THE MASS FUNCTION OF AN X-RAY FLUX-LIMITED SAMPLE OF GALAXY CLUSTERS

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ABSTRACT

A new X-ray-selected and X-ray flux-limited galaxy cluster sample is presented. Based on the ROSAT All-Sky Survey, the 63 brightest clusters with galactic latitude $|b_{II}| \ge 20^{\circ}$ and flux $f_X(0.1-2.4 \text{ keV}) \ge 2 \times 10^{-11}$ ergs s⁻¹ cm⁻² have been compiled. Gravitational masses have been determined utilizing intracluster gas density profiles, derived mainly from ROSAT PSPC pointed observations, and gas temperatures, as published mainly from ASCA observations, assuming hydrostatic equilibrium. This sample and an extended sample of 106 galaxy clusters is used to establish the X-ray luminosity-gravitational mass relation. From the complete sample the galaxy cluster mass function is determined and used to constrain the mean cosmic matter density and the amplitude of mass fluctuations. Comparison to Press-Schechter type model mass functions in the framework of cold dark matter cosmological models and a Harrison-Zeldovich initial density fluctuation spectrum yields the constraints $\Omega_m = 0.12^{+0.06}_{-0.04}$ and $\sigma_8 = 0.96^{+0.15}_{-0.12}$ (90% c.l.). Various possible systematic uncertainties are quantified. Adding all identified systematic uncertainties to the statistical uncertainty in a worst-case fashion results in an upper limit $\Omega_m < 0.31$. For comparison to previous results a relation $\sigma_8 = 0.43\Omega_m^{-0.38}$ is derived. The mass function is integrated to show that the contribution of mass bound within virialized cluster regions to the total matter density is small; i.e., $\Omega_{cluster} = 0.012^{+0.003}_{-0.004}$ for cluster masses larger than $6.4^{+0.7}_{-0.6} \times 10^{13} h_{50}^{-1} M_{\odot}$.

Subject headings: cosmological parameters — cosmology: observations — galaxies: clusters: general — intergalactic medium — large-scale structure of universe — X-rays: galaxies: clusters

On-line material: machine-readable tables

1. INTRODUCTION

The galaxy cluster mass function is the most fundamental statistic of the galaxy cluster population. It is determined by the initial conditions of the mass distribution set in the early universe in a relatively straightforward way, since the evolution of the large-scale matter distribution on scales comparable to the size of clusters and larger is linear and since the formation of clusters is governed by essentially only gravitational processes. Choosing a specific cosmological scenario provides definitive predictions about these initial conditions in a statistical sense. The overall process of the gravitational growth of the density fluctuations and the development of gravitational instabilities leading to cluster formation is comparatively easy to understand. It has been well described by analytical models (e.g., Press & Schechter 1974; Bond et al. 1991; Lacey & Cole 1993; Kitayama & Suto 1996; Schuecker et al. 2001a) and simulated in numerical gravitational N-body calculations (e.g., Efstathiou et al. 1988; Lacey & Cole 1994). Even though the slight deviations from the theoretical prescription found in recent simulations (e.g., Governato et al. 1999; Jenkins et al. 2001) seem to require further refinement in the theoretical framework, these deviations are too small to be significant for the current investigation, as will be shown later. For the precision needed here these large simulations therefore support the validity of the model predictions. Within this framework the observed cluster mass function provides the opportunity to test different cosmological models. The tests are particularly sensitive to the amplitude of the cosmic matter density fluctuations (at a scale of the order of 10 Mpc) as well as the normalized total matter density, Ω_m (e.g., Henry & Arnaud 1991; Bahcall & Cen 1992).

In addition to its importance in testing cosmological models, the integral of the mass function yields interesting information on the fractional amount of matter contained in gravitationally bound large-scale structures. Using one of the first attempts to construct a mass function over the mass range from giant elliptical galaxies to massive clusters by Bahcall & Cen (1993), Fukugita, Hogan, & Peebles (1998) obtain a mass fraction $\Omega_{\text{group}} = 0.12 \pm 0.02$ for $2 \times 10^{12} \ h_{50}^{-1} M_{\odot} \leq M \leq 2 \times 10^{14} \ h_{50}^{-1} M_{\odot}$ (where the mass fraction is expressed here in units of the critical density of the universe, ρ_c). This result is already close to the total matter density in some of the proposed cosmological scenarios. Therefore, a precise observational determination of the integral mass function is a very important task for astronomy.

Unfortunately, the galaxy cluster mass is not an easily and directly observable quantity (except for measurements of the gravitational lensing effect of clusters, which may play a large role in the construction of mass functions in the future), and one has to resort to the observation of other cluster parameters from which the cluster masses can be deduced.

X-ray astronomy has provided an ideal tool, first, to detect and select massive clusters by measuring their X-ray luminosity and, second, to perform mass determinations on individual clusters through X-ray imaging and X-ray spectroscopy.

In this paper we report the first rigorous application of these two approaches for the construction of the cluster mass function. Building on the *ROSAT* All-Sky X-Ray Survey (RASS; Trümper 1993; Voges et al. 1999), which has been well studied in the search of the brightest galaxy clus-

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ters through several survey projects (see references in § 2), we have compiled a new, highly complete sample of the X-ray-brightest galaxy clusters (HIFLUGCS, the HIghest X-ray FLUx Galaxy Cluster Sample).

Thanks to the numerous detailed galaxy cluster observations performed with the *ROSAT* and *ASCA* (Tanaka, Inoue, & Holt 1994) satellite observatories and accumulated in the archives, we can perform a detailed mass determination for most of these clusters and obtain a good mass estimate for the few remaining objects. From these data we first establish a (tight) correlation of the measured X-ray luminosity and the cluster mass. This relation assures that we have essentially sampled the most massive clusters in the nearby universe, which forms the basis of the construction of the cluster mass function.

Previous local galaxy cluster mass functions have been derived by Bahcall & Cen (1993), Biviano et al. (1993), Girardi et al. (1998), and Girardi & Giuricin (2000), for galaxy groups. Bahcall & Cen (1993) used galaxy richness to relate to cluster masses for optical observations and an X-ray temperature-mass relation to convert the temperature function given by Henry & Arnaud (1991) to a mass function. Biviano et al. (1993), Girardi et al. (1998), and Girardi & Giuricin (2000) used velocity dispersions for optically selected samples to determine the mass function. Here we use a different approach and construct the first mass function for X-rav-selected galaxy clusters based on the RASS using individually determined cluster masses. The mass function of this cluster sample is then used to determine the mass fraction in bound objects with masses above a minimum mass and to derive tight constraints on cosmological scenarios.

The paper is organized as follows: In § 2 the sample selection is described. The details of the determination of the observational quantities are given in § 3. The results are presented in § 4 and discussed in § 5. The conclusions are summarized in § 6.

Throughout, a Hubble constant $H_0 = 50 \ h_{50} \ \mathrm{km \ s^{-1}}$ Mpc⁻¹, $h_{50} = 1$, $\Omega_m = 1$, a normalized cosmological constant $\Omega_{\Lambda} \equiv \Lambda/(3H_0^2) = 0$, and a normalized curvature index $\Omega_k \equiv 1 - \Omega_m - \Omega_{\Lambda} = 0$ is used if not stated otherwise (present-day quantities). We note that the determination of physical cluster parameters has a negligible dependence on Ω_m and Ω_{Λ} for the small redshift range used here, as will be shown later. Therefore, it is justified to determine the parameters for an Einstein–de Sitter model but to discuss the results also in the context of other models.

2. SAMPLE

The mass function measures the cluster number density as a function of mass. Therefore, any cluster fulfilling the selection criteria and not included in the sample distorts the result systematically. It is then obvious that for the construction of the mass function it is vital to use a homogeneously selected and highly complete sample of objects, and, additionally, the selection must be closely related to cluster mass. In this work the RASS, where one single instrument has surveyed the whole sky, has been chosen as the basis for the sample construction. Using the X-ray emission from the hot intracluster medium for cluster selection minimizes projection effects, and the tight correlation between X-ray luminosity and gravitational mass convincingly demonstrates that X-ray cluster surveys have the important property of being mass selective (§ 4.1). Several cluster catalogs have already been constructed from the RASS with high completeness down to low flux limits (see references below). These we have utilized for the selection of candidates. Low thresholds have been set for selection in order not to miss any cluster because of measurement uncertainties. These candidates have been homogeneously reanalyzed, using higher quality ROSATPosition Sensitive Proportional Counter (PSPC) pointed observations whenever possible (§ 3). A flux limit well above the limit for candidate selection has then been applied to define the new flux-limited sample of the brightest clusters in the sky.

In detail, the candidates emerged from the following input catalogs. Table 1 lists the selection criteria and the number of clusters selected from each of the catalogs that are contained in the final sample.

1. The REFLEX (ROSAT-ESO Flux-Limited X-ray) galaxy cluster survey (Böhringer et al. 2001b) covers the southern hemisphere (declination $\delta \le +2^{\circ}5$; galactic latitude $|b_{\rm II}| \ge 20^{\circ}0$) with a flux limit $f_{\rm X,lim}(0.1-2.4 \text{ keV}) = 3.0 \times 10^{-12} \text{ ergs s}^{-1} \text{ cm}^{-2}$.

2. The NORAS (Northern *ROSAT* All-Sky) galaxy cluster survey (Böhringer et al. 2000) contains clusters showing extended emission in the RASS in the northern hemisphere ($\delta \ge 0.0^{\circ}$; $|b_{II}| \ge 20.0^{\circ}$) with count rates $C_{\rm X}(0.1-2.4 \text{ keV}) \ge 0.06 \text{ counts s}^{-1}$.

3. NORAS II (J. Retzlaff et al. 2002, in preparation) is the continuation of the NORAS survey project. It includes pointlike sources and aims for a flux limit $f_{X,lim}(0.1-2.4 \text{ keV}) = 2.0 \times 10^{-12} \text{ ergs s}^{-1} \text{ cm}^{-2}$.

4. The BCS (*ROSAT* Brightest Cluster Sample; Ebeling et al. 1998) covers the northern hemisphere ($\delta \ge 0^{\circ}0^{\circ}$; $|b_{II}| \ge 20^{\circ}0$) with $f_{X,lim}(0.1-2.4 \text{ keV}) = 4.4 \times 10^{-12} \text{ ergs}$ s⁻¹ cm⁻² and redshifts $z \le 0.3$.

5. The RASS 1 Bright Sample of Clusters of Galaxies (De Grandi et al. 1999) covers the south Galactic cap region in the southern hemisphere ($\delta < +2^{\circ}5$; $b_{\rm II} < -20^{\circ}0$) with an effective flux limit $f_{\rm X,lim}(0.5-2.0 \text{ keV})$ between ~ 3 and $4 \times 10^{-12} \text{ ergs s}^{-1} \text{ cm}^{-2}$.

6. XBACs (X-ray Brightest Abell-type Clusters of galaxies; Ebeling et al. 1996) is an all-sky sample of Abell (1958)/ACO (Abell, Corwin, & Olowin 1989) clusters limited to high galactic latitudes $|b_{II}| \ge 20^\circ$.0 with nominal ACO redshifts $z \le 0.2$ and X-ray fluxes $f_X(0.1-2.4 \text{ keV}) > 5.0 \times 10^{-12} \text{ ergs s}^{-1} \text{ cm}^{-2}$.

7. An all-sky list of Abell/ACO/ACO-supplementary clusters (H. Böhringer 1999, private communication) with count rates $C_{\rm X}(0.5-2.0 \text{ keV}) \ge 0.6 \text{ counts s}^{-1}$.

8. Early-type galaxies with measured RASS count rates from a magnitude-limited sample of Beuing et al. (1999) have been checked in order not to miss any X-ray-faint groups.

9. All clusters from the sample of Lahav et al. (1989) and Edge et al. (1990), where clusters had been compiled from various X-ray missions, have been checked.

The main criterion for candidate selection, a flux threshold 1.7×10^{-11} ergs s⁻¹ cm⁻², has been chosen to allow for measurement uncertainties in the input catalogs. For example, for REFLEX clusters with 1.5×10^{-11} ergs s⁻¹ cm⁻² $\leq f_X \leq 2.5 \times 10^{-11}$ ergs s⁻¹ cm⁻², the mean statistical flux error is less than 8%. With an additional mean systematic error of 6%, caused by underestimation of fluxes due to the comparatively low RASS exposure

TABLE 1
SELECTION OF CANDIDATES

	$C_{\rm X}$ (counts s ⁻¹)			f (10 ⁻¹¹ ergs			
CATALOG	0.1–2.4 keV	0.5–2.0 keV	0.64–2.36 keV	0.1–2.4 keV	0.5–2.0 keV	N_{Cl}	Reference
REFLEX		0.9		1.7		33	1
NORAS		0.7		1.7		25	2
NORAS II		0.7		1.7		4	3
BCS	1.0			1.7		1	4
RASS 1					1.0	0	5
XBACs	1.0			1.7		0	6
Abell/ACO		0.7				0	7
Early-type galaxies			0.7			0	8
Previous satellites ^a						0	9

Notes.—Only one of the criteria, count rate or flux, has to be met for a cluster to be selected as candidate. The catalogs are listed in search sequence; therefore, N_{Cl} gives the number of candidates additionally selected from the current catalog and contained in the final flux-limited sample. So in the case of NORAS, a cluster is selected as candidate if it fulfils $C_{\rm X}(0.5-2.0 \text{ keV}) \ge 0.7 \text{ counts s}^{-1}$ or $f_{\rm X}(0.1-2.4 \text{ keV}) \ge 1.7 \times 10^{-11} \text{ ergs s}^{-1} \text{ cm}^{-2}$ and has not already been selected from REFLEX. This candidate is counted under $N_{\rm Cl}$ if it meets the selection criteria for HIFLUGCS.

^a All clusters from this catalog have been flagged as candidates.

REFERENCES.—(1) Böhringer et al. 2001b. (2) Böhringer et al. 2000. (3) J. Retzlaff et al. 2002, in preparation. (4) Ebeling et al. 1998. (5) De Grandi et al. 1999. (6) Ebeling et al. 1996. (7) H. Böhringer 1999, private communication. (8) Beuing et al. 1999. (9) Lahav et al. 1989; Edge et al. 1990.

times,² the flux threshold 1.7×10^{-11} ergs s⁻¹ cm⁻² for candidate selection then ensures that no clusters are missed for a final flux limit $f_{X,lim} = 2.0 \times 10^{-11}$ ergs s⁻¹ cm⁻².

Almost none of the fluxes given in the input catalogs have been calculated using a measured X-ray temperature, but mostly using gas temperatures estimated from an L_X - T_X relation. In order to be independent of this additional uncertainty, clusters have also been selected as candidates if they exceed a count rate threshold that corresponds to $f_X(0.1-2.4 \text{ keV}) = 2.0 \times 10^{-11} \text{ ergs s}^{-1} \text{ cm}^{-2}$ for a typical cluster temperature, $T_{\text{gas}} = 4 \text{ keV}$, and redshift, z = 0.05, and for an exceptionally high column density, e.g., in the NORAS case, $n_{\text{H}} = 1.6 \times 10^{21} \text{ cm}^{-2}$.

Most of the samples mentioned above excluded the area on the sky close to the Galactic plane as well as the area of the Magellanic Clouds. In order to construct a highly complete sample from the candidate list, we applied the following selection criteria that successful clusters must fulfill:

1. redetermined flux $f_{\rm X}(0.1-2.4 \text{ keV}) \ge 2.0 \times 10^{-11} \text{ ergs} \text{ s}^{-1} \text{ cm}^{-2}$,

2. Galactic latitude $|b_{II}| \ge 20^{\circ}.0$,

3. projected position outside the excluded 324 deg^2 area of the Magellanic Clouds (see Table 2), and

4. projected position outside the excluded 98 deg² region of the Virgo galaxy cluster (see Table 2).³

These selection criteria are fulfilled by 63 candidates. The advantages of the redetermined fluxes over the fluxes from the input catalogs are summarized at the end of § 3.1. In

Table 1, one notes that 98% of all clusters in HIFLUGCS have been flagged as candidates in REFLEX, NORAS, or in the candidate list for NORAS II; these surveys are not only all based on the RASS but all use the same algorithm for the count rate determination, further substantiating the homogeneous candidate selection for HIFLUGCS.

The fraction of available ROSAT PSPC pointed observations for clusters included in HIFLUGCS equals 86%. The actually used fraction is slightly reduced to 75% because some clusters appear extended beyond the PSPC field of view and therefore RASS data have been used. The fraction of clusters with published ASCA temperatures equals 87%. If a lower flux limit had been chosen, the fraction of available PSPC pointed observations and published ASCA temperatures would have been decreased, thereby increasing the uncertainties in the derived cluster parameters. Furthermore, this value for the flux limit ensures that no corrections, due to low exposure in the RASS or high Galactic hydrogen column density, need to be applied for the effective area covered. This can be seen by the effective sky coverage in the REFLEX survey area for a flux limit

TABLE 2 Regions of the Sky Not Sampled in HIFLUGCS

Region	R.A. Range (deg)	Decl. Range (deg)	Area (sr)
LMC 1	58 to 103	-63 to -77	0.0655
LMC 2	81 to 89	-58 to -63	0.0060
LMC 3	103 to 108	-68 to -74	0.0030
SMC 1	358.5 to 20	-67.5 to -77	0.0189
SMC 2	356.5 to 358.5	-73 to -77	0.0006
SMC 3	20 to 30	-67.5 to -72	0.0047
Virgo	182.7 to 192.7	7.4 to 17.4	0.0297
Milky Way ^a	0 to 360 $(l_{\rm II})$	-20 to 20 ($b_{\rm II}$)	4.2980

NOTE.—Excised areas for the Magellanic Clouds are the same as in Böhringer et al. 2001b, because REFLEX forms the basic input catalog in the southern hemisphere.

^a Galactic coordinates.

² This has been measured by comparing the count rates determined using pointed observations of clusters in this work to count rates for the same clusters determined in REFLEX and NORAS. If count rates are compared also for fainter clusters, not relevant for the present work, the mean systematic error increases to about 9% (Böhringer et al. 2000).

³ The large-scale X-ray background of the irregular and very extended X-ray emission of the Virgo Cluster makes the undiscriminating detection/ selection of clusters in this area difficult. Candidates excluded because of this criterion are Virgo, M86, and M49.



FIG. 1.—Aitoff projection of the 63 HIFLUGCS galaxy clusters in galactic coordinates (*filled circles*). Additionally shown are 11 clusters above the flux limit but with $|b_{II}| < 20^{\circ}0$ (*open triangles*).

 $f_{\rm X,lim}(0.1-2.4 \text{ keV}) = 2.0 \times 10^{-11} \text{ ergs s}^{-1} \text{ cm}^{-2}$ and a minimum of 30 source counts, which amounts to 99%. The clear advantage is that the HIFLUGCS catalog can be used in a straightforward manner in statistical analyses, because the effective area is the same for all clusters and simply equals the covered solid angle on the sky.

The distribution of clusters included in HIFLUGCS projected onto the sky is shown in Figure 1. The sky coverage for the cluster sample equals 26,721.8 deg² (8.13994 sr), about two-thirds of the sky. The cluster names, coordinates, and redshifts are listed in Table 3. Further properties of the cluster sample are discussed in § 5.1.

For later analyses that do not necessarily require a complete sample, e.g., correlations between physical parameters, 43 clusters (not included in HIFLUGCS) from the candidate list have been combined with HIFLUGCS to form an "extended sample" of 106 clusters.

3. DATA REDUCTION AND ANALYSIS

This section describes the derivation of the basic quantities in this work, e.g., count rates, fluxes, luminosities, and mass estimates for the galaxy clusters. These and other relevant cluster parameters are tabulated along with their uncertainties.

3.1. Flux Determination

Measuring the count rate of galaxy clusters is an important step in constructing a flux-limited cluster sample. The count rate determination performed here is based on the growth curve analysis method (Böhringer et al. 2000), with modifications adapted to the higher photon statistics available here. The main features of the method as well as the modifications are outlined below.

The instrument used is the *ROSAT* PSPC (Pfeffermann et al. 1987), with a low internal background ideally suited for this study, which needs good signal-to-noise ratios of the outer, low surface brightness regions of the clusters. Mainly, pointed observations from the public archive at MPE have been used.⁴ If a cluster is extended beyond the PSPC field of view, making a proper background determination difficult, or if there is no pointed PSPC observation available, RASS data have been used. The *ROSAT* hard energy band (channels 52–201 \approx 0.5–2.0 keV) has been used for all count rate measurements because of the higher background in the soft band.

Two X-ray cluster centers are determined by finding the two-dimensional "center of mass" of the photon distribution iteratively for an aperture radius of 3' and 7.5 around the starting position. The small aperture yields the center representing the position of the cluster's peak emission and therefore probably indicates the position where the cluster's potential well is deepest. This center is used for the regional selection, e.g. $|b_{\rm II}| \ge 20^\circ$ 0. The more globally defined center with the larger aperture is used for the subsequent analysis tasks since for the mass determination it is most important to have a good estimate of the slope of the surface brightness profile in the outer parts of the cluster.

The background surface brightness is determined in a ring outside the cluster emission. To minimize the influence of discrete sources, the ring is subdivided into twelve parts of equal area, and a sigma clipping is performed. To determine the count rate, the area around the global center is divided into concentric rings. For pointed observations, 200 rings with a width of 15" each are used. Because of the lower photon statistics, a width of 30" is used for RASS data, and the number of rings depends on the field size extracted (100-300 rings for field sizes of $2 \times 2 \text{ deg}^2$ to $8 \times 8 \text{ deg}^2$). Each photon is divided by the vignetting and dead-timecorrected exposure time of the sky pixel where it has been detected, and these ratios are summed up in each ring, yielding the ring count rate. From this value the background count rate for the respective ring area is subtracted, yielding a source ring count rate. These individual source ring count rates are integrated with increasing radius, yielding the

⁴ See http://www.xray.mpe.mpg.de/cgi-bin/rosat/seq-browser.

TABLE 3Cluster Properties

Cluster	R.A.	Decl.	Ζ	n _H	C _x	Δ	r _x	$f_{\mathbf{X}}$	$L_{\mathbf{X}}$	$L_{\rm bol}$	Obs.	Ref.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
A0085	10.4632	-9.3054	0.0556	3.58	3.488	0.6	2.13	7.429	9.789	24.448	Р	2
A0119	14.0649	-1.2489	0.0440	3.10	1.931	0.9	2.68	4.054	3.354	7.475	Р	1
A0133	15.6736	-21.8806	0.0569	1.60	1.058	0.8	1.52	2.121	2.944	5.389	P	3
NGC 507	20.9106	33.2553	0.0165	5.25 5.52	1.093	1.3	0.88	2.112	0.247	0.320	P	5
A0400	44.4152	6.0170	0.0101	9.38	1.146	1.1	1.43	2.778	0.686	1.033	P	8
A0399	44.4684	13.0462	0.0715	10.58	1.306	5.4	3.18	3.249	7.070	17.803	R	6
A0401	44.7384	13.5796	0.0748	10.19	2.104	1.1	3.81	5.281	12.553	34.073	Р	6
A3112	49.4912	-44.2367	0.0750	2.53	1.502	1.1	2.18	3.103	7.456	16.128	Р	1
Fornax	54.6686	-35.3103	0.0046	1.45	5.324	5.6	0.53	9.020	0.082	0.107	P+R	4
2A 0335 7w III 54	55 3225	9.9713	0.0349	16.68	5.028 0.708	0.8	1.54	9.162	4./89	1 226	P R	10
A3158	55.7282	-53.6301	0.0511	1.06	1.909	1.5	1.94	3.794	5.638	1.220	P	1
A0478	63.3554	10.4661	0.0900	15.27	1.827	0.6	3.12	5.151	17.690	49.335	Р	6
NGC 1550	64.9066	2.4151	0.0123	11.59	1.979	5.4	0.71	4.632	0.302	0.407	R	13
EXO 0422	66.4637	-8.5581	0.0390	6.40	1.390	6.2	1.32	3.085	2.015	3.283	R	10
A3266	67.8410	-61.4403	0.0594	1.48	2.879	0.7	2.99	5.807	8.718	23.663	Р	4
A0496	68.4091	-13.2605	0.0328	5.68	3.724	0.7	1.78	8.326	3.837	7.306	Р	8
A3391	96 5925	- 53 6938	0.0433	5.01	0.999	1.4	2.80	2.430	2.174	4.077	P P	4
A3395s	96.6920	-54.5453	0.0498	5.42 8.49	0.836	3.8	1.45	2.009	2.001	4.471	P	4
A0576	110.3571	55.7639	0.0381	5.69	1.374	6.8	2.32	3.010	1.872	3.518	R	6
A0754	137.3338	-9.6797	0.0528	4.59	1.537	1.6	1.91	3.366	3.990	11.967	Р	6
Hydra A	139.5239	-12.0942	0.0538	4.86	2.179	0.6	1.66	4.776	5.930	11.520	Р	13
A1060	159.1784	-27.5212	0.0114	4.92	4.653	3.3	0.95	9.951	0.554	0.945	R	6
A1367	176.1903	19.7030	0.0216	2.55	2.947	0.8	1.55	6.051	1.206	2.140	Р	8
$MKW 4 \dots$	181.1124	1.8902	0.0200	1.80	1.1/3	1./	1.25	2.208	0.390 5.240	0.545	P D	10
NGC 4636	190.7084	2.6880	0.0037	1.75	3.102	7.2	0.39	4.085	0.023	0.027	R	13
A3526	192.1995	-41.3087	0.0103	8.25	11.655	2.2	1.64	27.189	1.241	2.238	R	15
A1644	194.2900	-17.4029	0.0474	5.33	1.853	5.1	1.85	4.030	3.876	7.882	R	8
A1650	194.6712	-1.7572	0.0845	1.54	1.218	6.6	3.17	2.405	7.308	17.955	R	6
A1651	194.8419	-4.1947	0.0860	1.71	1.254	1.2	2.03	2.539	8.000	18.692	Р	22
Coma	194.9468	27.9388	0.0232	0.89	17.721	1.4	4.04	34.438	7.917	22.048	R	8
NGC 5044	198.8530	- 10.38/9	0.0090	4.91 5.36	3.103	0.5	0.56	5.514 3.537	0.193	0.246	P	24 25
A3558	201.9921	-31.5017	0.0480	3.63	3.158	0.5	2.11	6.720	6.615	14.600	P	1
A3562	203.3984	-31.6678	0.0499	3.91	1.367	0.9	2.01	2.928	3.117	6.647	P	4
A3571	206.8692	-32.8553	0.0397	3.93	5.626	0.7	2.35	12.089	8.132	20.310	Р	21
A1795	207.2201	26.5944	0.0616	1.20	3.132	0.3	2.14	6.270	10.124	27.106	Р	6
A3581	211.8852	-27.0153	0.0214	4.26	1.603	3.2	0.64	3.337	0.657	0.926	Р	28
MKW 8	220.1596	3.4717	0.0270	2.60	1.255	8.4	1.90	2.525	0.789	1.355	R	29
A2029	227.7551	7 0211	0.0707	2.07	5.294 2.279	0.0	2.78	0.938	2 449	4 061	P P	6
MKW 3s	230.4643	7.7059	0.0340	3.15	1.578	1.0	1.39	3.299	2.865	5.180	P	10
A2065	230.6096	27.7120	0.0721	2.84	1.227	6.1	3.09	2.505	5.560	12.271	R	6
A2063	230.7734	8.6112	0.0354	2.92	2.038	1.3	2.13	4.232	2.272	4.099	Р	8
A2142	239.5824	27.2336	0.0899	4.05	2.888	0.9	3.09	6.241	21.345	64.760	Р	6
A2147	240.5628	15.9586	0.0351	3.29	2.623	3.2	1.87	5.522	2.919	6.067	Р	8
A2163	243.9433	-6.1436	0.2010	12.27	0.773	1.5	3.15	2.039	34.128	123.200	Р	31
A2199	247.1580	59.5477	0.0302	0.84 5.94	5.555 1 211	1.8	2.37	2 750	4.105	7.904 68.989	К Р	8 6
A2204	255.6749	34.0578	0.0970	2.07	1.034	2.1	2.64	2.130	8.468	21.498	P	6
A2256	255.9884	78.6481	0.0601	4.02	2.811	1.4	3.09	6.054	9.322	22.713	P	6
A2255	258.1916	64.0640	0.0800	2.51	0.976	1.2	3.22	2.022	5.506	13.718	Р	6
A3667	303.1362	- 56.8419	0.0560	4.59	3.293	0.7	2.81	7.201	9.624	24.233	Р	1
S1101	348.4941	-42.7268	0.0580	1.85	1.237	0.9	1.64	2.485	3.597	5.939	P	35
A2589	350.9868	16.7753	0.0416	4.39	1.200	1.3	1.46	2.591	1.924	3.479	Р	37
A2634	354 6201	- 12.1240	0.0852	2.50 5.17	1.074	1.2 1.6	1.43 1 70	2.213	0.882 1.008	13.520	r P	0
A2657	356,2334	9.1952	0.0312	5.27	1.148	0.9	1.52	2.535	1.771	3.202	P	8
A4038	356.9322	-28.1415	0.0283	1.55	2.854	1.3	1.35	5.694	1.956	3.295	P	4
A4059	359.2541	-34.7591	0.0460	1.10	1.599	1.3	1.72	3.170	2.872	5.645	Р	36

TABLE 3—Continued

Cluster	R.A. (2)	Decl.	<i>z</i> (4)	$n_{\rm H}$ (5)	$C_{\mathbf{x}}$	Δ (7)	$r_{\rm X}$ (8)	$f_{\mathbf{X}}$ (9)	$L_{\rm X}$ (10)	L_{bol} (11)	Obs. (12)	Ref. (13)
(-)		Clus	sters from the	he Extende	d Sample N	lot Inclu	ded in HI	FLUGCS	()	()	()	()
A2734	2.8389	-28.8539	0.0620	1.84	0.710	2.5	1.74	1.434	2.365	4.357	Р	1
A2877	17.4796	-45.9225	0.0241	2.10	0.801	1.2	1.06	1.626	0.405	0.714	Р	4
NGC 499	20.7971	33.4587	0.0147	5.25	0.313	2.5	0.30	0.479	0.045	0.051	Р	5
AWM 7	43.6229	41.5781	0.0172	9.21	7.007	2.0	1.58	16.751	2.133	3.882	R	7
Perseus	49.9455	41.5150	0.0183	15.69	40.723	0.8	3.30	113.731	16.286	40.310	R	9
S405	58.0078	-82.2315	0.0613	7.65	0.781	8.2	2.14	1.800	2.899	5.574	R	12
3C 129	72.5602	45.0256	0.0223	67.89	1.512	5.6	1.61	10.566	2.242	4.996	R	10
A0539	79.1560	6.4421	0.0288	12.06	1.221	1.3	1.37	3.182	1.135	1.935	Р	14
S540	85.0265	-40.8431	0.0358	3.53	0.788	5.0	0.84	1.611	0.887	1.353	R	4
A0548w	86.3785	-25.9340	0.0424	1.79	0.136	5.4	0.73	0.234	0.183	0.240	Р	15
A0548e	87.1596	-25.4692	0.0410	1.88	0.771	1.8	2.12	1.551	1.117	1.870	Р	15
A3395n	96.9005	- 54.4447	0.0498	5.42	0.699	3.9	1.37	1.555	1.650	3.461	Р	4
UGC 03957	115.2481	55.4319	0.0340	4.59	0.936	6.0	0.94	1.975	0.980	1.531	R	16
PKS 0745	116.8837	-19.2955	0.1028	43.49	1.268	1.0	2.44	6.155	27.565	70.604	Р	17
A0644	124.3553	-7.5159	0.0704	5.14	1.799	1.0	4.02	3.994	8.414	22.684	Р	6
S636	157.5151	-35.3093	0.0116	6.42	3.102	4.9	1.18	5.869	0.341	0.446	R	18
A1413	178.8271	23.4051	0.1427	1.62	0.636	1.6	2.39	1.289	11.090	28.655	Р	6
M49	187.4437	7.9956	0.0044	1.59	1.259	1.0	0.27	1.851	0.015	0.019	Р	15
A3528n	193.5906	-29.0130	0.0540	6.10	0.560	2.3	1.51	1.263	1.581	2.752	Р	1
A3528s	193.6708	-29.2254	0.0551	6.10	0.756	1.6	1.35	1.703	2.224	3.746	Р	20
A3530	193.9211	-30.3451	0.0544	6.00	0.438	2.8	1.55	0.987	1.252	2.317	Р	21
A3532	194.3375	-30.3698	0.0539	5.96	0.797	1.8	1.64	1.797	2.235	4.483	Р	21
A1689	197.8726	-1.3408	0.1840	1.80	0.712	1.1	2.36	1.454	20.605	60.707	Р	23
A3560	203.1119	-33.1355	0.0495	3.92	0.714	2.5	2.00	1.519	1.601	2.701	Р	26
A1775	205.4582	26.3820	0.0757	1.00	0.654	1.8	2.02	1.290	3.175	5.735	Р	27
A1800	207.3408	28.1038	0.0748	1.18	0.610	7.9	1.98	1.183	2.840	5.337	R	28
A1914	216.5035	37.8268	0.1712	0.97	0.729	1.4	2.35	1.454	17.813	56.533	Р	6
NGC 5813	225.2994	1.6981	0.0064	4.19	0.976	6.7	0.17	1.447	0.025	0.029	R	13
NGC 5846	226.6253	1.6089	0.0061	4.25	0.569	2.3	0.21	0.851	0.014	0.016	Р	13
A2151w	241.1465	17.7252	0.0369	3.36	0.754	1.9	1.46	1.568	0.917	1.397	Р	8
A3627	243.5546	-60.8430	0.0163	20.83	9.962	3.0	2.20	31.084	3.524	8.179	R	30
Triangulum	249.5758	-64.3557	0.0510	12.29	4.294	0.7	2.54	11.308	12.508	37.739	Р	32
Ophiuchus	258.1115	-23.3634	0.0280	20.14	11.642	2.0	2.29	35.749	11.953	37.391	R	33
Zw Cl 1742	266.0623	32.9893	0.0757	3.56	0.889	4.4	1.83	1.850	4.529	9.727	R	34
A2319	290.2980	43.9484	0.0564	8.77	5.029	1.0	3.57	12.202	16.508	47.286	Р	6
A3695	308.6991	-35.8135	0.0890	3.56	0.836	9.2	2.58	1.739	5.882	12.715	R	1
Zw II 108	318.4752	2.5564	0.0494	6.63	0.841	7.3	2.20	1.884	1.969	3.445	R	5
A3822	328.5438	-57.8668	0.0760	2.12	0.964	7.3	3.18	1.926	4.758	9.877	R	1
A3827	330.4869	- 59.9641	0.0980	2.84	0.953	5.8	1.78	1.955	7.963	20.188	R	1
A3888	338.6255	-37.7343	0.1510	1.20	0.546	2.4	1.52	1.096	10.512	30.183	Р	23
A3921	342.5019	-64.4286	0.0936	2.80	0.626	1.7	2.43	1.308	4.882	11.023	Р	12
HCG 94	349.3041	18.7060	0.0417	4.55	0.820	1.0	2.09	1.775	1.324	2.319	Р	36
RXJ 2344	356.0723	-4.3776	0.0786	3.54	0.653	1.4	1.61	1.385	3.661	7.465	Р	12

Notes.—Col. (1): Cluster name. Cols. (2)–(3): Equatorial coordinates of the cluster center used for the regional selection for the epoch J2000.0 in decimal degrees. Col. (4): Heliocentric cluster redshift. Col. (5): Column density of neutral galactic hydrogen in units of 10^{20} atoms cm⁻². Col. (6): Count rate in the channel range 52–201, which corresponds to about (the energy resolution of the PSPC is limited) the energy range 0.5–2.0 keV in units of counts s⁻¹. Col. (7): Relative 1 σ Poissonian error of the count rate, the flux, and the luminosity in percent. Col. (8): Significance radius in h_{50}^{-1} Mpc. Col. (9): Flux in the energy range 0.1–2.4 keV in units of 10^{-11} ergs s⁻¹ col. (10): Luminosity in the energy range 0.1–2.4 keV in units of h_{50}^{-2} 10⁴⁴ ergs s⁻¹. Col. (12): Indicates whether a RASS (R) or a pointed (P) *ROSAT* PSPC observation has been used. Col. (13): Code for the redshift reference decoded below. Table 3 is also available in machine-readable form in the electronic edition of the *Astrophysical Journal*.

REFERENCES.—(1) Katgert et al. 1996. (2) Mazure et al. 1996. (3) Median of nine galaxy redshifts compiled from Lauberts & Valentijn 1989; Merrifield & Kent 1991; Loveday et al. 1996; Way, Flores, & Quintana 1998. (4) Abell et al. 1989. (5) Huchra, Vogeley, & Geller 1999. (6) Struble & Rood 1987. (7) dell'Antonio, Geller, & Fabricant 1994. (8) Zabludoff et al. 1993. (9) Poulain, Nieto, & Davoust 1992. (10) NED Team 1992. (11) Böhringer et al. 2000. (12) De Grandi et al. 1999. (13) de Vaucouleurs et al. 1991. (14) Zabludoff, Huchra, & Geller 1990. (15) den Hartog & Katgert 1996. (16) Michel & Huchra 1988. (17) Yan & Cohen 1995. (18) Garcia 1995. (19) Ebeling et al. 1998. (20) Median of eight galaxy redshifts compiled from de Vaucouleurs et al. 1991; Quintana et al. 1995; Katgert et al. 1998. (21) Vettolani et al. 1990. (22) Allen et al. 1992. (23) Teague, Carter, & Gray 1990. (24) da Costa et al. 1998. (25) Dressler & Shectman 1988. (26) Melnick & Moles 1987. (27) Median of 13 galaxy redshifts compiled from Kirshner et al. 1993; Zabludoff et al. 1992; Davoust & Considere 1995; Oegerle, Hill, & Fitchett 1995. (28) Postman, Huchra, & Geller 1992. (29) Andersen & Owen 1994. (30) Kraan-Korteweg et al. 1996. (31) Elbaz, Arnaud, & Böhringer 1995. (32) Edge & Stewart 1991a. (33) Lahav et al. 1989. (34) Ulrich 1976. (35) Stocke et al. 1991. (36) Hickson et al. 1992. (37) Beers et al. 1991.



FIG. 2.—Cumulative source count rate as a function of radius (solid line) for the cluster A2029 (pointed observation). The vertical dashed line indicates the outer significance radius, r_x . The dashed lines just above and below the source count rate indicate the 1 σ Poissonian error bars.

(cumulative) source count rate for a given radius (Fig. 2). Obvious contaminating point sources have been excluded manually. The cut-out regions have then been assigned the average surface brightness of the ring. If a cluster has been found to be clearly made up of two components, for instance, A3395n/s, these components have been treated separately. This procedure ensures that double clusters are not treated as a single entity for which spherical symmetry is assumed. For the same reason, strong substructure has been excluded in the same manner as contaminating point sources. In this work the aim is to characterize all cluster properties consistently and homogeneously. Therefore, if strong substructure is identified, then it is excluded for the flux/luminosity and mass determination.

An outer significance radius of the cluster, r_x , is determined at the position from where on the Poissonian 1 σ error rises faster than the source count rate. Usually, the source count rate settles into a nearly horizontal line for radii larger than r_x . We have found, however, that in some cases the source count rate seems to increase or decrease roughly quadratically for radii larger than r_x , indicating a possibly under- or overestimated background (Fig. 3). We therefore fitted a parabola of the form $y = mx^2 + b$ to the source count rate for radii larger than r_x and corrected the measured background. An example for a corrected source count rate profile is shown in Figure 4.

Figure 5 shows for the extended sample (106 clusters) that the difference between measured and corrected source count rates is generally very small. Nevertheless, an inspection of each count rate profile has been performed, to decide whether the measured or corrected count rate is adopted as the final count rate, to avoid artificial corrections due to large-scale variations of the background (especially in the large RASS fields). The count rates are given in Table 3.

The conversion factor for the count rate-to-flux conversion depends on the hydrogen column density, $n_{\rm H}$, on the cluster gas temperature, $T_{\rm gas}$, on the cluster gas metallicity, on the cluster redshift, z, and on the respective detector responses for the two different PSPCs used. The $n_{\rm H}$ value is taken as the value inferred from 21 cm radio measurements for our Galaxy at the projected cluster position (Dickey &



FIG. 3.—Cumulative source count rate as a function of radius for the cluster EXO 0422, shown as an extreme example (RASS data). The parabolic dashed line indicates the best-fit parabola for count rates larger than $r_{\rm X}$.

Lockman 1990;⁵ included in the EXSAS software package [Zimmermann et al. 1998];⁶ photoelectric absorption cross sections are taken from Morrison & McCammon 1983). Gas temperatures have been estimated by compiling X-ray temperatures, T_X , from the literature, giving preference to temperatures measured with the ASCA satellite. For clusters where no ASCA measured temperature has been available, X-ray temperatures measured with previous X-ray satellites have been used. The X-ray temperatures and corresponding references are given in Table 4. For two clusters included in HIFLUGCS, no measured temperature has been found in the literature, and the $L_X(<2h_{50}^{-1} \text{ Mpc}) - T_X$ relation of Markevitch (1998) has been used. The relation for non cooling flow-corrected luminosities and cooling flow-corrected/emission-weighted temperatures has been

⁵ See http://imagelib.ncsa.uiuc.edu/project/document/95.FL.01.





FIG. 4.—Corrected cumulative source count rate as a function of radius for the cluster EXO 0422. The count rate correction is less than 5%.



FIG. 5.—Comparison of measured and corrected source count rates for the extended sample of 106 galaxy clusters. The solid line indicates equality.

chosen. Since the conversion from count rate to flux depends only weakly on T_{gas} in the ROSAT energy band for the relevant temperature range, a cluster temperature $kT_{\text{gas}} = 4$ keV has been assumed in a first step to determine $L_{\rm X} (<2 h_{50}^{-1} {\rm Mpc})$ for the clusters where no gas temperature has been found in the literature. With this luminosity the gas temperature has been estimated. The metallicity is set to 0.35 times the solar value for all clusters (e.g., Arnaud et al. 1992). The redshifts have been compiled from the literature and are given in Table 3 together with the corresponding references. With these quantities and the count rates given in Table 3, fluxes in the observer rest-frame energy range 0.1-2.4 keV have been calculated applying a modern version of a Raymond-Smith spectral code (Raymond & Smith 1977). The results are listed in Table 3. The flux calculation has also been checked using XSPEC (Arnaud 1996) by folding the model spectrum created with the parameters given above with the detector response and adjusting the normalization to reproduce the observed count rate. It is found that for 90% of the clusters the deviation between the two results for the flux measurement is less than 1%. Luminosities in the source rest-frame energy range 0.1–2.4 keV have then been calculated within XSPEC by adjusting the normalization to reproduce the initial flux measurements.

The improvements in the flux determination performed here compared to the input catalogs in general are now summarized: (1) Because of the use of a high fraction of pointed observations, the photon statistics are on average much better; e.g., for the 33 clusters contained in REFLEX and HIFLUGCS, one finds a mean of 841 and 19,580 source photons, respectively. Consequently, the cluster emission has been traced out to larger radii for HIFLUGCS. (2) The higher photon statistics have allowed a proper exclusion of contaminating point sources (stars, active galactic nuclei, etc.) and substructure and the separation of double clusters. (3) An iterative background correction has been performed. (4) A measured X-ray temperature has been used for the flux calculation in most cases.

Simulations have shown that even for the HIFLUGCS clusters with the lowest number of photons, the determined flux shows no significant trend with redshift in the relevant redshift range (Ikebe et al. 2001).

3.2. Mass Determination

A parametric description of the cluster gas density profile has been derived using the standard β -model (e.g., Cavaliere & Fusco-Femiano 1976; Gorenstein et al. 1978; Jones & Forman 1984). Assuming spherical symmetry the model

$$S_{\rm X}(R) = S_0 \left(1 + \frac{R^2}{r_{\rm c}^2} \right)^{-3\beta + 1/2} + B$$
 (1)

is fitted to the measured surface brightness profile (ring count rates per ring area), where R denotes the projected distance from the cluster center. This yields values for the core radius, r_c , the β -parameter, and the normalization, S_0 (and also a fitted value for the background surface brightness, B, since the fit is performed on the non-backgroundsubtracted data). The fit values have been used to construct the radial gas density distribution

$$\rho_{\rm gas}(r) = \rho_0 \left(1 + \frac{r^2}{r_{\rm c}^2} \right)^{-(3/2)\beta} \,. \tag{2}$$

The gravitational cluster mass, M_{tot} , has been determined assuming the intracluster gas to be in hydrostatic equilibrium and isothermal. Using equation (2) and the ideal gas equation under these assumptions leads to

$$M_{\rm tot}(<\!r) = \frac{3kT_{\rm gas} r^3 \beta}{\mu m_p G} \left(\frac{1}{r^2 + r_c^2}\right),\tag{3}$$

where μ (=0.61) represents the mean molecular weight, m_n the proton mass, and G the gravitational constant. Combined N-body/hydrodynamic cluster simulations have shown that this method generally gives unbiased results with an uncertainty of 14%-29% (e.g., Schindler 1996a; Evrard, Metzler, & Navarro 1996). Currently, there are contradictory measurements concerning the general presence of gas temperature gradients in clusters (e.g., Fukazawa 1997; Markevitch et al. 1998; Irwin, Bregman, & Evrard 1999; White 2000; Irwin & Bregman 2000). If there is a systematic trend in the sense that the gas temperature decreases with increasing radius in the outer cluster parts similar to that found by Markevitch et al., then the isothermal assumption leads to an overestimation of the cluster mass of about 30% at about six core radii (Markevitch et al. 1998). Finoguenov, Reiprich, & Böhringer (2001) determined masses by employing the assumption of isothermality and also using measured cluster gas temperature profiles. A comparison for 38 clusters included in their sample indicates that the latter masses are on average a factor of 0.80 smaller than the isothermal masses within r_{200} (this radius is defined in the next paragraph). Until the final answer on this issue is given by XMM-Newton,⁷ we retain the isothermal assumption. The influence of a possible overestimation of the cluster mass on the determination of cosmological parameters is investigated in § 5.3.2.

Having determined the integrated mass as a function of radius, a physically meaningful fiducial radius for the mass measurement has to be defined. The radius commonly used is either the Abell radius, r_{200} , or r_{500} . The Abell radius is fixed at $r_A \equiv 3 h_{50}^{-1}$ Mpc. The radius r_{200} or r_{500} is the radius within which the mean gravitational mass density $\langle \rho_{tot} \rangle =$

⁷ First results indicate that apart from the central regions the intracluster gas is isothermal out to at least $(1/2)r_{200}$ (e.g., T. H. Reiprich 2001, in Clusters of Galaxies and the High Redshift Universe Observed in X-Rays, available at http://www-dapnia.cea.fr/Conferences/Morion_astro_2001/ abs03/reiprich.html; M. Arnaud 2001, private communication).

TABLE 4Cluster Properties

Cluster (1)	β (2)	r_c (3)	<i>T</i> _X (4)	M ₅₀₀ (5)	r ₅₀₀ (6)	M ₂₀₀ (7)	<i>r</i> ₂₀₀ (8)	М _А (9)	Ref. (10)
	0.500±0.004	22+3	c a a ±0.40	c o (+0.66	(co ±0.05	(0.00+1.12	a c c + 0 00		
A0085	$0.532^{+0.004}_{-0.004}$	83^{+3}_{-3}	$6.90^{+0.40}_{-0.40}$	$6.84^{+0.00}_{-0.66}$	$1.68^{+0.03}_{-0.06}$	$10.80^{+1.12}_{-1.04}$	$2.66^{+0.09}_{-0.09}$	12.21	1
A0119	$0.6/5_{-0.023}$	$501 - \frac{26}{26}$	$5.60^{+0.30}_{-0.30}$	$0.23_{-0.76}$	$1.03_{-0.07}$	$10.76_{-1.39}$	$2.00_{-0.13}$	12.24	1
A0133	$0.530_{-0.004}$	45_{-2} 10^{+1}	$3.80^{+0.00}_{-0.90}$	$2.78_{-0.95}$	$1.24_{-0.16}$	4.41 - 1.52	$1.97_{-0.27}$ 1.04 $^{+0.04}$	0./1	9
NGC 507	$0.444_{-0.005}$	19_{-1}^{+12}	$1.20_{-0.07}$	$0.41_{-0.04}$	$0.00_{-0.02}$	$0.04_{-0.06}$	$1.04_{-0.04}$ 1.25+0.05	1.80	2
A0262	$0.443_{-0.017}$	42_{-10}^{+10}	$2.15_{-0.06}$	$0.90^{+0.10}_{-0.09}$	$0.86_{-0.03}$	$1.42_{-0.13}$	$1.35_{-0.04}$ 1.52 ± 0.08	3.17	2
A0400	$0.534_{-0.013}$ 0.712+0.137	154_{-9}^{-9}	$2.31_{-0.14}$	$1.28_{-0.15}$	$0.90_{-0.04}$	$2.07_{-0.25}$	$1.53_{-0.06}$	4.10	2
A0399	$0.713_{-0.095}$	430_{-100}^{+100}	$7.00_{-0.40}$	10.00 - 2.48 10.07 + 1.08	1.91 - 0.18 1.02 + 0.07	$10.04_{-4.32}$	$3.07_{-0.30}$	16.24	1
A0401	$0.013_{-0.010}$ 0.576+0.006	240_{-10}^{-10}	$8.00_{-0.40}$ 5 20 ± 0.70	10.27 - 0.93 5 17+1.17	$1.92_{-0.05}$ 1 52 + 0.11	10.39 - 1.62 8 22 + 1.79	$3.07_{-0.10}$	10.21	1
A5112	$0.370_{-0.006}$	01_{-3} 174^{+17}	$3.30_{-1.00}$ 1 20+0.04	$3.17_{-1.45}$	$1.33_{-0.16}$	$0.22_{-2.31}$ 1 42+0.36	$2.43_{-0.25}$ 1 25 ± 0.11	2 20	1
7 0111ax	$0.004_{-0.084}$ 0.575 $^{+0.004}$	$1/4_{-15}$ 22 ⁺¹	$1.20_{-0.04}$ 2 01 $^{+0.07}$	$0.07_{-0.16}$	$0.04_{-0.06}$ 1 15 ^{+0.02}	$1.42_{-0.27}$ 2 51 + 0.16	$1.53_{-0.09}$ 1 82+0.03	5.20	2
$2_{\rm TW}$ III 54	$0.373_{-0.003}$ 0.887 $^{+0.320}$	33_{-1}	$(2.16^{+0.07})$	2.21 - 0.09 2.36 + 2.22	1.13 - 0.02 1 18 + 0.29	3.01 - 0.15 3.03 + 3.83	$1.03_{-0.03}$ 1 80+0.48	632	11
Δ 3158	$0.661^{+0.025}$	269_{-73} 269^{+20}	$(2.10_{-0.30})$ 577 ^{+0.10}	$7.00^{+0.52}$	$1.10_{-0.17}$ $1.69^{+0.04}$	$11.29^{+0.95}$	$2.69^{+0.08}$	12.61	3
A0478	$0.601_{-0.022}$ $0.613^{+0.004}$	98^{+2}	$840^{+0.80}$	$11.32^{+1.78}$	$1.09_{-0.03}$ 1 99 ^{+0.10}	17.29 - 0.68 17.89 + 2.95	$2.09_{-0.06}$ 3 13 ^{+0.18}	17.12	1
NGC 1550	$0.513_{-0.004}$ $0.554^{+0.049}$	45 ⁺¹⁵	$143^{+0.04}$	$0.69^{+0.12}$	$0.78^{+0.04}$	$109^{+0.20}$	$1.23^{+0.07}$	2 64	5
EXO 0422	$0.33^{+0.037}_{-0.037}$	142^{+40}	$290^{+0.90}$	$289^{+2.39}$	$1.26^{+0.28}$	$463^{+3.84}$	$2.00^{+0.44}$	6.96	9
A 3266	$0.722_{-0.071}$ $0.796^{+0.020}$	564^{+21}	$8.00^{+0.50}$	$1417^{+1.94}$	$2.14^{+0.09}$	$23.76^{+3.23}$	$345^{+0.15}$	20.47	1
A0496	$0.484^{+0.003}$	30^{+1}	$4.13^{+0.08}$	$2.76^{+0.11}$	$1.24^{+0.02}$	$4.35^{+0.18}$	$1.96^{+0.03}$	6.66	2
A3376	$1.054^{+0.101}$	755^{+69}	$4.00^{+0.40}$	$6.32^{+2.11}$	$1.64^{+0.17}$	$11.96^{+3.82}$	$2.75^{+0.26}$	13.20	1
A3391	$0.579^{+0.026}$	234^{+24}	$5.40^{+0.60}$	$5.18^{+1.31}$	$1.53^{+0.12}$	8.41 + 2.13	$2.44^{+0.19}$	10.35	1
A3395s	$0.964^{+0.275}_{-0.167}$	604^{+173}_{-118}	$5.00^{+0.30}_{-0.30}$	$8.82^{+4.79}_{-2.61}$	$1.83^{+0.29}_{-0.20}$	$15.34^{+8.79}_{-1.74}$	$2.99^{+0.49}_{-0.35}$	15.42	1
A0576	$0.825^{+0.432}_{-0.167}$	394^{+221}	$4.02^{+0.07}_{-0.07}$	$5.36^{+4.42}$	$1.55^{+0.34}_{-0.12}$	8.96 ^{+8.01}	$2.50^{+0.60}$	10.86	3
A0754	$0.698 \substack{+0.027\\-0.024}$	239^{+17}_{-16}	$9.50^{+0.70}_{-0.40}$	$16.37^{+2.91}_{-1.84}$	$2.25^{+0.13}_{-0.09}$	$26.19^{+4.45}_{-2.95}$	$3.57^{+0.18}_{-0.15}$	21.94	1
Hydra A	$0.573^{+0.003}_{-0.003}$	50^{+1}_{-1}	$4.30^{+0.40}_{-0.40}$	$3.76^{+0.58}_{-0.55}$	$1.38^{+0.07}_{-0.07}$	$5.94^{+0.91}_{-0.84}$	$2.17^{+0.11}_{-0.10}$	8.21	1
A1060	$0.607^{+0.040}_{-0.034}$	94^{+15}_{-13}	$3.24^{+0.06}_{-0.06}$	$2.66^{+0.34}_{-0.28}$	$1.23^{+0.05}_{-0.04}$	$4.24^{+0.55}_{-0.47}$	$1.95^{+0.08}_{-0.08}$	6.54	2
A1367	$0.695^{+0.035}_{-0.032}$	383^{+24}_{-22}	$3.55^{+0.08}_{-0.08}$	$3.34^{+0.36}_{-0.32}$	$1.32^{+0.05}_{-0.04}$	$5.69^{+0.63}_{-0.56}$	$2.14^{+0.08}_{-0.07}$	8.08	2
MKW 4	$0.440^{+0.004}_{-0.005}$	11^{+1}_{-1}	$1.71^{+0.09}_{-0.09}$	$0.64^{+0.06}_{-0.06}$	$0.76^{+0.02}_{-0.03}$	$1.00^{+0.10}_{-0.09}$	$1.20^{+0.04}_{-0.03}$	2.51	2
Zw Cl 1215	$0.819^{+0.038}_{-0.034}$	431^{+28}_{-25}	$(5.58^{+0.89}_{-0.78})$	$8.79^{+3.00}_{-2.29}$	$1.83^{+0.19}_{-0.18}$	$14.52^{+4.92}_{-3.67}$	$2.93^{+0.30}_{-0.27}$	14.91	11
NGC 4636	$0.491^{+0.032}_{-0.027}$	$6^{+\frac{3}{2}}$	$0.76^{+0.01}_{-0.01}$	$0.22^{+0.03}_{-0.02}$	$0.53^{+0.02}_{-0.02}$	$0.35^{+0.04}_{-0.04}$	$0.85^{+0.03}_{-0.03}$	1.24	4
A3526	$0.495^{+0.011}_{-0.010}$	37^{+5}_{-4}	$3.68^{+0.06}_{-0.06}$	$2.39^{+0.15}_{-0.13}$	$1.18^{+0.02}_{-0.02}$	$3.78^{+0.23}_{-0.18}$	$1.87^{+0.04}_{-0.03}$	6.07	2
A1644	$0.579^{+0.111}_{-0.074}$	300^{+128}_{-92}	$4.70^{+0.90}_{-0.70}$	$4.10^{+2.64}_{-1.41}$	$1.42^{+0.26}_{-0.18}$	$6.73^{+4.54}_{-2.38}$	$2.27^{+0.43}_{-0.31}$	8.98	10
A1650	$0.704^{+0.131}_{-0.081}$	281^{+104}_{-71}	$6.70^{+0.80}_{-0.80}$	$9.62^{+4.91}_{-2.92}$	$1.88^{+0.28}_{-0.21}$	$15.60^{+8.08}_{-4.85}$	$3.01^{+0.45}_{-0.35}$	15.56	1
A1651	$0.643^{+0.014}_{-0.013}$	181^{+10}_{-10}	$6.10^{+0.40}_{-0.40}$	$7.45^{+1.00}_{-0.95}$	$1.73^{+0.07}_{-0.08}$	$11.91^{+1.60}_{-1.52}$	$2.75^{+0.12}_{-0.13}$	13.01	1
Coma	$0.654^{+0.019}_{-0.021}$	344^{+22}_{-21}	$8.38^{+0.34}_{-0.34}$	$11.99^{+1.28}_{-1.29}$	$2.03^{+0.07}_{-0.08}$	$19.38^{+2.08}_{-1.97}$	$3.22^{+0.11}_{-0.11}$	18.01	2
NGC 5044	$0.524^{+0.002}_{-0.003}$	11^{+1}_{-1}	$1.07^{+0.01}_{-0.01}$	$0.41^{+0.01}_{-0.01}$	$0.66^{+0.01}_{-0.01}$	$0.65^{+0.01}_{-0.01}$	$1.04^{+0.01}_{-0.01}$	1.87	2
A1736	$0.542^{+0.147}_{-0.092}$	374^{+178}_{-130}	$3.50^{+0.40}_{-0.40}$	$2.19^{+1.23}_{-0.74}$	$1.15^{+0.18}_{-0.15}$	$3.78^{+2.41}_{-1.34}$	$1.87^{+0.34}_{-0.25}$	6.22	1
A3558	$0.580^{+0.006}_{-0.005}$	224^{+5}_{-5}	$5.50^{+0.40}_{-0.40}$	$5.37^{+0.70}_{-0.64}$	$1.55^{+0.07}_{-0.06}$	$8.64^{+1.12}_{-1.03}$	$2.46^{+0.10}_{-0.10}$	10.56	1
A3562	$0.472^{+0.006}_{-0.006}$	99^{+5}_{-5}	$5.16^{+0.16}_{-0.16}$	$3.68^{+0.24}_{-0.23}$	$1.37^{+0.03}_{-0.03}$	$5.83^{+0.38}_{-0.36}$	$2.16^{+0.05}_{-0.04}$	8.10	3
A3571	$0.613\substack{+0.010\\-0.010}$	181^{+7}_{-7}	$6.90^{+0.20}_{-0.20}$	$8.33^{+0.56}_{-0.53}$	$1.79^{+0.04}_{-0.04}$	$13.31^{+0.90}_{-0.85}$	$2.85^{+0.06}_{-0.06}$	14.04	1
A1795	$0.596^{+0.003}_{-0.002}$	78^{+1}_{-1}	$7.80^{+1.00}_{-1.00}$	$9.75^{+2.01}_{-1.90}$	$1.89^{+0.12}_{-0.14}$	$15.39^{+3.17}_{-2.92}$	$2.99^{+0.19}_{-0.20}$	15.46	1
A3581	$0.543^{+0.024}_{-0.022}$	35^{+5}_{-4}	$1.83^{+0.04}_{-0.04}$	$0.96^{+0.09}_{-0.09}$	$0.87^{+0.02}_{-0.03}$	$1.52^{+0.16}_{-0.13}$	$1.38^{+0.05}_{-0.04}$	3.30	5
MKW 8	$0.511^{+0.098}_{-0.059}$	107^{+70}_{-42}	$3.29^{+0.23}_{-0.22}$	$2.10^{+0.86}_{-0.52}$	$1.14^{+0.13}_{-0.10}$	$3.33^{+1.45}_{-0.83}$	$1.79^{+0.24}_{-0.17}$	5.60	5
A2029	$0.582^{+0.004}_{-0.004}$	83^{+2}_{-2}	$9.10^{+1.00}_{-1.00}$	$11.82^{+2.14}_{-1.99}$	$2.01^{+0.11}_{-0.12}$	$18.79^{+3.40}_{-3.17}$	$3.20^{+0.18}_{-0.19}$	17.62	1
A2052	$0.526^{+0.003}_{-0.005}$	37^{+2}_{-2}	$3.03^{+0.04}_{-0.04}$	$1.95^{+0.07}_{-0.06}$	$1.10^{+0.02}_{-0.01}$	$3.10^{+0.09}_{-0.11}$	$1.75^{+0.01}_{-0.02}$	5.30	3
MKW 3s	$0.581_{-0.007}^{+0.007}$	66_{-3}^{+3}	$3.70^{+0.20}_{-0.20}$	$3.06_{-0.30}^{+0.52}$	$1.29_{-0.04}$	$4.84_{-0.47}^{+0.01}$	$2.03_{-0.07}^{+0.07}$	/.16	1
A2065	$1.162_{-0.282}$	690^{+300}_{-186}	$5.50^{+0.40}_{-0.40}$	13.44 - 5.17	$2.10^{+0.03}_{-0.31}$	$23.37_{-9.42}$	$3.43^{+1.05}_{-0.54}$	20.21	1
A2003	$0.501_{-0.011}$	110_{-6}	$3.08_{-0.11}$	$2.84_{-0.19}$	$1.23_{-0.03}$	$4.54_{-0.31}$	$1.99_{-0.04}$	0.80	2
A2142	$0.591_{-0.006}^{+0.006}$	154_{-6}^{+6}	$9.70^{+1.00}_{-1.10}$	$13.29_{-2.41}$	$2.10_{-0.14}$	$21.04_{-3.69}$	$3.31_{-0.20}$	19.05	1
A214/	$0.444_{-0.046}$	238_{-65}^{-65}	$4.91_{-0.28}$ 12 20 ± 0.64	$2.99_{-0.63}$	1.28 - 0.10 2.81 + 0.12	$4.84_{-1.03}$	$2.03_{-0.15}$	7.21 24.19	2
A2105	$0.790_{-0.028}$	120^{+10}	$13.29_{-0.64}$	31.03 - 3.74	2.01 - 0.11 1 42 + 0.04	$51.99_{-6.13}$ 672+0.52	$4.49_{-0.18}$	24.10 8.02	2
A2199	$0.033_{-0.021}$ 0.507 $^{+0.008}$	139_{-10} 67^{+3}	$4.10_{-0.08}$ 7 21 $^{+0.25}$	$4.21_{-0.29}$ 8 67 + 0.67	$1.43_{-0.03}$ $1.82^{+0.05}$	$0.73_{-0.51}$ 13 70 ^{+0.96}	$2.27_{-0.06}$ 2 80+0.06	0.92 14 34	2
A2204	$0.097_{-0.007}$ 0.607 ^{+0.016}	126^{+11}	$7.21_{-0.25}$ 7 10+5.00	$8.07_{-0.57}$ 865+11.47	$1.02_{-0.04}$ 1.82 ^{+0.59}	$13.79_{-1.00}$ 13.78 + 18.02	$2.09_{-0.08}$ 2 80 + 0.92	14.34	10
A2256	$0.007_{-0.015}$ $0.914^{+0.054}$	587^{+40}	$660^{+0.40}$	$1283^{+2.38}$	$207^{+0.12}$	$21.81^{+4.07}$	$2.09_{-0.52}$ 3 36 ^{+0.19}	19 34	10
A2255	$0.797^{+0.047}$	593+35	$6.87^{+0.20}$	$10.90^{+1.15}$	1.07 - 0.11 $1.96^{+0.07}$	$1865^{+2.01}$	$3.18^{+0.11}$	17.54	3
A3667	$0.541^{+0.008}$	279^{+10}	$7.00^{+0.60}$	$6.88^{+1.08}$	$1.68^{+0.08}$	$11.19^{+1.76}$	$2.69^{+0.13}$	12.50	1
S1101	$0.639^{+0.008}$	56^{+2}	$3.00^{+1.20}$	$2.58^{+1.76}$	$1.22^{+0.23}$	$4.08^{+2.78}$	$1.92^{+0.36}$	6 38	9
A2589	$0.596^{+0.013}$	118^{+8}	$3.70^{+2.20}$	$3.14^{+3.44}$	$1.29^{+0.37}$	$5.01^{+5.41}$	$2.06^{+0.56}$	7.33	9
A2597	$0.633^{+0.008}_{-0.008}$	58+2	$4.40^{+0.40}$	$4.52^{+0.72}$	$1.47^{+0.07}$	$7.14^{+1.14}$	$2.31^{+0.11}$	9.27	1
A2634	$0.640^{+0.051}_{-0.042}$	364^{+44}_{-30}	$3.70^{+0.28}_{-0.28}$	$3.15^{+0.78}_{-0.60}$	$1.29^{+0.10}_{-0.00}$	$5.35^{+1.34}_{-1.04}$	$2.10^{+0.17}_{-0.14}$	7.77	2
A2657	0.556+0.008	119^{+5}_{-2}	$3.70^{+0.30}_{-0.30}$	$2.83^{+0.43}_{-0.30}$	$1.25^{+0.06}_{-0.06}$	$4.52^{+0.68}_{-0.62}$	$1.99^{+0.10}_{-0.00}$	6.84	1
A4038	$0.541^{+0.009}_{-0.008}$	59^{+3}_{-4}	$3.15^{+0.03}_{-0.03}$	$2.16^{+0.09}_{-0.08}$	$1.14^{+0.02}_{-0.02}$	$3.41^{+0.14}_{-0.11}$	$1.80^{+0.03}_{-0.01}$	5.67	3
A4059	$0.582\substack{+0.010\\-0.010}$	$90^{+\frac{5}{5}}_{-5}$	$4.40^{+0.30}_{-0.30}$	$3.95^{+0.52}_{-0.48}$	$1.40^{+0.06}_{-0.06}$	$6.30^{+0.83}_{-0.76}$	$2.22^{+0.09}_{-0.09}$	8.52	1

MASS FUNCTION OF GALAXY CLUSTERS

Cluster (1)	β (2)	r _c (3)	<i>T</i> _x (4)	M ₅₀₀ (5)	r ₅₀₀ (6)	M ₂₀₀ (7)	r ₂₀₀ (8)	М _А (9)	Ref. (10)
		Cluster	s from the Exten	ded Sample Not	Included in HIF	FLUGCS			
A2734	$0.624^{+0.034}_{-0.029}$	212^{+26}_{-23}	$(3.85^{+0.62}_{-0.54})$	$3.49^{+1.25}_{-0.89}$	$1.34^{+0.15}_{-0.12}$	$5.67^{+1.98}_{-1.48}$	$2.14^{+0.22}_{-0.21}$	7.97	11
A2877	$0.566^{+0.029}_{-0.025}$	190^{+19}_{-17}	$3.50^{+2.20}_{-1.10}$	$2.61^{+3.32}_{-1.24}$	$1.22^{+0.39}_{-0.23}$	$4.24^{+5.28}_{-2.00}$	$1.95^{+0.60}_{-0.38}$	6.57	10
NGC 499	$0.722^{+0.034}_{-0.030}$	24^{+2}_{-2}	$0.72^{+0.03}_{-0.02}$	$0.36^{+0.05}_{-0.04}$	$0.63^{+0.03}_{-0.02}$	$0.58^{+0.08}_{-0.06}$	$1.00^{+0.04}_{-0.04}$	1.73	4
AWM 7	$0.671^{+0.027}_{-0.025}$	173^{+18}_{-15}	$3.75^{+0.09}_{-0.09}$	$3.79^{+0.38}_{-0.32}$	$1.38^{+0.05}_{-0.04}$	$6.08^{+0.62}_{-0.52}$	$2.19^{+0.08}_{-0.06}$	8.35	2
Perseus	$0.540^{+0.006}_{-0.004}$	64^{+2}_{-2}	$6.79^{+0.12}_{-0.12}$	$6.84^{+0.29}_{-0.26}$	$1.68^{+0.02}_{-0.02}$	$10.80^{+0.46}_{-0.41}$	$2.66^{+0.04}_{-0.04}$	12.20	2
S405	$0.664^{+0.263}_{-0.133}$	459^{+262}_{-159}	$(4.21^{+0.67}_{-0.59})$	$3.91^{+3.56}_{-1.57}$	$1.40^{+0.33}_{-0.22}$	$6.75^{+6.80}_{-2.81}$	$2.27^{+0.60}_{-0.37}$	9.09	11
3C 129	$0.601^{+0.260}_{-0.131}$	$318 ^{+178}_{-107}$	$5.60^{+0.70}_{-0.60}$	$5.68^{+5.58}_{-2.29}$	$1.58^{+0.40}_{-0.25}$	$9.30^{+9.51}_{-3.85}$	$2.53^{+0.67}_{-0.42}$	11.08	9
A0539	$0.561^{+0.020}_{-0.018}$	148^{+13}_{-12}	$3.24^{+0.09}_{-0.09}$	$2.33^{+0.21}_{-0.19}$	$1.18^{+0.03}_{-0.03}$	$3.74^{+0.35}_{-0.34}$	$1.87^{+0.05}_{-0.06}$	6.04	2
S540	$0.641^{+0.073}_{-0.051}$	130^{+38}_{-29}	$(2.40^{+0.38}_{-0.34})$	$1.83^{+0.83}_{-0.54}$	$1.08^{+0.14}_{-0.12}$	$2.93^{+1.34}_{-0.87}$	$1.72^{+0.23}_{-0.19}$	5.13	11
A0548w	$0.666^{+0.194}_{-0.111}$	198^{+90}_{-62}	$(1.20^{+0.19}_{-0.17})$	$0.63^{+0.48}_{-0.23}$	$0.76^{+0.16}_{-0.11}$	$1.06^{+0.84}_{-0.41}$	$1.23^{+0.26}_{-0.19}$	2.64	11
A0548e	$0.480^{+0.013}_{-0.013}$	118^{+12}_{-11}	$3.10^{+0.10}_{-0.10}$	$1.74^{+0.15}_{-0.15}$	$1.07^{+0.03}_{-0.04}$	$2.77^{+0.27}_{-0.23}$	$1.68^{+0.06}_{-0.05}$	4.95	3
A3395n	$0.981^{+0.619}_{-0.244}$	672^{+383}_{-203}	$5.00^{+0.30}_{-0.30}$	$8.70^{+9.53}_{-3.19}$	$1.82^{+0.51}_{-0.26}$	$15.47^{+18.82}_{-6.07}$	$2.99^{+0.92}_{-0.46}$	15.55	1
UGC 03957	$0.740^{+0.133}_{-0.086}$	142_{-33}^{+45}	$(2.58^{+0.41}_{-0.36})$	$2.51^{+1.50}_{-0.83}$	$1.20^{+0.21}_{-0.15}$	$4.02^{+2.41}_{-1.33}$	$1.91^{+0.33}_{-0.23}$	6.35	11
PKS 0745	0.608 + 0.006	71^{+2}_{-2}	$7.21^{+0.11}_{-0.11}$	$8.88^{+0.35}_{-0.28}$	$1.83^{+0.03}_{-0.01}$	$14.12^{+0.56}_{-0.53}$	$2.91^{+0.04}_{-0.04}$	14.58	3
A0644	$0.700^{+0.011}_{-0.011}$	$203^{+\frac{7}{7}}_{-7}$	$7.90^{+0.80}_{-0.80}$	$12.50^{+2.29}_{-2.11}$	$2.06^{+0.12}_{-0.12}$	19.83 + 3.79	$3.24^{+0.21}_{-0.17}$	18.33	1
S636	$0.752^{+0.217}_{-0.123}$	344^{+130}_{-86}	$(1.18^{+0.19}_{-0.17})$	$0.61^{+0.44}_{-0.22}$	$0.75^{+0.12}_{-0.10}$	$1.16^{+0.90}_{-0.44}$	$1.26^{+0.27}_{-0.18}$	2.93	11
A1413	$0.660^{+0.017}_{-0.015}$	179^{+12}_{-11}	$7.32^{+0.26}_{-0.24}$	$10.20^{+0.93}_{-0.82}$	$1.92^{+0.05}_{-0.05}$	$16.29^{+1.49}_{-1.31}$	$3.05^{+0.09}_{-0.08}$	16.03	3
M49	0.592 ± 0.007	11^{+1}_{-1}	$0.95^{+0.02}_{-0.01}$	$0.41^{+0.02}_{-0.01}$	$0.66^{+0.01}_{-0.01}$	$0.65^{+0.04}_{-0.02}$	$1.04^{+0.02}_{-0.01}$	1.87	4
A3528n	0.621 ± 0.034	178^{+17}_{-16}	$3.40^{+1.66}_{-0.64}$	$2.89^{+2.84}_{-0.04}$	$1.26^{+0.32}_{-0.15}$	$4.65^{+4.54}_{-1.48}$	$2.00^{+0.51}_{-0.23}$	7.00	8
A3528s	$0.463^{+0.013}_{-0.013}$	101^{+9}	$3.15^{+0.89}_{-0.59}$	$1.69^{+0.87}_{-0.50}$	$1.05^{+0.16}_{-0.12}$	$2.70^{+1.39}$	$1.67^{+0.25}_{-0.10}$	4.86	8
A3530	$0.773^{+0.114}_{-0.035}$	421^{+75}	$3.89^{+0.27}$	$4.52^{+1.52}$	$1.47^{+0.15}_{-0.12}$	$7.64^{+2.72}$	$2.36^{+0.26}_{-0.20}$	9.82	7
A3532	$0.653^{+0.034}$	282^{+27}	$4.58^{+0.19}$	$4.77^{+0.70}$	$1.49^{+0.07}$	$7.79^{+1.16}$	$2.38^{+0.12}$	9.88	7
A1689	$0.690^{+0.011}$	163^{+7}	$9.23^{+0.28}$	$15.49^{+1.18}$	$2.20^{+0.06}$	$24.68^{+1.70}$	$3.50^{+0.07}$	21.13	3
A3560	$0.566^{+0.033}_{-0.033}$	256^{+30}	$(3.16^{+0.51})$	$2.16^{+0.79}$	$1.14^{+0.12}$	$3.59^{+1.30}$	$1.84^{+0.20}$	5.92	11
A1775	$0.673^{+0.026}_{-0.026}$	260^{+19}	$3.69^{+0.20}$	$3.61^{+0.50}$	$1.36^{+0.06}$	$5.91^{+0.83}$	$2.17^{+0.09}$	8.21	3
A 1800	$0.766^{+0.308}$	392^{+223}	$(4.02^{+0.64})$	$475^{+4.64}$	1.30 = 0.05 1.49 + 0.38	797 + 8.31	2.17 = 0.07 2.39 + 0.65	10.08	11
A 1914	$0.751^{+0.018}$	231^{+11}	$1053^{+0.51}$	$21.43^{+2.39}$	$246^{+0.09}$	33 99 + 4.06	388+0.16	26.20	3
NGC 5813	$0.751_{-0.017}$ 0.766 ^{+0.179}	251_{-10} 25^{+9}	$(0.52^{+0.08})$	$0.24^{+0.17}$	$0.55^{+0.11}$	$0.38^{+0.27}$	$0.87^{+0.16}$	1 32	11
NGC 5846	$0.700_{-0.103}$ 0.599 ^{+0.016}	$\frac{23-6}{7+1}$	$0.82^{+0.01}$	$0.24_{-0.08}$ $0.33^{+0.02}$	$0.55_{-0.07}$ $0.61^{+0.01}$	0.50 ± 0.13 0.53 ± 0.03	$0.07_{-0.12}$ $0.97^{+0.02}$	1.52	4
A 2151w	$0.599_{-0.015}$ 0 564 $+ 0.014$	68^{+5}	$240^{+0.06}$	$152^{+0.12}$	$1.02^{+0.01}$	$242^{+0.18}$	$1.61^{+0.01}$	4 51	3
A2131W	$0.504_{-0.013}$ 0.555 $^{+0.056}$	200 + 56	$602^{+0.08}$	$563^{+0.95}$	$1.02_{-0.02}$ 1 57 ^{+0.09}	$0.70^{+1.61}$	$251^{+0.14}$	11.03	3
Triangulum	$0.555_{-0.044}$ 0.610 ^{+0.010}	279 - 49 279 + 11	$9.60^{+0.60}$	13.03 - 0.68 13.42 + 1.70	$210^{+0.09}$	$21.54^{+2.73}$	$2.31_{-0.10}$ $3.34^{+0.14}$	10.35	1
Ophiuchus	$0.010_{-0.010}$ $0.747^{+0.035}$	279 - 10 270 + 23	$10.26^{+0.32}$	15.42 - 1.55 20 25 + 2.51	$2.10_{-0.09}$ 2 $11^{+0.10}$	21.54 - 2.36 32.42 + 4.05	3.94 - 0.11 3.93 + 0.16	25 32	2
$7_{\rm W} \cap 1742$	$0.747_{-0.032}$ 0.717 ^{+0.073}	279_{-22} 232+46	$(5.23^{+0.84})$	20.25 - 2.10 6 88 + 3.06	2.41 - 0.08 1 68 + 0.22	52.45 - 3.38 11 05 + 4.93	$267^{+0.13}$	12.32	11
Zw CI 1/42	$0.717_{-0.053}$ 0.501+0.013	232 - 38 285 + 15	(3.23 - 0.73) 8 80 + 0.50	$1116^{+1.39}$	1.00 - 0.18 1 07 + 0.08	11.03 - 3.16 18 07 + 2.12	$2.07_{-0.28}$ 2 16 ^{+0.11}	12.42	11
A2519	$0.591_{-0.012}$ 0.642+0.259	200 + 254	$(5.00_{-0.50})$	557+5.29	$1.97_{-0.07}$ 1 57+0.39	0.07 - 2.06 0.22 + 9.56	$3.10_{-0.13}$ $3.52^{+0.67}$	11.17	11
A3093	$0.042_{-0.117}$	399_{-149}	$(3.29_{-0.74})$	$3.57_{-2.16}$	$1.37_{-0.24}$ 1.27 $^{+0.24}$	9.32 - 3.74 5 04 + 3.60	$2.33_{-0.40}$	7 47	11
ZW 11 100	$0.002_{-0.097}$	303_{-105} 251+160	$(3.44_{-0.48})$	$2.90_{-1.02}$	1.27 - 0.16 1 51 + 0.28	$3.04_{-1.80}$	$2.00_{-0.28}$	10.20	11
A3022	$0.039_{-0.093}$	551_{-111} 502^{+248}	$(4.90_{-0.69})$	$4.97_{-1.75}$	$1.31_{-0.20}$	$0.20_{-3.02}$	$2.43_{-0.34}$	10.29	11
AJ02/	$0.989_{-0.192}$	393 - 149 401 + 46	(1.00 - 0.99)	$10.33 - 6.76^{-1}$	2.23 - 0.37 2 49 + 0.31	2/.44 - 11.46 25 74 + 15.07	$3.02_{-0.60}$	22.44	11
A 2021	$0.928_{-0.066}$	401_{-40}	(0.04 - 1.24)	22.00 - 6.28	2.40 - 0.26	33./4 - 10.38 12.00 + 1.87	$3.90_{-0.44}$	20.83	11
A3921	$0.762_{-0.030}$	328_{-23}	5./5 - 0.23	8.40 - 0.96	$1.80_{-0.07}$	$13.80^{+1.59}_{-1.59}$	$2.89_{-0.12}^{+0.12}$	14.3/	5
HCG 94	$0.514_{-0.006}^{+0.007}$	80-4 201+20	5.43 - 0.30	2.28 - 0.34	$1.1/_{-0.06}$	$3.02_{-0.51}$	$1.84_{-0.09}^{+0.09}$	5.90	6
кхј 2344	0.807-0.030	301_{-18}^{+20}	(4./3 + 0.66)	6.91 - 1.69	$1.68_{-0.14}^{+0.17}$	11.27 - 2.80	$2.69_{-0.25}^{+0.25}$	12.58	11

NOTES.—Col. (1): Cluster name. Col. (2): β -parameter value and the corresponding 68% c.l. statistical uncertainty for two interesting parameters. Col. (3): Core radius in h_{50}^{-1} kpc and the corresponding uncertainty. Col. (4): X-ray temperature along with its error. For some references the temperature uncertainty is quoted at the 90% confidence level and therefore represents a conservative error estimate. Cols. (5), (7): M_{500} and M_{200} and their uncertainties in units of $10^{14} h_{50}^{-1} M_{\odot}$, calculated as described in § 3.2. Cols. (6), (8): r_{500} and r_{200} and their uncertainties in h_{50}^{-1} Mpc. Col. (9): $M_A \equiv M_{tot}(< r_A)$ in units of $10^{14} h_{50}^{-1} M_{\odot}$. Col. (10): Code for the temperature reference decoded below. Temperatures for codes 1–7 have been determined with ASCA, code 8 with ROSAT, code 9 with EXOSAT, code 10 with Einstein, and code 11 with a ROSAT-ASCA L_X - T_X relation. Temperatures for code 11 are enclosed in parentheses, and the corresponding errors have been calculated using the scatter in the L_X - T_X relation. Table 4 is also available in machine-readable form in the electronic edition of the Astrophysical Journal.

REFERENCES.—(1) Markevitch et al. 1998. (2) Fukazawa et al. 1998. (3) White 2000. (4) Matsushita 1997. (5) Ikebe et al. 2001. (6) Finoguenov et al. 2001. (7) This work. (8) Schindler 1996b. (9) Edge & Stewart 1991a. (10) David et al. 1993. (11) Estimated from the L_X - T_X relation given by Markevitch 1998.

 $200\rho_c$ or $500\rho_c$. The critical cosmic matter density is defined as $\rho_c \equiv 3H^2/(8\pi G)$, where $H^2 = H_0^2 E(z)^2$ and $E(z) = [\Omega_m(1 + z)^3 + \Omega_k(1 + z)^2 + \Omega_A]^{1/2}$. It has been shown that a correction for redshift is not necessary for the nearby clusters included in HIFLUGCS (Finoguenov et al. 2001), and we use the zero redshift value for all calculations, i.e., $\rho_c = 4.6975 \times 10^{-30}$ g cm⁻³, unless noted otherwise. Nevertheless, the influence of this approximation is tested in § 5.3.2 for the model $\Omega_m = 1$, $\Omega_A = 0$, where evolution is strong. In order to treat clusters of different size in a homogeneous way, we determine the cluster mass at a characteristic density but also give the mass determined formally at a fixed radius for comparison. Spherical collapse models predict a cluster virial density $\langle \rho_{\rm vir} \rangle \approx 178 \rho_c$ for $\Omega_m = 1$, $\Omega_{\Lambda} = 0$, so a pragmatic approximation to the virial mass is to use r_{200} as the outer boundary. Simulations performed by Evrard et al. (1996) have shown, however, that isothermal X-ray mass measurements may be biased toward high

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symbol sizes.

masses for $r > r_{500}$. Furthermore, for most of the clusters in HIFLUGCS (86%) up to r_{500} , no extrapolation outside the significantly detected cluster emission is necessary, i.e., $r_{500} < r_{\rm X}$, whereas the fraction is lower for r_{200} (25%) and $r_{\rm A}$ (17%). In summary, the most accurate results are expected for $M_{tot}(< r_{500}) \equiv M_{500}$, but for a comparison to predicted mass functions, M_{200} is the more appropriate value (§ 5.3.2). Results for all determined masses and their corresponding radii are given in Table 4. Masses for the cluster gas will be given in a subsequent paper.

A major source of uncertainty comes from the temperature measurements. However, this (statistical) error is less than 5% for one-third of the clusters; therefore, also other sources of error have to be taken into account. In particular, one cannot neglect the uncertainties of the fit parameter values when assessing the statistical errors of the mass measurements. Therefore, mass errors have been calculated by varying the fit parameter values, β and r_c , along their 68% confidence level error ellipse and using the upper and lower bound of the quoted temperature ranges. The statistical mass error range has then been defined between the maximum and minimum mass. Note that a simple error propagation applied to equation (3) would underestimate the uncertainty of M_{200} and M_{500} , since r_{200} and r_{500} also depend on T_{gas} , β , and (weakly) r_c . The individual mass errors have been used in subsequent calculations, unless noted otherwise.⁸ A mean statistical error of 23% for clusters included in HIFLUGCS and a mean error of 27% for the extended sample have been found.

4. RESULTS

In this section it is shown that a tight correlation exists between the gravitational cluster mass and the X-ray luminosity. This ensures that HIFLUGCS is essentially selected by cluster mass. In the second part of this section the cluster mass function is presented, including the proper treatment of the scatter in the $L_{\rm x}$ - $M_{\rm tot}$ relation.

4.1. Mass-Luminosity Relation

Since the aim is the construction of a mass function from a flux-limited sample, it is now important to test for a correlation between X-ray luminosity and gravitational mass. In Figure 6 L_x , given in the ROSAT energy band, is plotted as a function of M_{200} , showing clearly the existence of a tight (linear Pearson correlation coefficient = 0.92) correlation, as expected.

To quantify the mass-luminosity relation, a linear regression fit in log-log space has been performed. The method used allows for intrinsic scatter and errors in both variables (Akritas & Bershady 1996).⁹ Tables 7-11 in the Appendix give the results for different fit methods, where minimization has been performed in vertical, horizontal, and orthogonal direction, and the bisector result is given, which bisects the best-fit results of vertical and horizontal minimization. The fits have been performed using the form

$$\log\left[\frac{L_{\rm X}(0.1-2.4 \text{ keV})}{h_{50}^{-2} \, 10^{40} \text{ ergs s}^{-1}}\right] = A + \alpha \log\left(\frac{M_{200}}{h_{50}^{-1} \, M_{\odot}}\right).$$
(4)

⁸ In log space errors are transformed as $\Delta \log x = \log (e)(x^+ - x^-)/(e^+ - x^-)/(e^- - x^-)/(e^+ - x^-)/(e^- - x$ (2x), where x^+ and x^- denote the upper and lower boundary of the quantity's error range, respectively.

⁹ See http://www.astro.wisc.edu/~mab/archive/stats/stats.html.



We find, as noted in general by previous authors (e.g., Isobe et al. 1990), that the chosen fitting method has a significant influence on the best-fit parameter values.¹⁰ In this work the appropriate relation for the application under consideration is always indicated.

The difference between the fit results for 63 and 106 clusters may indicate a scale dependence of the L_X - M_{tot} relation, since the difference is slightly larger than the uncertainty evaluated with the bootstrap method. The small number of low-luminosity clusters in HIFLUGCS compared to the extended sample may be responsible for the less steep relation obtained using the HIFLUGCS clusters only. Note that only two out of the six clusters with $L_{\rm X} < h_{50}^{-2} 10^{43}$ ergs s⁻¹ are included in HIFLUGCS. To reliably detect any deviations from the power-law shape of the $L_{\rm X}$ - $M_{\rm tot}$ relation, however, more clusters with $M_{\rm tot} < 10^{14} h_{50}^{-1} M_{\odot}$ (and possibly $M_{\rm tot} > 3 \times 10^{15} h_{50}^{-1} M_{\odot}$) need to be sampled. Such work is in progress. As will be seen later, in the procedure used here for the comparison of observed and predicted mass functions the precise shape of the $L_{\rm X}$ - $M_{\rm tot}$ relation is not important.

When constructing the mass function, the overall (measurement plus intrinsic) scatter in the L_X - M_{tot} relation may become important (§ 4.2). After verifying that the scatter is approximately Gaussian in log space, the scatter has been measured as given in Table 12 in the Appendix. The scatter in log [$L_x(0.1-2.4 \text{ keV})$], log (M_{200}), and orthogonal to the best-fit line is given by $\sigma_{\log L_X}$, $\sigma_{\log M_{tot}}$, and $\sigma_{\log L/M}$, respectively.

4.2. Mass Function

The commonly used definition of the galaxy cluster mass function is analogous to the definition of the luminosity function (e.g., Schechter 1976): the mass function, $\phi(M)$,

¹⁰ This also implies that for a proper comparison of relations that have been quantified by many different authors, e.g., the L_X - T_X relation, one and the same fitting statistic ought to be used (e.g., Wu, Xue, & Fang 1999).



FIG. 7.—Gravitational mass functions for HIFLUGCS. The mass function plotted with filled circles has been determined using $V_{\max}(L_X)$, the ones with open symbols using $V_{\max}(M_{tot})$, where triangles correspond to the relation $L \mid M$ and squares to $M \mid L$, both for the flux-limited sample (see text, § 4.2). Vertical error bars correspond to the formal 1 σ Poisson errors; the horizontal bars indicate the mass intervals covered. Each bin contains 10 clusters, apart from the highest mass bin, which contains 11 clusters. The highest and lowest mass intervals.

denotes the number of clusters, N, per unit comoving volume, dV, per unit mass in the interval [M, M + dM]; i.e., $\phi(M) \equiv N(M)/(dV dM) \equiv dn(M)/dM$. Assuming constant density, the classical V_{max} estimator (e.g., Schmidt 1968; Felten 1976; Binggeli, Sandage, & Tammann 1988) can be used for estimation of luminosity functions; i.e., $\hat{\phi}(L) = 1/\Delta L \sum_{i=1}^{N} (1/V_{\text{max},i})$. The term V_{max} is the maximum comoving volume within which a cluster with given luminosity for a given survey flux limit and sky coverage could have been detected. As mentioned in § 2, the HIFLUGCS flux limit is constant over 99% of the covered area, which simplifies the calculation of V_{max} .

In the previous section it has been shown that the X-ray luminosity is closely correlated to cluster mass. Therefore, the V_{max} estimator can also be applied to estimate the mass function, V_{max} then being a function of mass. We employ different methods to correct for the scatter present in the $L_{\rm X}$ - $M_{\rm tot}$ relation. If $V_{\rm max}(L_{\rm X})$ is used instead of $V_{\rm max}(M_{\rm tot}) \equiv$ $V_{\text{max}}[L(M_{\text{tot}})]$, where $L(M_{\text{tot}})$ is the luminosity estimated from the L_X - M_{tot} relation using the determined cluster mass $M_{\rm tot}$, the scatter is automatically taken into account. This method has been widely used in the construction of X-ray temperature functions, recently, e.g., by Henry (2000). If $V_{\text{max}}(M_{\text{tot}})$ is used and the utilized $L_{\text{X}}-M_{\text{tot}}$ relation is assumed to be the "true" relation, then the scatter in this relation has to be taken into account explicitly. Therefore, following the method employed for the temperature function by Markevitch (1998) and Ikebe et al. (2001), the mass function may also be estimated by determining $V_{\max}^*(M_{tot})$, where the measured scatter in $\log L_{\rm X}$ is included. Specifically, we use

$$V_{\max}^{*}(M_{\text{tot}}) \equiv \int_{-\infty}^{\infty} V_{\max}(L) (2\pi\sigma_{\log L_{X}}^{2})^{-1/2} \\ \times \exp\left\{-\frac{\left[\log L - (A+40) - \alpha \log M_{\text{tot}}\right]^{2}}{2\sigma_{\log L_{X}}^{2}}\right\} d \log L ,$$
(5)

where A and α are the best-fit parameter values taken from the appropriate L_X - M_{tot} relation of the form (4) and $\sigma_{\log L_X}$ is the corresponding measured standard deviation in log L_X given in Table 12. However, we can also use the measured L_X - M_{tot} relation directly, i.e., $V_{max}(M_{tot})$, taking advantage of the fact that in our flux-limited sample there are fewer lowluminosity clusters for a given mass than high-luminosity ones, which results in a slightly increased normalization of the relation. Therefore, using this relation directly, the effect of the scatter and the resulting bias toward higher luminosity clusters is already included and thus directly accounted for.

The drawback of using $V_{max}(L_x)$ is that a small number of clusters per mass bin possibly does not represent the true scatter well. To minimize this effect we use at least 10 clusters per mass bin. The drawback of using $V_{max}^*(M_{tot})$ or $V_{max}(M_{tot})$, as noted, e.g., by Markevitch (1998) and Eke et al. (1998), is the reliance on the validity of the measured relation over the entire mass range. The first method and the method that accounts for the scatter explicitly (eq. [5]) have been tested by using Monte Carlo simulations for a precisely known L_X - T_X relation and scatter and have been shown to give accurate estimates of $\phi(T)$ for a large number of clusters in the study of the HIFLUGCS temperature function by Ikebe et al. (2001).

In Figure 7 HIFLUGCS mass functions are shown. As expected, the method employing $V_{max}(L_x)$ prompts a mass function exhibiting a larger scatter, because in this case the scatter is accounted for by the actual scatter of the 10 or 11 clusters in each mass bin. For comparison, the two extreme mass functions calculated using $V_{\text{max}}(M_{\text{tot}})$ are shown. Extreme is meant in the sense of using the steepest (1) and shallowest (2) L_X - M_{tot} relation for the HIFLUGCS sample, i.e., $M \mid L$ with $\alpha = 1.538$ and $L \mid M$ with $\alpha = 1.310$ (Table 8). At the low-mass end, (relation 1) predicts a lower luminosity for a given mass than (relation 2), resulting in a smaller V_{max} and therefore a higher dn/dM. At the high-mass side, the effect is opposite, resulting in a lower dn/dM for (relation 1). The differences of these mass functions from the mass function calculated using $V_{max}(L_x)$ can be understood in a similar way and are caused partly by the indication of a deviation from a power-law shape of the L_X - M_{tot} relation. Using $V_{\max}^*(M_{tot})$ results in similar mass functions, as shown by the open symbols in Figure 7, but the points lie systematically lower because the scatter is accounted for twice. For the comparison of the observational mass function to mass functions predicted by certain cosmological models, $V_{\text{max}}(L_{\text{X}})$ is used because it is independent of the precise shape of the L_X - M_{tot} relation and also because L_X has a much smaller measurement uncertainty than M_{tot} . The influence of the choice of the V_{max} calculation on the estimation of cosmological parameters is investigated in § 5.3.2.

5. DISCUSSION

A precise determination of distribution functions requires a high sample completeness. In § 5.1 several completeness tests for HIFLUGCS are discussed, indicating a high completeness. The observed L_X - M_{tot} relation is compared to expectations in § 5.2, and possible applications are indicated. The cluster mass function is compared to previous determinations and to predictions of cosmological models in § 5.3. The total gravitational mass contained in galaxy clusters is compared to the total mass in the universe in § 5.4.

5.1. Sample Completeness

The sample completeness is important for the accuracy of the mass function. The selection criteria detailed in § 2 are met by 63 clusters with mean redshift $\langle z \rangle = 0.05$ and with two clusters having z > 0.1. The sample is constructed from surveys with much deeper flux limits and high completenesses. A possible remaining incompleteness in these surveys is likely to be present at low fluxes close to their flux limits, which therefore would not effect HIFLUGCS. Nevertheless, four completeness tests have been performed and are described in this section; they all indicate a high completeness of HIFLUGCS. The log *N*-log *S* and L_X -*z* diagram are compared to expectations, the luminosity function is compared to luminosity functions of deeper surveys, and the V/V_{max} test is performed.

Figure 8 shows the integral number counts as a function of X-ray flux ("log N-log S"). The slope in the log N-log S diagram is very close to the value -1.5 expected in a static Euclidean universe for uniformly distributed clusters. Because of the small number of clusters (4), the deviation is not significant for $f_X \gtrsim 1 \times 10^{-10}$ ergs s⁻¹ cm⁻². Since the average redshift is smallest for the highest fluxes, large-scale structure is not completely washed out at the high-flux end; therefore, the slight bump visible around $f_X \sim$ 6×10^{-11} ergs s⁻¹ cm⁻² in Figure 8 suggests a deviation caused by cosmic variance. The effect of an expanding and finite universe on the log N-log S—flattening of the slope toward low fluxes—is small for the redshift range covered by the sample. The slope consistent with -1.5 toward the flux limit therefore indicates a high completeness of HIFLUGCS.



FIG. 8.—The log $N(>f_X)$ -log f_X diagram. Fluxes are measured in the ROSAT energy band (0.1–2.4 keV). The dashed line has a slope -1.5, expected for a uniform distribution of clusters in a static Euclidean universe ("three-halves law"); the line is normalized to produce the same cluster number at $f_X = 8 \times 10^{-11}$ ergs s⁻¹ cm⁻².

In Figure 9 the X-ray luminosity is plotted as a function of redshift. The flux limit is shown as a solid line.¹¹

One notes the increase in rare luminous systems with increasing redshift (volume). Because of the seeming underdensity of clusters in the redshift range 0.10 < z < 0.15, a comparison with the expected number of clusters as derived from N-body simulations has been performed. An open cold dark matter simulation, carried out for analysis of the power spectral densities of REFLEX clusters (Schuecker et al. 2001b), adjusted to the HIFLUGCS survey volume in the southern hemisphere (roughly half of the total volume sampled by HIFLUGCS) has been used. The simulation yields 39 clusters, while 33 HIFLUGCS clusters have been detected in this region. It is found that in fact not even one cluster with z > 0.1 is expected for this volume based on this simulation, and the HIFLUGCS subsample also does not contain any cluster with a redshift larger than 0.1. This is a further piece of evidence for the high completeness of the sample.

In Figure 10 the HIFLUGCS X-ray luminosity function is compared to luminosity functions of larger surveys in the southern (REFLEX; Böhringer et al. 2001a) and northern (BCS; Ebeling et al. 1997) hemisphere. These surveys have much deeper flux limits (§ 2) and contain many more clusters. Very good agreement is found, which shows the high completeness and homogeneous selection of HIFLUGCS.

The V/V_{max} test (e.g., Rowan-Robinson 1968; Schmidt 1968; Avni & Bahcall 1980; Peacock 1999, § 14.5) can be used to assess a possible sample incompleteness. Assuming a uniform distribution of clusters, a value $\frac{1}{2}$ is expected on average. For HIFLUGCS, $\langle V/V_{\text{max}} \rangle = 0.47 \pm 0.04$, which is consistent with the expectation, and we interpret this result as a clear sign that HIFLUGCS covers a large enough volume for most of the L_x range to be representative of the local universe with a high sample completeness.

¹¹ The correction $K(T_{\rm gas}, z)$ for converting observer rest-frame luminosities to source rest-frame luminosities depends on redshift *and* source spectrum ($T_{\rm gas}$). For source rest-frame luminosities, it is therefore not possible to plot the flux limit as one line in two dimensions (L_x , z), but rather as an area in three dimensions (L_x , z, $T_{\rm gas}$). For consistency we therefore give in this two-dimensional plot the observer rest-frame luminosity (the correction is less than 6% for 90% of the clusters anyway).



FIG. 9.—X-ray luminosity as a function of redshift. The flux limit is shown as a solid line.



FIG. 10.—X-ray luminosity function for HIFLUGCS compared to surveys with deeper flux limits in the northern (N) and southern (S) hemisphere. Vertical error bars correspond to the formal 1 σ Poisson errors (no cosmic variance); the horizontal bars indicate the luminosity intervals covered.

The local nature of HIFLUGCS becomes obvious by noting that the result of the comoving V/V_{max} test is almost identical to the result of the equivalent test assuming a Euclidean and nonexpanding space; i.e., $\langle (f_X/f_{X,\text{lim}})^{-3/2} \rangle = 0.46$.

5.2. Mass-Luminosity Relation

The close correlation between the X-ray luminosity and the gravitational mass found in § 4.1 is not surprising. Simple self-similar scaling relations predict $T_{\rm gas} \propto M_{\rm tot} R_{\rm ch}^{-1}$ and $M_{\rm tot} \propto R_{\rm ch}^3$, where $R_{\rm ch}$ is a characteristic radius, e.g., the virial radius. Combined with bremsstrahlung emission, $L_{\rm bol} \propto \rho_{\rm gas}^2 T^{1/2} R_{\rm ch}^3 [L_{\rm X}(0.1-2.4 \text{ keV}) \propto \rho_{\rm gas}^2 R_{\rm ch}^3]$, the relation $L_{\rm bol} \propto f_{\rm gas}^2 M_{\rm tot}^{4/3} [L_{\rm X}(0.1-2.4 \text{ keV}) \propto f_{\rm gas}^2 M_{\rm tot}]$ is predicted (e.g., Perrenod 1980), where the gas fraction $f_{\rm gas} \equiv M_{\rm gas} M_{\rm tot}^{-1}$ and $L_{\rm bol}$ is the bolometric luminosity.

Observationally, from the tight correlations between X-ray luminosity and temperature (e.g., Markevitch 1998) and temperature and mass (e.g., Finoguenov et al. 2001), a correlation between luminosity and mass clearly is expected. Also, correlations found between X-ray luminosity and galaxy velocity dispersion (e.g., Edge & Stewart 1991b) and X-ray luminosity and mean shear strength from weak lensing studies (e.g., Smail et al. 1997) indicate a correlation between L_X and M_{tot} .

The X-ray luminosity has been compared directly to gravitational mass estimates by Reiprich (1998), Reiprich & Böhringer (1999a, 2001), Schindler (1999), Jones & Forman (1999), Miller, Ledlow, & Batuski (1999), Ettori & Fabian (2000), and Borgani & Guzzo (2001), where good correlations have been found in all of these studies.

In order to compare the empirical $L_{\rm X}$ - $M_{\rm tot}$ relation with predictions, a quasi bolometric luminosity, $L_{\rm bol}$, has been calculated in the source rest-frame energy range 0.01–40 keV (for the relevant range of cluster gas temperatures, at least 99% of the flux is contained in this energy range). In Figure 11 this $L_{\rm bol}$ - M_{200} relation is compared to predicted relations. The solid line shows the best-fit relation for the 106 clusters in the extended sample, and the triple-dot-dashed line shows the best-fit relation determined using HIFLUGCS. Here the bisector fit results have been used in



FIG. 11.—Gravitational mass-bolometric X-ray luminosity relation compared to predicted relations. Shown are best-fit relation for the extended sample (*solid line*), best-fit relation determined using HIFLUGCS (*triple-dot-dashed line*), self-similar relation normalized by simulations of Navarro et al. (1995; *dot-dashed line*), and preheated relation given by Evrard & Henry (1991), using a normalization taken from the simulations of preheated clusters by Navarro et al. (1995; *dashed line*).

order to treat variables symmetrically, which is the appropriate method for a comparison to theory (e.g., Isobe et al. 1990). The dot-dashed line shows the self-similar relation $(L_{\rm bol} \propto M^{4/3})$ normalized by the simulations of Navarro, Frenk, & White (1995), and the dashed line shows the "preheated" relation given by Evrard & Henry (1991; $L_{\rm bol} \propto M^{11/6}$), using a normalization taken from the simulations of preheated clusters by Navarro et al. (1995). The idea of preheating is that the intracluster gas is not cold initially, as in the self-similar case, but is heated by some form of nongravitational heat input, e.g., from supernovae or active galactic nuclei, before or during cluster formation. Assuming the central regions of all clusters to have the same entropy yields the latter relationship. Figure 11 shows that measured and predicted relations are in rough agreement, the difference between the predicted relations being larger than the difference from the observed relations. Note, however, that the X-ray luminosity is one of the most uncertain quantities to be derived from simulations. Frenk et al. (1999) recently showed in a comparison of 12 different cosmological hydrodynamics codes that a factor of 2 uncertainty is a realistic estimate of the current accuracy. Including gas cooling in simulations worsens the situation (e.g., Balogh et al. 2001). The slopes of the observed relations are closer to the preheated relation. Observationally, the effect of preheating can also result in a decrease of the gas mass fraction for low-temperature systems. This has actually been observed for the clusters in our sample (Reiprich 1998; Reiprich & Böhringer 1999b). The possibility that winds from, e.g., supernovae-originally invoked to explain the apparent low gas content of elliptical galaxies (e.g., Mathews & Baker 1971; Larson 1974)-preheat and dilute the central gas and thereby break the self-similarity has been pointed out by various authors (e.g., Kaiser 1986). Such a process would work most efficiently on the least massive clusters (e.g., White 1991; Metzler & Evrard 1997; Ponman, Cannon, & Navarro 1999). The L_X - M_{tot} relation and other relevant relations between physical cluster

parameters of the extended sample have been discussed more thoroughly in this context in Reiprich (2001).

One may wonder about the origin of the scatter in the $M_{\rm tot}$ - $L_{\rm X}$ relation. Because of the use of pointed observations for most of the clusters and the local nature of HIFLUGCS, the statistical errors of $L_{\rm X}$ are negligible. The logarithmic mean mass measurement uncertainty has been measured as 0.12. The overall scatter in log mass of the data points compared to the best-fit relation is larger and has been measured as 0.21 (Table 12). This indicates a possible contribution of intrinsic scatter to the overall scatter in the $M_{\rm tot}$ - $L_{\rm X}$ relation. An obvious candidate to cause intrinsic scatter is the central excess emission (central surface brightness exceeding single β -model surface brightness) present in a number of clusters. This excess emission may have its physical origin either in cooling flows (e.g., Fabian 1994) or in the presence of cD galaxies (e.g., Makishima et al. 2001). A cooling flow analysis of the HIFLUGCS clusters is in progress, and first results indicate that indeed clusters with a high inferred mass deposition rate lie on the high- $L_{\rm X}$ side of the M_{tot} - L_X relation (Y. Chen et al. 2002, in preparation).

In Figure 12 the measured number of cluster member galaxies as taken from Abell et al. (1989) is compared to L_x as a gravitational mass tracer. It is clearly seen that a selection by X-ray luminosity is much more efficient than a selection by Abell richness in terms of mass. Even though only the X-ray surface brightness profile and neither its normalization nor the X-ray luminosity are directly used in the X-ray mass determination via the hydrostatic equation, it is nonetheless reassuring that a similar result is obtained when L_x and richness are compared to masses estimated from optical velocity dispersions (Borgani & Guzzo 2001).



FIG. 12.—Measured number of cluster member galaxies, N_{gx} , as taken from Abell et al. (1989), and X-ray luminosity of the same (66) clusters as a function of the gravitational mass. Double clusters, whose components have been treated separately here, e.g., A3528n/s, are removed. Above the dashed line all clusters have an Abell richness $R \ge 1$.

A wide range of possible applications becomes available with the quantification of the L_X - M_{tot} relation and its scatter. For large X-ray flux-limited cluster surveys, where individual mass determinations are currently not feasible, luminosities can be directly converted to masses. No combination of observations, simulations, and theory is then needed, like the frequently used approach of relating X-ray luminosities to X-ray temperatures by an observed relation and converting X-ray temperatures to masses using a relation where the slope is taken from theoretical arguments and the normalization from hydrodynamical simulations (e.g., Moscardini et al. 2000). The observational $L_{\rm X}$ - $M_{\rm tot}$ relation has first been applied directly in this sense in the power spectral analysis of REFLEX clusters (Schuecker et al. 2001b). An example of another direct application is given in § 5.3.3. The L_X - M_{tot} relation may also be applied to convert theoretical or simulated mass functions to luminosity functions for comparison with observations of X-ray flux-limited samples, which is currently being performed in the interpretation of the REFLEX luminosity function.

At this point it is important to note that even for the highest redshift cluster in our sample (z = 0.2), the dependence of the observational determination of M_{tot} and L_X on the chosen cosmological model is very weak. For instance, at z = 0.2 the increase in the luminosity distance, D_L , and the diameter distance, D_A , is less than 5% going from $\Omega_m = 1.0$, $\Omega_\Lambda = 0$ to $\Omega_m = 0.1$, $\Omega_\Lambda = 0$. From equation (3) one finds that $M_{tot}(< r) \propto r$ and therefore $M_{tot}(< r) \propto D_A$, implying an increase of M_{tot} by less than 5% for the two models above. For L_X one has an increase of less than 10%. This means that the L_X - M_{tot} relation given here can be used unchanged for various cosmological applications (unless redshift ranges are probed where evolution becomes important; in this case a model-dependent redshift correction has to be introduced). A similar calculation for V_{max} shows that for the extreme case $z_{max} = 0.2$, the increase in V_{max} is less than 14%, which is less than the size of the Poissonian error bars in Figure 14.

More detailed investigations on the shape of the relation are in progress, and it is also envisaged to construct a volume-limited sample, spanning a reasonably large range in luminosity and mass, to test how much the L_X - M_{tot} relation given here is affected by being estimated partly from a flux-limited sample.

5.3. Mass Function

5.3.1. Comparison to Previous Estimates

Bahcall & Cen (1993) give a mass function constructed (1) from optically selected clusters with masses determined from the galaxy richness and (2) from the cluster X-ray temperature function given by Henry & Arnaud (1991). Very good agreement is found for masses determined within $r_{\rm A}$ between the Bahcall & Cen and HIFLUGCS mass functions (Fig. 13).

White, Efstathiou, & Frenk (1993a) constrain the cluster abundance by using published values for the abundance and median velocity dispersion of richness class $R \ge 1$ Abell clusters. It is not surprising that their density is significantly higher than the HIFLUGCS density since they have intentionally used conservative mass estimates, which are overestimates of the true cluster masses.

Biviano et al. (1993) and Girardi et al. (1998) have determined the cluster mass function using optically selected cluster samples with masses determined from published



FIG. 13.—Cumulative gravitational mass functions for HIFLUGCS using three different outer radii. Vertical error bars give the Poissonian errors. Horizontal bars indicate the individual bin sizes. Each bin contains 10 clusters, apart from the highest mass bin, which contains 13 clusters. The abundances from previous works are determined for M_A .

line-of-sight velocity dispersions of cluster galaxies. At the completeness limit given by Girardi et al. (1998; Fig. 13, triangle), the cluster density given by Biviano et al. (1993) is about a factor of 2 higher than the density given by Girardi et al. (1998). The latter authors explain this by their on average 40% smaller mass estimates due to an improved technique for removing interlopers and the use of a surfacecorrection term in the virial theorem. The value given by Girardi et al. itself lies significantly higher than the comparable HIFLUGCS density. The reason could lie in the fact that their optically estimated masses are in general slightly larger than the X-ray masses, or that the external normalization for $R \ge 1$ ($N_{gx} \ge 50$) clusters that they applied to their mass function is intrinsically higher than the normalization obtained through HIFLUGCS directly, or both. By comparing the mass estimates for a common subsample of 42 clusters, it has been found that the virial masses determined by Girardi et al. (1998) are on average 25% larger than the masses determined in this work. This difference might be smaller if masses out to the Abell radius were compared, since common radii would have been used in this case. Increasing the X-ray masses artificially by 25%, the diamonds in Figure 13 shift toward higher masses, but the shift is too small to account for the difference from the triangle. The large scatter in the N_{gx} - M_{tot} diagram (Fig. 12) makes a reliable estimate of a best-fit relation between these two quantities very difficult. Nevertheless, in order to get a rough idea of the mass within r_A that corresponds to N_{ex} = 50, we have performed fits using the minimization methods outlined earlier and find $5.1 \times 10^{14} h_{50}^{-1} M_{\odot} \leq M_A(N_{gx} = 50) \leq 8.8 \times 10^{14} h_{50}^{-1} M_{\odot}$. Note that this range is in agreement with the ranges $(5-8) \times 10^{14} h_{50}^{-1} M_{\odot}$ and $(5-7) \times 10^{14} h_{50}^{-1} M_{\odot}$ and $(5-7) \times 10^{14} h_{50}^{-1} M_{\odot}$ and $(5-7) \times 10^{14} h_{50}^{-1} M_{\odot}$. 7) × 10¹⁴ h_{50}^{-1} M_{\odot} given by Bahcall & Cen (1993) and Girardi et al. (1998), respectively, for $N_{gx} = 50$. This mass range corresponds to a cumulative number density obtained through HIFLUGCS in the range $1.7 \times 10^{-7} h_{50}^3$ $Mpc^{-3} \le n(>M_A) \le 8.7 \times 10^{-7} h_{50}^3 Mpc^{-3}$. The external density estimate applied to normalize the Girardi et al. mass function therefore is a factor 1.2-6.2 higher than the corresponding estimate obtained here. It is therefore concluded that both effects (masses and normalization) are important but the latter factor is responsible for a larger fraction of the discrepancy. Assuming both normalizations to be determined from samples that are highly complete and representative of the local universe, this may indicate that either X-ray and optical clusters are drawn from different populations or that projection effects (e.g., line-of-sight galaxy overdensities, which do not form a bound structure in three dimensions) possibly bias optically determined normalizations high.

Girardi & Giuricin (2000) recently extended the mass function to loose galaxy groups, finding $n(>1.8 \ h_{50}^{-1} \times 10^{13} \ M_{\odot}) = (1.6-2.4) \times 10^{-4} \ h_{50}^{3} \ Mpc^{-3}$, which is outside the mass range we can test. They find that the group mass function can be described by a smooth extension of the cluster mass function by Girardi et al. (1998). Consistently, this abundance is higher than the abundance given by Bahcall & Cen (1993) at that mass scale.

Carlberg et al. (1997) have compiled and partially reestimated abundances for local cluster samples (Henry & Arnaud 1991; Mazure et al. 1996; White et al. 1993a; Eke, Cole, & Frenk 1996) for comparison with higher redshift samples (the multiplication sign in Fig. 13 shows the density for a sample with higher mean redshift, and therefore it cannot be compared directly). Note that Borgani et al. (1999a) find that the reestimate of the mass limit of the Mazure et al. sample by Carlberg et al. (the plus sign at $n \approx 3 \times 10^{-7} h_{50}^3 \text{ Mpc}^{-3}$) may lead to an underestimated mass limit. In general, the comparison to the HIFLUGCS mass function shows better agreement than the abundances given by Girardi et al. (1998) and White et al. (1993a).

The obvious importance of the definition of the cluster outer radius for the cluster mass function can be directly appreciated by noting the large differences between the mass functions determined for HIFLUGCS for M_{500} , M_{200} , and M_A in Figure 13, especially toward lower masses. Since for self-similar clusters the mass scales with the third power of the characteristic radius (§ 5.2), determination of the mass within a characteristic overdensity is the natural choice. We mainly give the formally determined M_A mass function for the comparison with previous mass functions and recall again that especially for the low-mass systems the assumption of hydrostatic equilibrium may not be justified out to r_A , and therefore our mass estimates of M_{500} and M_{200} should be considered much more precise than the estimates for M_A .

5.3.2. Comparison to Predicted Mass Functions

We use the standard formalism based on the Press-Schechter (PS) prescription to predict cluster mass functions for given cosmological models (see, e.g., Borgani et al. 1999b). To allow easier comparison with the theoretical literature on this subject, in this paragraph $h_{100} = h_{50}/2$ is used. The mass function is then given by

$$\frac{dn(M)}{dM} = \sqrt{\frac{2}{\pi}} \frac{\bar{\rho}_0}{M} \frac{\delta_c(z)}{\sigma(M)^2} \left| \frac{d\sigma(M)}{dM} \right| \exp\left[-\frac{\delta_c(z)^2}{2\sigma(M)^2} \right]$$
(6)

(Press & Schechter 1974; Bond et al. 1991; see, e.g., Schuecker et al. 2001a for a compilation of published extensions of the PS mass function). Here M represents the halo (cluster) virial mass and $\bar{\rho}_0 = 2.7755 \times 10^{11}$ $\Omega_m h_{100}^2 M_{\odot}$ Mpc⁻³ is the present-day mean matter density. The linear overdensity computed at present $\delta_c(z) = \delta_c^v(z)D(0)D(z)^{-1}$, where the linear overdensity at the time of virialization, $\delta_c^v(z)$, is computed using the spherical collapse model summarized in Kitayama & Suto (1996), for $\Omega_m = 1$ using equation (A2) and for $\Omega_m < 1 \land \Omega_k = 0$ using equations (A6) and (A7); the linear growth factor D(z) = $2.5 \ \Omega_m E(z) \int_z^{\infty} (1 + z') E(z')^{-3} dz'$ and E(z) have been defined in § 3.2. As mentioned earlier, because of the low redshift range spanned by HIFLUGCS, the effect of a redshift correction is very small, and we therefore set z = 0 for all calculations, unless noted otherwise. The variance of the cosmic mass density fluctuations

$$\sigma(M)^{2} = \sigma_{8}^{2} \frac{\int_{0}^{\infty} k^{2+n} T(k)^{2} |W[kR(M)]|^{2} dk}{\int_{0}^{\infty} k^{2+n} T(k)^{2} |W(k8 \ h_{100}^{-1} \ \text{Mpc})|^{2} dk}, \quad (7)$$

where σ_8 represents the amplitude of density fluctuations in spheres of radius 8 h_{100}^{-1} Mpc. Recent measurements of the cosmic microwave background (CMB) anisotropies indicate that the primordial power spectral index, *n*, has a value close to 1 (e.g., Balbi et al. 2000; Jaffe et al. 2001; Pryke et al. 2002; Wang, Tegmark, & Zaldarriaga 2001; de Bernardis et al. 2001) and is therefore set to 1, unless noted otherwise. For the transfer function we use the fitting formula for cold dark matter (CDM) cosmologies provided by Bardeen et al. (1986) for $q(k) = k/(\Gamma h_{100} \text{ Mpc}^{-1})$:

$$T(k) \equiv T[q(k)] = \ln(1 + 2.34q)/(2.34q)$$

× $[1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4]^{-1/4}$, (8)

where the shape parameter is given by (modified to account for a small normalized baryon density $\Omega_b > 0$; Sugiyama 1995)

$$\Gamma = \Omega_m h_{100} \left(\frac{2.7 \text{ K}}{T_0} \right)^2 \exp\left(-\Omega_b - \sqrt{\frac{h_{100}}{0.5}} \frac{\Omega_b}{\Omega_m} \right). \quad (9)$$

The temperature of the CMB $T_0 = 2.726$ K (Mather et al. 1994), and $\Omega_b h_{100}^2 = 0.0193$ (Burles & Tytler 1998); for the latter equation and equation (9), $h_{100} = 0.71$ has been used (Mould et al. 2000). The comoving filter radius $R(M) = [3M/(4\pi\bar{\rho}_0)]^{1/3}$ for the top-hat filter function $W(x) = 3(\sin x - x \cos x)/x^3$, which is adopted in this analysis, because the HIFLUGCS masses have been determined with a top-hat filter too.¹² Since the PS recipe as outlined above assumes virial masses based on the spherical collapse model, we use M_{200} as an approximation to the virial mass (§ 3.2).

Similarly to the work of Ikebe et al. (2001), the statistical uncertainty in the mass determination is incorporated in the model mass function as

$$\frac{d\tilde{n}(M)}{dM} \equiv \frac{1}{V_{\max}(M)} \int_{-\infty}^{\infty} \frac{dn(M')}{dM'} V_{\max}(M')$$
$$\times (2\pi\bar{\sigma}_{M_{\text{tot,log}}}^2)^{-1/2} \exp\left[\frac{-(\log M' - \log M)^2}{2\bar{\sigma}_{M_{\text{tot,log}}}^2}\right] d\log M' ,$$
(10)

where $\bar{\sigma}_{M_{\text{tot},\log}} = 0.12$ represents the logarithmic mean mass measurement uncertainty. Note that since HIFLUGCS is flux limited and not volume limited, the weighting has to be performed on the mass distribution, N(M)/dM, rather than



FIG. 14.—HIFLUGCS mass function compared to the best fit model mass function with $\Omega_m = 0.12$ and $\sigma_8 = 0.96$ (solid line). Also shown are the best fit model mass functions for fixed $\Omega_m = 0.5$ ($\Rightarrow \sigma_8 = 0.60$; dashed line) and $\Omega_m = 1.0$ ($\Rightarrow \sigma_8 = 0.46$; dotted line).

on the mass function itself. The effect of this weighting on the model mass function is a slight amplitude increase at the high-mass end.

For the modeling to be independent of the precise knowledge of the L_X - M_{tot} relation, the quantitative comparison has been performed using a standard χ^2 procedure on the differential binned mass function given in Figure 14 (rather than using a maximum likelihood approach on the mass distribution). After identifying the crude position where χ^2 is minimal in a large $\Omega_m - \sigma_8$ parameter space region, the χ^2 values have been calculated in a fine grid of $200 \times 200 \ \Omega_m$ - σ_8 values in the range $0.05 \le \Omega_m \le 0.26$ and $0.65 \le \sigma_8 \le 0.26$ 1.30. A flat cosmic geometry has been assumed; i.e., Ω_m $+ \Omega_{\Lambda} = 1$. The cosmological constant enters the calculation only through δ_c , however, and therefore has a negligible influence here. The minimum and statistical error ellipses for some standard confidence levels (c.l.'s) are given in Figure 15. The tight constraints obtained show that with HIFLUGCS we can go beyond determining an Ω_m - σ_8 relation and put limits on Ω_m and σ_8 individually. It is found

FIG. 15.—Statistical confidence contours for the χ^2 procedure. The cross indicates the position of the minimum, χ^2_{min} . Ellipses indicate the 68%, 90%, 95%, and 99% confidence levels for two interesting parameters, i.e., $\Delta\chi^2 \equiv \chi^2 - \chi^2_{min} = 2.30, 4.61, 6.17, and 9.21$, respectively.



¹² This approach follows the custom of disregarding the inconsistency of using top-hat masses, while the PS mass function with the correct normalization has been derived for the sharp k-space filter (Bond et al. 1991); see Schuecker et al. (2001a) for a generalization to more realistic filter functions.

that

$$\Omega_m = 0.12^{+0.06}_{-0.04}, \quad \sigma_8 = 0.96^{+0.15}_{-0.12} \tag{11}$$

(90% c.l. statistical uncertainty for two interesting parameters), indicating a relatively low value for the density parameter. The large covered mass range, the specific region in $\Omega_m \sigma_8$ parameter space, and the assumption of CDM cosmological models with given H_0 , Ω_b , *n*, and T_0 allow to derive these tight constraints from a local cluster sample. For comparison for a given σ_8 value the Ω_m value that minimizes χ^2 is calculated. In the interval shown in Figure 15 these pairs can then roughly be described by a straight line in log space given by

$$\sigma_8 = 0.43 \Omega_m^{-0.38} \,. \tag{12}$$

In Figure 14 we also plot the best-fit model mass functions for given $\Omega_m = 0.5$ and $\Omega_m = 1.0$, and one notes immediately that these value pairs give a poorer description of the shape of the mass function. The slight differences between the best-fit values for σ_8 and the ones expected from equation (12) are caused by the fact that the simple power law is only an approximation to the real shape of the error ellipse and by the extrapolation involved. Previous estimates have generally yielded a combination of slightly higher $\Omega_m \sigma_8$ values (e.g., Peacock 1999, § 17.2). It has to be noted, however, that for instance in the important work of White et al. (1993a), who find $\sigma_8 \approx 0.57 \Omega_m^{-0.56}$, the authors explicitly state that their estimates of σ_8 are probably biased high because of their conservative mass estimates.

Before exploring sources of systematic errors, let us evaluate our treatment of the shape parameter Γ . By applying the Bardeen et al. (1986) fitting formula to calculate the transfer function, we have adopted the assumption of a CDM universe; i.e., $\Omega_m - \Omega_b = \Omega_{CDM}$. Within this model Γ and Ω_m are not independent. For a small Ω_b/Ω_m ratio, Γ is given by equation (9). However, to test its influence we have set Γ to a fixed value of 0.19 as implied by large-scale structure (LSS) measurements of galaxies and clusters (e.g., Szalav et al. 2001; Schuecker et al. 2001b; see also the discussion at the end of this section). The best-fit values for Ω_m and σ_8 change significantly, the Ω_m value becoming smaller and the σ_8 value larger. The Ω_m - σ_8 relation changes slightly to about $\sigma_8 = 0.40 \Omega_m^{-0.46}$ for the Ω_m range [0.04–0.30] (note that for this range relation [12] translates to about $\sigma_8 = 0.39 \Omega_m^{-0.43}$, the steepening being mainly due to a strong increase of σ_8 for the very lowest Ω_m values). The error ellipse in general becomes much longer when Γ is set to a fixed value, preventing the possibility to obtain tight individual constraints on Ω_m and σ_8 , even though $\Omega_m = 1.0$ is still excluded at least at the 90% c.l. for all $\Gamma \ge 0.03$. The decreased sensitivity is due to the loss of the amplification of a change of Ω_m through Γ (to first order $\Omega_m \propto \Gamma$; eq. [9]), since both a larger (smaller) Ω_m and a larger (smaller) Γ result in steeper (less steep) model mass functions. Having found a dependence of the results on Γ , we tried to constrain Γ directly from the observations, as done by previous authors (e.g., Borgani et al. 1999b). We have set Γ to 31 different fixed values in the range $0.01 \le \Gamma \le 0.50$ and calculated a χ^2_{\min} for each of those by varying Ω_m and σ_8 . The result is shown in Figure 16. The conclusion is that the data do not contain enough information to provide an independent constraint on Γ . Therefore, we have taken advantage of the dependence of Γ on Ω_m within the CDM framework



FIG. 16.—Minimal χ^2 values calculated by varying Ω_m and σ_8 for a variety of fixed values of Γ instead of using eq. (9). The dashed line represents the 68% c.l. statistical uncertainty for one interesting parameter. This line actually underestimates the uncertainty since the procedure involves three interesting parameters. Obviously, Γ cannot be constrained significantly.

and have continued to calculate Γ with equation (9). Notice that this procedure requires additional knowledge about H_0 and Ω_b . The influence of the uncertainty of these two parameters is tested along with various other possible systematic uncertainties in the following. It is important to note that the best-fit value for Ω_m is not determined solely through equation (9) but also directly through $\bar{\rho}_0$ in equation (6) and through the filter radius R(M).

The quoted error ranges have been calculated from the χ^2 procedure. Most of the remaining part of this section is devoted to an identification and quantification of possible systematic uncertainties. The quantitative results are summarized in Table 5.

Because of the large given ranges of several orders of magnitude in mass and especially density, the χ^2 values have been determined from the comparison of models to data naturally in log space. However, we have verified that the same calculation in linear space yields best-fit values lying within the 68% error ellipse.

In § 4.2 arguments have been given for why we have used $V_{\max}(L_X)$ for the determination of the mass function. Nevertheless, if $V_{\max}(M_{tot})$ is used instead (see Fig. 7), consistent results are obtained. Since in this case we want to estimate L from M, the relation $L \mid M$ is the appropriate one (e.g., Isobe et al. 1990). Performing a fit to a mass function constructed with $V_{\max}(M_{tot})$ results in best-fit values $\Omega_m = 0.14$ and $\sigma_8 = 0.86$, which is consistent with the error range given in equation (11).

The data point in Figure 14 that may be affected most by cosmic variance is the one at the lowest mass, since the maximum search volume is smallest for the clusters in this bin. We therefore tested the sensitivity of the best-fit results on this last point by ignoring it. The decrease in the covered mass range of course increases the resulting error ellipse, but the best-fit values vary only within the (smaller) 68% error ellipse. It may be worth noting that leaving out the highest mass bin or leaving out the highest and lowest mass bin changes the best-fit values only within the 90% error ellipse.

Simulations seem to indicate that the mass density profile of virialized halos follows the NFW profile (Navarro,

TABLE 5Systematic Uncertainties

Ω_m	σ_8	$\Delta\Omega_m$	$\Delta \sigma_8$	Test
0.116	0.928	± 0.000	-0.026	1
0.137	0.865	+0.021	-0.094	2
0.150	0.897	+0.034	-0.062	3
0.092	1.082	-0.024	+0.124	4
0.083	1.050	-0.034	+0.091	5
0.150	0.860	+0.034	-0.099	6
0.107	0.907	-0.009	-0.052	7
0.132	1.004	+0.016	+0.046	8
0.106	0.826	-0.011	-0.133	9
0.108	1.242	-0.008	+0.281	10
0.116	0.959	± 0.000	± 0.000	11
0.118	0.956	+0.002	-0.003	12
0.153	0.861	+0.037	-0.098	13
0.108	0.991	-0.008	+0.033	14
0.106	1.001	-0.011	+0.042	15
0.128	0.923	+0.012	-0.036	16
0.148	0.874	+0.032	-0.085	17
0.094	1.056	-0.022	+0.098	18

Notes.—See text for a more detailed description of the tests. The differences to the best-fit results have been calculated with more digits than given here. Tests are (1) χ^2 calculated in linear space, (2) $V_{\rm max}(M_{\rm tol})$ used with relation $L \mid M$ for HIFLUGCS (Table 8), (3) lowest mass bin ignored, (4) highest mass bin ignored, (5) r_{200} and M_{200} calculated assuming NFW profile, (6) universal mass function, (7) M_{200} reduced by factor 0.80, (8) M_{200} increased by factor 1.25, (9) M_{500} used, (10) M_{87} used, (11) redshift correction applied for cluster mass determination and model mass function calculated using Eisenstein & Hu fitting formula, (13) h_{100} set to 0.5, (14) h_{100} set to 0.8, (15) Ω_b set to 0.0100 h_{100}^{-2} , (16) Ω_b set to 0.0300 h_{100}^{-2} , (17) *n* set to 0.8, and (18) *n* set to 1.2.

Frenk, & White 1996, 1997; but see, e.g., Kravtsov et al. 1998). As a test, therefore, gravitational masses have been recalculated assuming the dark matter density to follow an NFW profile based on the measured values for β and r_c . The approximation $r_s = r_c/0.22$ and $b = \beta/0.9$ given by Makino, Sasaki, & Suto (1998) has been used combined with their equations (7) and (9). The values for r_{200} and M_{200} have been redetermined based on this model, and the resulting mass function has been compared to model mass functions. On average, it is found that the NFW masses are a factor of 0.90 lower, the difference being smaller for highmass clusters than for low-mass ones. The values for Ω_m and σ_8 for which χ^2 is minimal lie within the 90% statistical error ellipse shown in Figure 15. Therefore, the choice of mass calculation does not affect the results significantly.

As shown in § 3.2, it is possible that the assumption of isothermality leads to an overestimate of the cluster masses on average. Therefore, the robustness of the results has been tested by multiplying the isothermal cluster masses by 0.80 and recalculating the minimum. As expected, values for both Ω_m and σ_8 are found to be lower. But the new minimum is contained well within the error ranges given in equation (11). On the other hand, in § 5.3.1 it has been shown by comparison with optical virial mass measurements that masses could possibly be underestimated. After increasing all cluster masses by 25%, a fit shows that both Ω_m and σ_8 become slightly larger, but again they lie well within the range (11). Therefore, these tests indicate that the constraints obtained here are fairly insensitive against systematic uncertainties in the mass measurements. Note that systematic effects that show a mass dependence have a higher potential to affect the individual best-fit values of Ω_m

and σ_8 than effects that simply shift all data points by about the same amount in the same direction. However, no indications for a mass dependence of the differences between the above mass estimations have been found.

It has been mentioned that masses calculated within r_{500} are less affected by systematic uncertainties because less extrapolation beyond $r_{\rm X}$ is needed and because according to simulations the assumption of hydrostatic equilibrium can be more safely applied. Nevertheless, we have used M_{200} for the comparison to predicted mass functions because it is a better approximation to the cluster virial mass. However, for comparison the best-fit values have also been determined using M_{500} , knowing that virial masses are probably underestimated this way. The result is that the best-fit value for Ω_m changes only marginally. The best-fit σ_8 value is slightly lower than allowed by the statistical error. On the other hand, the spherical collapse model implies that one may need to extrapolate even further than r_{200} for low-density universes. For instance, in a flat universe $\Omega_m = 0.2$ implies that the virial radius is close to r_{87} . Therefore, a mass function has been constructed using M_{87} as cluster masses, knowing that these masses are likely to be rather uncertain. Nevertheless, the resulting best-fit Ω_m value again varies only insignificantly, whereas the σ_8 value becomes significantly larger. Note that for both tests the best-fit value for Ω_m hardly changes.

As mentioned in § 3.2, it has been shown that the small range of low redshifts covered here ensures that no redshift corrections need to be applied. Nevertheless, we have tested whether or not the best-fit parameter values change if M_{200} is calculated using $\rho_c = \rho_c(z)$ for the extreme (strong evolution) case $(\Omega_m = 1, \Omega_\Lambda = 0)$; i.e., $\rho_c = 4.6975 \times 10^{-30}(z+1)^3$ g cm⁻³ for each cluster redshift. The model mass function is then calculated for the mean redshift of HIFLUGCS, $\langle z \rangle = 0.05$, using the formulae outlined in the beginning of this section. We have found that within our grid the best-fit values do not change at all and also the error ellipses are almost not affected, thereby confirming that the application of redshift corrections does not affect the results.

The value $H_0 = 71 \text{ km s}^{-1} \text{ Mpc}^{-1}$ has been adopted for the calculation of model mass functions based on the recent combination of constraints obtained using the *Hubble Space Telescope* (Mould et al. 2000). Setting the Hubble constant to their lower limit, $H_0 = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$, does not affect the best-fit parameter values significantly. Using even $H_0 = 60 \text{ km s}^{-1} \text{ Mpc}^{-1}$ changes the results for Ω_m and σ_8 only well within the 68% error ellipse. In Table 5 results have also been listed for $H_0 = 50$ and 80 km s⁻¹ Mpc⁻¹ showing that even variations within this range change the Ω_m and σ_8 values only within their statistical uncertainties given in equation (11). Therefore, our constraints on these cosmological parameters do not depend significantly on the specific choice of H_0 .

A baryon density based on measurements of the primordial deuterium-to-hydrogen ratio combined with standard nucleosynthesis theory has been used in this work. The used value also agrees with recent CMB measurements (e.g., Pryke et al. 2002). However, we have also calculated the minimum for the values $\Omega_b \ h_{100}^2 = 0.0100$ and 0.0300, finding that the position of the minimum in each case does not deviate significantly from our original result.

The primordial power spectral index has been set to n = 1 based on measurements of the CMB fluctuations. The influence of a change in this parameter has been estimated

by calculating the minimum for the values n = 0.8 and n = 1.2. The differences in the best-fit parameters are smaller than the statistical uncertainties given in equation (11).

Since we have found that for the estimate of the statistical errors we need to explore ranges $\Omega_m < 0.1$ for $\Omega_b \sim 0.04$, we regarded it necessary to check if the approximation to the transfer function as given in the beginning of this section is still applicable. Recently, Eisenstein & Hu (1998) derived a fitting function that includes, e.g., also the oscillations induced by the baryons, which gives a better description of transfer functions computed with CMBFAST (Seljak & Zaldarriaga 1996) than fitting functions for zero or small baryon contribution to the total matter density derived previously. Therefore, we incorporated this improved version of an analytic transfer function in the χ^2 procedure. We have found that within our grid there is only a very weak shift of the minimum, the choice of the Bardeen et al. (1986) fitting function combined with the shape parameter given by Sugiyama (1995) therefore seems to be accurate enough for our purposes. However, the confidence contours toward low Ω_m are getting compressed when the Eisenstein & Hu (1998) fitting function is used, thereby slightly decreasing the area of the error ellipse for a given confidence level. Since we regard the latter statistical error ellipse as more realistic, we show this one in Figure 15.

We have also tested whether or not the recently found deviations of the PS formalism compared to large N-body simulations (e.g., Governato et al. 1999; Jenkins et al. 2001) have a significant influence on the results obtained here. We have compared the best-fit PS model ($\Omega_m = 0.12, \sigma_8 = 0.96$) to the model obtained using the "universal" mass function (fitted to N-body simulations; Jenkins et al. 2001) for the same parameter values. These two models agree well for $M \lesssim 10^{15} \ h_{50}^{-1} \ M_{\odot}$. The differences become larger than the size of the Poissonian error bars (Fig. 14) for $M \gtrsim$ $2 \times 10^{15} h_{50}^{-1} M_{\odot}$, in the sense that the Jenkins et al. mass function predicts higher cluster abundances than PS. For larger values of Ω_m , the differences become comparable to the size of the error bars at lower masses, e.g., for $\Omega_m = \sigma_8 = 0.5$ around $M \sim 5 \times 10^{14} h_{50}^{-1} M_{\odot}$. To estimate the influence of these differences on the best-fit values derived using the PS mass function, we adjusted the parameter values of the Jenkins et al. model to reproduce the PS mass function, finding $\Omega_m = 0.15$ and $\sigma_8 = 0.86$. The value for Ω_m becomes slightly larger, but the combination of both values is still contained within the 90% error ellipse. We therefore conclude that the differences between the model mass functions do not significantly affect the interpretation of the HIFLUGCS mass function. Moreover, we regard this test as confirmation of the validity of the PS mass function for the accuracy needed here.

Almost all identified systematics give rise to smaller uncertainties than the statistical uncertainties (eq. [11]). However, it is not impossible that several systematics combined have a significant effect. In order to determine the highest Ω_m value allowed by a conspiracy of all identified uncertainties, all positive $\Delta\Omega_m$ values from Table 5 have been added to the positive statistical uncertainty for Ω_m , and this sum has been added to the Ω_m value that minimizes $\chi^2 (\Omega_m = 0.12)$. In this worst-case procedure the upper limit

$$\Omega_m < 0.31 \tag{13}$$

has been found.

Using $\Omega_b = 0.19 \pm 0.08 \ h_{50}^{-3/2} \ \Omega_m$, as derived from the mean gas mass fraction within r_{200} for the 106 clusters in the enlarged sample, results in model mass functions very similar to the ones calculated using the baryon fraction given by Burles & Tytler (1998). It is therefore not surprising that the best-fit values for Ω_m and σ_8 vary only well within the 68% error ellipse if the former Ω_b determination is used. Moreover, it is worth noting that combining these two measurements of Ω_b gives further evidence for a low value for Ω_m by yielding an estimate $\Omega_m = 0.34^{+0.22}_{-0.10}$ using $H_0 = 71$ km s⁻¹ Mpc⁻¹ (Mould et al. 2000), where the error has been determined from the standard deviation of Ω_b given above. This value for Ω_m is an upper limit since baryons, e.g., contained in the cluster galaxies or forming part of the dark matter, have been neglected. A low value for Ω_m has been indicated by this method for smaller cluster samples by several works previously (e.g., White & Frenk 1991; Briel, Henry, & Böhringer 1992; Böhringer 1993; White et al. 1993b; White & Fabian 1995; Ettori & Fabian 1999; but see Sadat & Blanchard 2001). One has to keep in mind, however, that this estimate extrapolates the gas fraction from cluster scale to cosmic scales. For the clusters in our sample we have found that the gas fraction is not constant but varies with radius and cluster mass (Reiprich & Böhringer 1999b); therefore, further observational tests of this assumption may be useful.

Let us briefly discuss the role of Γ again. The constraints (11) together with $h_{100} = 0.71$, $\Omega_b h_{100}^2 = 0.0193$, and equation (9) imply $\Gamma = 0.06^{+0.04}_{-0.05}$, which is significantly lower than values for Γ determined through LSS studies of galaxies (e.g., Szalay et al. 2001 have found $\Gamma = 0.188 \pm 0.04$) or clusters (e.g., Schuecker et al. 2001b have found $\Gamma = 0.195 \pm 0.055$). Adopting the worst-case upper limit on Ω_{m} found above yields an upper limit $\Gamma < 0.18,$ consistent with the LSS results. However, also many previous studies of cluster abundances have yielded low values for Γ . For instance, Eke et al. (1998) have found $\Gamma \approx 0.09 \pm 0.08$; Henry (2000) has found $\Gamma \approx 0.05 \pm 0.22$; Oukbir & Arnaud (2001) have found $\Gamma = 0.05\Omega_m^{-0.71+0.89\Omega_m}$, which, e.g., for $\Omega_m = 0.3$ yields $\Gamma \approx 0.09$; judging from their Figure 3 and their best-fit values for a spatially flat universe, Borgani et al. (2001) have found $\Gamma \approx 0.1$ (however, not significantly constrained). It is unclear whether these values indicate a significant discrepancy, but it is interesting to note that cluster abundances seem to favor lower values for Γ than LSS studies do. If significant, this could possibly imply either unidentified systematic errors in one or both measurements-from the cluster abundance side one could for instance imagine a higher merger rate for more massive clusters resulting in overestimated X-ray luminosities, temperatures, and masses for the massive clusters, resulting in less steep L_X , T_X , and M_{tot} functions and therefore in a decreased value for Γ (and possibly Ω_m)—or that the description of the power spectrum using a single Γ parameter on all scales is not accurate (e.g., Eisenstein & Hu 1998; Saslaw 2000, § 33.3.3).

Even though we made a conservative estimate by neglecting the possible presence of gas temperature gradients, previous estimates obtained from cluster abundances generally yielded higher values for Ω_m and σ_8 (e.g., White et al. 1993a; Girardi et al. 1998). For the latter two works one could have expected this already from Figure 13, and possible reasons have been discussed in § 5.3.1. Recently, a number of constraints on Ω_m and σ_8 have been obtained using different

kinds of distribution functions assuming relations between the measured quantities and mass. Some of them incorporated additional dynamical information from the evolution of the corresponding distribution function. Especially, intracluster gas temperature functions have been constructed and used frequently. In Table 6 some results, which allowed a simple comparison for a given $\Omega_m = 0.3$, are listed. If there was a choice, results assuming flat cosmological models and isothermal clusters are quoted. This table is not a complete summary of recent results but merely shows that previous results yielded higher values for σ_8 than obtained here from a newly constructed sample (even using M_{87} instead of M_{200} would increase the σ_8 value only up to 0.82). The reasons may be manifold. Some have been discussed above. Other reasons may include different sources of temperature estimates. For instance, Pierpaoli, Scott, & White (2001) used cooling flow-corrected temperatures given by White (2000), which are generally higher than temperatures derived previously. Using theoretical/ simulated M_{tot} - T_{gas} relations (e.g., Evrard et al. 1996; Bryan & Norman 1998) to connect observed temperature functions with theoretical mass functions will on average also result in higher values for σ_8 , since these relations have been shown to have higher normalizations than observed X-ray mass-temperature relations (e.g., Horner, Mushotzky, & Scharf 1999; Nevalainen, Markevitch, & Forman 2000; Finoguenov et al. 2001). Assuming low-temperature groups to be affected by nongravitational heat input, temperature functions, unlike X-ray mass functions, which incorporate the additional information of the gas density profile, would show an additional artificial increase toward low temperatures. If not taken into account, e.g., by use of a modified M_{tot} - T_{gas} relation, this effect would result in artificially increased values for Ω_m (see Fig. 14). Also, the completeness and representativeness of the used cluster samples obviously play an important role for the Ω_m - σ_8 determination. Here advantage has been taken of the currently best available local sample in the sense of homogeneous X-ray selection, completeness, size, and availability of high-quality observations. The results obtained here are, however, in

good agreement with the results from the power spectral analysis of the 452 REFLEX clusters. Schuecker et al. (2001b) find for a given Λ CDM model ($\Omega_m = 0.3$) that $\sigma_8 =$ 0.7 represents the data well, which is very close to the $\sigma_8 =$ 0.68 expected using relation (12) found here. Moreover, the (1 σ) range $0.17 \le \Omega_m \le 0.37$ (using h = 0.71 in their eq. [18]) quoted for Ω_m directly is also consistent with the 90% range determined here. Furthermore, Ikebe et al. (2001), who analyzed the HIFLUGCS temperature function using temperatures from homogeneously reanalyzed ASCA data, find $\Omega_m = 0.18^{+0.08}_{-0.05}$ and $\sigma_8 = 0.96^{+0.11}_{-0.09}$ (90% c.l. statistical uncertainty; assuming an open cosmology) by comparison with Press-Schechter models, which is in good agreement with our results. Moreover, after this paper had been submitted another paper appeared on astro-ph (Borgani et al. 2001), where the authors also find Ω_m - σ_8 values that agree with values calculated using relation (12) presented here. After this had been accepted, two more papers appeared on astro-ph finding very similar $\Omega_m - \sigma_8$ relations from cluster abundances (Seljak 2001; Viana, Nichol, & Liddle 2001).

5.3.3. Mass Function Estimated Using L_X - M_{tot} Relation

To show consistency and the power of the L_X - M_{tot} relation, we have also performed fits to "mass" functions, where masses have been estimated from the measured X-ray luminosity. Relations for the flux-limited sample have been used to get the best mass estimate for the cluster luminosities included in HIFLUGCS. Mass functions for the two extreme relations $M \mid L$ with $\alpha = 1.538$ and $L \mid M$ with $\alpha = 1.310$ are shown in Figure 17. First of all, one notes the fairly good agreement between the three mass functions. In detail, the differences between the two mass functions estimated from different luminosity-mass relations can be understood by considering that at the low luminosity end the steeper relation predicts a higher mass for a given luminosity than the shallower relation, resulting in a shift toward higher masses of the mass function. At the highmass side the effect is opposite, resulting in a shift toward lower masses for the steeper relation. On average, the points

TABLE 6	
Examples of Previous Results for Given Ω_{m}	

Ω_m	σ_8	Relation	Reference
0.30	1.12	$\sigma_8 = 0.57 \Omega_m^{-0.56}$	1
0.30	1.16	$\sigma_8 = 0.60 \Omega_m^{-0.59 + 0.16 \Omega_m - 0.06 \Omega_m^2}$	2
0.30	0.93	$\sigma_8 = 0.52 \Omega_m^{-0.52 + 0.13 \Omega_m}$	3
0.30	1.01	$\sigma_8 = 0.60 \Omega_m^{-0.46 + 0.09 \Omega_m}$	4
0.30	1.00	$\sigma_8 = 0.53 \Omega_m^{-0.53}$	5
0.30	0.78	с <i>ш</i>	6
0.30	0.99	$\sigma_8 = 0.56 \Omega_m^{-0.47}$	7
0.30	0.96	$\sigma_8 = 0.58 \Omega_m^{-0.47+0.16\Omega_m}$	8
0.30	0.96		9
0.30	1.02	$\sigma_8 = 0.495 \Omega_m^{-0.60}$	10
0.30	0.91	$\sigma_8 = 0.59 \Omega_m^{-0.57 + 1.45 \Omega_m - 3.48 \Omega_m^2 + 3.77 \Omega_m^3 - 1.49 \Omega_m^4}$	11
0.30	0.87	$\sigma_8 = 0.477 \Omega_m^{-0.3 - 0.17 \Omega_m^{0.34 - 0.13 \Omega_{\lambda}}}$	12
0.30	0.68ª	$\sigma_8 = 0.43 \Omega_m^{-0.38}$	13

^a For consistency the σ_8 value has been calculated using the relation; the directly determined best-fit value for $\Omega_m = 0.30$ is given by $\sigma_8 = 0.72$.

REFERENCES.—(1) White et al. 1993a. (2) Viana & Liddle 1996. (3) Eke et al. 1996. (4) Girardi et al. 1998. (5) Pen 1998. (6) Markevitch 1998. (7) Viana & Liddle 1999. (8) Borgani et al. 1999b. (9) Blanchard et al. 2000. (10) Pierpaoli et al. 2001. (11) Oukbir & Arnaud 2001. (12) Proty Wu 2001. (13) This work.



FIG. 17.—HIFLUGCS mass function (filled circles) compared to "mass" functions estimated using measured luminosities and luminosity-mass relations (open symbols). Squares have been calculated using the $M \mid L$ relation and triangles using the $L \mid M$ relation for HIFLUGCS clusters.

for the steeper relation lie higher, which is caused by the fact that a steeper relation results in a smaller dM on average, which gives rise to an increased dn/dM. The differences from the mass function calculated using the measured masses are again understood by a similar comparison and are partially caused by a possible deviation of the shape of the L_X - M_{tot} relation from a pure power law. Despite these small differences, performing an actual fit¹³ results in the (Ω_m, σ_8) values (0.14, 0.85) for $L \mid M$ and (0.22, 0.74) for $M \mid L$. The first case is consistent with the error range given in equation (11), and in the second case, the 90% statistical error ellipse overlaps with the 90% ellipse in Figure 15. From the above and Figure 14, it is clear that using steeper luminosity-mass relations results in higher values for Ω_m and lower values for σ_8 . Here we want to estimate M from L, and therefore M | L is the appropriate relation to use. This test shows that with a comparatively easy to obtain X-ray luminosity function of a statistical cluster sample and with the knowledge of the empirical $L_{\rm X}$ - $M_{\rm tot}$ relation (even if approximated as a simple power law) and its scatter as presented here, useful constraints on cosmological parameters can be set by construction of a "quasi mass function."

5.4. Total Gravitating Mass in Clusters

To estimate the fraction of the gravitational mass density relative to the critical density contained in galaxy clusters, $\Omega_{cluster}(>M_{tot,min}) = 1/\rho_c \int_{M_{tot,min}}^{\infty} M_{tot} \phi(M_{tot}) dM_{tot}$, the individual cluster masses divided by the corresponding maximum search volumes have been summed up for HIFLUGCS; i.e., $\Omega_{cluster} = 1/\rho_c \sum_i M_{200,i}/V_{max,i}$. Note that the determination of $\Omega_{cluster}$ is independent of the Hubble constant. The cumulative diagram for $\Omega_{cluster}(>M_{200})$ is shown in Figure 18. In order to perform a conservative error estimate, HIFLUGCS has been split into two parts with $b_{II} \ge +20^{\circ}$ and $b_{II} \le -20^{\circ}$, and the results for these subsamples are also shown in the figure. This estimate is conservative because HIFLUGCS is about twice as large as each subsample. Taking the second- and third-lowest mass



FIG. 18.—Mass density contained in galaxy clusters as a function of minimum mass. Filled circles indicate the complete HIFLUGCS, open triangles indicate the 34 clusters north of the Galactic plane, and open diamonds the 29 clusters at southern Galactic latitudes included in HIFLUGCS.

clusters together with the maximum mass range given by their individual uncertainties, we obtain

$$\Omega_{\text{cluster}} = 0.012^{+0.003}_{-0.004} \tag{14}$$

for masses larger than $6.4_{-0.6}^{+0.7} \times 10^{13} h_{50}^{-1} M_{\odot}$; i.e., the total gravitating mass contained within the virial radius of clusters amounts only to $1.2_{-0.4}^{+0.3}$ percent of the total mass in a critical density universe. Combined with our best estimate $\Omega_m = 0.12$, this implies that about 90% of the total mass in the universe resides outside virialized cluster regions above the given minimum mass. If galaxies trace mass, it also follows that by far most of the galaxies do not sit in clusters. This result is consistent with the general presumption that clusters are rare objects, rare peaks in the density distribution field.

Comparing the diagram to the mass fraction $\Omega_{cluster} = 0.028^{+0.009}_{-0.008}$ in clusters with masses larger than 2×10^{14} $h_{50}^{-1} M_{\odot}$ given by Fukugita et al. (1998), based on the mass function determined by Bahcall & Cen (1993), one finds that their estimate is a factor 4–5 higher. However, the Bahcall & Cen mass function is given for M_A , and we get a consistent result if we calculate $\Omega_{cluster}$ using our formally determined cluster masses within r_A . It needs to be pointed out that at $M_A \sim 2 \times 10^{14} h_{50}^{-1} M_{\odot}$ we find that the typical virial radius is $\sim 1 h_{50}^{-1}$ Mpc, and a mass determination at $3 h_{50}^{-1}$ Mpc based on the assumption of virial equilibrium may therefore be rather uncertain and possibly lead to overestimates of $\Omega_{cluster}$. This becomes more crucial if mass functions for M_A are extrapolated even down to galaxy masses. This way, Fukugita et al. (1998) find $\Omega_{group} = 0.12 \pm 0.02$ within the mass range 2×10^{12} to $2 \times 10^{14} h_{50}^{-1} M_{\odot}$, which, compared to our results from the previous section, would account already for almost all mass in the universe.

6. SUMMARY

An X-ray-selected and X-ray flux-limited sample comprising the 63 X-ray-brightest galaxy clusters in the sky (excluding the Galactic band; called HIFLUGCS) has been constructed based on the *ROSAT* All-Sky Survey. The flux limit has been set at 2×10^{-11} ergs s⁻¹ cm⁻² in the energy band 0.1–2.4 keV. It has been shown that a high complete-

 $^{^{13}}$ For the fit, the corresponding scatter in log M for the two relations $L \mid M$ and $M \mid L, \sigma_{\log M_{\rm tot}} = 0.22$ and 0.21, respectively (Table 12), has replaced the mass measurement error, $\bar{\sigma}_{M_{\rm tot}}$, $\log = 0.12$, in eq. (10).

ness is indicated by several tests. Because of the high flux limit, this sample can be used for a variety of applications requiring a statistical cluster sample without any corrections to the effective survey volume.

Mainly, high-quality pointed observations have been used to determine fluxes and physical cluster parameters. It has been shown that a tight correlation exists between the X-ray luminosity and the gravitational mass using HIFLUGCS and an extended sample of 106 galaxy clusters. The relation and its scatter have been quantified using different fitting methods. A comparison to theoretical and numerical predictions shows an overall agreement. This relation may be directly applied in large X-ray cluster surveys or dark matter simulations for conversions between X-ray luminosity and gravitating mass.

Using HIFLUGCS, the gravitational mass function has been determined for the mass interval $3.5 \times 10^{13} h_{50}^{-1} M_{\odot} < M_{200} < 5.2 \times 10^{15} h_{50}^{-1} M_{\odot}$. Comparison with Press-Schechter mass functions assuming CDM power spectra has yielded tight constraints on cosmological parameters. The large covered mass range has allowed to put constraints on relevant parameters individually. Specifically, we have found $\Omega_m = 0.12^{+0.06}_{-0.04}$ and $\sigma_8 = 0.96^{+0.15}_{-0.12}$ (90% c.l. statistical uncertainty). Various tests for systematic uncertainties have been performed, including comparison of the Press-Schechter mass function with the most recent results from large *N*-body simulations, almost always yielding uncertainties smaller than the statistical uncertainties. Combining all identified systematic uncertainties in a worst-case scenario results in an upper limit $\Omega_m < 0.31$. For comparison we have also determined the best-fit Ω_m values for fixed σ_8 values, yielding the relation $\sigma_8 = 0.43\Omega_m^{-0.38}$.

The mass function has been integrated to obtain the fraction of the total gravitating mass in the universe contained in galaxy clusters. Normalized to the critical density, we have found $\Omega_{cluster} = 0.012^{+0.003}_{-0.004}$ for cluster masses larger than $6.4^{+0.7}_{-0.6} \times 10^{13} h_{50}^{-1} M_{\odot}$. With the value for Ω_m determined here, this implies that about 90% of the mass in the universe resides outside virialized cluster regions.

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¹⁴ This and related papers, supplementary information, as well as electronic files of the tables given in this paper are available at http://www.xray.mpe.mpg.de/ \sim reiprich and http://www.astro.virginia.edu/ \sim thr4f.

APPENDIX A

TABLES 7–12

TABLE 7

FIT PARAMETER	VALUES
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Fit	α	Δα	A	ΔA
BCES(L M)	1.496	0.089	-17.741	1.320
Bootstrap	1.462	0.089	-17.238	1.327
BCES(<i>M</i> <i>L</i>)	1.652	0.085	-20.055	1.261
Bootstrap	1.672	0.086	-20.357	1.278
BCES-bisector	1.571	0.083	-18.857	1.237
Bootstrap	1.562	0.083	-18.717	1.239
BCES-orthogonal	1.606	0.086	-19.375	1.283
Bootstrap	1.609	0.088	-19.419	1.308

Notes.—Best-fit parameter values and standard deviations for the extended sample (106 clusters) for a fit of the form given in eq. (4). The rows denoted "bootstrap" give the results obtained for 10,000 bootstrap resamplings.

TABLE 8 Fit Parameter Values

Fit	α	Δα	A	ΔA
BCES(L M)	1.310	0.103	-14.935	1.526
Bootstrap	1.256	0.103	-14.146	1.531
$BCES(M \mid L) \dots$	1.538	0.105	-18.320	1.568
Bootstrap	1.584	0.113	- 18.995	1.681
BCES-bisector	1.418	0.097	-16.536	1.434
Bootstrap	1.407	0.096	-16.368	1.427
BCES-orthogonal	1.460	0.105	-17.157	1.559
Bootstrap	1.468	0.110	-17.274	1.633

NOTE.—Same as Table 7, but for the purely flux-limited sample (63 clusters).

TABLE 9

Fit Parameter Values				
Fit	α	Δα	Α	ΔA
BCES(<i>L</i> <i>M</i>)	1.756	0.091	-21.304	1.350
Bootstrap	1.719	0.090	-20.746	1.338
$BCES(M \mid L) \dots$	1.860	0.084	-22.836	1.246
Bootstrap	1.881	0.084	-23.144	1.250
BCES-bisector	1.807	0.084	-22.053	1.251
Bootstrap	1.797	0.084	-21.899	1.241
BCES-orthogonal	1.835	0.085	-22.473	1.260
Bootstrap	1.841	0.085	-22.563	1.270

NOTE.—Same as Table 7, but for L_{hol} .

TABLE 10 FIT PARAMETER VALUES

Fit	α	Δα	A	ΔA
BCES(<i>L</i> <i>M</i>)	1.504	0.089	-17.545	1.298
Bootstrap	1.469	0.089	-17.042	1.300
$BCES(M \mid L) \dots$	1.652	0.086	-19.708	1.254
Bootstrap	1.671	0.086	-19.992	1.260
BCES-bisector	1.575	0.084	-18.590	1.228
Bootstrap	1.565	0.083	-18.445	1.224
BCES-orthogonal	1.609	0.087	-19.075	1.274
Bootstrap	1.611	0.088	-19.109	1.290

NOTE.—Same as Table 7, but for M_{500} .

TABLE 11 FIT PARAMETER VALUES

Fit	α	Δα	A	ΔA
BCES(L M)	2.683	0.215	-35.675	3.219
Bootstrap	2.606	0.218	-34.525	3.265
$BCES(M L) \dots$	2.488	0.127	-32.761	1.902
Bootstrap	2.520	0.130	-33.239	1.946
BCES-bisector	2.583	0.154	-34.170	2.297
Bootstrap	2.560	0.157	-33.840	2.347
BCES-orthogonal	2.513	0.128	-33.137	1.915
Bootstrap	2.530	0.132	-33.388	1.975

NOTE.—Same as Table 7, but for M_A for comparison (the same relative mass errors as for M_{200} have been assumed here).

TABLE	12
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MEASURED	SCATTER
WIEASUKED	SCATIER

Scatter	$L \mid M$	$M \mid L$	Bisector		
106 Cluster	106 Clusters Included in the Extended Sample				
$\sigma_{\log M_{tot}}$	0.21	0.21	0.21		
$\sigma_{\log L_x}$	0.31	0.34	0.32		
$\sigma_{\log L/M}$	0.17	0.18	0.17		
63 Clusters Included in HIFLUGCS					
$\sigma_{\log M_{tot}}$	0.22	0.21	0.21		
$\sigma_{\log L_x}$	0.29	0.32	0.30		
$\sigma_{\log L/M}$	0.18	0.18	0.17		

NOTE.—Scatter measured for different relations.

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