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UNBIASED ESTIMATE OF DARK ENERGY DENSITY FROM TYPE Ia SUPERNOVA DATA

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ABSTRACT

Type Ia supernovae (SNe Ia) are currently the best probes of the dark energy in the universe. To constrain the nature of dark energy, we assume a flat universe and that the weak energy condition is satisfied, and we allow the density of dark energy, $\rho_X(z)$, to be an *arbitrary* function of redshift. Using simulated data from a space-based SN pencil-beam survey, we find that by optimizing the number of parameters used to parameterize the dimensionless dark energy density, $f(z) = \rho_X(z)/\rho_X(z = 0)$, we can obtain an unbiased estimate of both $f(z)$ and the fractional matter density of the universe, Ω_m. A plausible SN pencil-beam survey (with a square degree field of view and for an observational duration of 1 yr) can yield about 2000 SNe Ia with $0 \leq z \leq 2$. Such a survey in space would yield SN peak luminosities with a combined intrinsic and observational dispersion of $\sigma(m_{\rm int}) = 0.16$ mag. We find that for such an idealized survey, Ω_m can be measured to 10% accuracy, and the dark energy density can be estimated to ~20% to $z \sim 1.5$, and ~20%–40% to $z \sim 2$, depending on the time dependence of the true dark energy density. Dark energy densities that vary more slowly can be more accurately measured. For the anticipated *Supernova/Acceleration Probe* (*SNAP*) mission, Ω_m can be measured to 14% accuracy, and the dark energy density can be estimated to ~20% to $z \sim 1.2$. Our results suggest that *SNAP* may gain much sensitivity to the time dependence of the dark energy density and Ω_m by devoting more observational time to the central pencil-beam fields to obtain more SNe Ia at $z > 1.2$. We use both a maximum likelihood analysis and a Monte Carlo analysis (when appropriate) to determine the errors of estimated parameters. We find that the Monte Carlo analysis gives a more accurate estimate of the dark energy density than the maximum likelihood analysis.

Subject headings: cosmology: observations — cosmology: theory — dark matter — supernovae: general

1. INTRODUCTION

Supernova (SN) data suggest that most of the energy in the universe is unknown to us (Garnavich et al. 1998b; Riess et al. 1998; Perlmutter et al. 1999). Much theoretical effort has been devoted to exploring the possible candidates for the dark energy (see, e.g., Peebles & Ratra 1988; Frieman et al. 1995; Caldwell, Davé, & Steinhardt 1998; Sahni & Wang 2000) and investigating the constraints on the nature of the dark energy that can be derived from observational data (see, e.g., White 1998; Garnavich et al. 1998a; Steinhardt, Wang, & Zlatev 1999; Efstathiou 1999; Huterer & Turner 2000; Podariu & Ratra 2000; Sahni & Starobinsky 2000; Saini et al. 2000; Waga & Frieman 2000; Ng & Wiltshire 2001; Podariu, Nugent, & Ratra 2001; Weller & Albrecht 2001).

The most straightforward and promising probe of dark energy is the distance-redshift relation derived from observations of cosmological standard candles. At present, Type Ia supernovae (SNe Ia) are our best candidates for cosmological standard candles (Phillips 1993; Riess, Press, & Kirshner 1995; Phillips et al. 1999; Branch et al. 2001). Most workers have concentrated on constraining the equation of state of the dark energy w_X from SN data. However, without making specific assumptions about w_X (e.g., assuming it to be a constant), it is extremely hard to constrain w_X using SN data (Maor, Brustein, & Steinhardt 2001; Barger & Marfatia 2001). Recently, Wang & Garnavich (2001, hereafter WG01) showed that it is easier to extract constraints on the dark energy density $\rho_X(z)$, instead of $w_X(z)$, from data. This is because there are multiple integrals relating $w_X(z)$ to the luminosity distance of SNe Ia, $d_L(z)$, which results in a "smearing" that obscures the difference between different $w_X(z)$ (Maor et al. 2001). On the other hand, $\rho_X(z)$ is related to the time derivative of the comoving distance to SNe Ia, $r'(z)$; hence, it is less affected by the smearing effect. The advantage of measuring $\rho_X(z)$ over that of measuring $w_X(z)$ is confirmed by Tegmark (2001).

WG01 gave a proof of principle of an adaptive iteration technique for extracting the dimensionless dark energy density $f(z) = \rho_X(z)/\rho_X(z = 0)$ as an arbitrary function from future SN data. They found that feasible future SN data will allow us to clearly differentiate dark energy with density that changes with time from a cosmological constant Λ; however, estimates of Ω_m and $f(z)$ tend to be significantly biased for dark energy densities that vary substantially with time. In this Letter, we apply a significantly improved and optimized version of the adaptive iteration technique for obtaining an unbiased estimate of the dark energy density from SN Ia data.

2. DARK ENERGY DENSITY

The total energy density of the universe is

$$\rho(z) = \rho_m^0(1+z)^3 + \rho_k^0(1+z)^2 + \rho_X^0 f(z) = \rho_c^0[\Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_X f(z)], \quad (1)$$

where the superscript "0" indicates present values, $f(z = 0) = 1$, and $\Omega_k = 1 - \Omega_m - \Omega_X = -k/H_0^2$. If the unknown energy is due to a cosmological constant Λ, $f(z) = 1$. Clearly, $f(z) = \rho_X(z)/\rho_X(z = 0)$ is a very good probe of the nature of the unknown energy. Following WG01, we impose the weak energy condition (the energy density is nonnegative for any observer [Wald 1984]), which implies that $f'(z) \geq 0$ (see WG01).

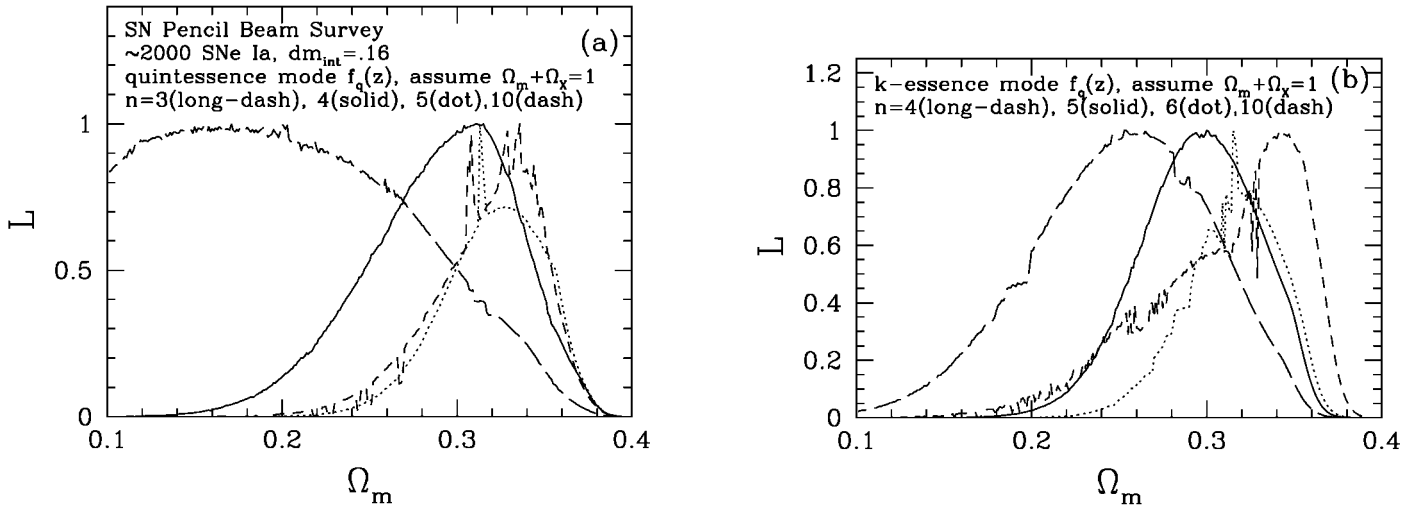


FIG. 1.—Likelihood function $\mathcal{L}(\Omega_m) \propto \exp[-\chi^2_{\min}(\Omega_m)/2]$ for the quintessence model $f_q(z)$ and k-essence model $f_k(z)$

The equation of state of the dark energy is

$$w_X(z) \equiv \frac{p_X(z)}{\rho_X(z)} = \frac{1}{3}(1+z)\frac{f'(z)}{f(z)} - 1. \tag{2}$$

A constant $w_X(z)$ corresponds to $f(z) \propto (1+z)^\alpha$, where α is a constant. The values $\alpha = 0$, $\alpha = 3$, and $\alpha = 4$ correspond to a cosmological constant, matter, and radiation, respectively. We parameterize $f(z)$ as

$$f(z) = \left(\frac{z_i - z}{z_i - z_{i-1}}\right) f_{i-1} + \left(\frac{z - z_{i-1}}{z_i - z_{i-1}}\right) f_i,$$

$$z_{i-1} < z \le z_i, \quad z_0 = 0, \quad z_n = z_{\max}; \quad f_0 = 1, \tag{3}$$

where f_i $(i = 1, 2, \ldots, n-1)$ are independent variables to be estimated from data. We let f_n be either an independent variable or linearly extrapolated from f_{n-1} and f_{n-2}, whichever gives the smaller χ^2 per degree of freedom for the same n.

The measured distance modulus for an SN Ia is $\mu_0^{(l)} = \mu_p^{(l)} + \epsilon^{(l)}$, where $\mu_p^{(l)}$ is the theoretical prediction $\mu_p^{(l)} = 5 \log [d_L(z_l)/\mathrm{Mpc}] + 25$ and $\epsilon^{(l)}$ is the uncertainty in the measurement, including observational errors and intrinsic scatter in the SN Ia absolute magnitudes. Denoting all the parameters to be fitted as $\boldsymbol{s}$, we can estimate $\boldsymbol{s}$ using a modified χ^2 statistic, which results from integrating the probability density function for parameters $\boldsymbol{s}$, $p(\boldsymbol{s}) \propto \exp(-\chi^2/2)$, over the Hubble constant H_0. We write (see WG01)

$$\tilde{\chi}^2 \equiv \chi^2_* - \frac{C_1}{C_2}\left(C_1 + \frac{2}{5}\ln 10\right), \tag{4}$$

where

$$\chi^2_* \equiv \sum_l \frac{1}{\sigma_l^2}(\mu_p^{*(l)} - \mu_0^{(l)})^2, \tag{5}$$

$$C_1 \equiv \sum_l \frac{1}{\sigma_l^2}(\mu_p^{*(l)} - \mu_0^{(l)}), \quad C_2 \equiv \sum_l \frac{1}{\sigma_l^2}, \tag{6}$$

$$\mu_p^* \equiv \mu_p(h = h^*) = 42.384 - 5\log h^* + 5\log [H_0 r(1+z)]. \tag{7}$$

We take $h^* = 0.65$. Our results are independent of the choice of h^*.

The current cosmic microwave background anisotropy measurements seem to indicate that we live in a flat universe (de Bernardis et al. 2000; Balbi et al. 2000). Cluster abundances strongly suggest a low matter density universe (Bahcall, Lubin, & Dorman 1995; Carlberg et al. 1996; Bahcall & Fan 1998a, 1998b). The best-fit model to current observational data is $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$. We will use $\Omega_m = 0.3$ and $\Omega_X = 0.7$ for our simulated data in the rest of this Letter.

In order to compare our results with those of WG01, we consider the same two hypothetical dark energy models given by

$$f_q(z) = \frac{e^{1.5z}}{(1+z)^{1.5}}, \quad w_q(z) = -1 + 0.5z,$$

$$f_k(z) = \exp[0.9(1 - e^{-z})], \quad w_k(z) = 0.3(1+z)e^{-z} - 1; \tag{8}$$

$f_q(z)$ and $f_k(z)$ represent quintessence ($dw_q/dz > 0$) and k-essence ($dw_k/dz < 0$) models, respectively (Caldwell et al. 1998; Armendariz-Picon, Mukhanov, & Steinhardt 2000), and they satisfy the weak energy condition $f'(z) \ge 0$.

3. RESULTS

A feasible SN pencil-beam survey (with a square degree field of view and for an effective observational period of 1 yr) can yield almost 2000 SNe Ia out to $z = 2$ (Wang 2000a). Let us combine the data from the SN pencil-beam survey with SN data at smaller redshifts, so that there are a minimum of 50 SNe Ia per 0.1 redshift interval at any redshift. This yields a total of 1966 SNe Ia for the quintessence model and 1898 SNe Ia for the k-essence model, up to $z = 2$ and for $\Omega_m = 0.3$, $\Omega_X = 0.7$. We assume intrinsic and observational dispersions that are Gaussian, with zero mean and a variance of 0.16 mag (for a space-based survey).

Figure 1 shows the likelihood function $\mathcal{L}(\Omega_m) \propto e^{-\chi^2_{\min}(\Omega_m)/2}$ for the quintessence model $f_q(z)$ and the k-essence model $f_k(z)$. Note that $\chi^2_{\min}(\Omega_m)$ is marginalized over n or $n-1$ independent parameters that parameterize the dimensionless dark energy density $f(z)$, f_i $(i = 1, 2, \ldots, n)$ (see eq. [3]). For the

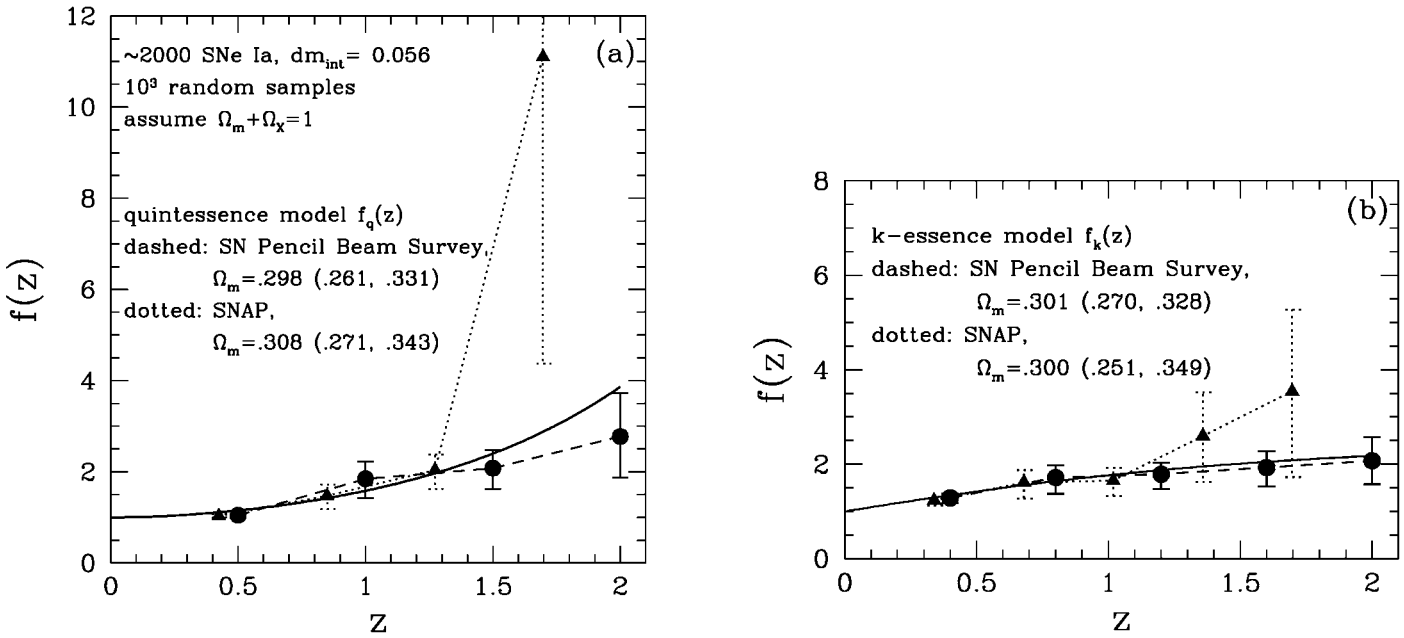


FIG. 2.—Dimensionless dark energy densities $f_q(z)$ and $f_k(z)$ estimated with $n = 4$ and $n = 5$, respectively (see Fig. 1). The dashed line and filled circles represent the estimate derived for an ideal space-based SN pencil-beam survey (Wang 2000a). The dotted lines and filled triangles represent the estimate derived for the *SNAP* mission.

quintessence model $f_q(z)$, we allow f_n to be an independent variable, and for the *k*-essence model $f_k(z)$, we linearly extrapolate f_{n-2} and f_{n-1} to obtain f_n (see text after eq. [3]). The curves (peak location from left to right) correspond to $n = 3, 4, 5, 10$ and $n = 4, 5, 6, 10$ for $f_q(z)$ and $f_k(z)$, respectively.

For the quintessence model $f_q(z)$, the χ^2 per degree of freedom decreases as we decrease n from $n = 10$ to $n = 4$, with a modest shift in the most likely value of Ω_m (see Fig. 1*a*); this is as expected because our estimate of Ω_m should be roughly independent of the parameterization of $f(z)$. There is a substantial shift in the most likely value of Ω_m as we change n from $n = 4$ to $n = 3$. Although the χ^2 per degree of freedom is smaller for $n = 3$, the optimal choice is $n = 4$ for this model since a greater degree of degeneracy between Ω_m and f_i ($i = 1, 2, \ldots, n$) sets in for $n = 3$, which renders the estimated value of Ω_m significantly biased. For real data, and the true value of Ω_m unknown, this sudden increase in degeneracy can be inferred via the substantial shift in the estimated value of Ω_m.

For the *k*-essence model $f_k(z)$, the χ^2 per degree of freedom decreases as we decrease n from $n = 10$ to $n = 4$, with the largest shift in the most likely value of Ω_m occurring as we change n from $n = 5$ to $n = 4$ (see Fig. 1*b*). Hence, $n = 5$ is the optimal choice for this model. Note that the transition that marks the sudden increase in degeneracy (at $n = 4$) is less dramatic than in the case of the quintessence model $f_q(z)$.

Figure 2 shows the dimensionless dark energy densities $f_q(z)$ and $f_k(z)$ estimated with $n = 4$ and $n = 5$, respectively (see Fig. 1). The dashed line and filled circles represent the estimate derived for an ideal space-based SN pencil-beam survey (Wang 2000a). The dotted lines and filled triangles represent the estimate derived for the *Supernova/Acceleration Probe* (*SNAP*)[1] mission. The errors are (16%, 84%)-intervals of the cumulative probability distribution of the estimated parameter for 10^3 Monte Carlo samples.

Note that Figures 1 and 2 differ significantly from Figure 2 of WG01, where the estimates of both Ω_m and f_i ($i = 1, 2, \ldots, n$) become significantly more biased as n is changed from $n = 10$ to $n = 6$. This work is greatly improved over that of WG01 as follows: (1) maximum likelihood analysis is used here to determine the optimal choice of n in parameterizing $f(z)$, which was not explored by WG01; (2) the endpoint of $f(z)$, f_n, is allowed to differ from f_{n-1} here, while $f_n = f_{n-1}$ was imposed in WG01; (3) the grid sizes of $\Delta\Omega_m = 0.001$, $\Delta f_i = 0.05$ are used here, compared with the grid sizes of $\Delta\Omega_m = 0.02$, $\Delta f_i = 0.1$ of WG01; and (4) the errors of the estimated parameters are derived via Monte Carlo analysis from 10^3 instead of 10^2 random samples as in WG01.

We have used a combination of maximum likelihood analysis and Monte Carlo technique (when appropriate) in this Letter. Figures 3*a* and 3*b* show the probability distribution functions of Ω_m and f_4 (see eq. [3]), respectively, for the quintessence model $f_q(z)$ with $n = 4$. The dotted curve is the likelihood function derived from a marginalized χ^2, while the histogram is the probability distribution derived from Monte Carlo analysis of 10^3 random samples. The arrows in each figure indicate the true value of the parameter. Clearly, Monte Carlo analysis gives a more accurate estimate of the dark energy density $f(z)$ than the maximum likelihood analysis for large $f(z)$.

4. DISCUSSION

Due to a substantial improvement in our adaptive iteration technique (see WG01), we are able to explore much greater ranges of possibilities in the parameterization of the dimensionless dark energy density $f(z) = \rho_X(z)/\rho_X(z = 0)$. As a result, we have developed a method for finding unbiased estimates of Ω_m together with $f(z) = \rho_X(z)/\rho_X(z = 0)$ (as an arbitrary function parameterized by its values at n equally spaced redshifts). The optimal choice of n corresponds to the smallest χ^2 per degree of freedom without significant shift in the estimated best-fit value of Ω_m (see Fig. 1). This leads to the unbiased estimate of Ω_m,

[1] See http://snap.lbl.gov.

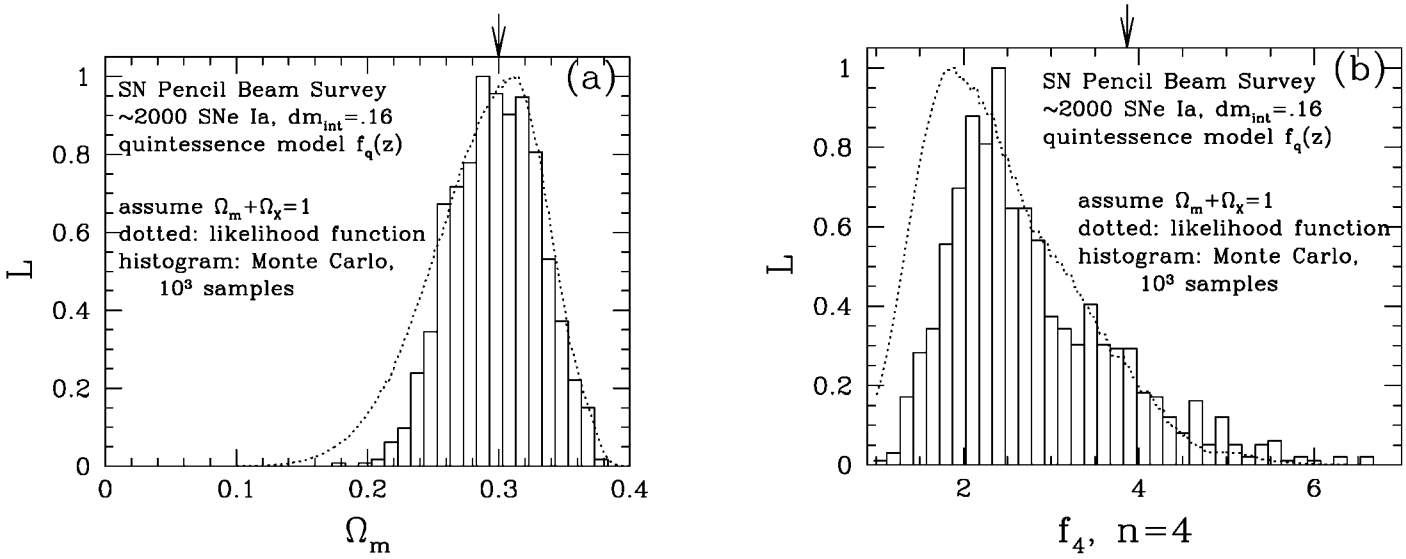


FIG. 3.—Probability distribution functions of (*a*) Ω_m and (*b*) f_4 for the quintessence model $f_q(z)$ with $n = 4$. The dotted curve is the likelihood function derived from a marginalized χ^2, while the histogram is the probability distribution derived from Monte Carlo analysis of 10^3 random samples.

which is crucial in deriving an unbiased estimate of the dimensionless dark energy density $f(z)$.

Our approach is more robust than constraining the dark energy equation of state $w_X(z)$ (as a linear function of z), assuming that Ω_m is known (Huterer & Turner 2000; Weller & Albrecht 2001), and complementary to the latter in probing the nature of the dark energy. We find that for an ideal SN pencil-beam survey (Wang 2000a) from space, Ω_m can be measured to 10% accuracy, and $f(z) = \rho_X(z)/\rho_X(z = 0)$ can be estimated to ~20% to $z \sim 1.5$, and 20%–40% to $z \sim 2$, depending on the time dependence of the true $\rho_X(z)$ (see Fig. 2). Dark energy densities that vary more slowly can be more accurately measured, as might have been expected, since it is easier in general to constrain slowly varying functions compared with rapidly varying functions.

For the anticipated *SNAP* mission, Ω_m can be measured to 14% accuracy, and $f(z) = \rho_X(z)/\rho_X(z = 0)$ can be estimated to ~20% to $z \sim 1.2$ (see Fig. 2). Compared with the idealized SN pencil-beam survey (Wang 2000a), the *SNAP* strategy is to obtain a larger number of SNe Ia (2408 vs. 2000 for a $1° \times 1°$ pencil beam) at $z \leq 1.2$ by devoting a significant fraction of the observational time on flanking $1° \times 1°$ fields that surround the two central $1° \times 1°$ pencil-beam fields, at the price of a sharply decreased number of SNe Ia at $z > 1.2$; this leads to large errors in estimated f_i for $z > 1.2$ (see Fig. 2). Our results suggest that *SNAP* may gain much sensitivity to the time dependence of $\rho_X(z)$ and Ω_m by devoting more observational time to the central pencil-beam fields to obtain more SNe Ia at $z > 1.2$.

The main systematic uncertainties of SNe Ia as cosmological standard candles are gray dust (Aguirre 1999), gravitational lensing (see, e.g., Kantowski, Vaughan, & Branch 1995; Wambsganss et al. 1997; Holz 1998; Wang 1999), and luminosity evolution (Drell, Loredo, & Wasserman 2000; Riess et al. 1999; Wang 2000b). These must be studied carefully before reliable cosmological constraints can be derived from SN data. A large number of SNe Ia at $z > 1$ from an SN pencil-beam survey are critical in resolving these important systematic uncertainties. The spectroscopic and broadband photometric data of SNe Ia from such a survey will enable us to constrain gray dust (Croft et al. 2000) and the SN luminosity evolution (Branch et al. 2001). Wang (2000b) has shown that flux averaging of calibrated SN Ia peak luminosities is effective in reducing the bias due to gravitational lensing. Therefore, we do not expect our results to change significantly in the presence of systematic uncertainties since the latter can be dealt with effectively given a large number of SNe Ia out to $z > 1$. Future SN pencil-beam surveys hold great promise for constraining the nature of dark energy.

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