

THE FADING OF SUPERNOVA REMNANT CASSIOPEIA A FROM 38 MHz TO 16.5 GHz FROM 1949 TO 1999 WITH NEW OBSERVATIONS AT 1405 MHz¹

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ABSTRACT

We report 1405 MHz measurements of the flux density of the ≈ 320 -year-old supernova remnant Cassiopeia A, relative to the flux density of Cygnus A, made between 1995 and 1999. When compared to measurements made between 1957 and 1976, we find that the rate at which Cassiopeia A has been fading at this and nearby frequencies has changed from $\approx 0.9\%$ yr⁻¹ in the 1960s to $\approx 0.6\%$ – 0.7% yr⁻¹ now. Furthermore, we have collected from the literature measurements of this fading rate at lower (38–300 MHz) and higher (7.8–16.5 GHz) frequencies. We show that the fading rate has dropped by a factor of ≈ 3 over the past 50 years at the lower frequencies, while remaining relatively constant at the higher frequencies, which is in agreement with the findings of others. Our findings at 1405 MHz, in conjunction with a measurement of the fading rate at the nearby frequency of 927 MHz by Vinyajkin, show an intermediate behavior at intermediate frequencies. We also find that Cassiopeia A, as of ≈ 1990 , was fading at about the same rate, $\approx 0.6\%$ – 0.7% yr⁻¹, at all of these frequencies. Future measurements are required to determine whether the fading rate will continue to decrease at the lower frequencies, or whether Cassiopeia A will now fade at a relatively constant rate at all of these frequencies.

Subject headings: ISM: individual (Cassiopeia A) — radio continuum: ISM — supernova remnants

1. INTRODUCTION: A HISTORY OF MEASUREMENTS OF CASSIOPEIA A'S FADING RATE

Cassiopeia A is thought to have exploded around A.D. 1681 ± 15 , based on radial velocity and proper-motion measurements of 15 of the supernova remnant's high-velocity [N II]-emitting knots, also called fast-moving flocculi or FMFs (Fesen, Becker, & Goodrich 1988). Indeed, Flamsteed (1725) observed a 6th magnitude star within 13' of Cassiopeia A's location in 1680 (Ashworth 1980) that later became a subject of debate when the star could not be found in the sky (Herschel 1798; Baily 1835). Cassiopeia A was rediscovered in 1943 at 160 MHz by Reber (1944), although a reanalysis of a day-long strip chart recording published by Jansky (1935) suggests that he unknowingly detected it as early as 1932 at 20.5 MHz (Sullivan 1978). Accurate measurements of Cassiopeia A's flux density—both absolute measurements and measurements relative to the flux density of the extragalactic radio source Cygnus A—began in 1949 at 81.5 MHz (Ryle & Elsmore 1951).

1.1. *The Fading Rate of Cassiopeia A around 1965*

Högbom & Shakeshaft (1961), with Ryle & Elsmore's (1951) 1949 measurement and two measurements of their own made in 1956 and 1960 also at 81.5 MHz, first showed that Cassiopeia A was fading; they measured a fading rate of $1.06\% \pm 0.14\%$ yr⁻¹, which was later refined to

$1.29\% \pm 0.08\%$ yr⁻¹ with the addition of four measurements made between 1966 and 1969, also at 81.5 MHz (Scott, Shakeshaft, & Smith 1969). Using data spanning 1957–1972, Baars & Hartsuijker (1972) measured significantly lower fading rates at 1420 and 3000 MHz, $0.89\% \pm 0.12\%$ and $0.92\% \pm 0.15\%$ yr⁻¹, respectively; they first suggested that Cassiopeia A was fading at different rates at different frequencies. By 1977, accurate fading rates had been determined at six different frequencies, spanning 81.5–9400 MHz. Baars et al. (1977; see also Dent, Aller, & Olsen 1974) collected these measurements, which we relist in Table 1 and plot in Figure 1, and determined the following empirical equation that describes the frequency dependence of Cassiopeia A's fading rate for the ≈ 1965 epoch:

$$-\frac{100}{F_\nu} \frac{dF_\nu}{dt} = 0.97(\pm 0.04) - 0.30(\pm 0.04) \log \nu_{\text{GHz}} \quad (1)$$

Using this equation and absolute measurements of Cassiopeia A's flux density made around 1965 over a frequency range that spans 22 MHz to 31 GHz, Baars et al. (1977) also determined an empirical equation that describes the absolute spectrum of Cassiopeia A for the 1965 epoch to an accuracy of $\approx 2\%$. Empirical equations that describe the absolute spectra of Cygnus A and Taurus A, and a semi-absolute spectrum for Virgo A, were also determined.

1.2. *The Decreasing Fading Rate of Cassiopeia A at Low Radio Frequencies (38–300 MHz)*

Equation (1) was first challenged by Erickson & Perley (1975), whose 1974 measurement of the flux density of Cassiopeia A at 38 MHz was brighter than the prediction of Baars et al. (1977) at the 3.5σ level. Subsequent measurements at 38 MHz by Read (1977a, 1977b), Walczowski & Smith (1985), Rees (1990), and Vinyajkin (1997) confirmed that the fading rate had decreased from $1.9\% \pm 0.5\%$ yr⁻¹ (determined using only the 1955–1966 measurements; Read

¹ The data presented here were obtained by participants of Educational Research in Radio Astronomy (ERIRA) 1992–1999, an outreach program that has received support from the National Radio Astronomy Observatory, the Ohio State University, the Pennsylvania State University, and the University of Pittsburgh at Bradford.

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TABLE 1
FADING RATES OF CASSIOPEIA A AROUND 1965

Frequency (MHz)	Measurement Epochs	Central Epoch	Fading Rate (% yr ⁻¹)	Reference
81.5	1949–1969	1959	1.29 ± 0.08	1
950	1964–1972	1968	0.85 ± 0.05	2
1420	1957–1971	1964	0.89 ± 0.12	3
1420	1957–1976	1966	0.86 ± 0.02	4
3000	1961–1972	1966	0.92 ± 0.15	3
3060	1961–1971	1966	1.04 ± 0.21	5
7800	1963–1974	1968	0.70 ± 0.10	6
9400	1961–1971	1966	0.63 ± 0.12	5

REFERENCES.—(1) Scott, Shakeshaft, & Smith 1969; (2) Stankevich, Ivanov, & Torkhov 1973b; (3) Baars & Hartsuijker 1972; (4) Read 1977a; (5) Stankevich et al. 1973a; (6) Dent, Aller, & Olsen 1974.

1977a) to 0.66% ± 0.17% yr⁻¹ (determined using all of the measurements through 1995; Vinyajkin 1997).⁴

Similar and more accurate results have been found at 81.5 MHz. As stated above, Scott et al. (1969) found a 1949–1969 fading rate of 1.29% ± 0.08% yr⁻¹ at this frequency. Hook, Duffett-Smith, & Shakeshaft (1992) found a significantly lower fading rate of 0.92% ± 0.16% yr⁻¹ when they included their 1989 measurement, and a fading rate of ≈ 0.63% yr⁻¹ when they considered only the measurements made after 1965. Agafonov (1996) found similar, although slightly higher, values: 1.25% ± 0.06% yr⁻¹ (1949–1985) and ~0.8% yr⁻¹ (1973–1985).

Consistent measurements have also been made at 102 MHz (Agafonov 1996), 151–152 MHz (Read 1977a; Agafonov 1996; Vinyajkin 1997), and 290–300 MHz (Baars & Hartsuijker 1972; Vinyajkin 1997). We plot the low-frequency (38–300 MHz) measurements of Cassiopeia A's

⁴ Rees (1990) suggested that the measurements at 38 MHz, as of 1987, could be treated as being consistent with a constant fading rate of ~0.8% yr⁻¹, particularly if one ignored the 1949–1969 measurements of Ryle & Elsmore (1951), Högbom & Shakeshaft (1961), and Scott, Shakeshaft, & Smith (1969), which imply a significantly higher fading rate, 1.29% ± 0.08% yr⁻¹, at the nearby frequency of 81.5 MHz. However, Rees (1990) did not rule out the possibility of a changing fading rate at 38 MHz, and indeed, the later measurement of this fading rate by Vinyajkin (1997) suggests that it has changed at the ≈ 2.3 σ confidence level.

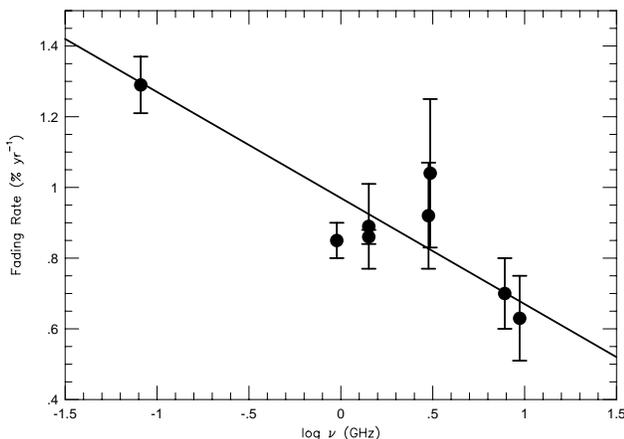


FIG. 1.—Fading rates of Cassiopeia A as a function of frequency for the ≈ 1965 epoch, from Baars et al. (1977; see Table 1). Eq. (1), also from Baars et al. (1977), is plotted as the solid line.

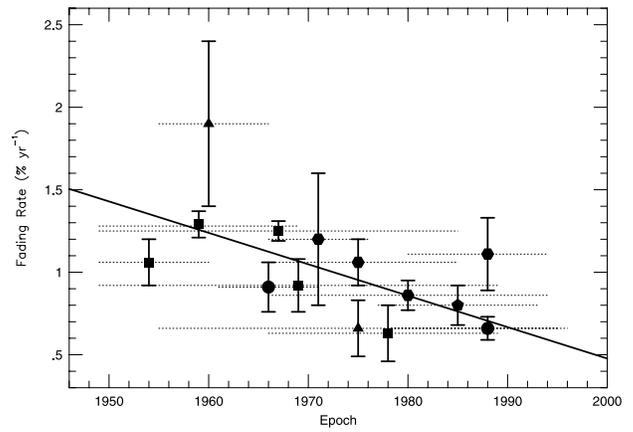


FIG. 2.—Fading rates of Cassiopeia A as a function of epoch for the low radio frequencies (38–300 MHz; see Table 2). The dotted lines indicate over what interval of time measurements were taken to determine each fading rate. Triangles denote 38 MHz measurements, squares denote 81.5 MHz measurements, pentagons denote 102 MHz measurements, hexagons denote 151–152 MHz measurements, and circles denote 290–300 MHz measurements. The solid line corresponds to a fading rate that is decreasing at a rate of ≈ 2% yr⁻¹ century⁻¹.

fading rate, as well as the intervals of time over which these measurements were made, in Figure 2, and we list this information in Table 2. Clearly, as is suggested by Hook et al. (1992), and advocated by Agafonov (1996), the rate at which Cassiopeia A is fading at low frequencies has decreased over the past 50 years.⁵ We find the fading rate to be decreasing by ≈ 2% yr⁻¹ century⁻¹ at these frequencies.

⁵ Hook, Duffett-Smith, & Shakeshaft (1992) suggested that instrumental uncertainties and/or flaring of Cassiopeia A's radio emission might systematically affect measurements of Cassiopeia A's fading rate at these low frequencies. However, they made arguments to the contrary, and Agafonov (1996) provided evidence to the contrary. Consequently, we do not further pursue these scenarios in this paper; instead, we refer the interested reader to these papers.

TABLE 2
FADING RATES OF CASSIOPEIA A AT LOW RADIO FREQUENCIES (38–300 MHz)

Frequency (MHz)	Measurement Epochs	Central Epoch	Fading Rate (% yr ⁻¹)	Reference
38	1955–1966	1960	1.9 ± 0.5	1
38	1955–1995	1975	0.66 ± 0.17	2
81.5	1949–1960	1954	1.06 ± 0.14	3
81.5	1949–1969	1959	1.29 ± 0.08	4
81.5	1949–1985	1967	1.25 ± 0.06	5
81.5	1949–1989	1969	0.92 ± 0.16	6
81.5	1966–1989	1978	≈ 0.63 ^a	6
102	1977–1993	1985	0.80 ± 0.12	5
151	1966–1976	1971	1.2 ± 0.4	1
152	1966–1985	1975	1.06 ± 0.15	5
151.5	1966–1994	1980	0.86 ± 0.09	2
151.5	1980–1994	1988	1.11 ± 0.22	2
300	1961–1971	1966	0.91 ± 0.15	7
290	1978–1996	1988	0.66 ± 0.07	2

^a We calculate an error of ± 0.17 from information provided in Hook, Duffett-Smith, & Shakeshaft 1992 and Scott, Shakeshaft, & Smith 1969.

REFERENCES.—(1) Read 1977a; (2) Vinyajkin 1997; (3) Högbom & Shakeshaft 1961; (4) Scott et al. 1969; (5) Agafonov 1996; (6) Hook, Duffett-Smith, & Shakeshaft 1992; (7) Baars & Hartsuijker 1972.

1.3. The Constant Fading Rate of Cassiopeia A at High Radio Frequencies (7.8–16.5 GHz)

However, different conclusions have been reached at significantly higher frequencies. O’Sullivan & Green (1999) compare four measurements of Cassiopeia A’s brightness relative to Cygnus A’s brightness that they made in 1994 and 1995 at 13.5, 15.5, and 16.5 GHz to the predictions of Baars et al. (1977). The measurements of O’Sullivan & Green (1999) are perfectly consistent with the $\approx 0.6\% \text{ yr}^{-1}$ predictions of equation (1),⁶ suggesting that the rate at which Cassiopeia A fades has not changed significantly over the course of the last half-century at these high frequencies. We plot the high-frequency (7.8–16.5 GHz) measurements of Cassiopeia A’s fading rate in Figure 3, and we list this information in Table 3.

These apparently contradictory behaviors at low and high radio frequencies suggests that the rate at which Cassiopeia A fades is changing in a frequency-dependent way: the fading rate is decreasing at the low frequencies (38–300 MHz) and is relatively constant at the high frequencies (7.8–16.5 GHz). In this paper we investigate and confirm this

⁶ Technically, eq. (1) is an extrapolation at frequencies greater than 9.4 GHz; however, this is not a large extrapolation.

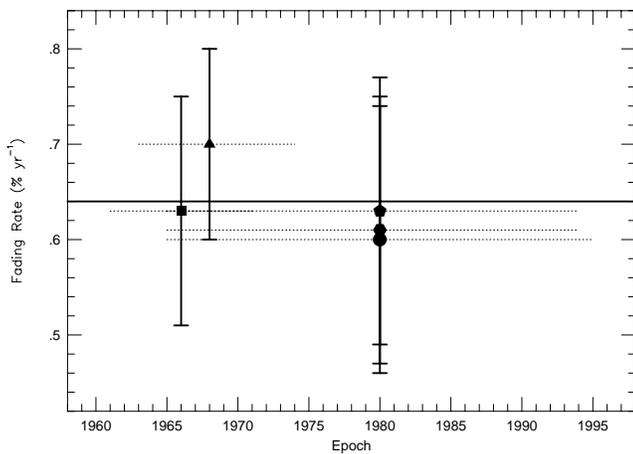


FIG. 3.—Fading rates of Cassiopeia A as a function of epoch for the high radio frequencies (7.8–16.5 GHz; see Table 3). The dotted lines are as defined in Fig. 2. Triangles denote 7.8 GHz measurements, squares denote 9.4 GHz measurements, pentagons denote 13.5 GHz measurements, hexagons denote 15.5 GHz measurements, and circles denote 16.5 GHz measurements. The solid line corresponds to a constant fading rate.

TABLE 3

FADING RATES OF CASSIOPEIA A AT HIGH RADIO FREQUENCIES (7.8–16.5 GHz)

Frequency (GHz)	Measurement Epochs	Central Epoch	Fading Rate (% yr ⁻¹)	Reference
7.8	1963–1974	1968	0.70 ± 0.10	1
9.4	1961–1971	1966	0.63 ± 0.12	2
13.5	1965–1994	1980	≈ 0.63 ^a	3
15.5	1965–1994	1980	≈ 0.61 ^a	3
16.5	1965–1995	1980	≈ 0.60 ^a	3

^a We estimate errors of ±0.14 from information provided in O’Sullivan & Green 1999.

REFERENCES.—(1) Dent et al. 1974; (2) Stankevich et al. 1973a; (3) O’Sullivan & Green 1999.

trend at the intermediate frequency of 1405 MHz with measurements of the flux density of Cassiopeia A relative to that of Cygnus A that we made between 1995 and 1999 with the 40 foot (12.2 m) telescope at the National Radio Astronomy Observatory in Green Bank, West Virginia. We analyze these measurements in § 2; we draw conclusions in § 3.

2. GREEN BANK MEASUREMENTS OF $F_{1405}^{\text{CasA}}/F_{1405}^{\text{CygA}}$

In August of 1995, 1996, 1997, and 1999, we took drift scans of Cassiopeia A and Cygnus A using the 40 foot telescope at the National Radio Astronomy Observatory in Green Bank. In 1995 we used a 70 MHz band centered about 1405 MHz; in 1996–1999 we used a 110 MHz band, also centered about 1405 MHz. The drift scans were $\approx 4^\circ$ in length, which is ≈ 4 times the resolution element of the telescope. We describe these observations and their analysis in greater detail below. In total, we measured the flux density of Cassiopeia A relative to that of Cygnus A, $F_{1405}^{\text{CasA}}/F_{1405}^{\text{CygA}}$, 5 times over a span of 5 years. We list these measurements in Table 4. If one models the flux density of Cassiopeia A as a constant over this interval of time, we find that $F_{1405}^{\text{CasA}}/F_{1405}^{\text{CygA}} = 1.266 \pm 0.023$ for a mean epoch of 1997.4. This implies a statistical measurement error of $\approx 1.8\%$, which is an upper limit, since Cassiopeia A is in reality fading over this time interval. Since we used only one of two orthogonal linears, we add in quadrature to this statistical error, systematic errors of $\leq 0.2\%$ and $\leq 0.5\%$, corresponding to the polarizations of Cassiopeia A and Cygnus A, respectively. This implies a total measurement error of $\leq 1.9\%$. Conservatively adopting a total measurement error of 1.9%, a value of $F_{1405}^{\text{CygA}} = 1581 \text{ Jy}$ from the absolute spectrum calibration of Cygnus A from Baars et al. (1977), and a 2% uncertainty in this calibration, we find that $F_{1405}^{\text{CasA}} = 2002 \pm 55 \text{ Jy}$ for this mean epoch.

Prior to the transit of each source, we manually adjusted the declination of the telescope, ensuring, with the aid of a strip chart recorder, that the pointing of the telescope contributed no more than $\approx 0.5\%$ of error to the flux density measurement. During transit and for the remainder of the observation, we left the telescope at this, its final declination. Afterward, we excised the data from the first half of the observation that we had taken at declinations other than the final declination. This allowed the background to be reliably modeled and subtracted. Across the $\approx 4^\circ$ lengths of the observations, the background appears to be linear, both in the case of the Cassiopeia A observations and in the case of the Cygnus A observations. Consequently, we modeled the background as linear and subtracted it, simultaneously removing any H I component to the emission. We

TABLE 4

GREEN BANK 40 FOOT TELESCOPE MEASUREMENTS OF THE FLUX DENSITY OF CASSIOPEIA A RELATIVE TO THAT OF CYGNUS A AT 1405 MHz FROM 1995 TO 1999

Epoch	$F_{1405}^{\text{CasA}}/F_{1405}^{\text{CygA}}$ (±1.9%)	F_{1405}^{CasA} (Jy)
1995.6	1.294	2046 ± 56
1996.6	1.267	2003 ± 55
1997.6	1.273	2013 ± 56
1997.6	1.265	2000 ± 55
1999.6	1.231	1946 ± 54

^a See § 2.

^b For $F_{1405}^{\text{CygA}} = 1581 \pm 32 \text{ Jy}$ (see § 2).

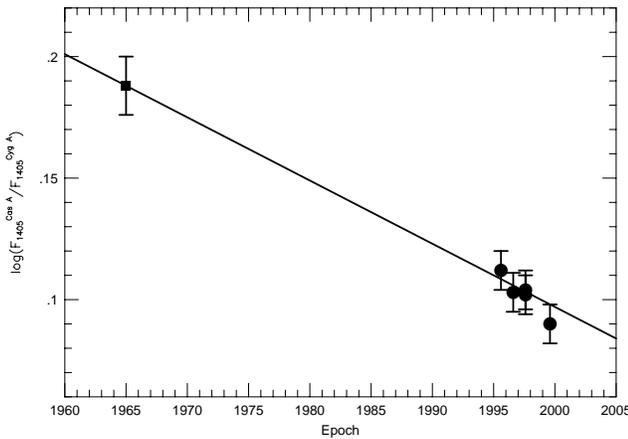


FIG. 4.—Light curve of measurements of the flux density of Cassiopeia A relative to that of Cygnus A at 1405 MHz. Circles denote our measurements (see Table 4); the square denotes the 1965 value from Baars et al. (1977). The solid line corresponds to a fading rate of $0.62\% \text{ yr}^{-1}$.

estimate that background subtraction contributes $\approx 1\%$ of error to each flux density measurement. Finally, before and after the ≈ 30 minute durations of the Cassiopeia A observations and the ≈ 20 minute durations of the Cygnus A observations, we took 2 minute long calibration readings. Each pair of readings agreed to better than $\approx 1\%$; nonetheless, we linearly interpolated between the readings, making this a negligible source of error. Consequently, and in total, we estimate that the measured flux density ratios, $F_{1405}^{\text{CasA}}/F_{1405}^{\text{CygA}}$, should be in error by no more than $\approx 1.6\%$; this is consistent with the statistical measurement error of $\lesssim 1.8\%$ that we determine above. Conservatively adopting total measurement errors of 1.9% , a 1965 value of $F_{1405}^{\text{CasA}}/F_{1405}^{\text{CygA}} = 1.542$ from the absolute spectrum calibrations of Cassiopeia A and Cygnus A from Baars et al. (1977), and 2% uncertainties in each of these calibrations, we find a 1965–1999 fading rate of $0.62\% \pm 0.12\% \text{ yr}^{-1}$ ($\chi^2 = 0.36$ for $\nu = 4$ degrees of freedom) at 1405 MHz. We plot the 1405 MHz light curve in Figure 4.

3. CONCLUSIONS: A DECREASING FADING RATE OF CASSIOPEIA A AT INTERMEDIATE RADIO FREQUENCIES (927–3060 MHz)

We plot the intermediate-frequency (927–3060 MHz) measurements of Cassiopeia A's fading rate in Figure 5, and we list this information in Table 5. When compared to measurements of the fading rate between 1957 and 1976, our measurement at 1405 MHz, in conjunction with a recent measurement at the nearby frequency of 927 MHz by Vinyajkin (1997), show that the fading rate is decreasing at a rate that is intermediate to the rates measured at the lower (38–300 MHz) and the higher (7.8–16.5 GHz) frequencies. We find the fading rate to be decreasing by $\approx 1\% \text{ yr}^{-1}$ century $^{-1}$ at these intermediate frequencies.

Furthermore, we find that around 1990, Cassiopeia A was fading at about the same rate, $\approx 0.6\%–0.7\% \text{ yr}^{-1}$, at all of these frequencies. The next decade of observations should reveal whether the fading rate will continue to decrease at

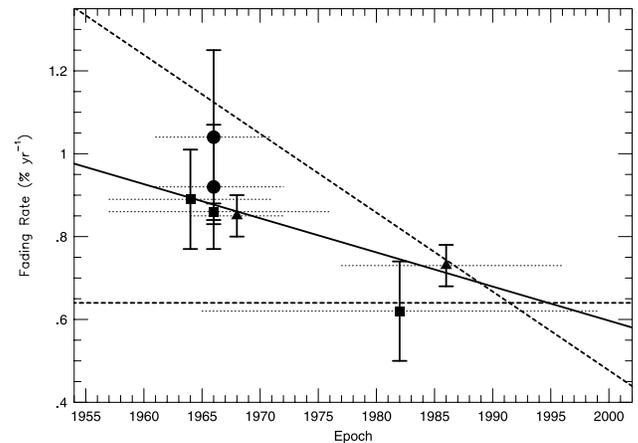


FIG. 5.—Fading rates of Cassiopeia A as a function of epoch for the intermediate radio frequencies (927–3060 MHz; see Table 5). The dotted lines are as defined in Fig. 2. Triangles denote 927–950 MHz measurements, squares denote 1405–1420 MHz measurements, and circles denote 3000–3060 MHz measurements. The solid line corresponds to a fading rate that is decreasing at a rate of $\approx 1\% \text{ yr}^{-1}$ century $^{-1}$. The dashed lines are the same as the solid lines in Figs. 2 and 3.

TABLE 5
FADING RATES OF CASSIOPEIA A AT INTERMEDIATE RADIO FREQUENCIES (927–3060 MHz)

Frequency (MHz)	Measurement Epochs	Central Epoch	Fading Rate (% yr $^{-1}$)	Reference
950	1964–1972	1968	0.85 ± 0.05	1
927	1977–1996	1986	0.73 ± 0.05	2
1420	1957–1971	1964	0.89 ± 0.12	3
1420	1957–1976	1966	0.86 ± 0.02	4
1405	1965–1999	1982	0.62 ± 0.12	5
3000	1961–1972	1966	0.92 ± 0.15	3
3060	1961–1971	1966	1.04 ± 0.21	6

REFERENCES.—(1) Stankevich, Ivanov, & Torkhov 1973b; (2) Vinyajkin 1997; (3) Baars & Hartsuijker 1972; (4) Read 1977a; (5) this paper; (6) Stankevich et al. 1973a.

the lower frequencies, or whether Cassiopeia A will now fade at a relatively constant rate at all of these frequencies.

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