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ON OPTICAL SYNCHROTRON EMISSION OF RADIO PULSARS

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ABSTRACT

It is well known that the distribution function of relativistic particles is one-dimensional at the pulsar surface. Any transverse momentum will be lost via synchrotron emission during a very short time ($\leq 10^{-20}$ s). However, the cyclotron instability causes an appearance of transverse momenta of relativistic particles in the outer parts of pulsar magnetospheres. We formulate the equation controlling such processes, and obtain the solution of this equation. It gives the distribution of pitch angles ψ for particles emitting the synchrotron radiation. We show that this radiation is generated near the light cylinder. The mean value of the pitch angle corresponds to optical frequencies in this region. For example, $\bar{\psi} \sim 10^{-6}$ for the radio pulsar PSR B0656+14 (in the observer's frame), and the frequency of the spectral peak is equal to 3×10^{14} Hz. The flux observed in the optical range requires for this pulsar a density of radiating electrons $\sim 10^{9}$ cm^{-3}. Some other models of synchrotron emission are discussed. We show that optical emission will be observed in pulsars with small angles β between the rotation and magnetic axes. Pulsars with shorter periods must have harder spectra.

Subject headings: plasmas — pulsars: general — radiation mechanisms: nonthermal

1. INTRODUCTION

Throughout the history of radio pulsar investigation, many attempts have been made to detect optical emission from them. Recently, new efforts have been undertaken to register a number of pulsars in the optical range. As a result, nine objects from more than 1000 radio pulsars have been found to emit an optical signal (Caraveo 1999). A list of these pulsars is given in Table 1. Some of the observed optical emission has a thermal nature and is connected to the hot neutron star surface or to the polar cap bombarded by the secondary electrons (or positrons) and heated by them. There is, however, an appreciable nonthermal component in the optical emission (Pavlov, Welty, & Córdova 1997). We describe this component here, and consider the possibility of generating synchrotron emission in the optical range.

Transverse momenta ($p_\perp$) of relativistic electrons and positrons are lost during a very short time ($\leq 10^{-20}$ s) via synchrotron emission in very strong magnetic fields at the surface of a neutron star. Therefore, the distribution function of these particles becomes one-dimensional. Generation of waves is possible during the further motion of relativistic outflows along dipole magnetic field lines if the condition of cyclotron resonance is fulfilled. The reason for the generation of such waves is an anisotropy of the distribution function (Sagdeev & Shafranov 1960). Therefore, a diffusion of particles arises during the quasi-linear stage of the instability development not only along but also across the magnetic field. Plasma particles acquire transverse momenta (pitch angles) and begin to rotate along Larmor orbits. The radius $r_{\rm L} = v_\perp \gamma/\omega_B$ of a such orbit is much more than the radius of particles on Landau's orbits, $r_l = c/\omega_B(l + 1/2)$. Here γ is the Lorentz factor of plasma particles, $\omega_B = eB/mc$, e is the charge of an electron, m is its mass, c is the speed of light, B is the value of the magnetic field, and l is the Landau quantum number. Synchrotron emission is generated as a result of the appearance of pitch angles. The synchrotron frequency must be higher than natural frequencies, or the wavelength λ must be much less than the value of $n^{-1/3}$, the distance between particles, where n is the density of plasma particles. In this case, the radiation does not "feel" a medium and emerges freely from the region of its generation. This mechanism was proposed by Lominadze, Machabeli, & Mikhailovskii (1979) and by Machabeli & Usov (1979). We consider the case in which the synchrotron frequencies are in the optical range, and it is necessary to have the cyclotron instability for beam particles with Lorentz factors $\gamma_b \sim 10^6$ and with corresponding pitch angles.

It is usually suggested that the pulsar magnetosphere is filled by a relativistic $e^{\pm}$ plasma. The distribution function of such a plasma is shown in Figure 1. The properties of this plasma have been investigated quite thoroughly (e.g., Hardee & Rose 1976; Volokitin, Krasnosel'skikh, & Machabeli 1985; Arons & Barnard 1986). A gyrotropy is absent in such a case, and there are only three types of waves. One of them (O mode) is superluminal, i.e., its phase velocity, $v_{\rm ph}$, is more than the light velocity. It cannot be excited by particles, and we do not consider it further. Two other modes are (1) the Alfvén mode (A mode), with an electric vector $\boldsymbol{E}$ lying in the plane ($\boldsymbol{k}$, $\boldsymbol{B}$), where $\boldsymbol{k}$ is the wavevector and $\boldsymbol{B}$ is the vector of the magnetic field; and (2) the X mode, for which $\boldsymbol{E}$ is perpendicular to the plane ($\boldsymbol{k}$, $\boldsymbol{B}$). The A mode also has a longitudinal component; it is a mixed (longitudinal-transverse) wave.

2. THE MODEL

Let us consider the possibility of generating transverse (t) waves with the spectrum (in the plasma rest frame)

$$\omega_t = k\,c(1 - \delta)\,, \qquad (1)$$

$$\delta = \frac{\omega_p^2}{4\omega_B^2 \gamma_T \gamma_p^2}\,,$$

TABLE 1

RADIO PULSARS WITH OPTICAL EMISSION

No.	PSR	P (ms)	$\dot{P}_{-14}$ (s s^{-1})	β (deg)	$-\log \psi_H$ (rad)	$\log \nu_m$ (Hz)
1.....	B0531+21	33	42.10	85? (6.5)	8.52	23.19–14.70
2.....	B0540−69	50	47.91	(7)	8.32	22.93–14.70
3.....	B0656+14	385	5.50	23.5	5.32–6.52	16.65–13.06
4.....	B0833−45	89	12.48	35.6	6.92–7.64	20.67–18.86
5.....	B0950+08	253	0.023	7	2.85–5.60	14.08–5.85
6.....	B1055−52	197	0.58	9	4.05–6.47	16.78–9.53
7.....	B1509−58	151	153.65	(11.4)	7.82	21.05–14.70
8.....	B1929+10	227	0.12	8	3.47–6.04	15.42–7.69
9.....	J0633+1746[a]	237	1.10	~10	4.20–6.48	16.76–9.91

NOTE.—Here β is the angle between magnetic and rotation axes. Values of β are given by Malov 1990. The angles β for PSR 0540−69 and PSR 1509−58 are still unknown. The values of periods P and $\dot{P}$ are taken from the catalog of J. H. Taylor et al. (1995, unpublished). Values of ψ_H and ν_m are calculated for distances from $r = r_{LC}$ to $r = r_{LC}/\sin \beta$.

[a] Geminga.

$$\omega_p^2 = \frac{4\pi n_p e^2}{m}, \tag{2}$$

where $\gamma_T \sim \gamma_p$ is the half-width of the distribution function of the bulk of the secondary plasma. For such a generation to take place, the cyclotron resonance condition must be satisfied.

This can be written in cylindrical coordinates (Fig. 2) in the form (Kazbegi, Machabeli, & Melikidze 1992)

$$\omega - k_\varphi V_\varphi - k_x u_x \pm \frac{\omega_B}{\gamma_r} = 0, \tag{3}$$

where $V_\varphi \approx c[1 - (u_x^2/c^2) - (1/2\gamma_r^2)]$, $k_\varphi^2 + k_\perp^2 = k^2$, $k_\perp^2 = k_x^2 + k_r^2$, γ_r is the Lorentz factor for the resonant particles, the drift velocity of the particles due to the curvature of the field lines is

$$u_x = \frac{cV_\varphi \gamma_r}{\rho \omega_B}, \tag{4}$$

and ρ is the radius of curvature of the field lines. Using these expressions, the resonance condition takes the form (cf. Lyutikov, Blandford, & Machabeli 1999)

$$\frac{1}{2\gamma_r^2} + \frac{(k_\perp/k_\varphi - u_x/c)^2}{2} + \frac{1}{2}\frac{k_r^2}{k_\varphi^2} - \delta = \pm \frac{\omega_B}{\gamma_r k_\varphi c}. \tag{5}$$

As shown by Machabeli & Usov (1979), t-waves are excited in the pulsar magnetosphere only by the anomalous Doppler effect (plus sign in eq. [5]). The minus sign of this equation corresponds to wave damping.

A similar expression can be written for potential-nonpotential (lt) waves, whose spectrum is

$$\omega_{lt} = k_\varphi c\left(1 - \delta - \frac{k_\perp^2 c^2}{16\omega_p^2 \gamma_p}\right). \tag{6}$$

These are excited under the condition

$$-\frac{1}{2}\frac{k_x^2}{k_\varphi^2} + \frac{1}{2\gamma_r^2} + \frac{1}{2}\left(\frac{k_x}{k_\varphi} - \frac{u_x}{c}\right)^2 - \delta - \frac{k_\perp^2 c^2}{16\omega_p^2 \gamma_p} = \pm \frac{\omega_B}{\gamma_r k_\varphi c}. \tag{7}$$

A minus sign in equation (7) corresponds to wave excitation and a plus sign to wave damping. Both excitation and damping of the waves lead to a redistribution of the par-

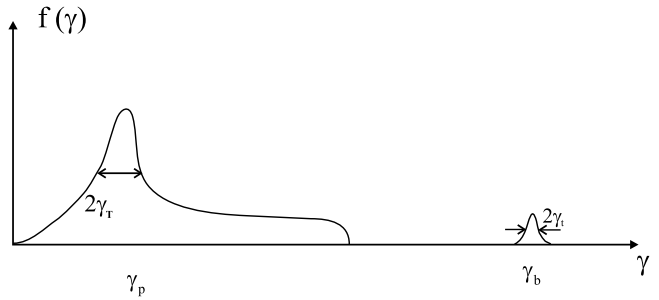


FIG. 1.—Distribution function of a one-dimensional plasma in the pulsar magnetosphere. Left corresponds to secondary particles, right to the primary beam.

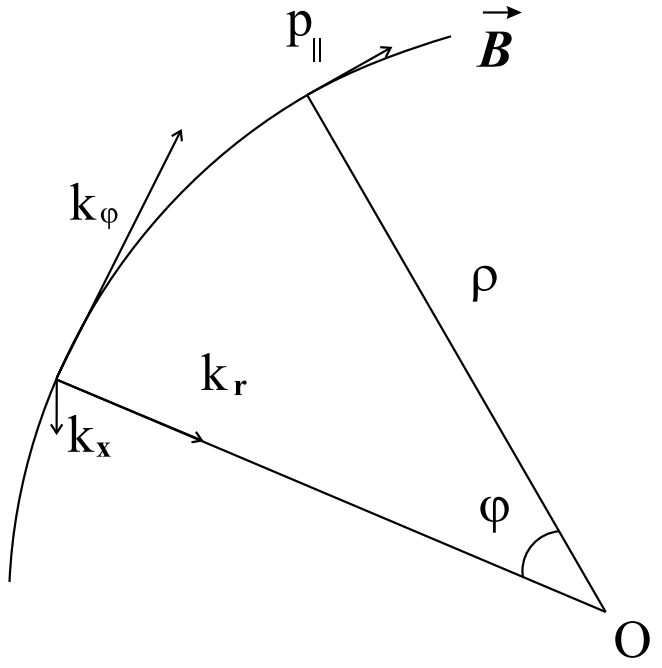


FIG. 2.—Coordinate system (ρ is a local radius of curvature)

ticles via quasi-linear diffusion. The damping can take place at the left slope of the distribution function of the bulk (Lominadze et al. 1986).

When a particle moves along a curved magnetic field, it is affected by the force responsible for conservation of the adiabatic invariant (Landau & Lifshitz 1971), $I = 3cp_\perp^2/2eB$ in a weakly nonuniform field ($[\lambda/B(r)][dB(r)/dr] < 1$, where λ is the wavelength of the excited waves).

The transverse and longitudinal components of this force with respect to the magnetic field B have the form (Sivukhin 1963)

$$G_\perp = -\frac{mc^2}{\rho}\,\gamma_r\psi\,, \quad G_\parallel = \frac{mc^2}{\rho}\,\gamma_r\psi^2\,, \tag{8}$$

where ψ is the pitch angle of relativistic particles. The force G was used in the kinetic equation for the first time by Machabeli & Usov (1979). In addition, if a particle radiates (which can be efficient when $\lambda < n^{-1/3}$), a radiative deceleration force appears. We assume that emission is produced by the synchrotron mechanism only. Then (Landau & Lifshitz 1971)

$$F_\perp = -\alpha\psi(1+\gamma_r^2\psi^2)\,,$$
$$F_\parallel = -\alpha\gamma_r^2\psi^2\,, \quad \alpha = \frac{2}{3}\frac{e^2\omega_B^2}{c^2}\,. \tag{9}$$

In addition to forces G and F, the reaction force of curvature radiation, $f_{\rm curv}$, will act on particles. However, this force has a longitudinal component only and does not play any role in the formation of pitch angles. The force $f_{\rm curv}$ must be taken into account if we consider quasi-linear diffusion along the magnetic field. Here we confine our investigation to the transverse diffusion only.

Let us consider the case of small pitch angles ($\psi \ll 1$). A number of interesting features then appear in the synchrotron radiation, as described by Epstein (1973). Let us compare the transverse and longitudinal components of the forces given by equations (8) and (9):

$$\frac{G_\perp}{F_\perp} = \frac{3m^3c^6}{2e^4B_s^2}\,\frac{\gamma}{\rho}\left(\frac{r}{R_*}\right)^6\frac{1}{1+\gamma^2\psi^2}\,,$$
$$\frac{G_\parallel}{F_\parallel} = -\frac{3m^3c^6}{2e^4B_s^2\,\gamma\rho}\left(\frac{r}{R_*}\right)^6. \tag{10}$$

If $\rho \approx 10^9$ cm, we obtain for the case of PSR 0656+14

$$\frac{G_\perp}{F_\perp} = 7\times10^{-16}\gamma\left(\frac{r}{R_*}\right)^6,$$
$$\left|\frac{G_\parallel}{F_\parallel}\right| = 7\times10^{-16}\gamma^{-1}\left(\frac{r}{R_*}\right)^6. \tag{11}$$

Using equations (5) or (7), we can estimate the distance at which the wave excitation occurs, if we assume that approximately

$$\delta = \frac{\omega_B}{\omega\gamma_r}\,. \tag{12}$$

Taking into account equation (2), we obtain for the dipole field

$$\frac{r}{R_*} = \left(2\times10^{39}\,\frac{\gamma_p^4 PB_{12}^2}{\gamma_b^2\,\omega}\right)^{1/6}. \tag{13}$$

If $\gamma_p \sim 10$ and $\gamma_b \sim 10^6$, this gives $r/R_* \sim 500$ for all frequencies in the optical range ($\omega \sim 10^{15}$ s^{-1}) and below. In other words, r is of the order of the radius of the light cylinder, $r_{\rm LC} = cP/2\pi$. Consequently, waves are excited in the peripheral regions of the magnetosphere. If $r = 500R_*$, it follows from equation (11) that $G_\perp \gg F_\perp$ and $G_\parallel \ll F_\parallel$. Taking into account the above relations and using the inequality $\psi = p_\perp/p_\parallel \ll 1$, we can write the kinetic equation for the distribution function of the beam particles in the form (Machabeli & Usov 1979)

$$\frac{\partial f^0}{\partial t} + \frac{1}{mc\gamma_b^2\psi}\,\frac{\partial}{\partial\psi}\,(\psi G_\perp f^0) + \frac{1}{mc}\,\frac{\partial}{\partial\gamma_b}\,(F_\parallel f^0)$$
$$= \frac{1}{m^2c^2\gamma_b^2}\,\frac{1}{\psi}\,\frac{\partial}{\partial\psi}\left(\psi D_{\perp\perp}\,\frac{\partial f^0}{\partial\psi}\right), \tag{14}$$

where

$$D_{\perp\perp} \approx \frac{\pi e^2\omega_p^2}{4c\omega_B^2}\,\gamma_p\,|E(k)|^2_{k=k_{\rm res}} \tag{15}$$

is the diffusion coefficient, $|E(k)|^2_{k=k_{\rm res}}$ is the energy density for the oscillations, and $k_{\rm res} = m\omega_B^2/p_\parallel\,\omega_p^2\,\gamma_p$. Diffusion arises through the influence of the generated waves back on resonant particles. We assume that $\partial/\partial\psi \gg \partial/\partial\gamma$, consider the quasi-stationary case ($\partial f/\partial t = 0$), and seek a solution of this equation of the form

$$\dot{f}(\boldsymbol{p}) = \chi(\psi)f(p)\,. \tag{16}$$

Under these assumptions, we find

$$\frac{1}{mc\gamma_b^2\psi}\,\frac{d}{d\psi}\,(\psi G_\perp f^0) = \frac{1}{m^2c^2\gamma_b^2}\,\frac{1}{\psi}\,\frac{d}{d\psi}\left(\psi D_{\perp\perp}\,\frac{df^0}{d\psi}\right). \tag{17}$$

This equation has the solution

$$\chi(\psi) = C\,\exp\left(\int\frac{G_\perp mc\gamma_b}{D_{\perp\perp}}\,d\psi\right) = CE^{-A\psi^2}\,, \tag{18}$$

where

$$A = \frac{2m^2c^4\gamma_b^2(\omega_B/\omega_p)^2}{\pi e^2\rho\,|E(k)|^2\gamma_p}\,. \tag{19}$$

The mean value of the angle ψ is

$$\bar{\psi} \approx \frac{\int_0^\infty C\psi e^{-A\psi^2}d\psi}{\int_0^\infty Ce^{-A\psi^2}d\psi} = \frac{1}{\sqrt{\pi A}}\,. \tag{20}$$

To evaluate $\bar{\psi}$, we use the quantity

$$|E(k)|^2 = \frac{1}{2}\,\frac{mc^2n_b\gamma_b c}{\omega}\,. \tag{21}$$

After substituting this expression into equation (20) and using the formula for the density of the primary beam (Goldreich & Julian 1969),

$$n_b = B/Pce\,, \tag{22}$$

the expression

$$\rho = r_{\rm LC} = cP/2\pi\,, \tag{23}$$

and the equality

$$n_b\gamma_b = 2n_p\gamma_p\,, \tag{24}$$

we obtain

$$\bar{\psi} = \frac{\sqrt{\pi} mc\gamma_b}{2\sqrt{2} eP B \gamma_p} = \frac{mc^4 \gamma_b P^2}{16\sqrt{2\pi}\pi^2 e R_*^3 \gamma_p B_s} . \quad (25)$$

Here we consider the pitch angle at the light cylinder. Using the values $R_* = 10^6$ cm, $\gamma_b = 10^6$, and $\gamma_p = 10$ for all pulsars, we have

$$\bar{\psi} = 4 \times 10^{-4} P^2 / B_{12} , \quad (26)$$

where $B_{12} = B_s/10^{12}$. One of the best-studied pulsars at optical frequencies is PSR 0656+14 (Pavlov et al. 1997; Kurt et al. 1998). By using the data from Table 1, we find from equation (25) that $\bar{\psi} = 1.3 \times 10^{-5}$.

Equation (24) characterizes the pitch angles in the rest frame of the plasma. Using the aberration formula (Landau & Lifshitz 1971),

$$\sin \psi_H = \frac{\sin \psi}{\gamma_p (1 + v \cos \psi / c)} , \quad (27)$$

we obtain in the observer's frame,

$$\psi_H \approx \frac{\psi}{2\gamma_p} , \quad (28)$$

which is equal to 6.5×10^{-7} (for $\gamma_p = 10$).

3. ESTIMATES FOR PSR 0656+14

As we can see, $\gamma_b \psi \approx 1$ for PSR 0656+14. In this case it follows from Epstein (1973) that the peak of the single-particle spectrum corresponds to the frequency

$$\nu_m = a(\gamma\psi) \frac{eB}{2\pi mc\gamma\psi^2} , \quad (29)$$

which is approximately 3.2×10^{14} Hz in PSR 0656+14 for $r/r_{\rm LC} = 1.6$. The boundary of the magnetosphere for this pulsar lies at $r = r_{\rm LC}/\sin \beta = 2.5 r_{\rm LC}$ (see Table 1), and such a level for the generation of optical emission is rather reasonable. Here β is the angle between the magnetic moment and the rotation axis, and $a(\gamma\psi)$ is a factor of ~ 1 for this pulsar. Beyond the maximum, the emission intensity decreases slowly toward higher frequencies. This picture is in agreement with the observed optical spectrum of PSR 0656+14 (Kurt et al. 1998; Pavlov et al. 1997) from 3×10^{14} to 8.9×10^{14} Hz.

To obtain further estimates, we used the value for the flux density at 3.5×10^{14} Hz (the I filter), equal to 6×10^{-30} erg cm^{-2} s^{-1} Hz^{-1}. The corresponding spectral luminosity can be written as

$$L_\nu = \frac{\pi^3 d^2 W_{0.5} S_\nu}{P} = 4.5 \times 10^{13} \text{ ergs s}^{-1} \text{ Hz}^{-1} . \quad (30)$$

We take the distance to PSR 0656+14, $d = 760$ pc, and the half-width of the pulsar profile, $W_{0.5} = 4.7 \times 10^{-2} P$ (J. H. Taylor et al. 1995, unpublished).[1] We assume that the width of the window in the optical range is the same as at radio frequencies, by analogy with PSR 0531+21 in the Crab Nebula (Smith 1977). On the other hand, for frequencies

$$\frac{\nu_B}{\gamma} \le \nu < \frac{\nu_B}{\gamma\psi^2} \quad (31)$$

up to the spectral peak, the power emitted by a single electron can be approximately written as (Epstein 1973)

$$P_\nu = \frac{2\pi^2 e^2 \psi^2 \nu}{c} . \quad (32)$$

To estimate the total power, this quantity must be multiplied by the number of emitting electrons, nV. We take for the volume V the value $0.1 r_{\rm LC}^3$. Then the particle number density must be $\sim 10^9$ cm^{-3} to produce the luminosity given in equation (30). This constraint on the density is quite severe. However, we emphasize that, according to the distribution equation (18), there are some particles with larger values of ψ. This means that the requirement placed on n could be weaker, since $P_\nu \propto \psi^2$.

The aim of the present paper was to investigate the fundamental possibility of explaining the presence of optical emission from radio pulsars in the framework of the synchrotron mechanism and the conditions under which this mechanism works. A quantitative theory can be developed after investigation of the structure of the outer layers in the magnetosphere, with detailed descriptions of the character of the magnetic field and the distributions of the density and Lorentz factors of relativistic particles.

4. DISCUSSION

1. The possibility of using the synchrotron mechanism to explain optical emission from pulsars was considered in early papers on these objects (see, e.g., Ginzburg, Zheleznyakov, & Zaitsev 1969; Shklovsky 1970). A comprehensive synchrotron model accounting for optical, X-ray, and gamma-ray emissions of PSR 0531+21 was put forward by Zheleznyakov & Shaposhnikov (1972). The last effort in this direction was made by Malov & Machabeli (1999).

Zheleznyakov & Shaposhnikov (1972) assumed that the pitch angles of emitting electrons were large ($\psi \sim 1$ rad). If the pitch angles are produced by the mechanism we have considered here, it is evident that this assumption is not valid ($\psi \ll 1$). Moreover, the synchrotron model proposed by Zheleznyakov & Shaposhnikov works for power-law or piecewise power-law electron-energy distribution functions only. The parameter

$$\xi = \frac{B^2}{8\pi w_p} , \quad (33)$$

where w_p is the energy density of the plasma, is above 2×10^3 at the light cylinder of PSR 0531+21; all other parameters for this pulsar have been expressed in terms of this parameter. In this case, the size of the radiation source,

$$D = 4.78 \times 10^7 \xi^{1/17} \text{ cm} , \quad (34)$$

proves to be half the radius of the light cylinder; i.e., it appears to be very large. In addition, it is not clear to what extent this model is applicable to other pulsars.

2. Consideration of processes leading to changes in the transverse momenta of relativistic particles in a pulsar magnetosphere shows that the quasi-stationary distribution of particle pitch angles, ψ, is formed by these processes. The mean pitch angle for PSR 0656+14 is $\bar{\psi} \sim 10^{-6}$. This pulsar has been detected at several optical frequencies, and we have used it here to illustrate the results obtained.

3. If the pitch angle of a relativistic particle $\psi \ll 1$, it emits synchrotron radiation with specific properties. The peak

[1] Available at ftp://pulsar.princeton.edu/pub/catalog.

frequency of this emission for PSR 0656+14 is in the optical range ($\nu \sim 3 \times 10^{14}$ Hz).

4. We consider the optical radiation to be associated with the primary beam particles with Lorentz factor $\gamma_b = 10^6$. Here we assume that the energy distribution in the beam has a Gaussian shape, with $\gamma_m = \gamma_b = 10^6$ and $\sigma = \gamma_t = 10^2$:

$$N(\gamma) = N_0 \exp\left[-\frac{(\gamma - \gamma_b)^2}{\gamma_t^2}\right], \quad (35)$$

where $N_0 = n_b/\sqrt{\pi}\gamma_t$, and $n_b = B/Pce$ is the density of primary-beam particles, equal to the Goldriech-Julian density. Since the thermal width $\gamma_t \ll \gamma_b$, this distribution is very close to a δ-function. Consequently, in the case of incoherent synchrotron radiation, which is, as we believe, the origin of the optical radiation, under the assumptions indicated above, the electron distribution can be taken as monoenergetic. In this case, we can use the power of a single electron multiplied by the number of emitting particles to estimate the total luminosity, L_ν, and then compare it with the observed one (eq. [30]).

5. The optical flux densities for the pulsars under investigation are very similar (10^{-30}–10^{-29} erg s^{-1} Hz^{-1}; Thompson 1996; Bignami et al. 1996). The spectrum of PSR 0531+21 is harder than that of PSR 0656+14, as expected in the proposed model. In fact, substituting $B = [8\pi^3 B_s R_*^3 (r_{LC}/r)^3]/(c^3 P^3)$ and $B_s(G) = 6.4 \times 10^{19}(P\dot{P})^{1/2}$ into equation (25), we find

$$\bar{\psi} \approx 6 \times 10^{-17} \frac{\gamma_b P^{3/2}(r/r_{LC})^3}{\gamma_p \dot{P}_{-14}^{1/2}}, \quad (36)$$

or

$$\psi_H = 3 \times 10^{-10} \frac{\gamma_b P^{3/2}(r/r_{LC})^3}{\gamma_p^2 \dot{P}_{-14}^{1/2}}, \quad (37)$$

where $\dot{P}_{-14} = \dot{P}/10^{-14}$. In this case, as follows from equation (29),

$$\nu_m = 1.8 \times 10^{27} a(\gamma\psi) \frac{\gamma_p^4 \dot{P}_{-14}^{3/2}(r/r_{LC})^9}{P^{11/2}\gamma_b^3}, \quad (38)$$

i.e., the shorter the pulsar's period, the harder its spectrum.

6. As we can see from Table 1, the condition $\gamma_b \psi_H \leq 1$ is fulfilled at the light cylinder for all pulsars, and here equation (37) can be used. We must take into account that $\sin\beta \leq r_{LC}/r \leq 1$. Using the values P and $\dot{P}$, in this case we obtain for $a(\gamma_b\psi_H) \approx 1$ the estimates of the peak frequencies ν_m listed in the last column of Table 1. These estimates show that the proposed mechanism can produce optical emission in pulsars PSR B0656+14, B1055−52, B1929+10, and J0633+1746 at distances $r \geq r_{LC}$. The spectral peak in PSR B0833−45 occurs at frequencies above 10^{18}–10^{19} Hz, and optical emission can be observed in this pulsar on the low-frequency tail of the synchrotron spectrum only. There is no synchrotron optical radiation in PSR B0950+08, and its emission at optical frequencies must have a thermal nature. PSR B0531+21, PSR B0540−69, and PSR B1509−58 can have the maximum of the synchrotron spectrum in the optical range ($\nu_m = 5 \times 10^{14}$ Hz) for very small values of the angle β (these values are shown in Table 1 in brackets) and at distances $r \approx r_{LC}/\sin\beta$. The alternative possibility is to suggest that their optical emission is the low-frequency tail of the synchrotron spectrum (similar to the pulsar PSR B0833−45).

7. Equation (37) can be used if $\gamma_b \psi_H \leq 1$. However, as we can see from Table 1, $\gamma_b \psi_H \gg 1$ for some distances in a number of pulsars. In this case, we must use formulae for usual synchrotron emission. For example, the frequency of the maximum of the synchrotron spectrum will be equal to (Pacholczyk 1970)

$$\nu_m = 0.3\nu_c = \frac{0.9}{4\pi} \frac{eB \sin\psi \gamma_b^2}{mc}. \quad (39)$$

Using our results, we can obtain in this case the next very simple formula,

$$\nu_m = \frac{0.9\gamma_b^3}{8\sqrt{2\pi}\gamma_p P} = \frac{4.5 \times 10^{15}}{P}, \quad (40)$$

where the values $\gamma_b = 10^6$ and $\gamma_p = 10$ are used. The maximum in such spectra will be in the ultraviolet range for pulsars with $P \sim 1$ s and in the soft X-ray diapason if $P \leq 0.1$ s. The data of Table 1 show that radio pulsars with observed optical emission have short periods ($\bar{P} = 180$ ms) and small values of the angle β ($\bar{\beta} = 15.^\circ5$ for six pulsars).[2] If optical emission is generated in these pulsars by the usual synchrotron mechanism, it will be connected with the low-frequency tail of the synchrotron spectrum.

8. To explain observed optical fluxes in PSR 0656+14, we must assume that the concentration of emitting particles is $\leq 10^9$ cm^{-3}. The wavelength of the observed optical emission ($\lambda \sim 10^{-5}$–10^{-4} cm) for such concentration is considerably less than $n^{-1/3}$. Therefore, the incoherent emission condition $\lambda \ll n^{-1/3}$ noted in the formulation of the kinetic equation (14) is indeed satisfied.

9. We have taken $\gamma_p = 10$ in this paper. If the magnetic field is dipolar near the surface of a neutron star, this value must be much more ($\gamma_p \geq 10^3$). However, the maximum of the distribution function of the secondary plasma must occur at Lorentz factors ≤ 10 (Machabeli & Usov 1989) in the more complex field, realized as it is believed to exist in radio pulsars.

10. The additional argument for the synchrotron nature of optical emission in radio pulsars comes from the correlation between the total optical luminosity, L_B, and the magnetic field at the light cylinder (Shearer, Golden, & Beskin 2000). Moreover, this correlation shows that such optical radiation is generated near the light cylinder, as we argue here.

11. The proposed model can be verified using its prediction of a higher frequency for the spectral peak in pulsars with shorter periods.

12. In this paper we take into account the reaction force of synchrotron emission on any individual particle, This can be done because synchrotron frequencies are in the high-frequency (optical) range, where $\lambda \ll n^{-1/3}$, and we can consider radiation of individual particles.

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[2] We do not know β for PSR 0540−69 and 1509−58, and we exclude the Crab pulsar, because its β could be incorrect (see Malov 1999).

REFERENCES

Arons, J., & Barnard, J. J. 1986, ApJ, 302, 120
Bignami, G. F., Caraveo, P. A., Mignani, R., Edelstein. J., & Bowyer, S. 1996, ApJ, 456, L111
Caraveo, P. A. 1999, in ASP Conf. Ser. 202, Pulsar Astronomy—2000 and Beyond, ed. M. Kramer, N. Wex, & R. Wielebinski (San Francisco: ASP), 289
Epstein, R. I. 1973, ApJ, 183, 593
Ginzburg, V. L., Zheleznyakov, V. V., & Zaitsev, V. V. 1969, Usp. Fiz. Nauk, 98, 201
Goldreich, P., & Julian, W. H. 1969, ApJ, 157, 869
Hardee, P. E., & Rose, W. K. 1978, ApJ, 219, 274
Kazbegi, A. Z., Machabeli, G. Z., & Melikidze, G. I. 1992, in Proc. IAU Colloq. 128, The Magnetospheric Structure and Emission Mechanisms of Radio Pulsars, ed. T. H. Hankins, J. M. Rankin, & J. A. Gil (Zielona Gora: Pedagogical Univ. Press), 232
Kurt, V. G., Sokolov, V. V., Zharikov, S. A., Pavlov, G. G., & Komberg, B. V. 1998, A&A, 333, 547
Landau, L. D., & Lifshitz, E. M. 1971, Classical Theory of Fields (London: Pergamon)
Lominadze, D. G., Machabeli, G. Z., Melikidze, G. I., & Pataraya, A. D. 1986, Soviet J. Plasma Phys., 12, 712
Lominadze, D. G., Machabeli, G. Z., & Mikhailovskii, A. B. 1979, Fiz. Plaz., 5, 1337
Lyutikov, M., Blandford, R. D., & Machabeli, G. 1999, MNRAS, 305, 338
Machabeli, G. Z., & Usov, V. V. 1979, AZh Pis'ma, 5, 445
———. 1989, AZh Pis'ma, 15, 910
Malov, I. F. 1990, AZh, 67, 377
———. 1999, AZh, 76, 825
Malov, I. F., & Machabeli, G. Z. 1999, AZh, 76, 788
Pacholczyk, A. G. 1970, Radio Astrophysics (San Francisco: W. H. Freeman)
Pavlov, G. G., Welty, A. D., & Cordova, F. A. 1997, ApJ, 489, L75
Sagdeev, R. S., & Shafranov, V. D. 1960, Zh. Eksp. Teor. Fiz., 39, 181
Shearer, A., Golden, A., & Beskin, G. 2000, in ASP Conf. Ser. 202, Pulsar Astronomy—2000 and Beyond, ed. M. Kramer, N. Wex, & R. Wielebinski (San Francisco: ASP), 307
Shklovsky, I. S. 1970, ApJ, 159, L77
Sivukhin, D. V. 1963, Voprosi Teorii Plasmy (Moscow: Gosatomizdat) 1, 7
Smith, F. G. 1977, Pulsars (Cambridge : Cambridge Univ. Press)
Thompson, D. J. 1996, in ASP Conf. Ser. 105, Pulsars: Problem and Progress, ed. S. Johnston, M. A. Walker, & D. M. Bailes (San Francisco: ASP), 307
Volokitin, A. S., Krasnosel'skikh, V. V., & Machabeli, G. Z. 1985, Fiz. Plazmy, 11, 310
Zheleznyakov, V. V., & Shaposhnikov, V. E. 1972, Ap&SS, 18, 141