

POSTBOUNCE EVOLUTION OF CORE-COLLAPSE SUPERNOVAE: LONG-TERM EFFECTS OF THE EQUATION OF STATE

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ABSTRACT

We study the evolution of a supernova core from the beginning of the gravitational collapse of a $1.5 M_{\odot}$ star up to 1 s after core bounce. We present results of spherically symmetric simulations of core-collapse supernovae by solving general relativistic ν -radiation hydrodynamics in the implicit time differencing. We aim to explore the evolution of shock waves in the long term and investigate the formation of proto-neutron stars together with supernova neutrino signatures. These studies are done to examine the influence of the equation of state (EOS) on the postbounce evolution of shock waves in the late phase and the resulting thermal evolution of proto-neutron stars. We compare two sets of EOSs, namely, those by Lattimer and Swesty (LS-EOS) and by Shen et al. (SH-EOS). We found that, for both EOSs, the core does not explode and the shock wave stalls similarly in the first 100 ms after bounce. A revival of the shock wave does not occur even after a long period in either case. However, the recession of the shock wave appears different beyond 200 ms after bounce, having different thermal evolution of the central core. A more compact proto-neutron star is found for LS-EOS than SH-EOS with a difference in the central density by a factor of ~ 2 and a difference of ~ 10 MeV in the peak temperature. The resulting spectra of supernova neutrinos are different to an extent that may be detectable by terrestrial neutrino detectors.

Subject headings: equation of state — hydrodynamics — neutrinos — stars: neutron — supernovae: general

1. INTRODUCTION

Understanding the explosion mechanism of core-collapse supernovae is a grand challenge that requires conducting numerical simulations of ν -radiation hydrodynamics with the best knowledge of particle and nuclear physics. Three-dimensional simulations of ν -radiation hydrodynamics, which are currently formidable, and better determinations of the nuclear equation of state of dense matter and the neutrino-related reaction rates are mandatory. One has to advance step by step by developing numerical methods and examining microphysics and its influence in various stages of supernovae.

Even with the extensive studies in recent years with currently available computing resources, the numerical results have not made the explosion mechanism clear. On one hand, recent multi-dimensional supernova simulations with approximate neutrino transport schemes have revealed the importance of asymmetry such as rotation, convection, magnetic fields, and/or hydrodynamic instability (Blondin et al. 2003; Buras et al. 2003; Kotake et al. 2003, 2004; Fryer & Warren 2004; Walder et al. 2005). On the other hand, spherically symmetric supernova simulations of

late have removed the uncertainty of neutrino transport and clarified the role of neutrinos in core-collapse and shock propagation (Rampp & Janka 2000, 2002; Liebendörfer et al. 2001; Mezzacappa et al. 2001; Thompson et al. 2003). In this study, we focus on spherically symmetric simulations that are advantageous for examining the role of microphysics without the ambiguity of neutrino transport.

Almost all authors have reported that neither prompt explosion nor delayed explosion occurs under spherical symmetry. This conclusion is commonly reached by simulations with Newtonian (Rampp 2000; Rampp & Janka 2000; Mezzacappa et al. 2001; Thompson et al. 2003), approximately relativistic (Rampp & Janka 2002), and fully general relativistic (Liebendörfer et al. 2001) gravity, together with standard microphysics, i.e., the equation of state (EOS) by Lattimer & Swesty (1991) and the weak reaction rates by Bruenn (1985). The influence of nuclear physics inputs has been further assessed by employing extended neutrino reactions (Thompson et al. 2003; see also § 3.2) and more up to date electron capture rates on nuclei (Hix et al. 2003). The dependence on the progenitor models (Liebendörfer et al. 2002, 2004; Thompson et al. 2003; Janka et al. 2005) and the sets of

physical EOS (Janka et al. 2005) has been studied very recently. These simulations so far have shown that the collapse of iron cores leads to a stalled shock after bounce without a successful explosion.

In the current study, we explore the influence of EOS in a time period that has not been studied very well in the previous studies. Most recent numerical simulations have been performed until about 300 ms after bounce. This is due to the severe limitation on time steps by the Courant condition for explicit time-differencing schemes. A typical time step in the explicit method is about 10^{-6} s after the formation of dense compact objects. However, in the implicit method that we employ in this study, the time step is not restricted by the Courant condition. This is advantageous for a long-term evolution. In the studies by Liebendörfer et al. (2002, 2004), who also adopted the implicit method, the postbounce evolution has been followed up to about 1 s for a small number of models with Lattimer & Swesty EOS. Historically, the idea of the delayed explosion was proposed by Wilson's simulations that followed more than several hundred milliseconds after bounce. In some cases, the revival of the shock wave occurred even beyond 0.5 s (e.g., Bethe & Wilson 1985). It is still interesting to explore this late phase in the light of the possible influence of microphysics.

The progress of the supernova EOS put an additional motivation to the study of this late postbounce phase. Only recently have the sets of physical EOS, which cover a wide range of density, composition and temperature in a usable and complete form, become available for simulations. A table of EOSs was made for the first time by Hillebrandt & Wolff (1985) within the Skyrme Hartree-Fock approach and applied to some simulations (Hillebrandt et al. 1984; Suzuki 1990, 1993, 1994; Sumiyoshi et al. 1995c; Janka et al. 2005). Another set of EOSs has been provided as a numerical routine by Lattimer & Swesty (1991) utilizing the compressible liquid drop model. This EOS has been used since then as a standard.

Recently, a new complete set of EOSs for supernova simulations has become available (Shen et al. 1998a, 1998b). The relativistic mean field (RMF) theory with a local density approximation was applied to the derivation of the table of supernova EOSs. This EOS is different in two important aspects from previous EOSs. One is that the Shen EOS is based on the relativistic nuclear many-body framework whereas the previous ones are based on nonrelativistic frameworks. The relativistic treatment is known to affect the behavior of the EOS at high densities (i.e., stiffness; Brockmann & Machleidt 1990) and the size of the nuclear symmetry energy (Sumiyoshi et al. 1995b). The other aspect is that the Shen EOS is based on the experimental data of unstable nuclei, which have become available recently. The data of neutron-rich nuclei, which are close to the astrophysical environment, were used to constrain the nuclear interaction. The resulting properties of isovector interaction are generally different from their nonrelativistic counterparts, and the size of the symmetry energy is different. Significant differences in stiffness and composition during collapse and bounce have been shown between the Shen EOS and Lattimer-Swesty EOS by hydrodynamic calculations (Sumiyoshi et al. 2004). Therefore, it would be exciting to explore supernova dynamics with the new set of EOSs. Such an attempt was made recently by Janka et al. (2005), and no explosion was reported up to 300 ms after bounce.

Our aim in the current study is, therefore, the comparison of the postbounce evolutions beyond 300 ms for the first time. We perform core-collapse simulations adopting the two sets of EOS, that is, the Shen EOS (SH-EOS) and Lattimer-Swesty EOS (LS-EOS). We follow the evolution of the supernova core for a

long period. We explore the fate of the stalled shock up to 1 s after bounce. In this time period, one can also see the birth of a proto-neutron star as a continuous evolution from the collapsing phase together with the long-term evolution of neutrino emissions. Although the supernova core does not display a successful explosion, as we will see, the current simulations may provide some aspects of the central core leading to the formation of a proto-neutron star or black hole. This information is also helpful for envisaging the properties of supernova neutrinos in the first second, since the simulations of proto-neutron star cooling done so far usually start from several hundred milliseconds after bounce for some given profiles. As a whole, we aim to clarify how the EOS influences the dynamics of the shock wave, the evolution of the central core, and supernova neutrinos.

2. NUMERICAL METHODS

A new numerical code of general relativistic, ν -radiation hydrodynamics under spherical symmetry has been developed (Yamada 1997; Yamada et al. 1999) for supernova simulations. The code solves a set of equations of hydrodynamics and neutrino transfer simultaneously in the implicit way, which enables us to have substantially longer time steps than explicit methods. This is advantageous for the study of long-term behaviors after core bounce. The implicit method has been also adopted by Liebendörfer et al. (2004) in their general relativistic ν -radiation hydrodynamics code. However, they have taken an operator splitting method so that hydrodynamics and neutrino transfer could be treated separately.

2.1. Hydrodynamics

The equations of Lagrangian hydrodynamics in general relativity are solved by an implicit, finite differencing. The numerical method is based on the approximate linearized Riemann solver (Roe-type scheme) that captures shock waves without introducing artificial viscosities. Assuming spherical symmetry, the metric of Misner & Sharp (1964) is adopted to formulate hydrodynamics and neutrino transport equations. A set of equations for the conservation of baryon number, lepton number, and energy momentum are solved together with the metric equations and the entropy equation. Details of the numerical method of hydrodynamics can be found in Yamada (1997), where standard numerical tests of the hydrodynamics code have also been reported.

2.2. Neutrino Transport

The Boltzmann equation for neutrinos in general relativity is solved by a finite difference scheme (S_N method) implicitly together with the above-mentioned Lagrangian hydrodynamics. The neutrino distribution function, $f_i(t, m, \mu, \epsilon_\nu)$, as a function of time t , Lagrangian mass coordinate m , neutrino propagation angle μ , and neutrino energy ϵ_ν , is evolved. Finite differencing of the Boltzmann equation is mostly based on the scheme by Mezzacappa & Bruenn (1993a). However, the update of variables in a time step is done simultaneously with the hydrodynamics. The reactions of neutrinos are explicitly calculated in the collision terms of the Boltzmann equation with incident/outgoing neutrino angles and energies taken into account. Detailed comparisons with the Monte Carlo method have been made to validate the Boltzmann solver and to examine the angular resolution (Yamada et al. 1999).

2.3. ν -Radiation Hydrodynamics

The whole set of finite-differenced equations described above are solved by the Newton-Raphson iterative method. The Jacobian

matrix forms a block tridiagonal matrix, in which dense block matrices arise from the collision terms of the transport equation. Since the inversion of this large matrix is most costly in the computing time, we utilize a parallel algorithm of block cyclic reduction for the matrix solver (Sumiyoshi & Ebisuzaki 1998). In the current simulations, we adopt 255 nonuniform spatial zones for the Lagrangian mass coordinate. We discretize the neutrino distribution function with six angle zones and 14 energy zones for ν_e , $\bar{\nu}_e$, $\nu_{\mu/\tau}$, and $\bar{\nu}_{\mu/\tau}$, respectively.

2.4. Rezoning

The description of the long-term evolution of accretion in Lagrangian coordinates is a numerically tough problem. In order to keep enough resolution during the accretion phase, rezoning of accreting materials is done long before they accrete onto the surface of a proto-neutron star and become opaque to neutrinos. At the same time, dezoning of the hydrostatic inner part of the proto-neutron star is done to avoid the increase of grid points.

When we have tried simulations without rezoning, neutrino luminosities oscillate greatly in time due to the intermittent accretion of coarse grid points, and this sometimes leads to erroneous dynamics (even explosions). Therefore, we have checked that the resolution of the grid points is high enough by refining the initial grid points and rezoning during the simulations. Even then, there are still slight oscillations in luminosities and average energies of neutrinos in the last stage of calculations. There are also transient kinks sometimes when the grid size in mass coordinates changes during accretion as we see in § 4.5. These slight modulations of neutrino quantities, however, do not affect the overall evolution of proto-neutron stars with accretion once we have enough resolution.

3. MODEL DESCRIPTIONS

As an initial model, we adopt the profile of the iron core of a $15 M_{\odot}$ progenitor from Woosley & Weaver (1995). This progenitor has been widely used in supernova simulations. The computational grid points in mass coordinates are nonuniformly placed to cover the central core, shock propagation region, and accreting material with enough resolution.

3.1. Equation of State

The new complete set of EOS for supernova simulations (SH-EOS; Shen et al. 1998a, 1998b) is derived by the relativistic mean field (RMF) theory with a local density approximation. The RMF theory has been a successful framework for reproducing the saturation properties, masses, and radii of nuclei, and proton-nucleus scattering data (Serot & Walecka 1986). We stress that the RMF theory (Sugahara & Toki 1994) is based on the relativistic Brückner-Hartree-Fock (RBHF) theory (Brockmann & Machleidt 1990), which is a microscopic and relativistic many-body theory. The RBHF theory has been shown to be successful in reproducing the saturation of nuclear matter starting from the nucleon-nucleon interactions determined by scattering experiments. This is in good contrast with nonrelativistic many-body frameworks, which can account for the saturation only with the introduction of extra three-body interactions.

The effective interactions in the RMF theory have been determined by least-squares fittings to reproduce the experimental data of masses and radii of stable and unstable nuclei (Sugahara & Toki 1994). The determined parameters of interaction, TM1, have been applied to many studies of nuclear structures and experimental analyses (Sugahara et al. 1996; Hirata et al. 1997). One of the stringent tests on the isovector interaction is passed by

excellent agreement between the theoretical prediction and the experimental data on neutron and proton distributions in isotopes including neutron-rich ones with neutron skins (Suzuki et al. 1995; Ozawa et al. 2001). The RMF theory with parameter set TM1 provides uniform nuclear matter with an incompressibility of 281 MeV and a symmetry energy of 36.9 MeV. The maximum mass of the neutron star is $2.2 M_{\odot}$ for the cold neutron star matter in the RMF with TM1 (Sumiyoshi et al. 1995a). The table of EOSs covers a wide range of density, electron fraction, and temperature for supernova simulations and has been applied to numerical simulations of the r -process in neutrino-driven winds (Sumiyoshi et al. 2000), prompt supernova explosions (Sumiyoshi et al. 2001), and other simulations (Sumiyoshi et al. 1995c, 2004; Rosswog & Davies 2003; Janka et al. 2005).

For comparison, we also adopt the EOS by Lattimer & Swesty (1991). The LS-EOS is based on the compressible liquid drop model for nuclei together with dripped nucleons. The bulk energy of nuclear matter is expressed in terms of density, proton fraction, and temperature. The values of nuclear parameters are chosen according to nuclear mass formulae and other theoretical studies with the Skyrme interaction. Among various parameters, the symmetry energy is set to be 29.3 MeV, which is smaller than the value in the relativistic EOS. As for the incompressibility, we use 180 MeV, which has been used frequently for recent supernova simulations. In this case, the maximum mass of the neutron star is estimated to be $1.8 M_{\odot}$. This choice enables us to make comparisons with previous works, although 180 MeV is smaller than the standard value as is discussed below. The sensitivity to the incompressibility of the LS-EOS has been studied by Thompson et al. (2003) using the choices of 180, 220, and 375 MeV. The numerical results of core collapse and bounce with different incompressibilities turn out to be similar up to 200 ms after bounce. The differences in luminosities and average energies of emergent neutrinos are within 10% and do not significantly affect the postbounce dynamics on a timescale of 100 ms. The influence of different incompressibilities in LS-EOS on a timescale of 1 s remains to be seen as an extension of the current study. For densities below 10^7 g cm^{-3} , the subroutine of the Lattimer-Swesty EOS runs into numerical troubles; therefore, we adopt Shen's EOS in this density regime instead. This is mainly for numerical convenience. In principle, it is preferable to adopt the EOS, which contains electrons and positrons at arbitrary degeneracy and relativity, photons, nucleons, and an ensemble of nuclei as nonrelativistic ideal gases (see, e.g., Timmes & Arnett 1999; Thompson et al. 2003). One also has to take into account non-NSE abundances determined from the preceding quasi-static evolutions. Note that we are chiefly concerned with the effect of EOS at high densities, and this pragmatic treatment does not have any significant influence on the shock dynamics.

Here we comment on the nuclear parameters of the EOS and its consequences for the astrophysical applications considered here. The value of incompressibility of nuclear matter has been considered to be within 200–300 MeV from experimental data and theoretical analyses. The value recently obtained within the nonrelativistic approaches (Colo & Van Giai 2004) is 220–240 MeV. The corresponding value extracted within the relativistic approaches is known to be higher than its nonrelativistic counterpart and is 250–270 MeV (Vretenar et al. 2003; Colo & Van Giai 2004). It is also known that the determination of incompressibility is closely related to the size of the symmetry energy and its density dependence. The incompressibility of the EOS in the RMF with TM1 is slightly higher than those standard values, and the SH-EOS is relatively stiff. The neutron stars with SH-EOS are, therefore, less compact with lower central densities

and have higher maximum masses than those obtained by LS-EOS with an incompressibility of 180 MeV. The adiabatic index of SH-EOS at the bounce of the supernova core is larger than that of LS-EOS (Sumiyoshi et al. 2004).

The value of the symmetry energy at the nuclear matter density is known to be around 30 MeV by nuclear mass formulae (Möller et al. 1995). The recent derivation of the symmetry energy in a relativistic approach gives higher values of 32–36 MeV together with the above-mentioned higher incompressibility (Dieperink et al. 2003; Vretenar et al. 2003). The symmetry energy in the RMF with TM1 is still a bit larger than the standard values. We note that the symmetry energy in the RMF is determined by the fitting of masses and radii of various nuclei including neutron-rich ones. The large symmetry energy in SH-EOS leads to large proton fractions in cold neutron stars, which may lead to a possible rapid cooling by the direct URCA process, as well as the stiffness of neutron matter (Sumiyoshi et al. 1995a). The difference between neutron and proton chemical potentials is large and leads to different compositions of free protons and nuclei (Sumiyoshi et al. 2004). The consequences of these differences in incompressibility and symmetry energy will be discussed in the comparison of numerical simulations in § 4.

3.2. Weak Reaction Rates

The weak interaction rates regarding neutrinos are evaluated by following the standard formulation by Bruenn (1985). For the collision term in the Boltzmann equation, the scattering kernels are explicitly calculated in terms of angles and energies of incoming and outgoing neutrinos (Mezzacappa & Bruenn 1993b). In addition to the Bruenn standard neutrino processes, the plasmon process (Braaten & Segel 1993) and the nucleon-nucleon bremsstrahlung process (Friman & Maxwell 1979; Maxwell 1987) are included in the collision term. The latter reaction has been shown to be an important process for determining the supernova neutrinos from the proto-neutron star cooling (Suzuki 1993; Burrows et al. 2000) as a source of $\nu_{\mu/\tau}$. The conventional *standard* weak reaction rates are used for the current simulations to single out the effect of the EOS and to compare with previous simulations. Recent progress of neutrino opacities in nuclear matter (Burrows et al. 2005) and electron capture rates on nuclei (Langanke & Martinez-Pinedo 2003) will be examined along with the updates of the EOS in future studies.

4. COMPARISON OF RESULTS

We present the results of two numerical simulations performed with the Shen EOS and Lattimer-Swesty EOS. They are denoted by SH and LS, respectively.

4.1. Shock Propagation

Figure 1 shows the radial trajectories of mass elements as a function of time after bounce in model SH. The trajectories are plotted for each $0.02 M_{\odot}$ in mass coordinate up to $1.0 M_{\odot}$ and for each $0.01 M_{\odot}$ for the rest of the outer part. Thick solid lines denote the trajectories for 0.5 , 1.0 , and $1.5 M_{\odot}$. One can see that the shock wave is launched up to 150 km and stalled there within 100 ms. The shock wave recedes down to below 100 km afterward, and neither a revival of the shock wave nor any sign of it is found, even after 300 ms.

Instead, a stationary accretion shock is formed at several tens of kilometers. As the central core gradually contracts, a proto-neutron star is born at the center. The material, which was originally located in the outer core, accretes onto the surface of the proto-neutron star. The accretion rate is about $0.2 M_{\odot} \text{ s}^{-1}$ on

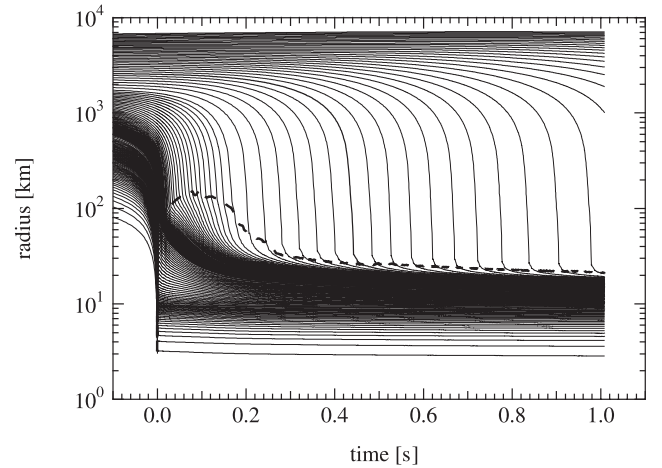


FIG. 1.—Radial trajectories of mass elements of the core of a $15 M_{\odot}$ star as a function of time after bounce in model SH. The location of shock wave is displayed by a thick dashed line.

average and gradually decreases from 0.25 to $0.15 M_{\odot} \text{ s}^{-1}$. This behavior is similar in model LS. At 1 s after bounce, the baryon mass of the proto-neutron star is $1.60 M_{\odot}$ for both cases.

The trajectories of the shock waves in models SH and LS are compared in Figure 2. The propagation of the shock wave in each model is similar in the first 200 ms (*left panel*). We note that slight fluctuations in the curves are due to a numerical artifact in the procedure for determining the shock position. Note that we have rather low resolution in the central part in order to have higher resolution in the accreting material. Except for the discrepancy due to the different numerical methods (e.g., approximate general relativity, Eulerian, etc.), zoning, and resolutions, the current simulations up to 200 ms are consistent with the results (Fig. 3, *middle*) by Janka et al. (2005) having similar maximum radii and timing of recession. The difference shows up from 200 ms after bounce and becomes more apparent in the later phase (Fig. 2, *right*). After 600 ms, the shock position in model LS is less than 20 km, and it is clearly different from that in model SH. This difference originates from the faster contraction of the proto-neutron star in model LS. We discuss the evolution of the proto-neutron star in § 4.4.

4.2. Collapse Phase

The initial propagation of the shock wave is largely controlled by the properties of the inner core during the gravitational collapse. We have found noticeable differences in the behavior of core collapse in the two models. However, they did not change the initial shock energy drastically, which then leads to the similarity of the early phase of shock propagation that we see above.

First of all, it is remarkable that the compositions of dense matter during the collapse are different. In Figure 3, the mass fraction is shown as a function of mass coordinate when the central density reaches $10^{11} \text{ g cm}^{-3}$. The mass fraction of free protons in model SH is smaller than that in model LS by a factor of ~ 5 . This is caused by the larger symmetry energy in SH-EOS, where the proton chemical potential is lower than the neutron chemical potential as discussed in Sumiyoshi et al. (2004). The smaller free proton fraction reduces the electron captures on free protons. Note that the electron capture on nuclei is suppressed in the current simulations due to the blocking above $N = 40$ in Bruenn's prescription. This is in accordance with the numerical results by Bruenn (1989) and Swesty et al. (1994), who studied the influence of the free proton fraction and the symmetry energy.

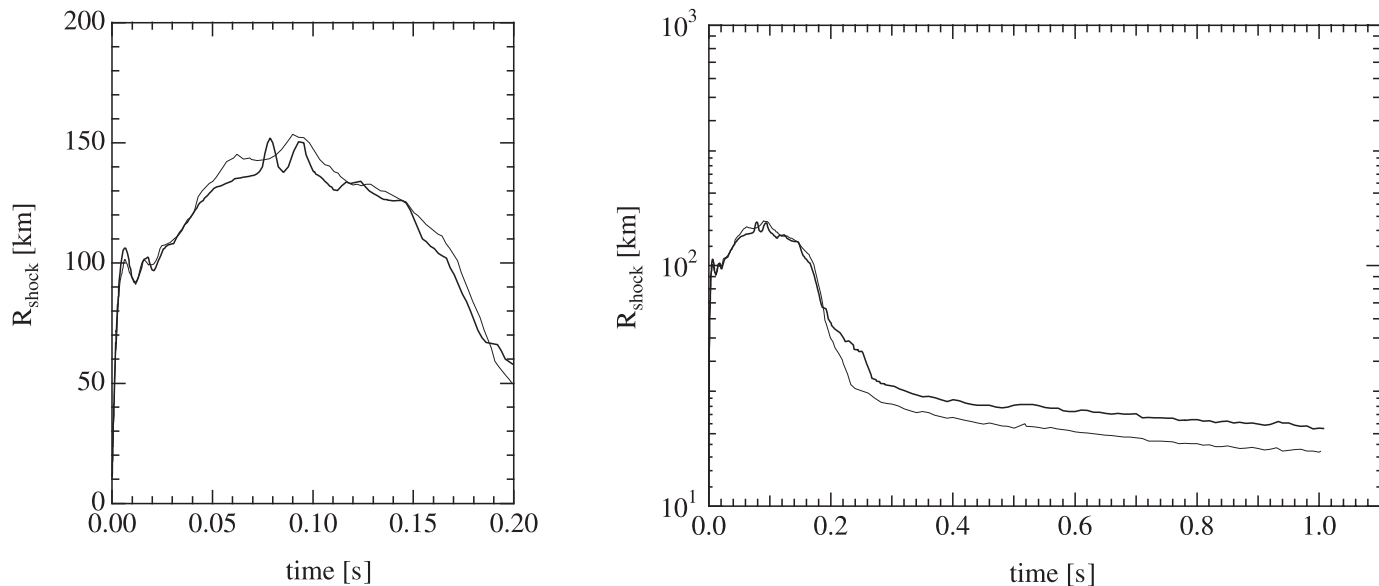


FIG. 2.—Radial positions of shock waves in models SH (*thick lines*) and LS (*thin lines*) as a function of time after bounce. The evolution at early (*left*) and late (*right*) times is shown. Small fluctuations in the curves are due to a numerical artifact in the procedure for determining the shock position from a limited number of grid points.

However, there is also negative feedback in the deleptonization during collapse (Liebendörfer et al. 2002). Smaller electron capture rates keep the electron fraction high, which then leads to an increase of the free proton fraction and consequently to electron captures after all. The resultant electron fraction turns out to be not significantly different as we see below.

It is also noticeable that the mass fraction of alpha particles differs substantially and the abundance of nuclei is slightly reduced in model SH. This difference of alpha abundances in the two models persists during the collapse and even in the post-bounce phase. The nuclear species appearing in the central core during collapse are shown in the nuclear chart (Fig. 4). The nuclei in model SH are always less neutron-rich than those in model LS by more than several neutrons. This is also due to the effect of the symmetry energy, which gives nuclei closer to the stability line in model SH. The mass number reaches up to ~ 80 and ~ 100 at the central density of $10^{11} \text{ g cm}^{-3}$ (*filled circles*) and $10^{12} \text{ g cm}^{-3}$ (*open circles*), respectively. In the current simula-

tions, the electron capture on nuclei is suppressed beyond $N = 40$ due to the simple prescription employed here and a difference in species does not make any difference. However, results may turn out differently when more realistic electron capture rates are adopted (Hix et al. 2003). It would be interesting to see whether the difference found in two EOSs leads to differences in central cores using recent electron capture rates on nuclei (Langanke & Martinez-Pinedo 2003). Further studies are necessary to discuss the abundances of nuclei and the influence of more updated electron capture rates for the mixture of nuclear species beyond the approximation of single species in the current EOSs.

The profiles of lepton fractions at bounce are shown in Figure 5. The central electron fraction in model SH is $Y_e = 0.31$, which is slightly higher than $Y_e = 0.29$ in model LS. The central lepton fractions including neutrinos for models SH and LS are rather close to each other, being $Y_L = 0.36$ and 0.35 , respectively. The difference of lepton fraction results in a different size

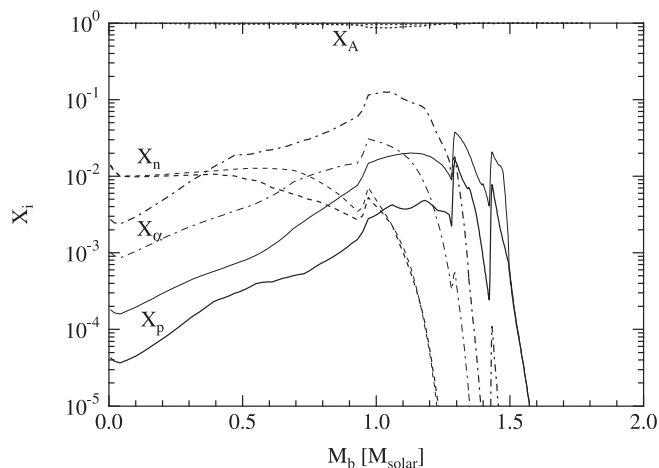


FIG. 3.—Mass fractions in the supernova cores as a function of baryon mass coordinate at the time when the central density reaches $10^{11} \text{ g cm}^{-3}$. Solid, dashed, dotted, and dot-dashed lines show mass fractions of protons, neutrons, nuclei, and alpha particles, respectively. The results for models SH and LS are shown by thick and thin lines, respectively.

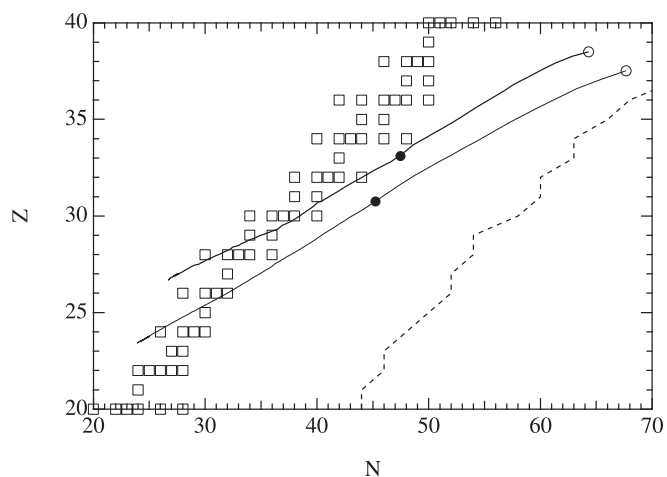


FIG. 4.—Nuclear species appearing in supernova cores plotted on the nuclear chart. Stable nuclei and the neutron drip line (Horiguchi et al. 2000) are shown by open square symbols and a dashed line, respectively. Nuclear species at the center of the core are marked by filled circles ($\rho_c = 10^{11} \text{ g cm}^{-3}$) and open circles ($\rho_c = 10^{12} \text{ g cm}^{-3}$). The results for models SH and LS are shown by thick and thin lines, respectively.

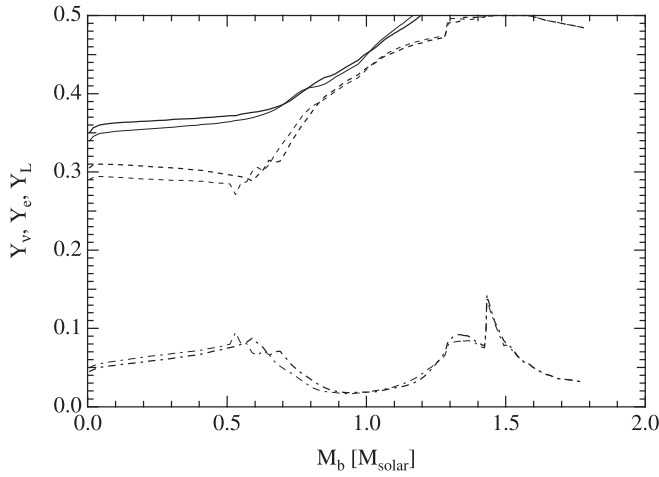


FIG. 5.—Lepton (solid lines), electron (dashed lines), and neutrino (dot-dashed lines) fractions at bounce as a function of baryon mass coordinate for models SH (thick lines) and LS (thin lines).

of the inner core. The larger lepton fraction in model SH leads to a larger inner core of $0.61 M_{\odot}$, whereas it is $0.55 M_{\odot}$ in model LS. Here the inner core is defined by the region inside the position of velocity discontinuity, which is the beginning of the shock wave. Figure 6 shows the velocity profile at bounce. We define the bounce ($t_{\text{pb}} = 0$ ms) as the time when the central density reaches maximum, which is similar to other definitions such as using the peak entropy height. The central density reaches 3.4×10^{14} and $4.4 \times 10^{14} \text{ g cm}^{-3}$ in models SH and LS, respectively. The difference of stiffness in the two EOSs leads to a lower peak central density in model SH than that in model LS. Because of this difference, the radial size of the inner core at bounce is ~ 1 km larger for model SH than that for model LS.

The initial shock energy, which is roughly estimated by the gravitational binding energy of the inner core at bounce, turns out not to be drastically different because of the increases in both mass and radial size of the inner core in model SH. A clearer difference appears at later stages when the proto-neutron star is formed having a central density much higher than the nuclear matter density. This is one of the reasons why we are interested in the late phase of the supernova core, where the difference of the EOS appears more clearly and its influence on the supernova dynamics could be seen.

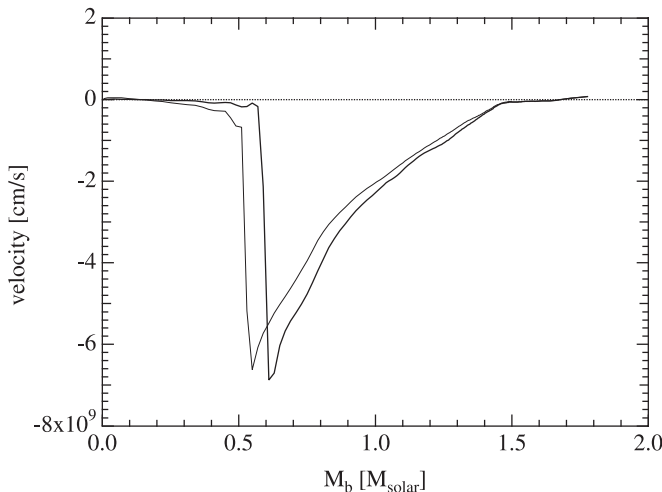


FIG. 6.—Velocity profiles at bounce as a function of baryon mass coordinate for models SH (thick lines) and LS (thin lines).

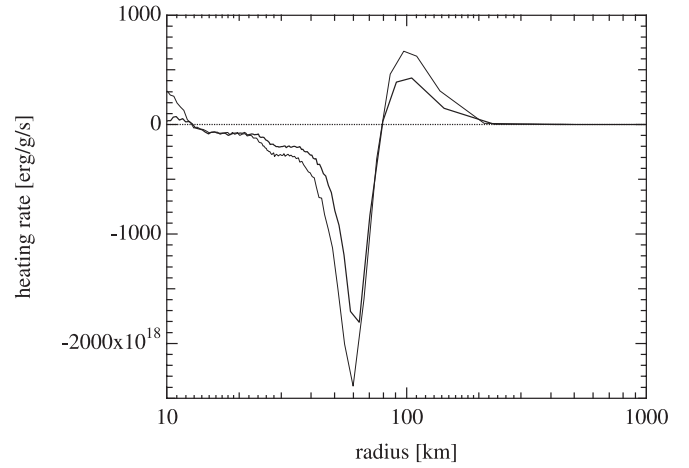


FIG. 7.—Heating rates at $t_{\text{pb}} = 150$ ms in two models as a function of radius. Notation is the same as in Fig. 6.

We remark here that the numerical results with LS-EOS at bounce are in good agreement with previous simulations such as the reference models by Liebendörfer et al. (2005). For example, the profiles of model LS shown in Figures 5 and 6 are in accord with the profiles of their model G15. The behavior after bounce is also qualitatively consistent with the reference models up to 250 ms (see also § 4.4).

4.3. Postbounce Phase

The postbounce phase is interesting in many aspects, especially in clarifying the role of the EOS in the neutrino heating mechanism and the proto-neutron star formation. As we see in § 4.1, the stall of the shock wave occurs in a similar manner in two models and the difference appears in a later stage. Here we discuss the similarities and the differences in terms of the effect of the EOS.

The evolution of the shock wave after it stalls around 100 km is controlled mainly by the neutrino heating behind the shock wave. The neutrinos emitted from the neutrinosphere in the nascent proto-neutron star contribute to the heating of material just behind the shock wave through absorption on nucleons. Whether the shock wave revives or not depends on the total amount of heating, hence more specifically, on the neutrino spectrum, luminosity, amount of targets (nucleons), mass of the heating region, and time duration.

The heating rates of material in the supernova core in the two models at $t_{\text{pb}} = 150$ ms are shown in Figure 7 as a function of radius. The heating rate in model SH is smaller than that in model LS around 100 km. The cooling rate (negative value in the heating rate) in model SH is also smaller than that in model LS. The smaller heating (cooling) rate in model SH is caused by lower neutrino luminosities and smaller free proton fractions. Figures 8 and 9 show the radial profiles of neutrino luminosities and mass fractions of dense matter around the heating region. The luminosities in model SH are lower than those in model LS for all neutrino flavors. The mass fraction of free protons, which are the primary target of neutrino heating, is slightly smaller in model SH around the heating region. These two combinations lower the heating rate in model SH. It is also interesting that other compositions (alpha and nuclei) appear different in this region.

The lower luminosities in model SH are related to the lower cooling rate. The temperature of the proto-neutron star in model SH is generally lower than that in model LS as shown in Figure 10. The peak temperature, which is produced by the shock

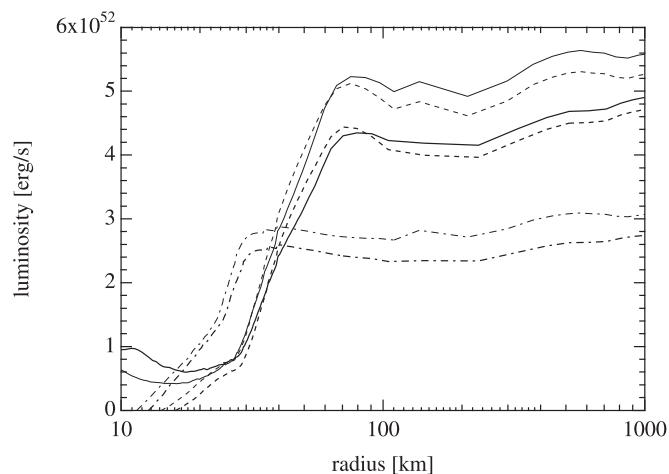


FIG. 8.—Luminosities of ν_e (solid lines), $\bar{\nu}_e$ (dashed lines), and $\nu_{\mu/\tau}$ (dot-dashed lines) around the heating region as a function of radius at $t_{\text{pb}} = 150$ ms for models SH (thick lines) and LS (thin lines).

heating and the contraction of the core, is lower in model SH than in model LS. This difference also exists in the surface region of the proto-neutron star, where neutrinos are emitted via cooling processes. The temperature at the neutrinosphere in model SH is lower, and, as a result, the cooling rate is smaller. The difference of temperature becomes more evident as the proto-neutron star evolves as we see in the next section.

4.4. Proto-Neutron Star

The thermal evolution of the proto-neutron star formed after bounce is shown in Figure 11 for both models. Snapshots of the temperature profile at $t_{\text{pb}} = 20, 50, 100, 200, 300, 400, 500, 600, 700, 800, 900$, and 1000 ms are shown. The temperature increase is slower in model SH than in model LS. The peak temperature at $t_{\text{pb}} = 1$ s is 39 and 53 MeV in models SH and LS, respectively. The temperature difference arises mainly from the stiffness of the EOS. The proto-neutron star contracts more in model LS and has a higher central density than in model SH. At $t_{\text{pb}} = 1$ s, the central density in model SH is $4.1 \times 10^{14} \text{ g cm}^{-3}$, whereas that in model LS is $7.0 \times 10^{14} \text{ g cm}^{-3}$, which means rapid contraction in model LS. Since the profiles of the entropy per baryon are similar to each other, lower density results in lower temperature. The rapid contraction also gives rise to the rapid recession of the shock wave down to 20 km in model LS.

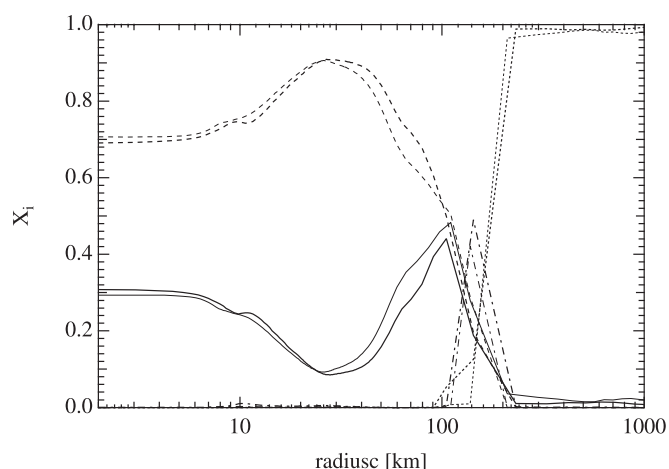


FIG. 9.—Mass fractions in dense matter around the heating region as a function of radius at $t_{\text{pb}} = 150$ ms. Notation is the same as in Fig. 3.

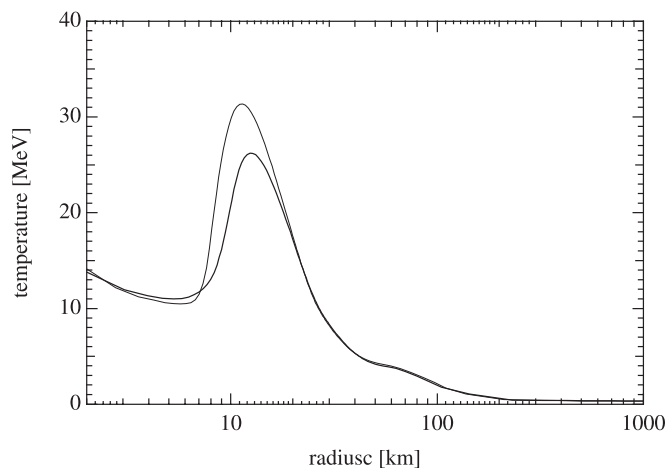


FIG. 10.—Temperature profiles at $t_{\text{pb}} = 150$ ms for two models as a function of radius. Notation is the same as in Fig. 6.

Here we discuss the effective mass. In SH-EOS, the effective mass of nucleons is obtained from the attraction by scalar mesons in the nuclear many-body framework. The effective mass at the center is reduced to 440 MeV at $t_{\text{pb}} = 1$ s. On the other hand, the nucleon mass is fixed to be the free nucleon mass in the LS-EOS.

The temperature difference within 1 s may, as we have found, affect the following evolution of the proto-neutron star up to several tens of seconds, during which the majority of supernova neutrinos are emitted. Although our models do not give a successful explosion, the obtained profiles will still give a good approximation to the initial setup for the subsequent proto-neutron star cooling. Since we have followed the continuous evolution of the central core from the onset of gravitational collapse, the calculated proto-neutron star contains the history of matter and neutrinos during the prior stages. This is much better than the situation so far for calculations of proto-neutron star cooling, where the profiles from other supernova simulations were adopted for the initial model. It would be interesting to study the cooling of a proto-neutron star for the two models obtained here. Even if such evolutions of a proto-neutron star are not associated with a successful supernova explosion, it will still be interesting for the collapsar scenario of gamma-ray burst and/or black hole formation. Exploratory studies on various scenarios for the fate of compact objects with continuous accretion of matter are fascinating and currently under way, but these are beyond the scope of the present study.

In Figure 12, we display the profiles of entropy and the lepton fraction in model LS at $t_{\text{pb}} = 100, 250$, and 500 ms and 1 s. The distributions of entropy, as well as other quantities (not shown here), at $t_{\text{pb}} = 100$ and 250 ms are consistent with the reference model G15 (Liebendörfer et al. 2005). We have found that the negative gradients in the profiles of entropy and lepton fraction commonly appear in the late phase for both models. As for entropy per baryon, the negative gradient appears after $t_{\text{pb}} = 100$ ms in the region between $\sim 0.7 M_{\odot}$ and the shock. The negative gradient of the lepton fraction appears first in the outer core behind the shock and then prevails toward the center until $t_{\text{pb}} = 1$ s. Since these regions are unstable against the convection according to the Ledoux criterion, the whole region of the proto-neutron star may be convective after core bounce. It has been pointed out that the sign of the derivative of the thermodynamical quantities ($\partial\rho/\partial Y_L|_{P,S}$) changes in the neutron-rich environment at high densities beyond $10^{14} \text{ g cm}^{-3}$ (Sumiyoshi et al. 2004), and the central core may be stabilized in model SH. Whether the convection

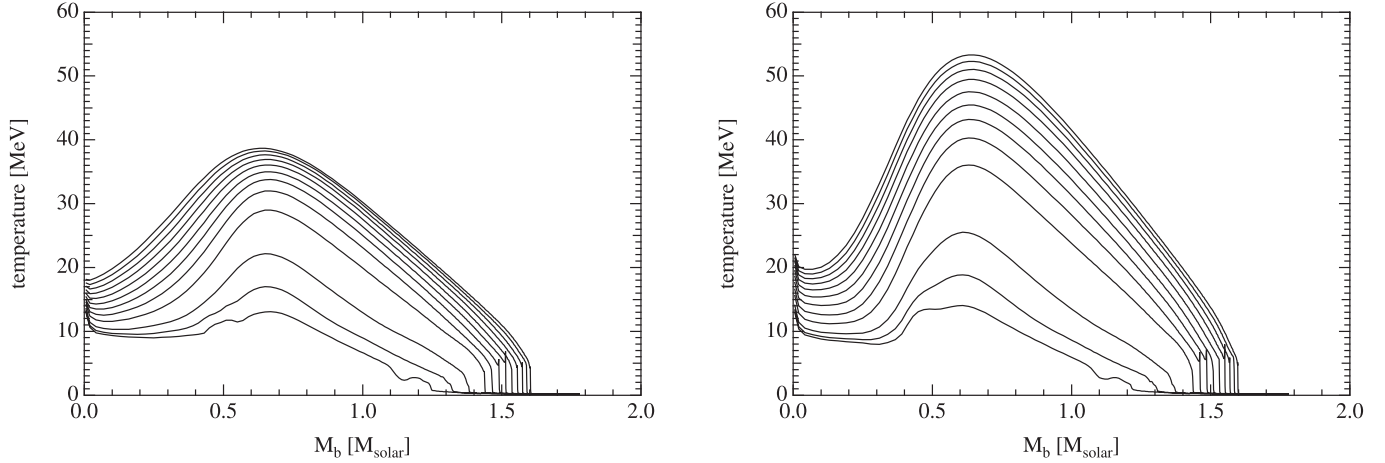


FIG. 11.—Snapshots of temperature profiles as a function of baryon mass coordinate from $t_{\text{pb}} = 20$ to 1000 ms in models SH (*left*) and LS (*right*). Note that small peaks around the central grid are artificial, resulting from the numerical treatment.

occurs efficiently enough to help the neutrino-driven mechanism for explosion remains to be studied in multidimensional ν -radiation hydrodynamics simulations with SH-EOS.

4.5. Supernova Neutrinos

The different temperature distribution could affect the neutrino luminosities and spectra. Here we discuss the properties of neutrinos emitted during the evolution of the supernova core up to 1 s. As we have already discussed in § 4.3, the luminosity of neutrinos in model SH is lower than that in model LS after bounce. This difference actually appears after $t_{\text{pb}} = 100$ ms as shown in Figure 13. The initial rise and peak of luminosities in the two models are quite similar to each other. The peak heights of the neutronization burst of electron-type neutrino are also similar. The difference, however, gradually becomes larger and apparent after $t_{\text{pb}} = 200$ ms. We remark here that the kinks around $t_{\text{pb}} = 500$ ms are a numerical artifact due to the rezoning of the mass coordinate as discussed in § 2.4. Except for this kink, the luminosities increase in time. For the last 150 ms, luminosities show oscillations numerically; therefore, we have plotted smoothed curves by taking average values. It is to be noted that we are interested in the relative differences of supernova neutrinos between the two models.

The difference in average energies of neutrinos appears in a similar manner to that in luminosities as seen in Figure 14. The average energy presented here is the rms average energy, $E_\nu = \langle \epsilon_\nu^2 \rangle^{1/2}$, at the outermost grid point (~ 7000 km). The average energies up to $t_{\text{pb}} = 100$ ms are almost identical in both models and become different from each other afterward. The average energies in model SH turn out to be lower than those in model LS. Kinks around $t_{\text{pb}} = 500$ ms appear for the same reason mentioned above, and the curves are smoothed around kinks and $t_{\text{pb}} \sim 1$ s to avoid artificial transient behaviors due to the rezoning. At $t_{\text{pb}} = 1$ s, the gap amounts to more than a few MeV and has a tendency to increase with time. The lower luminosity and average energy in model SH are due to the slow contraction of the proto-neutron star and, as a result, the slow rise of temperature as seen in Figure 11. Again, it would be interesting to see the subsequent cooling phase of the proto-neutron star up to ~ 20 s to obtain the main part of supernova neutrinos.

5. SUMMARY

We have performed numerical simulations of core-collapse supernovae by solving general relativistic ν -radiation hydrodynamics in spherical symmetry. We have adopted a relativistic

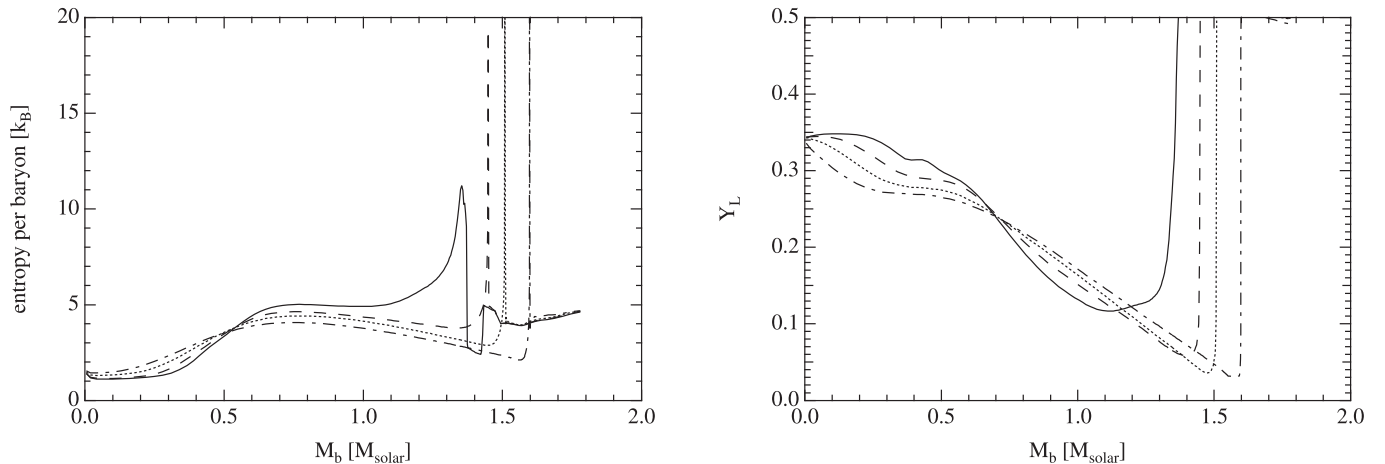


FIG. 12.—Snapshots of entropy (*left*) and lepton fraction (*right*) profiles in model LS as a function of baryon mass coordinate at $t_{\text{pb}} = 100$ ms (*solid line*), $t_{\text{pb}} = 250$ ms (*dashed line*), $t_{\text{pb}} = 500$ ms (*dotted line*), and $t_{\text{pb}} = 1000$ ms (*dot-dashed line*).

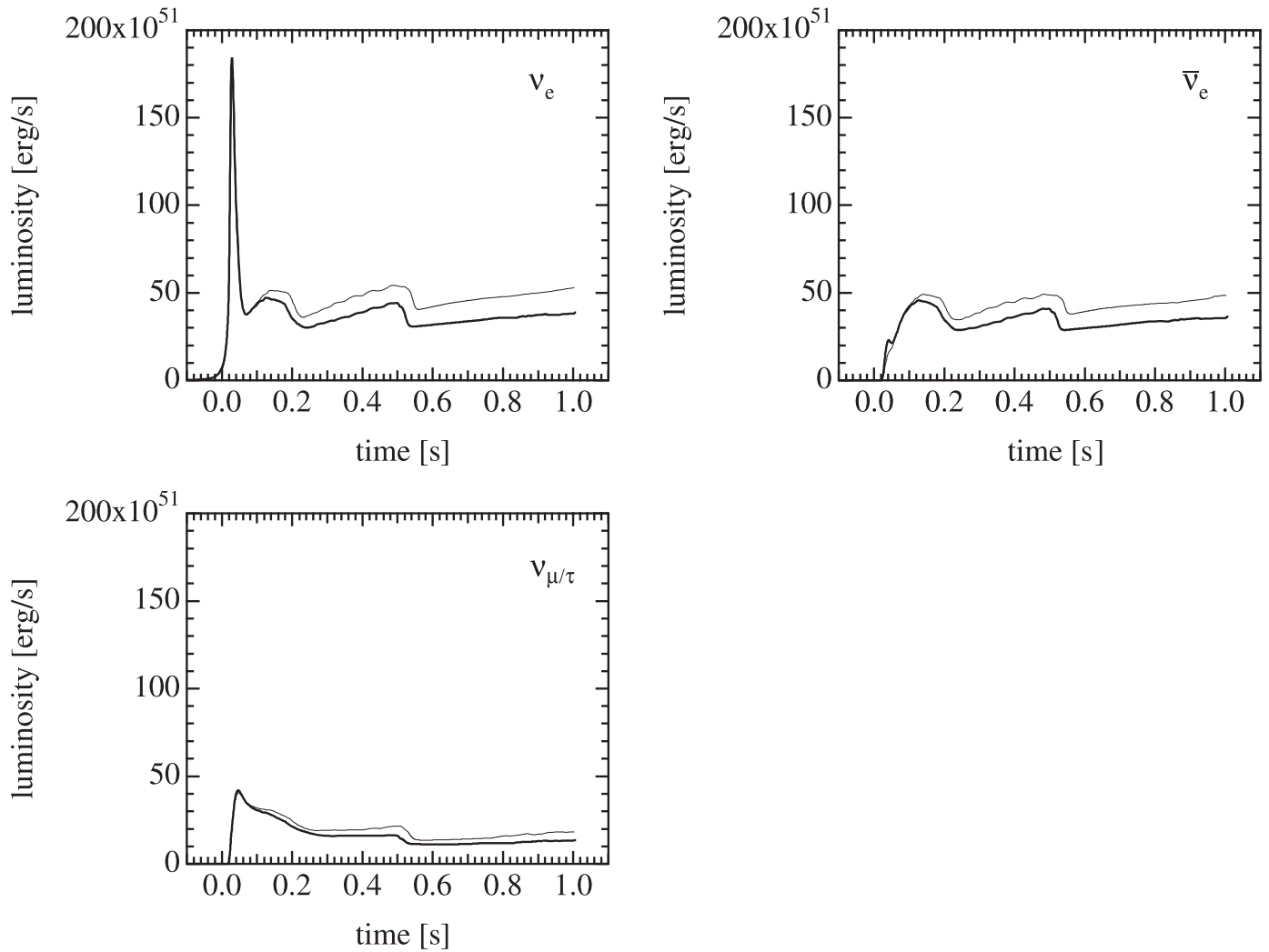


FIG. 13.—Luminosities of ν_e , $\bar{\nu}_e$, and $\nu_{\mu/\tau}$ as a function of time after bounce. Notation is the same as in Fig. 6. Kinks around $t_{\text{pb}} = 500$ ms are caused by a numerical artifact due to the rezoning of the mass coordinate. See the main text for details.

EOS table that is based on recent advancements in nuclear many-body theory, as well as recent experimental data of unstable nuclei, in addition to the conventional Lattimer-Swesty EOS. We have performed long-term simulations from the onset of gravitational collapse to the late phase far beyond 300 ms after bounce, which has not been well studied in previous studies due to numerical restrictions. This is meant to explore the chance of shock revival and the influence of the new EOS in this stage and is the first such attempt.

We have found that a successful explosion of the supernova core does not occur in either a prompt or a delayed way, even though we have followed the postbounce evolution up to 1 s with the new EOS table. The numerical simulation using the Lattimer-Swesty EOS shows no explosion either, which is in accord with other recent studies and in contrast to the finding by Wilson (Bethe & Wilson 1985). Note that Wilson incorporated convective effects into the spherical simulations to obtain a successful explosions. The shock wave stalls around 100 ms after bounce and recedes down to several tens of kilometers to form a stationary accretion shock.

Regardless of the outcome with no explosion, we have revealed the differences caused by two EOSs in many aspects, which might give some hints for a successful explosion. We have

seen the difference in composition of free protons and nuclei at the collapse phase of a supernova core in interesting manners. The difference in symmetry energy of the two EOSs caused this effect, which can change the electron capture rates and the resulting size of bounce cores. Although the early shock propagations turn out to be similar in the current simulations due to the counter-effect by the stiffness of the EOS and the neutrino heating, the implementation of up-to-date electron capture rates on nuclei remains to be done to obtain a more quantitatively reliable difference of composition during the collapse phase, which may then affect the initial shock energy.

During the postbounce evolution around 100 ms after bounce we have seen that the heating rates in the two models are different due to the different luminosities and compositions predicted by the two EOSs. Unfortunately, the merit of the larger inner core found in the model with SH-EOS is mostly canceled by the smaller heating rate, and the behaviors of the shock wave in the early post-bounce phase turn out to be similar in both simulations. In general, though, different heating rates by spectral change of neutrinos and compositional differences due to EOSs might contribute to the revival of the shock wave in the neutrino-driven mechanism.

One of the most important facts we have revealed in the comparison is that a larger difference actually appears from 200 ms

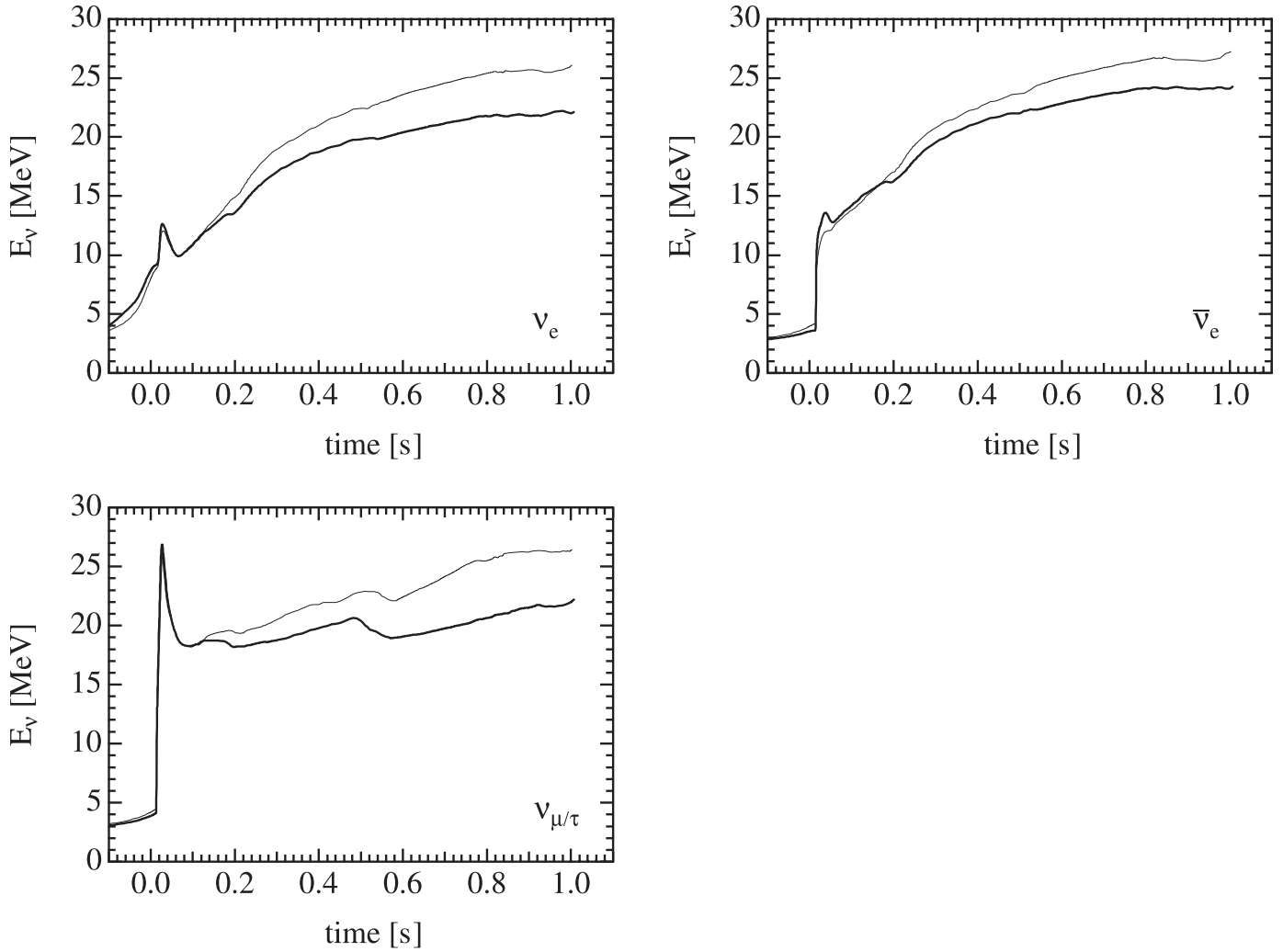


FIG. 14.—Same as Fig. 13, but for average energies of ν_e , $\bar{\nu}_e$, and $\nu_{\mu/\tau}$.

after bounce when the central core contracts to become a proto-neutron star. The temperature and density profiles display larger differences as the proto-neutron star shrinks further. It is in this late phase that we are interested in seeing possible influences of EOS on the shock dynamics, since the central density becomes high enough and the difference of EOS becomes more apparent. In the current study, we have not found any shock revival in either model. We have found, however, distinctly different thermal evolution of proto-neutron stars in the two models, and the resulting neutrino spectra are clearly different at this stage. This difference might have some influence on the accretion of matter. The following evolution of proto-neutron star cooling or the formation of a black hole or any other exotic objects will certainly be affected.

After all, the current numerical simulations of core-collapse supernovae in spherical symmetry have not produced successful explosions, even with a new EOS or after long-term evolution. One might argue that this situation indicates the necessity of breaking spherical symmetry, which is also suggested by some observations and has been supported by multidimensional simulations. However, before one jumps to the conclusion that the asymmetry is essential in the explosion mechanism, one also has to make efforts to find missing ingredients in microphysics (such as hyperons in the EOS, for example) in spherically symmetric simulations. Moreover, the spherical simulations serve as a re-

liable basis for multidimensional computations of ν -radiation hydrodynamics. Convection may somehow be taken into account effectively in spherical codes as in the stellar evolution codes. These extensions of simulations and microphysics are now in progress. The relativistic EOS table has recently been extended by including strangeness particles at high densities (Ishizuka 2005), and corresponding neutrino reactions in hyperonic matter are currently being implemented in ν -radiation hydrodynamics.

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REFERENCES

- Bethe, H. A., & Wilson, J. R. 1985, *ApJ*, 295, 14
- Blondin, J. M., Mezzacappa, A., & DeMarino, C. 2003, *ApJ*, 584, 971
- Braaten, E., & Segel, D. 1993, *Phys. Rev. D*, 48, 1478
- Brockmann, R., & Machleidt, R. 1990, *Phys. Rev. C*, 42, 1965
- Bruenn, S. W. 1985, *ApJS*, 58, 771
- . 1989, *ApJ*, 340, 955
- Buras, R., Rampp, M., Janka, H.-Th., & Kifonidis, K. 2003, *Phys. Rev. Lett.*, 90, 241101
- Burrows, A., Reddy, S., & Thompson, T. A. 2005, *Nucl. Phys. A*, in press
- Burrows, A., Young, T., Pinto, P., Eastman, R., & Thompson, T. A. 2000, *ApJ*, 539, 865
- Colo, G., & Van Giai, N. 2004, *Nucl. Phys. A*, 731, 15
- Dieperink, A. E. L., Dewulf, Y., Van Neck, D., Waroquier, M., & Rodin, V. 2003, *Phys. Rev. C*, 68, 064307
- Friman, B. L., & Maxwell, O. V. 1979, *ApJ*, 232, 541
- Fryer, C. L., & Warren, M. S. 2004, *ApJ*, 601, 391
- Hillebrandt, W., Nomoto, K., & Wolff, R. G. 1984, *A&A*, 133, 175
- Hillebrandt, W., & Wolff, R. G. 1985, in *Nucleosynthesis—Challenges and New Developments*, ed. W. D. Arnett & J. M. Truran (Chicago: Univ. Chicago Press), 131
- Hirata, D., Sumiyoshi, K., Tanihata, I., Sugahara, Y., Tachibana, T., & Toki, H. 1997, *Nucl. Phys. A*, 616, 438
- Hix, W. R., Messer, O. E. B., Mezzacappa, A., Liebendoerfer, M., Sampaio, J., Langanke, K., Dean, D. J., & Martínez-Pinedo, G. 2003, *Phys. Rev. Lett.*, 91, 201102
- Horiguchi, T., Tachibana, T., Koura, H., & Katakura, J. 2000, *Chart of the Nuclides* (Tokai: Japan Atomic Energy Research Inst.)
- Ishizuka, C. 2005, Ph.D. thesis, Hokkaido Univ.
- Janka, H.-Th., Buras, R., Kitaura Joyanes, F. S., Marek, A., & Rampp, M. 2005, in *Proc. 12th Workshop on Nuclear Astrophysics*, in press (astro-ph/0405289)
- Kotake, K., Sawai, H., Yamada, S., & Sato, K. 2004, *ApJ*, 608, 391
- Kotake, K., Yamada, S., & Sato, K. 2003, *ApJ*, 595, 304
- Langanke, K., & Martínez-Pinedo, G. 2003, *Rev. Mod. Phys.*, 75, 819
- Lattimer, J. M., & Swesty, F. D. 1991, *Nucl. Phys. A*, 535, 331
- Liebendörfer, M., Messer, O. E. B., Mezzacappa, A., Bruenn, S. W., Cardall, C. Y., & Thielemann, F.-K. 2004, *ApJS*, 150, 263
- Liebendörfer, M., Messer, O. E. B., Mezzacappa, A., Hix, W. R., Thielemann, F.-K., & Langanke, K. 2002, in *Proc. 11th Workshop on Nuclear Astrophysics*, ed. W. Hillebrandt & E. Müller (Garching: Springer), 126
- Liebendörfer, M., Mezzacappa, A., Thielemann, F.-K., Messer, O. E. B., Hix, W. R., & Bruenn, S. W. 2001, *Phys. Rev. D*, 63, 103004
- Liebendörfer, M., Rampp, M., Janka, H.-Th., & Mezzacappa, A. 2005, *ApJ*, 620, 840
- Maxwell, O. V. 1987, *ApJ*, 316, 691
- Mezzacappa, A., & Bruenn, S. W. 1993a, *ApJ*, 405, 669
- . 1993b, *ApJ*, 410, 740
- Mezzacappa, A., Liebendörfer, M., Messer, O. E. B., Hix, W. R., Thielemann, F.-K., & Bruenn, S. W. 2001, *Phys. Rev. Lett.*, 86, 1935
- Misner, C. W., & Sharp, D. H. 1964, *Phys. Rev. B*, 136, 571
- Möller, P., Nix, J. R., Myers, W. D., & Swiatecki, W. J. 1995, *At. Data Nucl. Data Tables*, 59, 185
- Ozawa, A., et al. 2001, *Nucl. Phys. A*, 691, 599
- Rampp, M. 2000, Ph.D. thesis, Max-Planck Inst. for Astrophysics
- Rampp, M., & Janka, H.-Th. 2000, *ApJ*, 539, L33
- . 2002, *A&A*, 396, 361
- Rosswog, S., & Davies, M. B. 2003, *MNRAS*, 345, 1077
- Serot, B. D., & Walecka, J. D. 1986, in *Advances in Nuclear Physics Vol. 16*, ed. J. W. Negele & E. Vogt (New York: Plenum), 1
- Shen, H., Toki, H., Oyamatsu, K., & Sumiyoshi, K. 1998a, *Nucl. Phys. A*, 637, 435
- . 1998b, *Prog. Theor. Phys.*, 100, 1013
- Sugahara, Y., Sumiyoshi, K., Toki, H., Ozawa, A., & Tanihata, I. 1996, *Prog. Theor. Phys.*, 96, 1165
- Sugahara, Y., & Toki, H. 1994, *Nucl. Phys. A*, 579, 557
- Sumiyoshi, K., & Ebisuzaki, T. 1998, *Parallel Computing*, 24, 287
- Sumiyoshi, K., Kuwabara, H., & Toki, H. 1995a, *Nucl. Phys. A*, 581, 725
- Sumiyoshi, K., Oyamatsu, K., & Toki, H. 1995b, *Nucl. Phys. A*, 595, 327
- Sumiyoshi, K., Suzuki, H., Otsuki, K., Terasawa, M., & Yamada, S. 2000, *PASJ*, 52, 601
- Sumiyoshi, K., Suzuki, H., & Toki, H. 1995c, *A&A*, 303, 475
- Sumiyoshi, K., Suzuki, H., Yamada, S., & Toki, H. 2004, *Nucl. Phys. A*, 730, 227
- Sumiyoshi, K., Terasawa, M., Mathews, G. J., Kajino, T., Yamada, S., & Suzuki, H. 2001, *ApJ*, 562, 880
- Suzuki, H. 1990, Ph.D. thesis, Univ. Tokyo
- . 1993, in *Proc. Int. Symp. on Neutrino Astrophysics: Frontiers of Neutrino Astrophysics*, ed. Y. Suzuki & K. Nakamura (Tokyo: Universal Academy), 219
- . 1994, in *Physics and Astrophysics of Neutrinos*, ed. M. Fukugita & A. Suzuki (Tokyo: Springer), 763
- Suzuki, T., et al. 1995, *Phys. Rev. Lett.*, 75, 3241
- Swesty, F. D., Lattimer, J. M., & Myra, E. S. 1994, *ApJ*, 425, 195
- Thompson, T. A., Burrows, A., & Pinto, P. 2003, *ApJ*, 592, 434
- Timmer, F. X., & Arnett, D. 1999, *ApJS*, 125, 277
- Vretnar, D., Niksic, T., & Ring, P. 2003, *Phys. Rev. C*, 68, 024310
- Walder, R., Burrows, A., Ott, C. D., Livne, E., Lichtenstadt, I., & Jarrah, M. 2005, *ApJ*, 626, 317
- Woosley, S. E., & Weaver, T. 1995, *ApJS*, 101, 181
- Yamada, S. 1997, *ApJ*, 475, 720
- Yamada, S., Janka, H.-Th., & Suzuki, H. 1999, *A&A*, 344, 533