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THE DETERMINATION OF THE TOTAL INJECTED POWER IN SOLAR FLARE ELECTRONS

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ABSTRACT

We here review the procedure by which the injected power in bremsstrahlung-producing electrons is determined from the hard X-ray spectra that they produce. In particular, we note that low-energy photons are produced in large part by electrons with energies that do not greatly exceed the thermal energy of the target with which they interact, so that the commonly used assumption of "cold-target" energy loss is not applicable over the entire energy range. We show that this significantly reduces the inferred energy content of the injected electron distribution and even makes the oft-dwelt-upon concept of a "lower cutoff energy" in the injected electron spectrum moot. Convenient formulae are provided for the total power in the injected electrons.

Subject heading: Sun: flares

1. INTRODUCTION

The *Reuven Ramaty High Energy Solar Spectroscopic Imager* (*RHESSI*) instrument has produced many high-quality hard X-ray spectra from solar flares. These spectra are of sufficient quality that the corresponding electron distribution function can be accurately computed. Holman et al. (2003) and Piana et al. (2003) have used such hard X-ray spectra to calculate the mean source *electron* spectrum $\bar{F}(E)$ (see Brown, Emslie, & Kontar 2003) for a series of times throughout the 2002 July 23 event. Analysis of the form of $\bar{F}(E)$ allows us, through modeling of the behavior of the electrons in the source, to calculate the total power in the *injected* electrons.

Hard X-ray spectra in solar flares are typically steep (with a power-law spectral index $\gamma \gtrsim 3$). The electron spectra derived from these are correspondingly steep (e.g., in the standard collisional thick-target model of Brown 1971, the spectral index, δ_0, of the injected electron flux spectrum $F_0(E_0)$ is given by $\delta_0 = \gamma + 1$). Since $\delta_0 > 2$, the injected energy flux $\int E_0 F_0(E_0) dE_0$ diverges at low energies. To keep the injected energy flux finite, authors have usually applied a low-energy "cutoff" to the integral, leading to results of the form "so many ergs cm^{-2} s^{-1} injected above a certain (arbitrary) reference energy."

Hard X-ray spectra typically show two regions: a steep (thermal) spectrum at low energies ($\lesssim$30 keV) and a flatter power-law spectrum at higher energies. Since these two regimes merge smoothly together, the application of an abrupt "low-energy cutoff" in the injected electron spectrum is clearly inappropriate. Moreover, as we show here, it is also *unnecessary*. The standard relation between the observed hard X-ray spectrum and the injected electron flux spectrum (e.g., Brown 1971) assumes that the injected electrons steadily lose energy in a "cold" target (i.e., one with thermal energy $kT \ll E$). However, electrons with injected energy $E \gtrsim kT$ lose energy at a rate significantly less than in a cold target, and below about $E \simeq kT$, the electrons are part of a thermally relaxed distribution, as likely to *gain* energy as lose it. Therefore, the power in the injected electron distribution is obtained by integrating only over energies $E \gtrsim kT$, with an appropriate "warm-target energy-loss reduction factor" included in the integrand. This results in a finite (and straightforwardly calculable) value for the *total* injected electron power, rather than the amount of power above an (arbitrary) "lower cutoff energy."

2. ENERGETICS OF THE INJECTED ELECTRONS

The hard X-ray flux observed at the Earth (in units of photons cm^{-2} s^{-1} keV^{-1}) from the injection of a beam of electrons with an energy spectrum $F_0(E_0)$ (in units of electrons cm^{-2} s^{-1} keV^{-1}) into a thick target can be written as (Brown & MacKinnon 1985)

$$I(\epsilon) = \frac{A}{4\pi R^2} \int_\epsilon^\infty \frac{Q(\epsilon, E)}{G_c(E)} H(E) dE, \tag{1}$$

where A is the flare area, R is the Earth-Sun distance, $Q(\epsilon, E)$ (in units of cm^2 keV^{-1}) is the cross section for emission of a photon of energy ϵ by an electron of energy E, $G_c(E) = K/E$ [in units of keV (cm^{-2})$^{-1}$] is the energy-loss rate per unit column density in a fully ionized, collisionally cold target (cf. Emslie 1978),

$$H(E) = \frac{1}{g(E)} \int_E^\infty F_0(E_0) dE_0, \tag{2}$$

and $g(E)$ is the ratio of the actual energy-loss rate $G(E)$ to $G_c(E)$. Brown et al. (2003) have pointed out that the only characteristic of the electron population that can be inferred *solely* from fitting (Holman et al. 2003) or inverting (Piana et al. 2003) the hard X-ray spectrum using a known bremsstrahlung cross section is the *mean electron spectrum* $\bar{F}(E)$ in the source, defined through

$$I(\epsilon) = \frac{1}{4\pi R^2} \bar{n} V \int_\epsilon^\infty Q(\epsilon, E) \bar{F}(E) dE, \tag{3}$$

where $\bar{n}$ is the mean source density and V is the emitting volume. Comparing equations (1) and (3), we see that

$$H(E) = \frac{\bar{n}V}{A} \bar{F}(E) G_c(E) = \frac{\bar{n}V}{A} K \frac{\bar{F}(E)}{E}. \tag{4}$$

Multiplying both sides of equation (2) by $g(E)$, differenti-

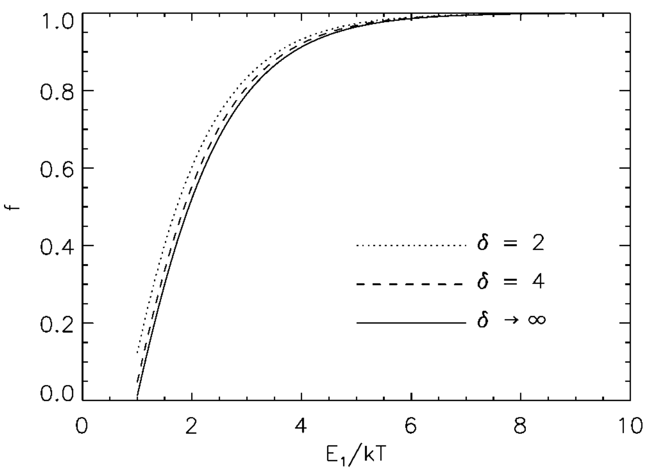

FIG. 1.—Ratio of injected electron energy flux above reference energy E_1 relative to the same quantity inferred through a cold-target ($kT \ll E_1$) analysis (see eq. [9]). Curves are shown for various values of δ, which is the power-law spectral index of $\bar{F}(E)$, the instantaneous mean electron flux spectrum in the source.

ating with respect to E, and then substituting for $H(E)$ from equation (4) give

$$F_0(E_0) = -\frac{\bar{n}V}{A} K \frac{d}{dE}\left[\frac{\bar{F}(E)}{E} g(E)\right]_{E=E_0}. \quad (5)$$

Equation (5) is the general relation between the mean source electron spectrum $\bar{F}(E)$ and the injected spectrum $F_0(E_0)$. For a cold target, $g(E) = 1$, and we recover equation (11) of Brown & Emslie (1988), viz.,

$$F_0(E_0) = \frac{\bar{n}V}{A} K \frac{\bar{F}(E_0)}{E_0^2}\left(1 - \frac{d \ln \bar{F}}{d \ln E}\right)_{E=E_0}. \quad (6)$$

For a power law $\bar{F}(E) \sim E^{-\delta}$, this gives $F_0(E_0) \sim E_0^{-\delta_0}$, where the injected spectral index $\delta_0 = \delta + 2$. However, for $g(E) \neq 1$, the actual form for $F_0(E_0)$ can differ substantially from this power-law form. Specifically, if $g(E)$ is less than unity and an increasing function of energy, it can readily be shown from equation (5) that $F_0(E_0)$ will be everywhere less than the value appropriate to a collisional cold target—physically, this is because the decrease in energy-loss rate with decreasing energy leads to a smaller number of electrons required at low energies. This has significant implications for the energy content of the injected electron distribution, as we shall see below.

The injected energy flux (in units of ergs cm^{-2} s^{-1}) above the reference energy E_1 is $\mathcal{F}_1 = \int_{E_1}^{\infty} E_0 F_0(E_0) dE_0$. Substituting from equation (5) for $F_0(E_0)$ and integrating by parts, we obtain

$$\mathcal{F}_1 = \frac{\bar{n}V}{A} K \left[\bar{F}(E_1) g(E_1) + \int_{E_1}^{\infty} \frac{\bar{F}(E)}{E} g(E) dE\right]. \quad (7)$$

The electron energy flux $\mathcal{F}_1$ thus depends on a combination of $\bar{F}(E)$ and a weighted integral of it; it is therefore fairly insensitive to the detailed form of $\bar{F}(E)$. Hence, for illustrative purposes, we now assume that $\bar{F}(E)$ has been fitted with a power-law form $\bar{F}(E) = CE^{-\delta}$ [although the analysis to follow can be straightforwardly generalized to arbitrary forms of $\bar{F}(E)$]. This gives

$$\mathcal{F}_1 = \frac{\bar{n}V}{A} KC \left[E_1^{-\delta} g(E_1) + \int_{E_1}^{\infty} E^{-\delta-1} g(E) dE\right]. \quad (8)$$

For a fully ionized cold target, $g(E) = 1$, and we obtain $\mathcal{F}_1 = (\bar{n}V/A)KC\,[(\delta+1)/\delta]\,E_1^{-\delta}$. This expression clearly diverges as $E_1 \to 0$, requiring that we impose an arbitrary finite value for E_1 in order to keep the total injected energy flux finite. However, using the substitution $x = E_1/E$ in equation (8), we find that the *actual* injected energy flux above energy E_1 differs from the cold-target–inferred value by the factor

$$f(E_1;\ \delta) = \frac{\delta}{\delta+1}\left[g(E_1) + \int_0^1 x^{\delta-1} g(E_1/x) dx\right], \quad (9)$$

which is equal to 1 for $g(E) \equiv 1$ and less than unity if $g(E) < 1$ everywhere. (Reduced energy-loss rates for the bremsstrahlung-producing electrons require fewer electrons for a given hard X-ray yield.)

As pointed out by Brown & MacKinnon (1985), there are several factors that can influence the expression for $g(E)$. For example, a varying degree of target ionization renders the energy-loss rate $G(E)$ a function of position in the target (Brown 1972) and hence implicitly a function of E. The primary interest here, however, is the determination of the total energy content in the injected electrons. Since the bulk of the electron energy content is contained in low-energy electrons, which interact principally with the ionized (coronal) regions of the flare volume, the principal factor controlling $g(E)$ is the effect of a "warm" target (i.e., $E_1 \sim kT$), through equation (7) or equation (8), on $\mathcal{F}_1$. Spitzer (1962, eq. [5-24]) gives the formula for the energy-loss rate in a target of temperature T, from which we obtain the expression for the energy-loss rate relative to a fully ionized cold target:

$$g_{\rm th}(E) = \operatorname{erf}\left(\sqrt{\frac{E}{kT}}\right) - 2\sqrt{\frac{E}{kT}}\operatorname{erf}'\left(\sqrt{\frac{E}{kT}}\right), \quad (10)$$

where erf(x) is the error function. Figure 1 shows $f(E_1/kT)$ for various values of δ. For $(E_1/kT) \gtrsim 5$, $f \simeq 1$, and a cold-target analysis is indeed appropriate. However, for lower values of E_1/kT, there is a substantial reduction in the injected electron energy flux relative to that inferred from a cold-target analysis.

For values of $E/kT \lesssim 1$, the energy-loss rate given by equation (10) is *negative*: particles gain more energy from the high-energy tail of the ambient Maxwellian distribution than they lose to the bulk. The energy regime $E \lesssim kT$ is more appropriately described as a thermally relaxed ensemble of particles, with no secular energy losses. The *total* injected power $\mathcal{P}_{\rm tot} = \mathcal{F}_{\rm tot}A$ therefore approaches a *fixed* value

$$\mathcal{P}_{\rm tot} = \bar{n}VKC \int_{0.98kT}^{\infty} E_0^{-\delta-1} g_{\rm th}(E_0) dE_0, \quad (11)$$

where $0.98kT$ is the value of E_1 for which $g(E_1) = 0$. The product $\bar{n}VC$ is directly determined from the observed photon spectrum (see eq. [1] of Brown et al. 2003); hence, we can evaluate $\mathcal{P}_{\rm tot}$ directly from the observed photon spectrum. We

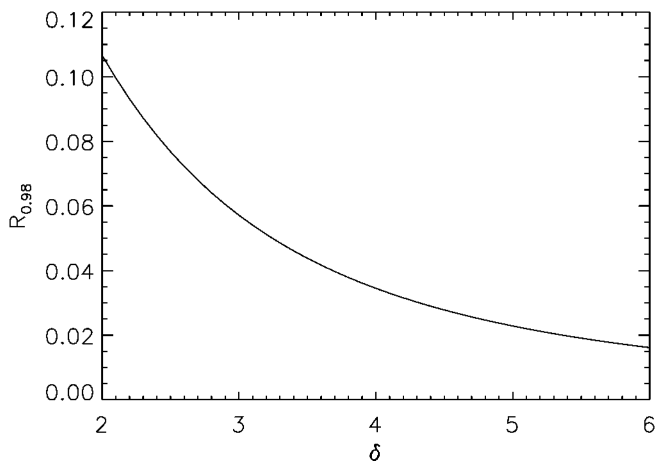

FIG. 2.—Ratio of *total* injected electron power to the power above $0.98kT$ inferred through a cold-target analysis (see eq. [12]), as a function of the mean electron flux spectral index δ.

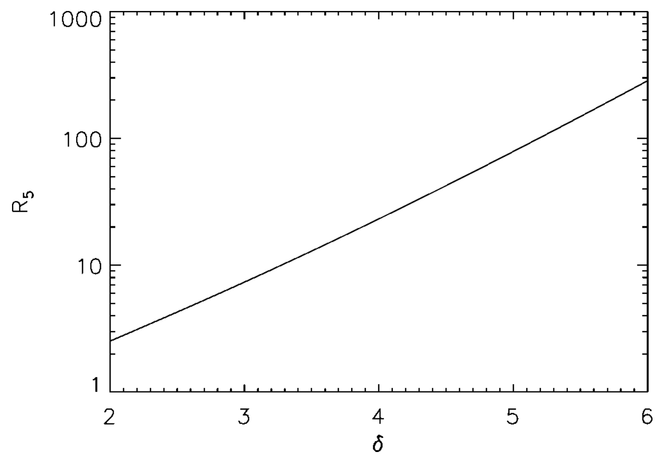

FIG. 3.—Ratio of *total* injected electron power to the power above $5kT$ inferred through a cold-target analysis, as a function of the mean electron flux spectral index δ.

stress that this lower limit on the integral in equation (11) is *not* an arbitrary reference value; it is imposed by the physics of electron energy loss in the target.

The ratio of $\mathcal{P}_{\rm tot}$ (eq. [11]) to the "cold-target" injected power above reference energy $E_1 = 0.98kT$ (viz., $\mathcal{P}_1 = \bar{n}VKC\,[(\delta+1)/\delta]\,(0.98kT)^{-\delta}$) is (by eq. [9])

$$R_{0.98}(\delta) = \frac{\delta}{\delta+1}\int_0^1 x^{\delta-1} g\left(\frac{0.98}{x}\right) dx. \qquad (12)$$

Figure 2 shows the behavior of $R_{0.98}(\delta)$. The *total* injected power in the electrons is only some 2%–10% of the "cold-target" injected power above $0.98kT$. This may seem like a very small percentage; however, it is based on a very low value of E_1. A more informative quantity would be R_5, the ratio of the total injected power to the cold-target power in the truly cold-target regime $E > 5kT$ (Fig. 1). Since the cold-target power scales as $E_1^{-\delta}$, $R_5(\delta) = (5/0.98)^{\delta} R_{0.98}$. Figure 3 shows $R_5(\delta)$. For example, for $\delta = 3$, the total power over the entire electron distribution $\mathcal{P}_{\rm tot}$ is 7.4 times the cold-target injected electron power above $5kT$. [Equivalently, we could state that the total power is equivalent to the cold-target power above an "effective cutoff" energy E_{00}, where $E_{00}^{-\delta} = R_5(\delta)(5kT)^{-\delta}$; i.e., $E_{00} = 5kTR_5^{-1/\delta}$. For $\delta = 3$, this gives an effective cutoff energy of $2.57kT$.]

For the 2002 July 23 flare, Holman et al. (2003) estimate the temperature of the thermal source to vary between 2 and 3.5×10^7 K, corresponding to kT in the range of 2–3 keV. It is reasonable to assume that this is representative of the target with which the nonthermal electrons interact. The column density required to stop a 30 keV ($>10kT$, well into the cold-target regime) electron is only of order 10^{20} cm^{-2} (cf. Emslie 1978), less than the column density of the flaring corona several minutes into the main phase of the flare. Thus, the electrons carrying the bulk of the injected power (i.e., those with energies in the range from $\simeq kT$ to $\simeq 5kT$) principally interact with the hot coronal regions of the flare.

For power-law forms $F(E) \sim E^{-\delta}$, the total injected power $\mathcal{P}_{\rm tot}$ can be obtained from multiplying the cold-target–inferred injected power above $5kT$ by $R_5(\delta)$. The total energy content for the main phase of the 2002 July 23 flare has been estimated by Holman et al. (2003) using such a procedure. For more general forms of $F(E)$ (see Piana et al. 2003), equation (7), with $E_1 = 0.98kT$, can be used.

3. SUMMARY AND CONCLUSIONS

We have shown that consideration of energy loss by bremsstrahlung-producing electrons in solar flares leads naturally to a finite total injected electron power. There is no need to impose an arbitrary low-energy cutoff to the injected electron distribution. Electrons with energies $\lesssim kT$ essentially lose no energy in the target, while those with energies in the range from $\sim kT$ to $\sim 5kT$ suffer a significantly smaller energy-loss rate than they would if the target were cold. Hence, the total injected power $\mathcal{P}_{\rm tot}$ can be calculated using an expression (eq. [7]) that is similar to that for a cold-target analysis but that incorporates a factor $g(E_0)$ that is close to zero at energies $E_0 \lesssim kT$, thereafter increasing smoothly toward unity at energies $E_0 \gtrsim 5kT$. Contrary to the behavior in the cold-target case, this integral is finite and provides the total power in the injected electron distribution.

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