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COSMIC RAYS FROM GAMMA-RAY BURSTS IN THE GALAXY

Charles D. Dermer[1] and Jeremy M. Holmes[2,3]
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ABSTRACT

The rate of terrestrial irradiation events by Galactic gamma-ray bursts (GRBs) is estimated using recent standard-energy results. We assume that GRBs accelerate high-energy cosmic rays, and we present results of three-dimensional simulations of cosmic rays moving in the Galactic magnetic field and diffusing through pitch-angle scattering. An on-axis GRB extinction event begins with a powerful prompt γ-ray and neutron pulse, followed by a longer lived phase from cosmic-ray protons and neutron-decay protons that diffuse toward Earth. Our results force a reinterpretation of reported $\sim 10^{18}$ eV cosmic-ray anisotropies and offer a rigorous test of the model in which high-energy cosmic rays originate from GRBs, and this model will soon be tested using the Auger Observatory.

Subject headings: astrobiology — cosmic rays — gamma rays: bursts

1. INTRODUCTION

Observations link GRBs—brief flashes of γ-ray light emitted by sources at cosmological distances—with star-forming galaxies and high-mass stars (van Paradijs et al. 2000). These results indicate that GRBs are produced by a rare type of supernova in which the evolved core of a massive star collapses to a black hole. Normal supernovae, in which the stellar core collapses to form a neutron star, are thought to produce nonrelativistic supernova remnant shocks that accelerate cosmic rays with energies $\lesssim 10^{14}$ eV. The origin of higher energy cosmic rays is controversial. One possibility is that high-energy ($\gtrsim 10^{14}$ eV) cosmic rays are accelerated by the relativistic shocks formed by GRB explosions in the Galaxy and throughout the universe (Vietri 1995; Waxman 1995; Dermer 2002; Wick et al. 2004), dating back to earlier suggestions of a Galactic origin of high-energy cosmic rays (Kulikov et al. 1969).

If GRBs accelerate high-energy cosmic rays, then Galactic GRBs could be detected as cosmic-ray sources from neutron β-decay emissions (Ioka et al. 2004). GRBs with their jets oriented toward the Earth have potentially lethal consequences and may have contributed to past extinction episodes (Dar et al. 1998; Melott et al. 2004). The sources of the $\approx 10^{18}$ eV cosmic-ray excesses reported from measurements made with the SUGAR (Sydney University Giant Air Shower Recorder) and AGASA (Akeno Giant Air Shower Array) detectors (Bellido et al. 2001; Hayashida et al. 1999; see Nagano & Watson 2000 for review) have been proposed to result from past GRBs in the Galaxy (Biermann et al. 2004).

To investigate these ideas, the rate of GRB events at different fluence levels is estimated, based on recent findings about beaming in GRBs (Frail et al. 2001; Bloom et al. 2003). A three-dimensional propagation model is used to simulate the sequence of irradiation events that occurs when a GRB jet is pointed toward Earth. The results suggest that a GRB jet could have produced radiations that contributed to the Ordovician extinction event (Melott et al. 2004). Our results are contrary to the claim that the SUGAR excess could be produced by a GRB in the Galaxy. If the Auger Observatory confirms the SUGAR cosmic-ray point source, then a model of high-energy cosmic rays from Galactic GRBs is incomplete.

2. RATE OF GRB EVENTS AT DIFFERENT FLUENCE LEVELS

Most of the electromagnetic radiation from a GRB is emitted during the prompt and early afterglow phases on timescales of minutes to hours. We define the bolometric photon fluence $\varphi = \mathcal{S}\varphi_\odot$ with reference to the solar energy fluence $\varphi_\odot = 1.4 \times 10^6 \mathcal{S}$ ergs cm^{-2} received at Earth in 1 s. Significant effects on atmospheric chemistry through the formation of nitrous oxide compounds and the depletion of the ozone layer is found when $\mathcal{S} \gtrsim 10^2$–10^3 (Ruderman 1974; Thorsett 1995; Gehrels et al. 2003; Thomas et al. 2005), taking into account the very hard incident radiation spectrum of GRBs. Reprocessing of incident GRB radiation into biologically effective 200–320 nm UV radiation (i.e., with $1/e$ lethality) on eukaryotes occurs when $\mathcal{S} \gtrsim 10$–10^2 (Scalo & Wheeler 2002; Smith et al. 2004).

Achromatic beaming breaks in GRB optical/IR afterglow light curves (Stanek et al. 1999), if due to jetted GRBs, imply typical GRB jet opening half-angles $\langle\theta_j\rangle \cong 0.1$. Analyses (Frail et al. 2001; Bloom et al. 2003) show that long-duration GRBs have a standard total energy $\mathcal{E} \cong \theta_j^2 E_{\gamma,\,\rm iso}/2 \cong 10^{51}\mathcal{E}_{51}$ ergs, with $\mathcal{E}_{51} \cong 1.33$, a variance by a factor of 2.2, and a low $E_{\gamma,\,\rm iso}$ population. Here $E_{\gamma,\,\rm iso}$ is the apparent isotropic γ-ray energy release inferred directly from observations. If the cone of emission from a GRB at distance R intercepts the line of sight to Earth, then the radiant fluence is given by $\varphi = E_{\gamma,\,\rm iso}/4\pi R^2$. Thus, the maximum sampling distance R_s of a GRB with apparent isotropic γ-ray energy release $E_{\gamma,\,\rm iso}$ to be detected at the fluence level $\varphi > \varphi_{\rm th} = \mathcal{S}\varphi_\odot$ is

$$R_s = \sqrt{\frac{E_{\gamma,\,\rm iso}}{4\pi\varphi_{\rm th}}} \cong \frac{1.1\ \rm kpc}{(\theta_j/0.1)}\sqrt{\frac{\mathcal{E}_{51}}{(\mathcal{S}/10^3)}}. \qquad (1)$$

When $\mathcal{S} \lesssim 10^5$, the sampling distance to a typical GRB exceeds the $\approx$100 pc disk scale height of molecular clouds and OB associations, and we can approximate the distribution of GRBs in the Galaxy by a uniform disk of radius $R_{\rm MW} \cong$

[1] E. O. Hulburt Center for Space Research, Code 7653, Naval Research Laboratory, Washington, DC 20375-5352; dermer@gamma.nrl.navy.mil.
[2] Thomas Jefferson High School for Science and Technology, 6560 Braddock Road, Alexandria, VA 22312.
[3] Current address: Florida Institute of Technology, 150 West University Boulevard, Melbourne, FL 32901.

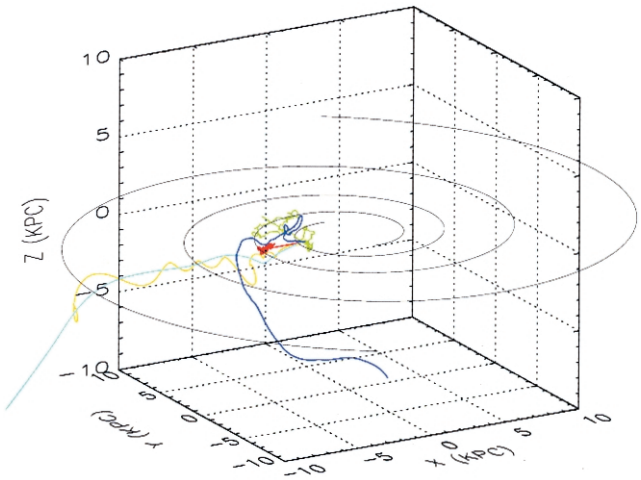


FIG. 1.—Trajectories of cosmic rays ejected outward from a cosmic-ray source located 3 kpc from the center of the Galaxy. The spiral curves show the peak magnetic field values in the disk of the galaxy for the magnetic field model of Alvarez-Muñiz et al. (2002). The orange and turquoise curves show trajectories taken by cosmic-ray protons with Lorentz factors $\gamma = 10^9$ and 3×10^9, respectively, with diffusive scattering omitted, and the green and blue curves illustrate the effects on protons with respective energies when diffusive pitch-angle scattering is included. The red curve shows the path taken by a cosmic-ray neutron with $\gamma = 3 \times 10^8$ and mean decay length of 3 kpc, including scattering effects on the path of the neutron-decay proton.

$15R_{15}$ kpc. The fluence size distribution of GRBs in this approximation is given by

$$\dot{N}(>\mathcal{S}) \simeq \dot{N}_{\rm GRB}\left(\frac{R_s}{R_{\rm MW}}\right)^2 \mathcal{P}(\theta_j), \tag{2}$$

where $\dot{N}_{\rm GRB}$ is the rate of GRBs in the Milky Way and $\mathcal{P}(\theta_j) \cong \theta_j^2/2$ is the probability that the Earth lies within the emission cone of a two-sided jet. If GRBs follow the star formation rate history of the universe, then one GRB occurs every $10^5 t_5$ yr in the Milky Way (Wick et al. 2004; Dermer 2002), with $t_5 \simeq 0.1$–1. Thus,

$$\dot{N}(>\mathcal{S}) \simeq \frac{0.3}{R_{15}^2}\frac{\mathcal{E}_{51}}{(\mathcal{S}/10^3)\mathcal{T}_5}\ {\rm Gyr}^{-1}. \tag{3}$$

Equations (1) and (3) show that a GRB at a distance ≈1 kpc with $\mathcal{S} \gg 10^2$ takes place about once every gigayear and more frequently if $t_5 \cong 0.1$.

3. COSMIC-RAY PROPAGATION IN THE GALAXY

We have developed a numerical model in which cosmic rays move in response to a large-scale magnetic field that traces the spiral arm structure of the Galaxy and diffuse through pitch-angle scattering due to magnetic turbulence. The magnetic field $\boldsymbol{B}$ of the Galaxy is modeled as a bisymmetric spiral for the Galaxy's disk and a dipole magnetic field for the Galaxy's halo (Alvarez-Muñiz et al. 2002). The evolution of the particle momentum $\boldsymbol{p} = m\gamma\boldsymbol{\beta}c$ is found by solving the Lorentz force equation $d\boldsymbol{p}/dt = q\boldsymbol{\beta} \times \boldsymbol{B}$, where q and m are the particle's charge and mass, respectively, $\boldsymbol{\beta}c$ is its velocity, and $\gamma = (1 - \beta^2)^{-1/2}$.

Magnetic turbulence causes a particle to change its pitch angle by ≈$\pi/2$ when traveling the mean free path λ. The energy dependence of λ is obtained by extrapolating the expression for λ in a diffusion-model fit (Wick et al. 2004) to the measured ionic flux near the knee of the cosmic-ray spectrum (Kampert et al. 2001) to high energies. Our approach assumes isotropic turbulence that is uniform in the disk and halo of the Galaxy; anisotropic turbulence is more realistic (Goldreich & Sridhar 1997) but increases the number of free parameters in the model. The particle's azimuth and cosine angle are randomly chosen between 0 and 2π and between $\mu_{\rm min}$ and 1, respectively, after every $\lambda[(1 - \mu_{\rm min})/2]^2/c\Delta t$ steps, where Δt is the time interval of each step in the numerical integration, which is set equal to a small fraction of the local gyroperiod.

The propagation calculations are performed in the test-particle approximation, assuming that the cosmic rays do not affect their surrounding environment. The energy density of the cosmic-ray shell can, however, greatly exceed the magnetic field energy density of the interstellar medium (ISM), in which case the protons will sweep up material, causing strong adiabatic losses and rapid deceleration within a Sedov length scale of ~1 pc. This forms a shock and a second phase of cosmic-ray acceleration, which will require a hydrodynamic simulation to treat. The cosmic-ray neutrons decay over the entire jet volume. Comparing the ISM magnetic field energy density with the energy density of the neutron-decay protons over their decay volume shows that the propagation of neutrons with

$$\gamma \gtrsim \frac{2 \times 10^7}{(B/3\ \mu{\rm G})^{0.62}(1 - \cos\theta_j)^{0.31}} \tag{4}$$

is unaffected by the energy injection of cosmic rays into the ISM. Thus, our subsequent discussion of ~10^{18} eV cosmic-ray neutrons is accurately described in the test-particle approximation.

The propagation of cosmic-ray protons and neutrons, with and without the effects of diffusive scattering, is illustrated in Figure 1. Cosmic-ray neutrons travel ≈$10(E/10^{18}$ eV) kpc before decaying. Cosmic-ray protons with energies $\gtrsim 3 \times 10^{18}$ eV and cosmic-ray neutrons with energies $\gtrsim 10^{18}$ eV escape almost directly into intergalactic space. Protons with energies $\lesssim 3 \times 10^{17}$ eV diffusively escape from the Galaxy through a combination of Larmor motions and pitch-angle scatterings.

Figure 2 displays the cosmic-ray halo that surrounds a GRB source ~4×10^{11} s after the event. The GRB is modeled by radially oriented jets with $\theta_j = 0.1$. A conical shell with an effective angular extent larger than 0.1 radians forms as a result of directly accelerated protons and neutron-decay protons with $E \gtrsim 10^{18}$ eV. The turbulent wave spectrum that is resonant with these high-energy cosmic rays is poorly known. In a simulation with pitch-angle scattering absent, energy-dependent features and wall-like structures are formed, as shown in the right panel of Figure 2.

4. DISCUSSION AND SUMMARY

About once every several hundred million years, the Earth is illuminated by prompt GRB photon and neutral radiations of sufficient intensity to have significant effects on the biota through erythema from cascade UV flux (Scalo & Wheeler 2002; Smith et al. 2004) and destruction of the ozone layer catalyzed by the formation of nitric oxide and NO_y "odd nitrogen" compounds (Ruderman 1974; Thorsett 1995; Thomas et al. 2005). The destruction of the ozone layer causes the solar UV fluence to greatly exceed the GRB cascade UV fluence when the net effects of the GRB radiation on the atmosphere are considered (Thomas et al. 2005). Such an event might have been responsible for trilobite extinction in the Ordovician era through the destruction of plankton (Melott et al. 2004).

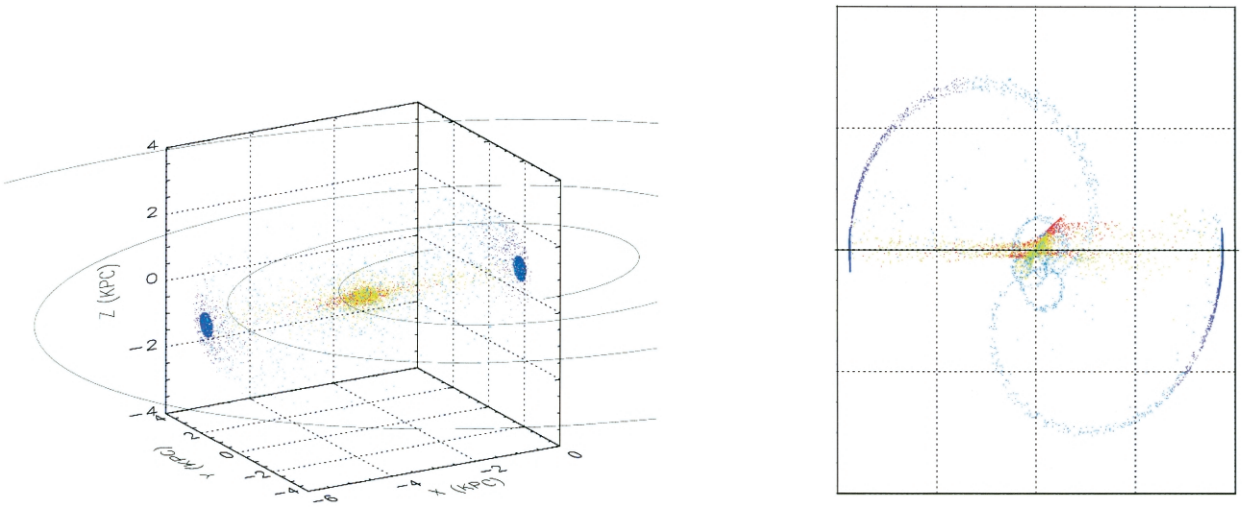


FIG. 2.—*Left*: Cosmic-ray halo formed 12,000 years after a GRB that took place 3 kpc from the center of the Galaxy. For clarity, equal numbers of cosmic rays are injected per decade, with cosmic rays color-coded by energy. Cosmic-rays protons with Lorentz factors γ in the ranges 10^7–10^8, 10^8–10^9, 10^9–10^{10}, and 10^{10}–10^{11} are red, green, turquoise, and purple, respectively. Cosmic-ray neutrons and neutron-decay protons are dark red, yellow, magenta, and dark blue in the respective energy ranges. *Right*: A side view of the cosmic-ray halo when diffusive scattering is turned off (grid lines are 2 kpc in separation).

If high-energy cosmic-ray production accompanies GRBs, then Galactic events could also have affected biological evolution due to DNA radiation damage by the elevated ground-level muon fluxes induced by $\approx 10^{17}$ eV ($\gamma_8 = \gamma/10^8 \cong 1$) cosmic-ray neutrons (Dar et al. 1998). The prompt energy fluence of neutrons is

$$\varphi_n(\gamma, r) = m_p c^2 \gamma^2 \frac{dN_n}{d\gamma_n} \frac{\exp(-r/r_n)}{f_b 4\pi r^2} \cong \frac{4 \times 10^8 \eta_n \mathcal{E}_{52}}{f_{-2} r_{\rm kpc}^2} \frac{\exp(-r_{\rm kpc}/\gamma_8)}{\gamma_8^{-1.2}} \ \text{ergs cm}^{-2}, \quad (5)$$

where an on-axis GRB at a distance of $r_{\rm kpc}$ kpc accelerates $10^{52}\eta_n\mathcal{E}_{52}$ ergs of nonthermal neutrons above $\approx$100 TeV into a two-sided jet with beaming factor f_{-2}%. The prompt muon number flux due to this type of event is at the level

$$\varphi_\nu(> E_\mu) \cong \frac{4 \times 10^{11} \sec\theta}{E_\mu^{1.76}(\text{GeV})\gamma_8^{0.44}} \frac{\eta_n \mathcal{E}_{52}}{f_{-2} r_{\rm kpc}^2} \exp(-r_{\rm kpc}/\gamma_8) \quad (6)$$

(Gaisser 1990, p. 206) for GRBs within $\pi/3$ of the zenith. This value is above the level $\varphi_\nu(> 3\ \text{GeV}) \sim 10^{10}$ for 50% mortality of human beings. The situation could be worse because of various leptonic and hadronic radiation pathways in the GRB leading to significant fluxes of >100 MeV–TeV radiation and enhanced cascade UV flux (Dar & De Rújula 2002).

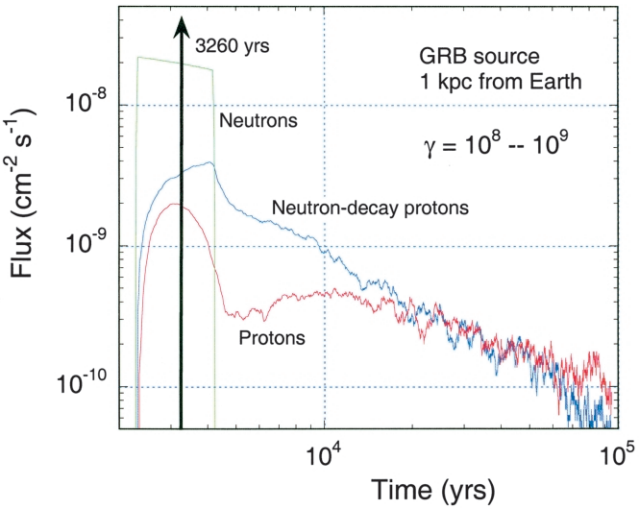


FIG. 3.—Fluxes of cosmic rays with $10^8 \le \gamma \le 10^9$ received at Earth from an on-axis GRB that occurred 1 kpc away. The fluxes of cosmic-ray neutrons, neutron-decay protons, and protons are shown by the curves that decline, rise, and peak during the initial phase. The prompt flux actually lasts for some minutes to hours, rather than the 2000 yr indicated by the numerical simulation, which reflects the approximations of the numerical method.

Deflection of charged particles by the Galactic magnetic field means that cosmic-ray protons arrive after the delay time $\delta t \simeq 2r_{\rm L}(\theta - \sin\theta) \simeq r_{\rm L}\theta^3/3c$ for $\theta \ll 1$, where the characteristic Larmor radius $r_{\rm L} \cong 0.1\gamma_8/B_{\mu\rm G}$ kpc is assumed to be much larger than the source distance and θ is a characteristic deflection angle. For a GRB 1 kpc away, $\gtrsim 4 \times 10^{18}$ eV protons arrive within angle θ of the source direction over $\approx 1000\theta^3$ yr, taking $B_{\mu\rm G} \approx 4$. Diffusive scattering spreads out the arrival times and directions compared to this estimate, and implies a "chirping" behavior in which higher energy particles arrive first. From the preceding expression, the energy dependence of the time delay for a proton with Lorentz factor $10^9\gamma_9$ is given by $\Delta t \cong 10^2 d_{\rm kpc}^3 B_{\mu\rm G}/\gamma_9$ s for $\gamma_9 \gtrsim B_{\mu\rm G} d_{\rm kpc}$ and $\Delta t \cong 10^2 d_{\rm kpc}^2/\gamma_9$ s for $10^{-2} \lesssim \gamma_9 \lesssim 1$, using the form of $\lambda(\gamma)$ from Wick et al. (2004).

Figure 3 shows the time dependence of the flux of cosmic-ray neutrons and protons with Lorentz factors $10^8 \le \gamma \le 10^9$ received from a GRB source located 1 kpc from the Earth. The received flux of prompt neutrons with this range of Lorentz factors would be $dN_n/dA\,dt \cong 1250/[(\theta_j/0.1)^2 t_{\rm dur}(\text{s})]$ cm^{-2} s^{-1}, where $t_{\rm dur}$(s) is the mean duration of the cosmic-ray emission event. Because of the numerical method, this duration has been artificially increased to 6.1×10^{10} s, so that if the actual duration of the event were 600 s, the flux would be 10^8 times greater, and the duration 10^8 times shorter, than shown in the figure, giving constant total fluence. The flux units shown in Figure 3 apply directly to the cosmic-ray protons and neutron-decay protons, and we see that the cosmic-ray fluence in the

prompt and extended phases are roughly equal. The measured energy density of 10^{17}–10^{18} eV cosmic rays is $\approx 10^{-18}$ ergs cm^{-3}, so Figure 3 shows that the cosmic-ray energy density at these energies would be ≈4 orders of magnitude larger than the currently measured energy density during a period of $\approx 10^4$ yr following such an event.

By depositing larger cosmic-ray fluence during a much shorter interval, the prompt flux has, however, much greater lethality and effect. The occurrence of the sequence of extinction events in the Ordovician due to a long-lasting ~1 Myr ice age (Melott et al. 2004) could happen if the prompt blast induced a long-term change in the climate or if the delayed cosmic rays induce a glaciation (Shaviv 2003). On-axis events are considerably more damaging than off-axis events that, though more numerous, release $\lesssim 10^{17}$ eV cosmic rays that slowly diffuse toward Earth over periods of thousands of years and longer.

The hypothesis (Biermann et al. 2004) that the $\gtrsim 10^{18}$ eV cosmic-ray excesses detected with the AGASA and SUGAR arrays (Hayashida et al. 1999) are cosmic-ray neutrons from a GRB is not, however, supported by our simulations. The relativistic blast waves in a GRB accelerate the highest energy cosmic rays over timescales of weeks or less (Zhang & Mészáros 2004), and the high-energy neutrons therefore arrive on this same timescale. Cosmic-ray protons with energies $\sim 10^{19}$ eV are delayed over a timescale $\approx 10{,}000\theta^3/B_{\mu\mathrm{G}}$ yr from a source at the distance of the Galactic center. For the SUGAR excess, which is coincident on the subdegree ($\theta \lesssim 0.02$) angular scale with a point source, a GRB would have to take place within weeks of the observation for cosmic-ray protons to maintain their direction to the source. Including the requirement that the GRB jet was also pointed toward Earth means that an impulsive GRB origin is excluded because such an event is highly improbable. The greater (≈10°) extent of the AGASA excess does not conclusively exclude a GRB origin, but here the diffuse excess could simply reflect the greater path length for cosmic-ray proton collisions with spiral arm gas along the Cygnus arm.

Because the SUGAR point source does not admit an impulsive GRB solution, only cosmic rays from a persistent source, such as a microquasar, could make such an excess. The hypothesis (Dermer 2002; Wick et al. 2004) that GRBs are sources of $\gtrsim 10^{14}$ eV cosmic rays is therefore incompatible with such a source. This cosmic-ray origin hypothesis will soon be tested by results from the Auger Observatory[4] to confirm this source. If the source is real, then the GRB/cosmic-ray model is incomplete. Searches for neutron β-decay radiation in recent Galactic GRBs (Ioka et al. 2004) and around galaxies that host GRBs (Dermer 2002) provide further tests of the hypothesis that high-energy cosmic rays are accelerated by GRBs.

The Earth resides on the inner edge of a spiral arm and not in an OB association where high-mass stars, and therefore GRBs, are usually found. The greater likelihood for intense irradiation events by GRBs excludes OB associations from the Galactic habitable zone (Lineweaver et al. 2004), except for planets with very thick ($\gtrsim$1000 gm cm^{-2}) atmospheres (Smith et al. 2004) needed to limit the radiation effects.

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[4] See http://www.auger.org.

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