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ASYMMETRY AND FREQUENCIES OF LOW-DEGREE *p*-MODES AND THE STRUCTURE OF THE SUN'S CORE

T. TOUTAIN,[1] T. APPOURCHAUX,[2] C. FRÖHLICH,[3] A. G. KOSOVICHEV,[4] R. NIGAM,[4] AND P. H. SCHERRER[4]

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ABSTRACT

An accurate determination of the frequencies of low-degree solar *p*-modes is an important task of helioseismology. Using 679 days of solar oscillation data observed in Doppler velocity and continuum intensity from two *Solar and Heliospheric Observatory* instruments (the Michelson Doppler Imager and the SunPhotoMeter), we show that fitting the spectra with Lorentzian profiles leads to systematic differences between intensity and velocity frequencies as large as 0.1 μHz for angular degrees $l = 0$, 1, and 2 because of the opposite asymmetry between intensity and velocity. We use a physics-based asymmetrical line shape to fit *p*-mode lines, and we demonstrate that their asymmetry is statistically significant and that frequency differences are considerably reduced. These measurements provide more accurate estimates of the solar eigenfrequencies. We discuss inferences of the structure of the solar core.

Subject headings: methods: data analysis — Sun: interior — Sun: oscillations

1. INTRODUCTION

An accurate determination of the frequencies of solar acoustic (*p*) modes of low angular degree *l* is necessary to infer the structure and rotation of the solar core. Since the work of Duvall et al. (1993), which is based on oscillation spectra averaged over the azimuthal order *m*, the line shape of medium-*l p*-modes has been considered possibly asymmetric. Moreover, this asymmetry shows up with an opposite sign for modes observed in intensity and in velocity. In standard techniques of measuring solar frequencies (see, e.g., Anderson, Duvall, & Jefferies 1990), the asymmetry is not taken into account, and the line shape is assumed to be a Lorentzian. This leads to systematic errors in frequency measurements and thus to errors in helioseismic inferences of the structure of the Sun. Toutain et al. (1997) have observed frequency differences on the order of 0.1 μHz between intensity and velocity low-degree *p*-mode lines when a symmetric Lorentzian fit was employed. In this Letter, we show that these frequency differences can be reduced significantly by using our model of an asymmetrical line profile.

The first successful attempt to detect the asymmetry of low-*l* modes was made by Toutain (1993) using the intensity data from the Interplanetary Helioseismology by Irradiance (IPHIR) space experiment (Fröhlich et al. 1990). However, the IPHIR data, being the longest intensity time series available at that time, had a poor resolution and signal-to-noise ratio when trying to make reliable conclusions about the asymmetry and corresponding frequency corrections.

The oscillations of the Sun have been observed continuously for about 2 years with three experiments on board the *Solar and Heliospheric Observatory* (*SOHO*). Two of these experiments are the Variability of Solar Irradiance and Gravity Oscillations (VIRGO; Fröhlich et al. 1995), which is measuring oscillations of the solar irradiance with a triple SunPhotoMeter (SPM) and a 16 pixel imager (the Luminosity Oscillation Imager; Appourchaux et al. 1995), and the Michelson Doppler Imager (MDI), which is measuring velocity oscillations in the Ni λ6768 line with a CCD detector (Scherrer et al. 1995). We have used the longest available time series from both experiments, consisting of 679 days of integrated velocity and spectral irradiance (at 400, 500, and 862 nm), to measure accurately the parameters of the *p*-modes for degrees $l = 0$, 1, and 2 in a frequency domain ranging from 2000 to 4000 μHz.

We discuss the fact that the shape of *p*-mode lines departs significantly from a Lorentzian profile. We use a theoretical model by Nigam & Kosovichev (1998) to fit the asymmetrical shape found in the data. We fitted both intensity and velocity lines in order to determine the frequency dependence of the asymmetry and to present new estimates of the eigenfrequencies of low-degree modes that are particularly important for determining the structure of the Sun's energy-generating core.

2. INTEGRATED VELOCITY AND INTENSITY POWER SPECTRA

The power spectra of the 679 day time series are plotted in Figure 1 in a frequency range spanning from 1000 to 5000 μHz. Below 2000 μHz, we hardly see *p*-mode lines in the intensity data because of the granulation noise that steeply increases, and above 4000 μHz, mode line widths are such that modes of degrees 0 and 2 overlap. We therefore restrict ourselves to working in this frequency range in order to compare the results from the velocity and intensity data. Because of the stochastic nature of solar oscillations, the power of a line in each frequency bin is distributed with a large variance around its expectation value (Woodard 1984). A smoothing of the line is therefore necessary to reveal its fine structure. Line asymmetry is visible mainly on the edges of the power spectra. As an example, in Figure 2, we have plotted the $l = 0$, $n = 14$ line for both MDI and SPM (red) spectra. The lines are smoothed with a 1 μHz boxcar average in order to enhance the asymmetry of the profiles. The results of the fitting, using a model of the asymmetrical line shape, are plotted on top of the profiles, as described in the next section. The line asymmetry that is reversed between velocity and intensity power spectra is apparent.

[1] Département CASSINI, Observatoire de la Côte d'Azur, BP4229, F-06003 Nice, France; toutain@obs-nice.fr.

[2] Space Science Department, European Space Research and Technology Centre, Postbus 299, 2200 AG Noordwijk, The Netherlands; thierrya@so.estec.esa.nl.

[3] Physikalisch-Meteorologisches Observatorium Davos, World Radiation Center, 7260 Davos-Dorf, Switzerland; cfrohlich@obsun.pmod.ch.

[4] W. W. Hansen Experimental Physics Laboratory, Stanford University, Stanford, CA 94305-4085; akosovichev@solar.stanford.edu, rnigam@solar.stanford.edu, pscherrer@solar.stanford.edu.

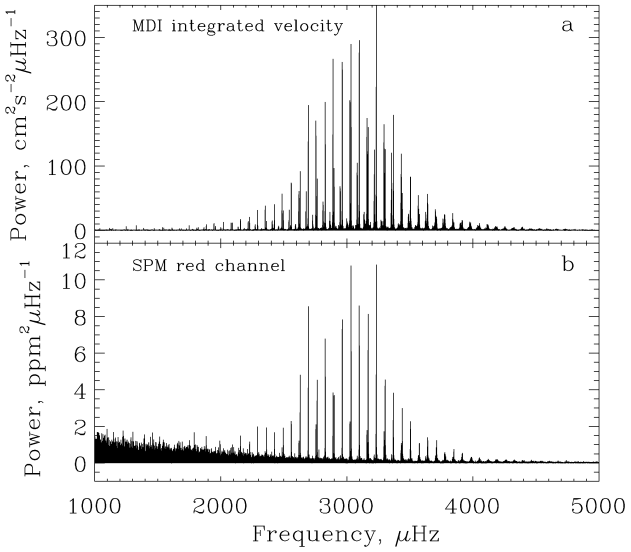


FIG. 1.—Oscillation power spectra of low angular degree solar *p*-modes obtained from the 679 day time series of (*a*) MDI-integrated velocity and (*b*) VIRGO/SPM intensity (red channel) measurements.

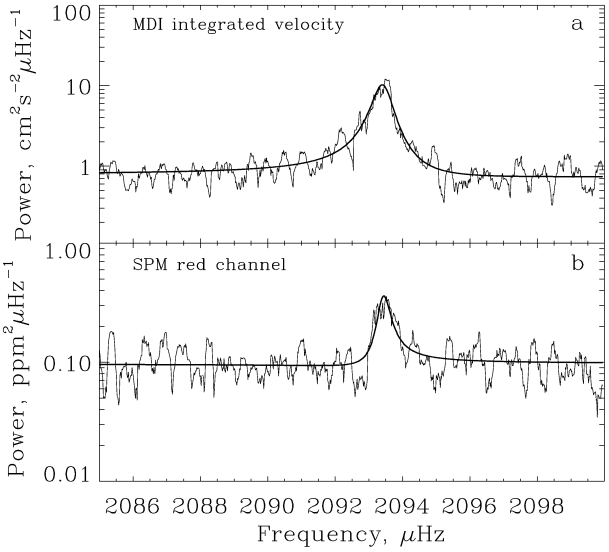


FIG. 2.—Smoothed $l = 0$, $n = 14$ line for (*a*) the MDI and (*b*) the SPM, with a boxcar average of 1 μHz.

3. PHYSICAL MODEL FOR *p*-MODE LINE ASYMMETRY AND FITTING FORMULA

Physically, asymmetry is a result of an interaction between an outward-directed wave from the source and a corresponding inward-directed wave that passes through the region of wave propagation (Duvall et al. 1993). As illustrated in Figure 2, the velocity power spectrum exhibits negative asymmetry (more power on the lower frequency end of the peak), whereas the intensity power spectrum shows positive asymmetry (more power on the higher frequency end of the peak). This reversal of asymmetry can be explained if the correlated noise is added to the amplitude spectra (Nigam et al. 1998).

The asymmetry in velocity and intensity power spectra is of the opposite sign because a component of the solar noise that is correlated with the source is present in the observed signal. The correlated component of the noise is below a certain threshold to preserve the asymmetry obtained by the model (Nigam et al. 1998) in the velocity power spectrum, but it is large enough (the correlated component is above a certain threshold) to reverse the asymmetry in the intensity spectrum. This is due to the fact that the correlated noise shifts the zero points in the amplitude spectra of the pressure perturbation. The uncorrelated noise plays no role in the reversal of asymmetry because it adds to the power, not to the amplitude spectrum.

The solar intensity and velocity oscillations are observed from variations in an absorption (Fraunhofer) line or, as in the case of the SPM, from intensity variations in relatively broad bands of the solar spectrum. These variations consist of two parts: solar oscillation modes and granulation noise. Because the oscillation modes are excited by granulation, the granulation signal (noise) is partially correlated with the oscillations. According to Goode et al. (1998), the solar oscillations are excited in the intergranular dark lanes. Thus, the observed intensity variations have a noise component that is caused directly by the dark lanes and that is correlated to the amplitude of the oscillations. Roxburgh & Vorontsov (1997) have suggested that the granulation overshoot forms the correlated component of the noise to the velocity signal. However, the observations show a higher level of noise in the intensity data and support the idea by Nigam et al. (1998) that the line asymmetry in the intensity power spectra is reversed because of the correlated noise in the intensity observations.

To measure the properties of solar oscillations, we employed an asymmetric fitting formula by Nigam & Kosovichev (1998) for the power, P:

$$P(x) = \frac{A[(1 + Bx)^2 + B^2]}{1 + x^2}, \qquad (1)$$

where $x = 2(\nu - \nu_0)/\gamma$, ν_0 is the mode eigenfrequency, γ is the line width, A is the power amplitude, and B is a parameter that controls the asymmetry. This parameter is positive for positive asymmetry and negative for negative asymmetry. It contains the effects of correlated noise and the properties of the excitation source (mainly, its location and type), the two factors that are responsible for asymmetry. To this formula, one adds the usual linear background while carrying out the fitting.

4. LINE ASYMMETRY

According to the line shape model described in the previous section, we have fitted $l = 0$–2 doublets and the $l = 1$ lines between 2000 and 4000 μHz. The technique of fitting is a standard maximum likelihood technique. Usually for a disk-averaged spectrum, we fitted simultaneously the line for $l = 0$ and 2 because they overlap. We therefore give a mean asymmetry for these degrees. The modes of $l = 1$ are fitted assuming two-component, rotationally split multiplets of $m = 1$ and -1, with both components having the same asymmetry coefficient. The asymmetry coefficient is plotted in Figures 3*a* and 3*b* as a function of frequency for the MDI and SPM data, respectively. It is clear from these results that, according to the definition of asymmetry we have, velocity lines exhibit negative asymmetry whereas intensity lines exhibit positive asymmetry. Moreover, the frequency dependence of the asymmetry is similar in both cases. Asymmetry seems to have a minimum at around 3000 μHz and appears to be smaller for the velocity than for the intensity, at least for low and high frequencies. Both effects can be explained easily if the correlation with

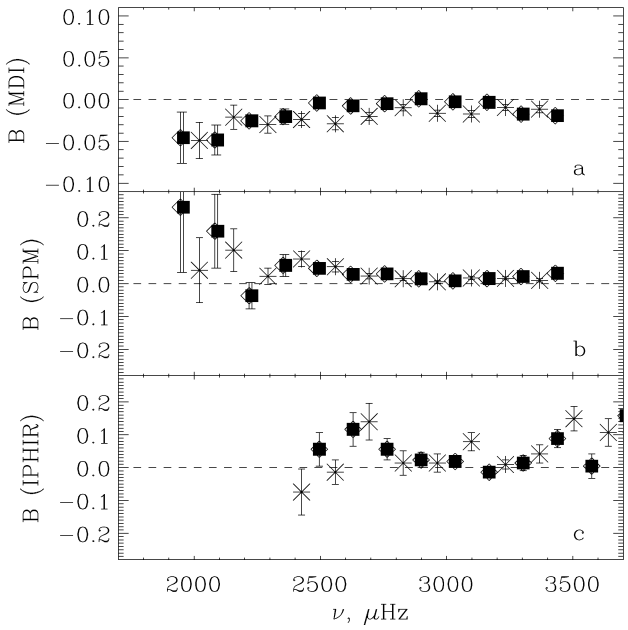


FIG. 3.—Asymmetry coefficient B for $l = 0$ (*squares*), $l = 1$ (*crosses*), and $l = 2$ (*diamonds*) modes as a function of mode frequency for (*a*) the MDI, (*b*) the SPM data, and (*c*) the IPHIR data.

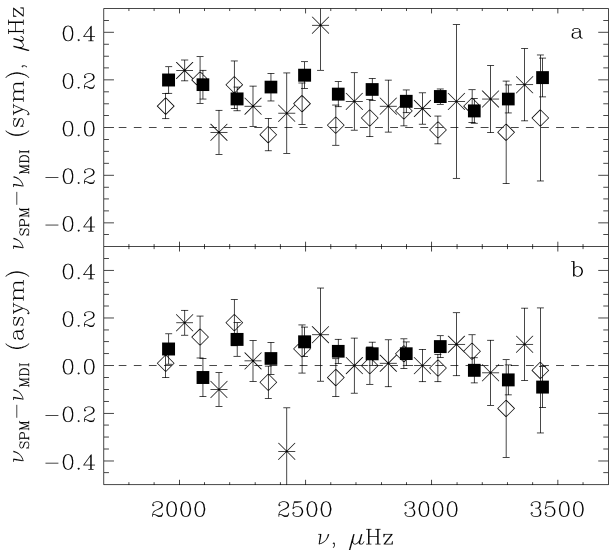


FIG. 4.—Frequency differences between SPM and MDI frequencies for $l = 0$ (*squares*), $l = 1$ (*crosses*), and $l = 2$ (*diamonds*) modes estimated using (*a*) a symmetrical Lorentzian profile and (*b*) the asymmetrical line profile (eq. [1]). The error bars are the 1 σ errors.

noise, along with the source exciting the modes, is the factor of asymmetry (Nigam et al. 1998). In this case, for a given correlation, the asymmetry is a function of the square root of the inverse of the signal-to-noise ratio, which is largest at around 3000 μHz and bigger in general for the velocity spectra than for the intensity spectra. These arguments also explain why lines in the red spectrum have slightly larger asymmetries compared with those from the SPM green channel spectrum. This is true for the following reason: the signal-to-noise ratio in the red is about 10% lower than in the green. On the other hand, blue and green signals are very well correlated and have the same signal-to-noise ratios leading to the same asymmetry.

It is interesting to compare these asymmetries with those obtained with the IPHIR space experiment (Fröhlich et al. 1990), which is an instrument similar to the SPM/VIRGO instrument. IPHIR lines were fitted with an empirical line shape formula (Toutain 1993), which describes well the asymmetrical profile shown by Duvall et al. (1993). Compared with SPM, IPHIR data have a signal-to-noise ratio that is 3 times smaller in power. If the noise were to be solar noise, then, according to our model of the noise-source correlation, we would expect the asymmetries with IPHIR to be 2 times larger than with SPM, and that is what they are. It is interesting to note that an increase in solar noise for a fixed line amplitude should mask the line asymmetry, but it turns out to be the opposite with IPHIR. The asymmetries from the IPHIR data are generally larger than the SPM asymmetries. It is therefore consistent to think that IPHIR noise in the 5 minute range is solar noise and that the *p*-mode amplitudes are correlated with the noise.

5. FREQUENCY DIFFERENCES

The frequency differences from our symmetrical (Lorentzian) fits to the MDI and SPM red channel data are plotted in Figure 4*a*. There is a systematic shift for both $l = 0$–2 and $l = 1$ frequencies. The frequencies inferred from the SPM intensity spectrum are systematically higher than the frequencies from the MDI velocity spectrum, as one would expect owing to the reversal of line asymmetry. These differences are on the order of 0.1 μHz.

Using the asymmetrical formula given in equation (1), we have recomputed the mode frequencies. The results are plotted in Figure 4*b*. Obviously, the differences are reduced significantly when an asymmetrical profile is used. We have tested the statistical significance of the asymmetry using the likelihood ratio test (Appourchaux et al. 1995), and we find that it is highly significant in most cases. Table 1 shows mean frequency differences between intensity and velocity data obtained with both symmetrical and asymmetrical fits. The close agreement between frequencies from the asymmetrical fits means that these frequencies provide a more accurate approximation to the solar mode eigenfrequencies than the frequencies previously measured with a symmetrical Lorentzian profile. In Table 2, we provide a list of the frequencies obtained from the asymmetrical fits to the MDI velocity spectra in the range from 1000 to 4000 μHz.

6. IMPLICATIONS FOR THE SOLAR CORE STRUCTURE

To determine the hydrostatic structure of the Sun, we combined the frequencies of the low-degree modes with the frequencies of $l = 3$–300 obtained by Rhodes et al. (1997) from the MDI medium-l data. The medium-l frequencies that constrain the structure outside the solar core have not been corrected for the asymmetry effect. The spherically symmetric structure of the Sun was determined by using a version of the optimally localized averaging inversion method of Gough &

TABLE 1

MEAN FREQUENCY DIFFERENCES (IN MICROHERTZ) BETWEEN THE INTENSITY (SPM) AND VELOCITY (MDI) DATA

l	$\overline{\nu_{SPM}-\nu_{MDI}}$(sym)	$\overline{\nu_{SPM}-\nu_{MDI}}$(asym)
0	0.151 ± 0.044	0.029 ± 0.066
1	0.136 ± 0.116	0.002 ± 0.143
2	0.056 ± 0.084	0.002 ± 0.122

TABLE 2

FREQUENCIES (IN MICROHERTZ) OF LOW-DEGREE MODES[a]

n	$l = 0$	$l = 1$	$l = 2$
6	...	1039.463 ± 0.014	1104.768 ± 0.004
7	...	1185.336 ± 0.185	1250.563 ± 0.006
8	1263.187 ± 0.019	1329.645 ± 0.004	1394.678 ± 0.005
9	1407.460 ± 0.021	1472.827 ± 0.019	1535.863 ± 0.017
10	1548.289 ± 0.019	1612.719 ± 0.023	1674.549 ± 0.011
11	1686.578 ± 0.022	1749.269 ± 0.015	1810.298 ± 0.014
12	1822.254 ± 0.039	1885.107 ± 0.040	1945.799 ± 0.016
13	1957.417 ± 0.040	2020.812 ± 0.024	2082.139 ± 0.019
14	2093.478 ± 0.042	2156.827 ± 0.026	2217.701 ± 0.020
15	2228.711 ± 0.052	2292.018 ± 0.025	2352.267 ± 0.020
16	2362.780 ± 0.060	2425.586 ± 0.026	2485.934 ± 0.022
17	2496.113 ± 0.054	2559.224 ± 0.029	2619.623 ± 0.021
18	2629.723 ± 0.047	2693.424 ± 0.027	2754.460 ± 0.020
19	2764.116 ± 0.046	2828.199 ± 0.027	2889.560 ± 0.019
20	2898.955 ± 0.047	2963.307 ± 0.025	3024.731 ± 0.019
21	3033.710 ± 0.045	3098.188 ± 0.028	3159.874 ± 0.040
22	3168.641 ± 0.053	3233.267 ± 0.030	3295.048 ± 0.026
23	3303.425 ± 0.063	3368.782 ± 0.040	3430.708 ± 0.040
24	3439.049 ± 0.082	3504.431 ± 0.063	3566.817 ± 0.057
25	3574.778 ± 0.129	3641.038 ± 0.095	3703.365 ± 0.086
26	3710.645 ± 0.186	3777.834 ± 0.187	3840.296 ± 0.140
27	3848.251 ± 0.691	3914.816 ± 0.302	...

[a] Obtained with the asymmetrical line profile from the 679 day MDI time series.

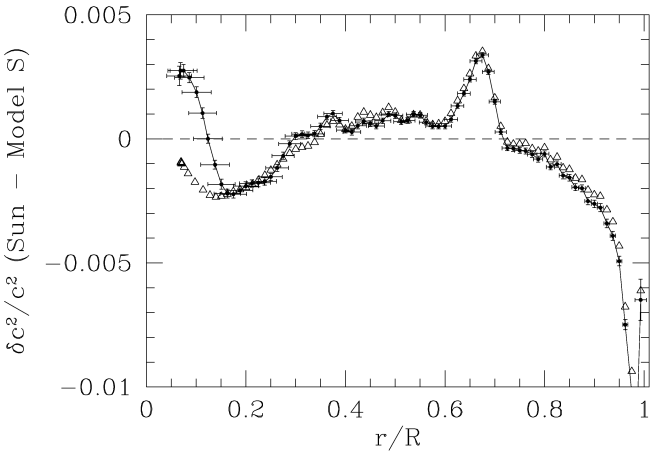


FIG. 5.—The differences in the squared sound speed between the Sun and the solar model S inferred from the frequencies of $l = 0$–2 modes and the medium-l frequencies of Rhodes et al. (1997). The solid curve with the crosses shows the results obtained with the asymmetrically fitted low-degree modes; the triangles represent the result of the symmetrical (Lorentzian) fits to the $l = 0$–2 mode lines of the oscillation power spectrum. The vertical error bars show the formal error estimates, and the horizontal bars show the characteristic width of the localized averages.

Kosovichev (1988). Figure 5 shows the relative difference between the square of the sound speed in the Sun and the standard solar model of Christensen-Dalsgaard et al. (1996). In the central core, the sound speed that was determined using the frequencies from the asymmetrical fits (*solid curve with error bars*) is approximately 0.2%–0.3% higher than that inferred from the symmetrical frequency fits. This shift of the sound speed profile does not result in any dramatic changes in our knowledge of the solar core structure. However, it may be important for understanding the difference between the Sun and the standard model.

It has been noted that the sound speed perturbation profile in the central core is consistent with partial mixing (Gough & Kosovichev 1988). The change of slope at $0.2R$ may indicate the boundary of mixing. Of course, at this stage, other possible explanations, e.g., variations of opacity and nuclear reaction rates, are not ruled out. Also, we note that our inversion results are not fully consistent with the recent results from the Global Oscillations at Low Frequencies (Turck-Chiéze et al. 1997), in which, however, the asymmetry effect was not taken into account.

7. CONCLUSIONS

We have studied p-mode frequency differences between velocity and intensity data using 679 days of MDI-integrated velocity and VIRGO/SPM intensity observations. Fitting $l = 0$–2 p-mode lines with commonly used Lorentzian profiles leads to frequency differences on the order of 0.1 μHz between intensity and velocity and also, to a smaller extent, between different wavelengths in intensity. Using a model of an asymmetrical line profile instead, which is based on the excitation of solar oscillations by a localized source and its correlation with the solar noise in the observed velocity and intensity signals, gives frequency differences that are significantly smaller than the differences resulting from Lorentzian fits. The asymmetrical line profile provides more accurate estimates of the eigenfrequencies of solar oscillations. The asymmetry effect results in small (0.2%–0.3%) but significant changes in the inferred sound speed inversion in the energy-generating core, which may be important for understanding the physical processes in the core.

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