence of the electron number density and of the corresponding electric field.

Finally, we compare the generalized Schwinger pair production rates and the corresponding flux with the expressions proposed by Usov (1998a, 1998b) and given by equation (2). We adopt a typical model for the electrosphere with an electron number density n_e of the order of $n_e = 2 \times 10^{-5}$ fm$^{-3} \approx 154$ MeV^3. The chemical potential of the electrons, μ_e, which is also equal to the Fermi energy of the electrons $\epsilon_{\rm F}$, is given by $\mu_e = (3\pi^2 n_e)^{1/3} \approx 16.58$ MeV. The number density of the pairs created by the electric field, which in this model is taken to be equal to the density of electronic empty states with energies below the pair creation threshold at thermodynamical equilibrium, is given by $\Delta n_\pm^{\rm (Usov)} = (3T/\mu_e)\exp(-2m_e/T)n_e \approx 27.8T \exp(-2m_e/T)$. By assuming that the electron spectrum is thermalized due to electron-electron collisions, the electron-positron pair flux from the electrosphere can be written as $F_\pm^{\rm (Usov)} = 0.01673T^3(m_e + T)\exp(-2m_e/T)J(\xi)$, where $\xi = 1.59/T$. For $T = 0.1$ MeV, $F_\pm^{\rm (Usov)} \approx 7.4 \times 10^{-9}$ MeV^4; for $T = 10$ MeV, $F_\pm^{\rm (Usov)} \approx 0.53$ MeV^4; and for $T = 40$ MeV, $F_\pm^{\rm (Usov)} \approx 3.45$ MeV^4. As one can see from Figures 7 and 9, there is a very large difference between the thermalized electron flux and the "pure" Schwinger flux due to pair creation by the electric field.

However, this comparison is not correct in the concrete physical framework of the quark star surface, since in the Usov (1998a, 1998b, 2001) model the electron thermalization time is used to calculate the flux, while the Schwinger flux is estimated in terms of the pair creation time in vacuum, which is of the order of $\Delta t \approx 1/(eE)^{1/2}$ (Nikishov 1970). In order to compare our results with the previous estimations of the electron-positron pair rate at the surface of strange stars, we have to systematically take into account the number of available (free) electron states at the surface of the star and the thermalization effects in both models.

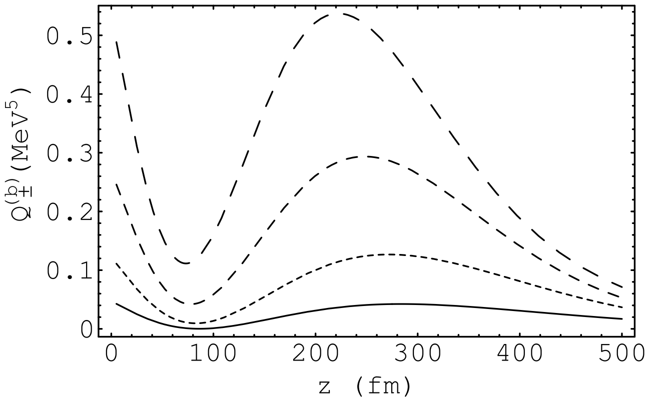

FIG. 8.—Vacuum electron-positron pair emissivity $Q^{(b)}_\pm$ of the electrosphere of the quark stars, with the boundary effects included, as a function of the distance z (in fermis) for different values of the temperature: $T = 0.5$ MeV (*solid curve*), $T = 1.5$ MeV (*dotted curve*), $T = 2.5$ MeV (*short-dashed curve*), and $T = 3.5$ MeV (*long-dashed curve*). In all cases the surface electrostatic potential of the quark star is $V_q = 15$ MeV.

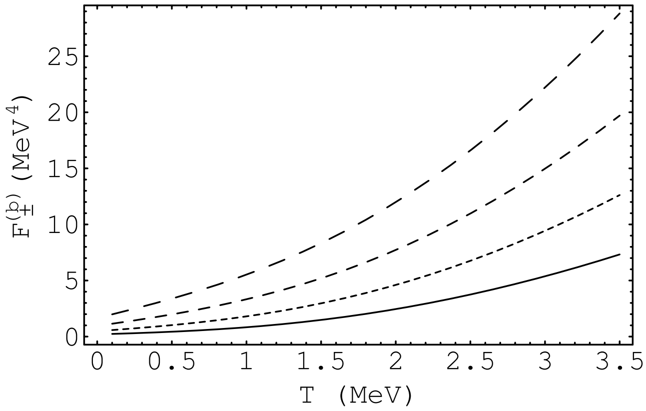

FIG. 9.—Vacuum electron-positron pair flux $F_{\pm}^{(b)}$ of the electrosphere of the quark stars in the presence of the boundary effects as a function of the temperature T (in MeV) for different values of the surface electrostatic potential: $V_q = 8$ MeV (*solid curve*), $V_q = 10$ MeV (*dotted curve*), $V_q = 12$ MeV (*short-dashed curve*), and $V_q = 14$ MeV (*long-dashed curve*).

To compare the two mechanisms we start with the discussion of the created particle number densities in the two models. In the Schwinger mechanism in vacuum, the particle number density is $\Delta n_{\pm}^0 = \dot{n}(z,T)\Delta t = \dot{n}(z,T)/(eE)^{1/2}$, which, in the approximation of the constant electric field, can be written as

$$\Delta n_{\pm}^0 = \frac{1}{4\pi^3} m_e^3 \left(\frac{E}{E_{\rm cr}}\right)^{3/2} \exp\left(-\pi \frac{E_{\rm cr}}{E}\right). \tag{33}$$

For a fixed value of the electric potential in the electrosphere V_q, which for simplicity we take as the electric potential at the surface of the quark star, the electric field is given by $E = (4/3\pi)^{1/2} V_q (V_q^2/2 + \pi^2 T^2)^{1/2}$, and the electron number density is $n_e = V_q^3/3\pi^2 + V_q T^2/3$. The temperature dependence of $\Delta n_{\pm}$ is only through the temperature dependence of the electric field E, which is significant only for values of the temperature so that $T \geq V_q$.

However, the number of free electron states at the quark star surface is given by $\Delta n_{\pm}^{(\rm Usov)} = 3T \exp(-2m_e/T)(V_q^2/3\pi^2 + T^2/3)$. From a physical point of view, $\Delta n_{\pm}^{(\rm Usov)}$ represents the number of available quantum states for pair creation (Usov 1998a, 1998b, 2001). Moreover, these states are not necessarily filled because of the finite value of the electric field. Hence, the number of created electron-positron pairs is also finite, and this number can be less than the number of available quantum states $\Delta n_{\pm}^{(\rm Usov)}$. Therefore, electron-positron pair creation is possible once the condition

$$\Delta n_{\pm}^{(\rm Usov)} \geq \Delta n_{\pm}^0 \tag{34}$$

is fulfilled. Generally, this condition will be satisfied for temperatures so that $T \geq T_{\rm cr}$, where the critical temperature can be obtained from the condition

$$3T_{\rm cr} \exp\left(-\frac{2m_e}{T_{\rm cr}}\right)\left(\frac{V_q^2}{3\pi^2} + \frac{T_{\rm cr}^2}{3}\right) = \frac{1}{4\pi^3} m_e^3 \left[\frac{E(T_{\rm cr})}{E_{\rm cr}}\right]^{3/2} \exp\left[-\pi \frac{E_{\rm cr}}{E(T_{\rm cr})}\right]. \tag{35}$$

The value of $T_{\rm cr}$ depends on the electrostatic properties of the quark star surface. The critical temperature is represented, as a function of the electrostatic potential V_q at the quark star surface, in Figure 10. The critical temperature is increasing with increasing V_q. Hence,

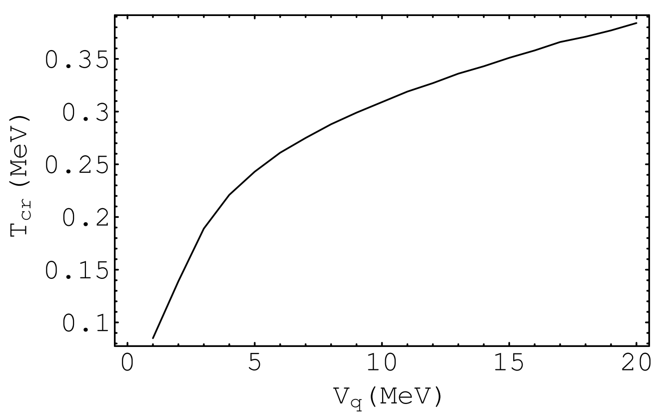

FIG. 10.—Critical temperature $T_{\rm cr}$ as a function of the surface electrostatic potential of the quark star.

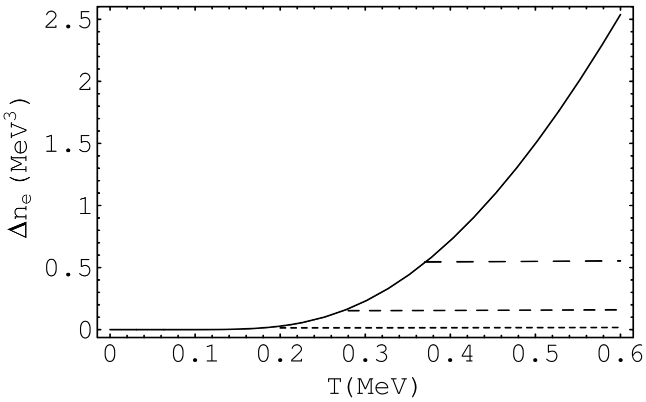

FIG. 11.—Temperature dependence of the electron-positron number density $\Delta n_{\pm}^{(\mathrm{Usov})}$ in the Usov mechanism (*solid curve*) and the particle number density $\Delta n_{\pm}$ in the Schwinger mechanism for different values of the surface electrostatic potential: $V_q = 5$ MeV (*dotted curve*), $V_q = 10$ MeV (*short-dashed curve*), and $V_q = 15$ MeV (*long-dashed curve*). For the Usov mechanism we have assumed that the Fermi energy of the electrons is $\varepsilon_{\mathrm{F}} = 20$ MeV and the thickness of the electrosphere is $\Delta r_E = 1000$ fm.

when $T \leq T_{\mathrm{cr}}$ all quantum states are filled, whereas for $T > T_{\mathrm{cr}}$ the number of available quantum states may exceed the number of electron-positron pairs created by the electric field.

Therefore, by taking into account the number of free electron states we define the electron-positron pair number density $\Delta n_{\pm}$ as

$$\Delta n_{\pm} = \begin{cases} \Delta n_{\pm}^{(\mathrm{Usov})}, & T < T_{\mathrm{cr}}, \\ \Delta n_{\pm}^{0}, & T \geq T_{\mathrm{cr}}. \end{cases} \quad (36)$$

The variation of the particle number densities in the two models is presented in Figure 11. For the chosen values of V_q the condition $\Delta n_{\pm}^{(\mathrm{Usov})} \geq \Delta n_{\pm}^{0}$ holds for temperatures $T \gtrsim 0.1$ MeV for $V_q = 5$ MeV and $T \gtrsim 0.4$ MeV for $V_q = 15$ MeV. It should be noted that despite the fact that the number of available free electron states increases very rapidly with the temperature, due to the temperature independence of the Schwinger process, the number of the created electron-positron pairs is basically determined by a single parameter, the quark star surface potential.

In the model of Usov (1998a, 2001) the thermalized electron-positron pair flux is given by

$$F_{\pm}^{(\mathrm{th})} = 4\pi R^2 \Delta r_E \Delta n_{\pm}^{(\mathrm{Usov})} t_{\mathrm{th}}^{-1}, \quad (37)$$

where R is the radius of the star, Δr_E is the thickness of the emitting region, and t_{th}^{-1} is the characteristic time of thermalization of the electrons. For t_{th}^{-1} the expression $t_{\mathrm{th}}^{-1} \approx (3/2\pi)(\alpha/\sqrt{\pi})(T^2/\epsilon_{\mathrm{F}})J(\xi)$, with $\xi = 2(\alpha/\pi)^{1/2}(\epsilon_{\mathrm{F}}/T)$, was assumed (Usov 1998a, 2001). The function $J(\xi)$ is defined by equation (3).

In order to obtain a more realistic description of the thermalized electron-positron flux of the electrosphere that takes into account the inhomogeneities of the electron and electric field distributions, we assume that due to its dependence on the Fermi energy $\epsilon_{\mathrm{F}} \approx \mu_e = V(z, T)$, the thermalization time is also a function of the distance z to the quark star surface.

Therefore, we define the thermalized electron-positron flux from the quark star's surface in the generalized emission model as

$$F_{\pm}^{(\mathrm{th})} = 4\pi R^2 \int_0^{\infty} \Delta n_{\pm}(z, T) t_{\mathrm{th}}^{-1}(z, T)\, dz \approx 4\pi R^2 \frac{3\alpha}{2\pi^{3/2}} T^2 \int_0^{\infty} \Delta n_{\pm}(z, T) \frac{J\left[2\sqrt{\alpha/\pi}V(z, T)/T\right]}{V(z, T)}\, dz. \quad (38)$$

The variations of the thermalized electron-positron fluxes in the Usov model and in the generalized electron-positron emission mechanism are presented in Figure 12.

In the limit of low temperatures the condition $T/V_q \to 0$ holds with a very good approximation. By assuming that the number density of the e^-e^+ pairs can be approximated by equation (33), in which all the parameters are estimated near the quark star surface $z \approx 0$, the electron-positron thermalized flux from the electrosphere of the quark stars can be represented in an approximate form as

$$F_{\pm}^{(\mathrm{th})} \approx 4\pi R^2 \Delta r_E V_q \left\{ \frac{\alpha \exp\left[-\sqrt{3/2}\pi^{3/2}\left(E_{\mathrm{cr}}/V_q^2\right)\right] m_e^3 \left(V_q^2/E_{\mathrm{cr}}\right)^{3/2}}{83^{3/4}(2\pi)^{1/4}} \left(\frac{T}{V_q}\right)^2 \right.$$
$$\left. -0.7165\sqrt{\alpha} \exp\left(-\sqrt{\frac{3}{2}}\pi^{3/2}\frac{E_{\mathrm{cr}}}{V_q^2}\right) m_e^3 \left(\frac{V_q^2}{E_{\mathrm{cr}}}\right)^{3/2} \left(\frac{T}{V_q}\right)^3 + \cdots \right\}. \quad (39)$$

For $T = 0$ the electron-positron flux from the quark star surface is zero. Generally, $F_{\pm}$ is a function of the electrostatic potential at the quark star surface V_q and of the temperature T.

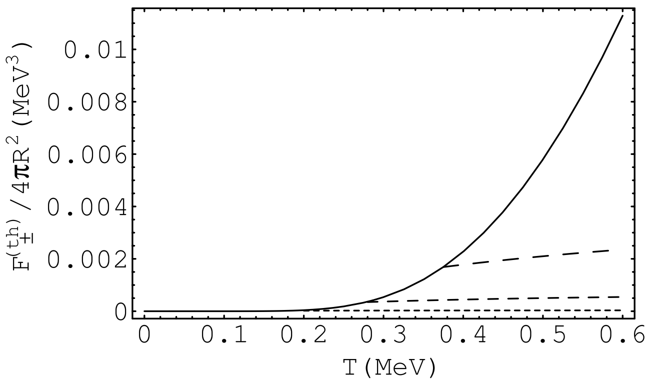

FIG. 12.—Temperature dependence of the thermalized electron-positron flux $F_{\pm}^{(\mathrm{th})}/4\pi R^2$ in the Usov mechanism (*solid curve*) and in the generalized electron-positron emission mechanism for different values of the quark star surface electrostatic potential: $V_q = 5$ MeV (*dotted curve*), $V_q = 10$ MeV (*short-dashed curve*), and $V_q = 15$ MeV (*long-dashed curve*). For the Usov mechanism we have assumed that the Fermi energy of the electrons is $\varepsilon_{\mathrm{F}} = 20$ MeV and the thickness of the electrosphere is $\Delta r_E = 1000$ fm.

The presence of a surface magnetic field H can also enhance the pair production rate (Nikishov 1970). The magnetic field increases the pair production rate by a factor of $\delta_H = \pi H/E \coth(\pi H/E)$. If $H \gg E$, there will be a significant increase in the pair production rate. The electric field of the electrosphere could be as high as $E = 40E_{\mathrm{cr}} \approx 120$ MeV^2. On the other hand, the estimated magnetic fields at the surface of the quark stars could be of the order of $H \approx 10^{15}$–10^{17} G $\approx$ 20–2000 MeV^2 (1 G $= 1.953 \times 10^{-14}$ MeV^2).

Magnetic fields with such high values may be present in very young quark stars. Assuming equipartition of energy, the energy of the differential rotation can be converted into magnetic energy, so that $I\Omega^2(\Delta\Omega/\Omega) \approx (4\pi/3)R^3(H^2/8\pi)$, where I is the moment of inertia of the star, R is its radius, and Ω and $\Delta\Omega$ are the angular velocity and the variation of the angular velocity, respectively. Therefore, the magnetic field of a young quark star can be approximated as $H \approx 10^4(\Delta\Omega/\Omega)^{1/2}$ MeV^2. By assuming that $\Delta\Omega/\Omega \approx 0.03$, we can obtain values of the magnetic field as high as $H \approx 2000$ MeV^2. Of course magnetic fields of such strength are not stable because they will be pushed to and through the surface by buoyant forces and then reconnect (Kluzniak & Ruderman 1998). For a magnetic field of the order of $H \approx 2000$ MeV^2 we have $\delta_H \approx 53$.

Therefore, strong magnetic fields can significantly increase the electron-positron pair production rate and, consequently, the luminosity of the electrosphere of quark stars.

6. CONCLUSIONS

In this paper we have reconsidered the electron-positron pair emission from the electrosphere of quark stars, as originally proposed by Usov (1998a, 1998b), by pointing out the important role the boundary effects and the inhomogeneity in the distribution of the electric field may play in the pair creation process. At zero temperature, there are no available free energy states in the electron plasma at the strange star's surface. Therefore, at low temperatures $T \leq T_{\mathrm{cr}} \approx 0.1$ MeV (corresponding to a quark star surface electric potential of $V_q = 5$ MeV), the pair production mechanism by the strong electric field of the electrosphere is severely limited by the quantum effects and the exclusion principle specific to the Fermi-Dirac statistics. At high temperatures $T \geq T_{\mathrm{cr}} \approx 0.1$ MeV, the pair creation rate is controlled by the electric field E and not by the temperature because such a process is essentially a quantum process. Once the number of available electron states becomes higher than the Schwinger pair production rate, electron-positron pairs can be freely created by the electric field at the surface of strange stars. This happens at a critical temperature T_{cr}, which strongly depends on the electrostatic properties of the quark star surface. The critical temperature increases with the increase of the electrostatic potential V_q. At high enough temperatures, the pair creation process is almost independent of the temperature and is controlled exclusively by the electric field. On the other hand, the actual thermalized pair creation rate, which from an astrophysical and observational point of view is the most relevant quantity, depends on the temperature through the thermalized timescale. The number density of pairs can be accurately evaluated by using the Schwinger formalism and by taking into account the inhomogeneities in the electric field distribution. However, the boundary effects also induce large quantitative and qualitative deviations of the particle production rate from what one deduces with the Schwinger formula and its generalization for the inhomogeneous electric field of the electrosphere.

Due to all these effects, we estimate that at high temperatures the energy flux due to e^-e^+ pair production could be lower than in the initial proposal of Usov (1998a, 1998b). However, this flux could still be the main observational signature of a quark star. On the other hand, the presence of a strong magnetic field at the quark star surface may significantly enhance the electron-positron flux.

The possible astrophysical and observational implications of the direct pair production effect will be considered in a future publication.

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