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GAMMA-RAY BURST ENERGETICS

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ABSTRACT

We estimate the fraction of the total energy in a gamma-ray burst (GRB) that is radiated in photons during the main burst. Random internal collisions among different shells limit the efficiency for converting bulk kinetic energy to photons. About 1% of the energy of explosion is converted to radiation, in the 10–10^3 keV energy band in the observer frame, for long-duration bursts (lasting 10 s or more); the efficiency is significantly smaller for shorter duration bursts. Moreover, about 50% of the energy of the initial explosion could be lost to neutrinos during the early phase of the burst if the initial fireball temperature is ∼10 MeV. If isotropic, the total energy budget of the brightest GRBs is $\gtrsim 10^{55}$ ergs, a factor of $\gtrsim 20$ larger than previously estimated. Anisotropy of explosion, as evidenced in two GRBs, could reduce the energy requirement by a factor of 10–100. Putting these two effects together, we find that the energy release in the most energetic bursts is about 10^{54} ergs.

Subject headings: gamma rays: bursts — gamma rays: theory

1. INTRODUCTION

The short (millisecond) time variability of gamma-ray bursts is believed to arise in internal shocks, i.e., when faster moving ejecta from the explosion collides with slower moving material ejected at an earlier time (Narayan, Paczyński, & Piran 1992; Paczyński & Xu 1994; Rees & Mészáros 1994; Sari & Piran 1997). The optical identification and measurement of redshifts for five gamma-ray bursts (GRBs) have determined their distances and the amount of energy that would be radiated in an isotropic explosion (e.g., Metzger et al. 1997; Kulkarni et al. 1998; Kelson et al. 1999; Piran 1999; and references therein). In three of these cases (GRB 971214, GRB 980703, and GRB 990123), the total isotropic energy radiated is estimated to be in excess of 10^{53} ergs. For GRB 990123, the isotropic energy in the gamma-ray burst is estimated to be 3.4×10^{54} ergs. However, the steepening of the falloff of the optical light curve, ∼2 days after the explosion, suggests that the explosion was not isotropic, and the total radiated energy might only be $\sim 6 \times 10^{52}$ ergs (Kulkarni et al. 1999; Mészáros & Rees 1999). There is little evidence for beaming in the other two cases.

The energy radiated in photons in gamma-ray bursts is only a fraction of the total energy released in the explosion. Collisions of shells or ejecta from the central source, believed to produce the highly variable gamma-ray burst emission, converts but a small fraction of the kinetic energy of the ejecta into thermal energy which is shared among protons, electrons, and magnetic field. If the initial temperature of the fireball is larger than a few MeV, then a fraction of the fireball energy is lost to neutrinos. Thus, a significantly larger amount of energy than "observed" must be released in these explosions. The purpose of this Letter is to provide an estimate for the radiative efficiency of GRBs in the framework of the internal shock model (§ 2). Some aspects of the work presented here have been previously considered by Kobayashi, Piran, & Sari (1997) and Daigne & Mochkovitch (1998). The main points are summarized in § 3.

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A fraction of the kinetic energy of ejecta in GRBs is converted into photons as a result of internal collision during the main burst. This efficiency factor is calculated below in § 2.1. Just after the explosion, when the adiabatic cooling is small and the temperature of the fireball is several MeV, neutrinos are copiously produced and carry away a fraction of the energy of the explosion. The fraction of energy lost to neutrinos is calculated in § 2.2.

2.1. *Efficiency of Internal Shocks*

The efficiency of conversion of the kinetic energy of ejecta to radiation via internal shocks has been considered by Kobayashi et al. (1997) and Daigne & Mochkovitch (1998). There are several differences between the calculation presented here and previous works. One is that we calculate synchrotron emission from forward and reverse shocks in colliding shells and Compton upscattering of photon energy, by solving appropriate equations for shock and radiation, to determine the observed fluence in the 10–10^3 keV energy band. We also take into consideration that about one-third of the total thermal energy produced in colliding shells is taken up by electrons, and only this fraction is available to be radiated away. Finally, we treat in a consistent manner energy radiation in shell collisions when the fireball is optically thick to Thomson scattering. In this case, photons do not escape the expanding ejecta but instead deposit their energy back into shells and increase the kinetic energy of ejecta. Most of this kinetic energy is not converted back to thermal energy until some later time when interstellar material is shocked. The reason for this is that shell mergers reduce the relative Lorentz factor of remaining shells and their subsequent mergers produce less thermal energy. The optical depth is important for bursts of duration 10 s or less (hereafter referred to as short-duration bursts).

We model the central explosion as resulting in random ejection of discrete shells, each carrying a random amount of energy (ϵ_i) and with a random Lorentz factor (γ_i). The baryonic mass of ith shell (m_i) is set by its energy (ϵ_i) and γ_i; $m_i = \epsilon_i/(c^2\gamma_i)$. The time interval between the ejection of two consecutive shells is taken to be a random number with mean time interval such as to give the desired total burst duration. The Lorentz factor of shells is taken to be uniformly distributed between a minimum ($\gamma_{\min} = 5$) and a maximum ($\gamma_{\max}$) value. The energy conversion efficiency is more or less independent of the number of shells ejected in the explosion so long as the number of shells is greater than a few.

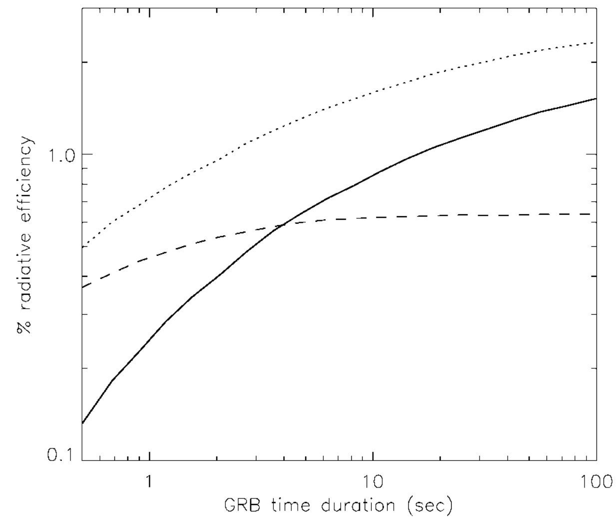

FIG. 1.—Efficiency for the conversion of the energy of explosion to radiation, in the energy band 10–10^3 keV, via internal shocks (η*100) is shown as a function of the time duration of GRBs. The energy lost to neutrinos is highly temperature dependent and has not been included in this calculation. The continuous curve corresponds to the maximum Lorentz factor of the ejected shells to be 200, and for the dotted curve the maximum Lorentz factor is 500. The minimum value of the Lorentz factor in both these cases was taken to be 5. The minimum Lorentz factor for the dashed curve was taken to be 50 and $\gamma_{\max} = 200$. Each point on the curve was calculated by averaging 250 realizations of "explosions" in which 50 shells were randomly expelled as described in § 2.1. The total energy in each of the explosions was taken to be 10^{52} ergs, which was independent of the burst duration. The radiative efficiency is almost independent of the number of shells ejected so long as the number is larger than a few.

When two cold shells i and j collide and merge, the thermal energy produced is

$$\Delta E = \gamma_f[(m_i^2 + m_j^2 + 2m_i m_j \gamma_r)^{1/2} - (m_i + m_j)]c^2,$$

where $\gamma_r = \gamma_i\gamma_j(1 - v_i v_j)$ is the Lorentz factor corresponding to the relative speed of collision and $\gamma_f = (m_i\gamma_i + m_j\gamma_j)(m_i^2 + m_j^2 + 2m_i m_j\gamma_r)^{-1/2}$ is the final Lorentz factor of the merged shells. The energy ΔE is shared among protons, electrons, and magnetic field. In equipartition, electrons take up one-third of the total energy, which is available to be radiated. In collisions involving two equal-mass shells with $\gamma_r = 2$, 6% of the energy can be radiated away, whereas collisions with $\gamma_r = 10$ result in a loss of 19% of the energy. The average relative Lorentz factor of shell collisions is about 2 if shells are randomly ejected in a relativistic explosion. Thus, the average bolometric radiative efficiency of internal shocks is about 6%. Approximately one-fourth of the total radiative energy lies in the 10–10^3 keV energy band, and therefore the effective radiative efficiency of internal shocks, in the observed energy band, is about 1%. More precise results from numerical simulations are presented below.

The timescale for the transfer of energy from protons to electrons due to Coulomb collisions, even when the number density of protons is $\sim 10^{13}$ cm^{-3} at a time when the fireball is just becoming optically thin, is much longer than the dynamical time, and so we assume that there is little transfer of energy from protons to electrons on the timescale of interest for internal shocks.

The synchrotron cooling time t_s is typically much less than the dynamical time within the first few minutes of the burst and does not limit the efficiency of GRBs. In any case, we include the effect of finite synchrotron cooling time on the radiative efficiency. We also include the inverse Compton cooling of electrons to calculate the spectrum and the fraction of thermal energy radiated away in internal shocks.

Following each shell collision, we calculate the thermodynamical state of the shocked gas and the emergent photon spectrum resulting from synchrotron emission plus the inverse Compton scattering. The optical depth for emergent photons to Thomson scattering is calculated by following their trajectory along with the trajectory of shells. If the optical depth of the fireball is greater than a few, the photon energy gets converted back to the energy of bulk motion via adiabatic expansion and the momentum deposit by photons. The energy δE_j and momentum δP_j incident on a shell j (as measured in its rest frame) by photons created in a colliding shell an optical depth τ_j away, which is moving with a relative velocity v_{cj} toward the jth shell, are given by

$$\delta E_j = \eta_j \gamma_{cj}(1 + v_{cj})^2$$

and

$$\delta P_j = \frac{\eta_j}{c(v_{cj}\gamma_{cj})^3} \times [\gamma_{cj}^4(1 + v_{cj})^2 - 4\gamma_{cj}^2(1 + v_{cj})v_{cj} + 2\ln \gamma_{cj}^2(1 + v_{cj}) - 1],$$

where $\eta_j = \Delta E \exp(-\tau_j)[1 - \exp(-\delta\tau_j)]/(6\gamma_f)$ is the energy incident on the jth shell if it were stationary with respect to the center of momentum of the colliding shells, γ_f is the Lorentz factor of merged shells, and $\delta\tau_j$ is the optical depth of the jth shell. For τ_j dominated by scattering opacity, the flux from a steady source is attenuated by a factor of $\sim 1/\tau_j$ instead of $\exp(-\tau_j)$ given above. However, the energy/momentum received from a transient source on the short, photon transit time is reduced by a factor of $\exp(-\tau_j)$. The remainder of the energy/momentum is received on a longer timescale, of order photon diffusion time, and is included in our numerical computation where appropriate. For the elastic Thomson scattering by cold electrons, the incident photon energy is only partially absorbed in optically thick shells as a result of the adiabatic expansion of the shell. The momentum intercepted by a shell that is moving away from the energy producing shell is smaller by a factor γ_{cj}^4 and is given by

$$\delta P_j = \frac{\eta_j}{c(v_{cj}\gamma_{cj})^3}\left[\frac{(1 + v_{cj})^2 - 1}{(1 + v_{cj})^2} - \frac{4v_{cj}}{1 + v_{cj}} + 2\ln(1 + v_{cj})\right].$$

The energy and momentum absorbed by the shell determines the change to its bulk velocity and its expansion, which we include in our numerical simulation to determine the radiative efficiency of internal collisions. Also included in our calculation is the conversion of the thermal energy of protons and the magnetic field to bulk motion as a result of adiabatic expansion.

The radiative efficiency η of a burst is defined as the total energy radiated in the 10–10^3 keV energy band, during a time interval in which shell collisions take place, divided by the total energy released in the explosion. Figure 1 shows a plot of η as a function of burst duration. The total energy in bursts in all of the cases shown in the figure was taken to be 10^{52} ergs, independent of the burst duration. The value of η is found be about 1% for long-duration bursts. The bolometric radiative efficiency of random internal shocks is found to be larger by a factor of about 4. The efficiency decreases with decreasing duration (for a fixed $\gamma_{\max}$). Internal shocks are very inefficient for short-duration bursts because of photon trapping, since a number of shell collisions occur when the shell radii are small and the fireball is optically thick. For instance, the radiative efficiency for bursts of 1 s duration is about 0.2% if $\gamma_{\max} = 200$. The radiative efficiency for short-duration bursts can increase significantly if the Lorentz factor of ejecta is larger in shorter duration bursts (see Fig. 1). Choice of a different distribution function for the Lorentz factor of ejecta has little effect on the efficiency of long-duration bursts. However, the efficiency of short-duration bursts can increase significantly if the width of the distribution function is taken to be small so that shells collide at larger radii, enabling photons to escape freely; for instance, in the case in which $\gamma_{\min} = 50$ and $\gamma_{\max} = 200$, the radiative efficiency is nearly constant ($\eta \approx 0.006$) for bursts of duration 1 s and longer (see Fig. 1). The efficiency for short-duration bursts is also enhanced if they are less energetic than longer duration bursts, thereby requiring smaller baryonic loading.

2.2. *Energy Loss Due to Neutrino Production*

Some fraction of gamma-ray bursts display variability on a millisecond timescale, if not smaller. The energy of explosion in these cases is expected to be generated in a region of size about 100 km. If the total energy release in an explosion underlying a GRB is E and it involves ejection of N shells, each of which have an initial radius of r_0, then the mean initial temperature of shells is $T_0 = [3E_n/(4Na\pi r_0^3)]^{1/4} = 20.6$ MeV $E_{53}^{1/4} r_{100}^{-3/4} N^{-1/4}$, where a is the radiation constant, E_{53} is energy in units of 10^{53} ergs, and $r_{100} = r_0/100$ km. We note that the energy of the explosion (E) is greater than the observed energy in the gamma-ray emission by a factor of at least 10 because of the inefficiency of photon production discussed in § 2.1. Moreover, the value of E that should be used in calculating the temperature is the total isotropic energy of explosion and not the reduced energy due to the finite opening angle of jet, so long as the jet was produced in the initial explosion and not by some collimation effect of the surrounding medium subsequent to a spherical explosion. Thus, $E \approx 10^{53}$ ergs is a reasonable value for the five GRBs with known redshift distance.

Neutrinos produced by e^--e^+ annihilation and the decay of muons and pions result in a loss of a fraction of the energy of explosions. The energy-loss rate due to e^--e^+ annihilation is given by

$$\frac{dE_n}{dt} = -2n_e c\sigma_e \epsilon_e (4\pi r^2 r_0 n_e),$$

where $E_n = E/N$, n_e is the number density of electrons, ϵ_e is the mean thermal energy of electrons, $4\pi r^2 r_0$ is the volume of the shell in its comoving frame when the shell has expanded to a radius r (the shell thickness r_0 is very nearly constant in the initial acceleration phase), and $\sigma_e = 2 \times 10^{-44}(\epsilon_e/1\ \mathrm{MeV})^2\ \mathrm{cm}^2$ is the effective cross section for e^+ and e^- annihilation to produce neutrinos of all different flavors. Since $E_n \approx 12\pi r^2 r_0 n_e \epsilon_e \gamma$, $n_e = 2.34 \times 10^{34} T_{10}^3\ \mathrm{cm}^{-3}$ ($T_{10} = T/10$ MeV), and $\epsilon_e = 3.15kT$, we find

$$\frac{d\ln E}{dt} = -\frac{9.5 \times 10^3}{\gamma} T_{10}^5.$$

Initially the Lorentz factor of shells (γ) increases linearly with their radius and the temperature declines as the inverse of the radius. Using these relations, we can integrate the above equation and find that

$$\ln\left[\frac{E(2t_0)}{E(t_0)}\right] = -1.9 \times 10^3 t_0 \left(\frac{T_0}{10\ \mathrm{MeV}}\right)^5,$$

where t_0 is the larger of r_0/c and the time when the shell becomes optically thin to neutrinos; shells become optically thin to electron neutrinos when $T_0 \leq 10.2$ MeV. A neutrino propagating outward sees the mean electron energy and density decrease, and therefore the opacity for scattering in an expanding medium is smaller than a corresponding static shell. For $r_0 = 10^7$ cm and $T_0 = 7$ MeV, we find that 10% of the energy of the explosion is lost to neutrinos from e^+-e^- annihilation, and for $T_0 = 10$ MeV, 50% of the energy is lost.

We next calculate the fraction of energy carried away by neutrinos produced by the decay of muons and pions. Let us consider an unstable particle ($\mu^\pm$ or $\pi^\pm$) of mass m_d that has a lifetime of t_d, number density n_d, and an amount of energy carried by neutrinos when it decays ϵ_ν. In the temperature range of interest to us, these particles are created by $e^\pm$ interaction on a timescale short compared to their decay time, and so their number density is given by the thermal distribution, i.e.,

$$n_d = 10.5\ T^3 \left(\frac{m_d}{kT}\right)^{3/2} \exp(-m_d c^2/kT)\ \mathrm{cm}^{-3}.$$

The rate of loss of energy of the explosion to escaping neutrinos produced by the decay of these particles is given by

$$\frac{dE}{dt} = -\frac{8\pi r^2 r_0 n_d \epsilon_\nu}{t_d} \approx -\frac{E\epsilon_\nu}{8t_d kT}\left(\frac{m_d c^2}{kT}\right)^{3/2} \exp(-m_d c^2/kT).$$

This equation can be easily integrated to yield[1]

$$\ln\left[\frac{E(2t_0)}{E(t_0)}\right] = -\frac{t_0}{t_d}\frac{\epsilon_\nu}{8kT}\left(\frac{m_d c^2}{kT_0}\right)^{1/2} \exp(-m_d c^2/kT).$$

For the muons $m_d = 105.66$ MeV, $t_d = 2.2 \times 10^{-6}$ s, and $\epsilon_\nu \approx 70$ MeV. Thus the fraction of energy lost by the decay of $\mu^\pm$ for $T_0 = 10$ MeV and $t_0 = 3.3 \times 10^{-4}$ s is 0.5%, whereas at $T_0 = 15$ MeV, 10% of the energy of the fireball is lost to neutrinos from muon decay.

For pions $m_d = 139.6$ MeV, $t_d = 2.55 \times 10^{-8}$ s, and $\epsilon_\nu \approx 29$ MeV. The fraction of energy lost by the decay of $\pi^\pm$ if we take $T_0 = 10$ MeV and $t_0 = 3.3 \times 10^{-4}$ s is 2%, whereas at $T_0 = 15$ MeV, 50% of the energy of the explosion is lost to neutrinos from pion decay.

The energy of these pre-GRB neutrinos is about 10–30 MeV, and they are undetectable from a typical GRB source at $z \sim 1$.

3. SUMMARY AND DISCUSSION

We find that the efficiency for internal shocks to convert the energy of explosion to radiation in the 10–10^3 keV energy band is of order 1% if electrons are in equipartition with protons and magnetic field. The efficiency is smaller if the electron energy is less than the equipartition value as suggested by

[1] The ν_μ's produced in these decays find the shell to be optically thin so long as the shell temperature is less than about 15 MeV. For $T_0 \gtrsim 15$ MeV, the ν_μ's are trapped in the fireball and their distribution is thermal in equilibrium with $e^\pm$. In this case, roughly 50% of the fireball energy is lost to neutrinos.

analysis of afterglow emission. Energy loss due to neutrino production at initial times, when the fireball temperature is ∼10 MeV for short-duration bursts, could be significant, further reducing the energy available for radiation by a factor of ∼2. The bolometric radiative efficiency of random internal shocks is found to be a factor of about 4 larger. A recent work of Panaitescu, Spada, & Mészáros (1999) finds the radiative efficiency of internal shocks in the 50–300 keV band to be about 1% and is consistent with our result.

For GRB 971214, GRB 980703, and GRB 990123, the total isotropic energy radiated in the BATSE energy band has been estimated from their observed redshifts and fluences and found to be 3×10^{53}, 2×10^{53}, and 3.5×10^{54} ergs, respectively. The flux in higher energy photons could increase the total energy budget by a factor of ∼2. These three bursts are the most energetic of the five bursts for which redshifts (or lower limits to z) are known. These energies should of course be corrected for beaming and the efficiency for photon production.

It has been suggested that the energy for GRB 990123, 3.5×10^{54} ergs for isotropic explosion, is reduced by a factor of about 50 due to finite beaming angle (Kulkarni et al. 1999; Mészáros & Rees 1999). However, the inefficiency of producing radiation raises the energy budget by a factor of about 100, so the energy in the explosion is more than 10^{54} ergs even if beaming is as large as suggested. For GRB 980703 (at $z = 0.966$), for which there is no evidence for beaming, the energy in the explosion is also of order 10^{54} ergs. So it appears that the total energy of explosion for the most energetic bursts is close to or possibly greater than 10^{54} ergs. This energy is greater than what one can realistically hope to extract from a neutron star–mass object.

The efficiency for gamma-ray production is significantly increased if photons during the main burst are produced in both internal and external shocks. However, since it is very difficult to get short time variability in external shocks (Sari & Piran 1997), only a small fraction of energy in highly variable bursts can arise in external shocks. The energy requirement is also reduced if shells ejected in explosions are highly inhomogeneous. This will be discussed in a future paper.

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