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ACCURATE DETERMINATION OF THE LAGRANGIAN BIAS FOR THE DARK MATTER HALOS

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ABSTRACT

We use a new method, the cross-power spectrum between the linear density field and the halo number density field, to measure the Lagrangian bias for dark matter halos. The method has several important advantages over the conventional correlation function analysis. By applying this method to a set of high-resolution simulations of 256^3 particles, we have accurately determined the Lagrangian bias, over 4 mag in halo mass, for four scale-free models with the index $n = -0.5$, -1.0, -1.5, and -2.0 and three typical cold dark matter models. Our result for massive halos with $M \geq M_*$ (M_* is a characteristic nonlinear mass) is in very good agreement with the analytical formula of Mo & White for the Lagrangian bias, but the analytical formula significantly underestimates the Lagrangian clustering for the less massive halos, $M < M_*$. Our simulation result, however, can be described satisfactorily, with an accuracy better than 15%, by the fitting formula of Jing for Eulerian bias under the assumption that the Lagrangian clustering and the Eulerian clustering are related with a linear mapping. It implies that it is the failure of the Press-Schechter theories for describing the formation of small halos that leads to the inaccuracy of the Mo & White formula for the Eulerian bias. The nonlinear effect in the mapping between the Lagrangian clustering and the Eulerian clustering, which was speculated as another possible cause for the inaccuracy of the Mo & White formula, must be negligible compared with the linear mapping. Our result indicates that the halo formation model adopted by the Press-Schechter theories must be improved, as independently stressed by Porciani et al.

Subject headings: cosmology: theory — dark matter — galaxies: formation — large-scale structure of universe

1. INTRODUCTION

Galaxies and clusters of galaxies are believed to form within the potential wells of virialized dark matter (DM) halos. Understanding the clustering of DM halos can provide important clues to understanding the large-scale structures in the universe. Therefore, a number of studies have been carried out to obtain the two-point correlation function ξ_{hh} of DM halos. Two distinctive approaches are widely adopted. One is analytical and is based on the Press-Schechter (PS) theories (see, e.g., Kashlinsky 1987, 1991; Cole & Kaiser 1989; Mann, Heavens, & Peacock 1993; Mo & White 1996, hereafter MW96; Catelan et al. 1998a; Porciani et al. 1998b). The other is numerical and is based on *N*-body simulations (see, e.g., White et al. 1987; Bahcall & Cen 1992; Jing et al. 1993; Watanabe, Matsubara, & Suto 1994; Gelb & Bertschinger 1994; Jing, Börner, & Valdarnini 1995; MW96; Mo, Jing, & White 1996; Jing 1998; Ma 1999; Porciani, Catelan, & Lacey 1998a). The up-to-date version of the analytical studies is given by MW96, which states that $\xi_{hh}(r, M)$ of halos with a mass M is proportional to the DM correlation function $\xi_{mm}(r)$ on the linear clustering scale [$\xi_{mm}(r) \ll 1$], i.e., $\xi_{hh}(r, M) = b^2(M)\xi_{mm}(r)$, with the bias factor

$$b_{\rm mw}(M) = 1 + \frac{\nu^2 - 1}{\delta_c}, \tag{1}$$

where $\delta_c = 1.68$, $\nu \equiv \delta_c/\sigma(M)$, and $\sigma(M)$ is the linearly evolved rms density fluctuation of top-hat spheres containing, on average, a mass M (see MW96 and references therein for more details about these quantities). The subscript "mw" for $b(M)$ in equation (1) denotes that the result is analytically derived by MW96. On the other hand, the most accurate simulation result was recently presented by our recent work (Jing 1998), where we studied ξ_{hh} for halos in four scale-free models and three cold dark matter (CDM) models with the help of a large set of high-resolution *N*-body simulations of 256^3 particles. Our result shows unambiguously that while the bias is linear on the linear clustering scale, the bias factor given by MW96 significantly underestimates the clustering for small halos with $\nu < 1$. Our simulation results, for both the CDM models and the scale-free models, can be accurately fitted by

$$b_{\rm fit}(M) = \left(\frac{0.5}{\nu^4} + 1\right)\left(1 + \frac{\nu^2 - 1}{\delta_c}\right), \tag{2}$$

where n is the index of the linear power spectrum $P_m(k)$ at the halo mass M:

$$n = \left.\frac{d \ln P_m(k)}{d \ln k}\right|_{k=2\pi/R}; \quad R = \left(\frac{3M}{4\pi\bar{\rho}}\right)^{1/3}. \tag{3}$$

In the above equation, $\bar{\rho}$ is the mean density of the universe.

MW96 derived their formula in two steps. First they obtained the bias factor $b^{\rm L}_{\rm mw}(M)$ in the Lagrangian space using the PS theories. The Lagrangian bias reads

$$b^{\rm L}_{\rm mw}(M) = \frac{\nu^2 - 1}{\delta_c}. \tag{4}$$

But the bias that is observable is in the Eulerian space. MW96 obtained the Eulerian bias (eq. [1]) with a *linear mapping* from the Lagrangian clustering pattern, $b_{\rm mw}(M) = b^{\rm L}_{\rm mw}(M) + 1$ (cf. Catelan et al. 1998a). From their derivation, we conjectured in Jing (1998) that two possibilities could have failed the MW96 formula for small halos. The first possibility is that the PS theories are not adequate for describing the formation of small

halos. The halo formation in the PS theories is uniquely determined by the local peak height through the spherical collapse model, while in reality, halo formation, especially the formation of small halos, can be significantly influenced by the nonlocal tidal force (see, e.g., Katz, Quinn, & Gelb 1993 and Katz et al. 1994). A recent analysis by Sheth & Lemson (1998) for simulations of 100^3 particles also gave some evidence that the Lagrangian bias of small halos has already deviated from the MW96 prediction (eq. [4]). The possible invalidity of the linear mapping, the second possibility that fails the MW96 formula, was discussed recently by Catelan, Matarrese, & Porciani (1998b). They pointed out that the linear mapping might not be valid for small halos because of a large-scale nonlinear tidal force. It is important to find out if one or a combination of the two possibilities can *quantitatively* explain the failure of the MW96 formula.

In this Letter, we report our new determination for the Lagrangian bias factor $b^{\rm L}(M)$ using the simulations of Jing (1998). We use a novel method, which we call the cross-power spectrum between the linear density field and the halo number density field, to measure the bias factor. The method has several important advantages over the conventional correlation function (CF) estimator. Applying this method to our high-resolution simulations yields a very accurate determination of $b^{\rm L}(M)$ for a halo mass of over 4 mag both in scale-free models and in CDM models. Our result of $b^{\rm L}(M)$ can be represented accurately by $b_{\rm fit}(M) - 1$, which indicates quantitatively that it is the failure of the PS theories in describing the formation of small halos that results in the failure of the MW96 formula. This result has important implications for the PS theories.

The recent work by Porciani et al. (1998a) reports on an independent investigation that is highly related to the present one. They measured the Lagrangian bias for two scale-free simulations ($n = -1$ and $n = -2$) of 128^3 particles by fitting the halo two-point correlation function with the linear and second-order terms (corresponding to the coefficients $b_1^{\rm L}$ and $b_2^{\rm L}$ in eq. [5]). They concluded that the failure of the MW96 formula essentially exists in the Lagrangian space and that their result can be reasonably described by $b_{\rm fit}(M) - 1$. While our present study ($n = -1$ and $n = -2$) confirms their result in these aspects, our simulations have significantly higher resolution (a factor of 8 in mass) that is essential for a robust, accurate measurement of the clustering for small halos. Moreover, we explore a much larger model space and use a superior measurement method. In addition, there is some quantitative difference between their measured bias and ours, which will be discussed in § 3.

We will describe and discuss the cross-power spectrum method in § 2, where we will also briefly describe our halo catalogs. Our measurement results will be presented and compared, in § 3, with the analytical formula (eq. [4]) and the fitting formula (eq. [2]) for the Eulerian bias. In § 4, we will summarize our results and discuss their implications for the PS theories and for galaxy formation.

2. METHODS AND SIMULATION SAMPLES

We use the fluctuation field $\delta(\boldsymbol{r}) = \rho(\boldsymbol{r})/\bar{\rho} - 1$ to denote the density field $\rho(\boldsymbol{r})$, where $\bar{\rho}$ is the mean density. Smoothing both the halo number density field and the linear density field over scales much larger than the halo Lagrangian size, we assume that the smoothed halo field of $\delta_h(\boldsymbol{r})$ can be expanded in the smoothed linear density field $\delta_m(\boldsymbol{r})$ (Mo, Jing, & White 1997),

$$\delta_h(\boldsymbol{r}) = b_1^{\rm L}\delta_m(\boldsymbol{r}) + \frac{1}{2}\,b_2^{\rm L}\delta_m^2(\boldsymbol{r}) + \ldots . \tag{5}$$

This general assumption, especially to the first order, has been verified by previous simulation analyses (see, e.g., MW96; Mo et al. 1997; Jing 1998). However, this expansion is naturally expected in the PS theories, with the first coefficient $b_1^{\rm L} \equiv b^{\rm L}$ given by equation (4) and the higher order coefficients by MW96, Mo et al. (1997), and Catelan et al. (1998a).

By Fourier-transforming equation (5), multiplying both sides by $\delta_m^*(\boldsymbol{k})$, and taking an ensemble average, we get the cross-power spectrum $P_c(\boldsymbol{k}) \equiv \langle \delta_h(\boldsymbol{k})\delta_m^*(\boldsymbol{k})\rangle$. Because the bispectrum of the linear density field vanishes for Gaussian fluctuations, the second term on the right-hand side of is zero. Therefore, we have

$$P_c(k) = b^{\rm L}P_m(k) + (\text{third and higher order terms}), \tag{6}$$

where $P_m(k)$ is the linear power spectrum. This equation serves as a base for our measuring $b^{\rm L}$. The linear density field $\delta_m(\boldsymbol{k})$ is known when we set the initial condition for the simulations. The halo density field $\delta_h(\boldsymbol{k})$ can be easily measured for a sample of the DM halos with the fast Fourier transform method. The ensemble average in equation (6) can be replaced in measurement by the average over different modes within a fixed range of the wavenumber k. Thus, both $P_c(k)$ and $P_m(k)$ can be easily measured. The bias factor $b^{\rm L}$ is just the ratio of these two quantities, if higher order corrections are small and can be neglected.

This method has several important advantages over the conventional CF analysis. On the linear scale where the clustering of small halos is weak (about 10% of the mass correlation), the cross-power spectrum can be estimated accurately, because it does not suffer from the finite volume effect or the uncertainty of the mean halo number density. The errors of $P_c(k)$ and $P_m(k)$ are uncorrelated among different k-bins for a Gaussian field, which eases our error estimate for $b^{\rm L}$. The second-order correction (the $b_2^{\rm L}$-term) vanishes in the cross-power spectrum. The method yields a determination of $b_1^{\rm L}$, not the square of $b^{\rm L}$, and thus we can see if the bias factor $b_1^{\rm L}$ is positive for $M/M_* > 1$ and negative for $M/M_* < 1$ as equation (4) predicts, where M_* is the characteristic mass defined by $\sigma(M_*) = \delta_c$. All these attractive features are not present in the CF analysis. An additional interesting feature is that the shot noise of the finite number of halos is greatly suppressed in the estimate of $P_c(k)$, because the linear density field does not contain any shot noise and because the cross-power spectrum of this field with the shot noise (of a random sample) is zero in the mean. It is quite different from the (self) power spectrum estimate of the halos that we must correct for the shot noise $1/N$ (N is the number of halos). In the next section, we will show quantitatively that the shot noise effect is indeed negligible in our measurement of $b^{\rm L}$, even for a sample containing as many as ~100 halos.

The halo catalogs analyzed here are the same as those used in Jing (1998). The cosmological models, the simulations, and the halo catalogs were described in detail by Jing (1998). Here we only briefly summarize the features that are relevant to the present work. The catalogs were selected, with the friends-of-friends algorithm (with the linking length 0.2 times the mean particle separation), from a set of P^3M N-body simulations of

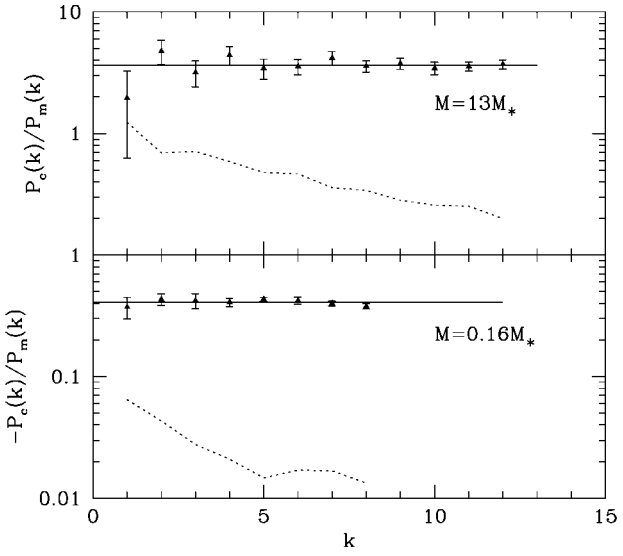


FIG. 1.—The ratio of the cross-power spectrum $P_c(k)$ to the linear density power spectrum $P_m(k)$ in the scale-free model of $n = -0.5$. The wavenumber k is in units of the fundamental wavenumber of the simulation. Only the linear clustering regime is considered here. The upper panel is for halo mass $M/M_* = 13$, and the lower one is for $M/M_* = 0.16$. In the lower panel, since the ratio $P_c(k)/P_m(k)$ is negative, we plot $-P_c(k)/P_m(k)$. The solid lines are the mean ratio averaged for different scales. The dotted lines are the 1 σ upper limits of the shot noise effect caused by the finite number of halos.

256^3 particles. The simulations cover four scale-free models, $P_m(k) \propto k^n$ with $n = -0.5$, -1.0, -1.5 and -2.0, and three typical CDM models, which are the standard (SCDM), the flat lower density (LCDM), and the open (OCDM) models, respectively. Each of the $n \geq -1.5$ scale-free simulations has seven outputs, and that of the $n = -2$ has eight outputs. The CDM models are simulated with two different box sizes, 100 and 300 h^{-1} Mpc. Three to four realizations were run for each model and for each box size in the case of the CDM models. In order to study the halo distribution in the Lagrangian space, we trace back each of the halo members to its initial position before the Zeldovich displacement, i.e., the position in the Lagrangian space. The position of a halo is defined as the center of the mass of its members in the Lagrangian space. In this way, our halo catalogs in the Lagrangian space are compiled.

3. THE LAGRANGIAN BIAS PARAMETER

We present our results for the linear clustering scales where the variance $\Delta^2(k) \equiv k^3 P_m(k)/2\pi^2$ is less than $\Delta^2_{\rm max}$. In this Letter, we take $\Delta^2_{\rm max} = 0.5$, but our results change little if we take $\Delta^2_{\rm max} = 0.25$ or $\Delta^2_{\rm max} = 1.0$. Figure 1 shows the ratio of the cross-power spectrum $P_c(k)$ to the linear power spectrum $P_m(k)$ as a function k for two halo masses in the $n = -0.5$ scale-free model. The ratios, i.e., the bias factors, do not depend on the scale k, so the linear bias approximation is valid. The bias factor is positive for the large halos of $M = 13M_*$ but negative for the small halos of $M = 0.16M_*$, which is consistent with the MW96 prediction (eq. [4]). Here we only show two examples, but the above features are found in all the models.

To examine quantitatively the shot noise effect that is caused by the finite number of the halos (§ 2), we repeat our calculation for random samples. For each halo sample, we generate 10 random samples. The mean bias factor that is calculated for the random samples is zero, as expected. More interesting is that the fluctuation of the bias factor between the random samples is always small compared with the halo bias factor. This is shown in Figure 1, where the dotted lines are the 1 σ upper limits of the shot noise. The catalog of $M = 13M_*$ contains only ~200 halos in each realization. Even in this case, the shot noise leads to an uncertainty of only ~10% of the halo bias at every k-bin (except for the first bin). The shot noise is indeed effectively suppressed in our cross-power spectrum measurement.

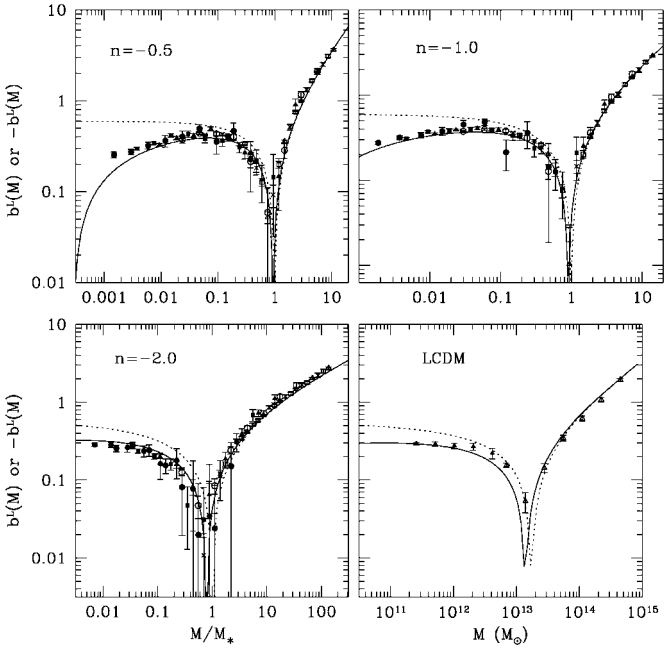


FIG. 2.—The Lagrangian bias parameter $b^{\rm L}$ as a function of the halo mass M. The mass is in units of M_* in the scale-free models and in units of solar mass ($M_\odot$) in the LCDM model. Because $b^{\rm L}$ is negative for $M/M_* < 1$, we plot $b^{\rm L}$ for $M/M_* > 1$ but $-b^{\rm L}$ for $M/M_* < 1$. For the scale-free models, the results at seven different evolutionary stages are plotted with different symbols. From the early to the late outputs, the symbols are, respectively, open triangles, open squares, crosses, open circles, solid triangles, solid squares, and solid circles. For $n = -2$, the result for a further output (more clustered) at the 1362th time step is added with hexagons. It is interesting to note that the results from different outputs agree remarkably well. For the LCDM model, the result is shown with open triangles for the simulation of box size 300 h^{-1} Mpc and with filled triangles for that of 100 h^{-1} Mpc. The dotted lines are the prediction of MW96 for the Lagrangian bias, which is in good agreement with the simulation results for $M/M_* \gtrsim 1$ while it significantly underpredicts (more negative) for $M/M_* \ll 1$. The solid lines are given by our fitting formula (eq. [2]) for the Eulerian bias under the linear mapping assumption, which can describe the simulation results remarkably well.

From Figure 1, we know that the Lagrangian bias $b^{\rm L}$, on the linear clustering scale, is a function of the halo mass M only. The self-similar nature of the scale-free models makes the bias depend only on M/M_*. Three panels of Figure 2 show our result of $b^{\rm L}_1(M/M_*)$ for the models $n = -0.5$, -1.0, and -2.0 at different output times. We do not plot, for the limited space, our result for the $n = -1.5$ model, but all of our following discussion still includes this model. The excellent scaling exhibited by $b^{\rm L}(M/M_*)$ at different outputs supports the fact that our measurement is not contaminated by any numerical artifacts. The bias factor is negative for $M < M_*$, zero for $M = M_*$, and positive for $M > M_*$, in agreement with the MW96 formula. More quantitatively, the MW96 formula describes very well the Lagrangian clustering for large halos, $M \gtrsim M_*$, but systematically underestimates (more negative bias) for small halos, $M \lesssim M_*$. If the linear mapping is valid, i.e., $b = b^{\rm L} + 1$, these results are fully consistent with our previous

finding (Jing 1998) that the MW96 formula systematically underestimates the Eulerian bias for small halos. More interestingly, with the linear mapping assumption and with our fitting formula (eq. [2]) for the Eulerian bias, we can get a Lagrangian bias $b^{\rm L}_{\rm fit} \equiv b_{\rm fit} - 1$ that agrees very well with our simulation results (the solid lines in Fig. 2). So the inaccuracy of the MW96 Eulerian bias formula (eq. [1]) already exists in their derivation for the Lagrangian bias. Our results for the $n = -1$ and $n = -2$ models are qualitatively in good agreement with Porciani et al. (1998a). Quantitatively, we note that their Lagrangian bias is significantly lower than the MW96 formula for $M > M_*$ in the $n = -1$ model, in contrast to the good agreement that we find with the MW96 formula for all models with $M > M_*$.

Our result for the LCDM model is shown in the lower right-hand panel of Figure 2. By comparing with the MW96 analytical formula (eq. [1]) and the fitting formula (eq. [2]), we get the same conclusion as that for the scale-free models: the analytical formula underestimates the Lagrangian bias value for small halos, and the fitting formula agrees quite well with the simulation results after the difference between the Eulerian and the Lagrangian spaces is considered with the linear mapping. The SCDM and OCDM models give essentially the same results, and we omit their plots.

Our fitting formula for the Eulerian bias can accurately describe the Lagrangian bias under the assumption of the linear mapping. The difference between the fitting formula and the simulation result is generally less than ~15% or 2 σ (σ is the error derived from the different realizations), except for one bin of the smallest halos in the $n = -0.5$ model. The simulation result is slightly higher in the $n = -0.5$ model, but lower in the $n = -2.0$ model (both about 15%), than the fitting formula for small halos ($M \ll M_*$). The difference could come from the possible effect of nonlinear mapping (Catelan et al. 1998b) and/or from higher order contributions that depend on measurement methods (for the Eulerian bias, we used the CF analysis). We will address this problem more closely in a future paper.

4. DISCUSSION AND CONCLUSIONS

In this Letter, we use a new method, the cross-power spectrum between the linear density field and the halo number density field, to determine the Lagrangian bias. The method has several apparent advantages over the conventional correlation function estimator in determining the bias factor. Applying this method to the halo catalogs of Jing (1998), we find that the Lagrangian bias is linear on the linear clustering scale. The Lagrangian bias $b^{\rm L}(M)$ is positive for halo mass $M > M_*$, zero for $M = M_*$, and negative for $M < M_*$, which is qualitatively consistent with the MW96 prediction for the Lagrangian bias. Quantitatively, our simulation results of $b^{\rm L}(M)$ are in good agreement with the MW96 formula for large halos, $M/M_* \gtrsim 1$, but the MW96 formula significantly underestimates the Lagrangian clustering for small halos, $M/M_* < 1$. Our measured Lagrangian bias can be described very well by our fitting formula for the Eulerian bias (eq. [2]) under the linear mapping assumption. Our results therefore unambiguously demonstrate that the inaccuracy of the MW96 formula for the Eulerian bias already exists in their derivation for the Lagrangian bias, in agreement with Porciani et al. (1998a).

A very subtle point is that there exists a small difference ($\lesssim$15%) between our measured Lagrangian bias and our fitting formula (eq. [2]) for Eulerian bias after the linear mapping is applied. This difference could be due to the difference of the measurement methods and/or the nonlinear mapping effect. Our result, however, assures us that the nonlinear effect in the mapping, if any, must be small compared with the linear mapping.

The result of this Letter has important implications for the Press-Schechter theories. The spherical collapse model that connects the halo formation with the density peaks has to be replaced with a model that can better describe the formation of small halos. This might be related to solving the long-standing problem that the halo number density predicted by the PS theories has a factor of a few difference from simulation results (see, e.g., Gelb & Bertschinger 1994; Lacey & Cole 1994; Ma & Bertschinger 1994; Y. P. Jing 1998, unpublished; Tormen 1998; Kauffmann et al. 1999; Somerville et al. 1998; Governato et al. 1998; Lee & Shandarin 1998; Jing, Kitayama, & Suto 1999). Because galaxies are believed to form within small halos ($M < M_*$), solving these problems is of fundamental significance for studying galaxy formation.

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