

REVISITING THE MODIFIED EDDINGTON LIMIT FOR MASSIVE STARS

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 Received 1997 August 11; accepted 1998 April 6

ABSTRACT

We have determined the location of the line-opacity modified Eddington limit for stars in the LMC using the most recent atmosphere models combined with a precise mapping to the H-R diagram through up-to-date stellar evolution calculations. While we find, in agreement with previous studies, that the shape of the modified Eddington limit *qualitatively* corresponds to the Humphreys-Davidson (HD) limit defined by the most luminous supergiants, the modified limit is actually *a full magnitude higher* than the upper luminosity limit observed for LMC stars. The observed limit is consistent with atmosphere models in which the maximum value of the ratio of the radiation force outward to the gravitational force inward ($Y_{\max}$) is ~ 0.9 , i.e., the photospheres of stars at the observed luminosity limit are bound. As massive stars evolve, they move to higher, and therefore less stable, values of $Y_{\max}$, so mass loss, either sporadic or continuous, may halt their natural redward evolution as they approach the observed $Y_{\max} \approx 0.9$ limit. We assess the metallicity dependence of this limit. If mass loss does limit the redward evolution of the most luminous stars, and if the value of $Y_{\max}$ corresponding to the luminosity limit in the LMC is universal, then the brightest supergiants of the SMC should be only marginally brighter (0.3 mag) than those of the LMC, in agreement with observations. Moreover, the brightest supergiants in M31 should be 0.75 mag fainter than those in the LMC.

Subject headings: Magellanic Clouds — stars: atmospheres — stars: fundamental parameters — stars: mass loss — supergiants

1. INTRODUCTION

The existence of a temperature-dependent upper luminosity limit for massive stars was first pointed out by Hutchings (1976) from observations of early-type stars in the Large Magellanic Cloud (LMC). Humphreys & Davidson (1979) subsequently determined the maximum luminosities of the coolest of the LMC's massive stars (the red supergiants) and defined a temperature-dependent maximum luminosity limit, often referred to as the “HD limit,” extending across the entire upper region of the H-R diagram. While estimates of the precise location of the limit have varied over the years, depending on the favored LMC distance modulus and the subjectivity in drawing upper limits on sparse data sets, newer observational studies have confirmed that the maximum luminosity observed for O-type stars is considerably higher, ~ 2 mag, than that seen for the most luminous M supergiants. This difference is much greater than can be explained by observational errors or uncertainties in the bolometric corrections and has had a profound impact on our understanding of the evolution of massive stars, indicating that stars with initial masses greater than $\sim 40 M_{\odot}$ spend their lives in the blue part of the H-R diagram without becoming red supergiants.

Evolution models for massive stars can be made to reproduce the observed H-R diagram if sufficiently high stellar mass-loss rates are assumed near the HD limit (e.g., see Schaller et al. 1992). High mass loss leads to the removal of the H-rich stellar envelopes, halts redward evolution, and ultimately produces Wolf-Rayet stars. The presence near the observed HD limit of the luminous blue variable stars (LBVs), with their occasional outbursts and extreme mass

ejections, seems to verify that mass loss—perhaps violent and episodic—is indeed the primary agent for shaping the properties of the upper H-R diagram (see Humphreys & Davidson 1994 for a detailed review of the LBVs).

One of the most fundamental unanswered questions regarding the evolution of massive stars is the nature of the underlying instability mechanism that induces mass-loss rates high enough to produce the outbursts observed in the LBVs and to carve the HD limit into the H-R diagram. One of the first suggestions was that stars become unstable near the HD limit due to radiation pressure (Humphreys & Davidson 1984; Lamers 1986; Appenzeller 1986). This model holds that as massive stars ($M > 40 M_{\odot}$) evolve away from the zero-age main sequence, their photospheres become decreasingly stable against radiation pressure and ultimately reach a critical point where the radiation pressure and gravity are balanced, leading to large mass loss and ending the redward evolution. Because of metal line opacity, the luminosity at which a stellar photosphere becomes unstable may be much lower than that predicted by the classical electron-scattering Eddington limit.

Quantitative studies of a “modified Eddington limit” were performed by Lamers & Fitzpatrick (1988) using low-gravity, line-blanketed, plane-parallel LTE model atmosphere calculations. By extrapolating from the low-gravity models to a point at which radiation pressure balances gravitational pressure, they determined that the modified Eddington limit was in reasonable agreement with the observed upper luminosity limit for hot stars ($> 10,000$ K) in the LMC. Lamers & Noordhoek (1993) extended this work to examine the metallicity dependence of the modified Eddington limit. In recent work, Lamers (1997) also examines the metallicity dependence of the Eddington limit and argues that LBVs are close to, but not at, the modified Eddington limit unless they are rapidly rotating. Achmad, de Jager, & Nieuwenhuijzen (1993) found that cool super-

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giants ($< 10,000$ K) are observationally excluded from the region of luminosity/temperature space predicted to be unstable from the modified Eddington limit approach (Gustafsson & Plez 1992).

Alternative explanations for the HD limit include instabilities of radial modes in massive stars (Glatzel & Kiriakidis 1993; Kiriakidis, Fricke, & Glatzel 1993); turbulent pressure (e.g., de Jager 1984); and binary star models (e.g., Kenyon & Gallagher 1985). Humphreys & Davidson (1994) critically review all of these proposed instability mechanisms and conclude that none, at least in the current state of development, is fully satisfactory. It is important to understand the nature of the mass-loss and instability mechanisms operating in the upper H-R diagram—not only to complete the theoretical picture of stellar evolution but also to aid in the interpretation of observations of massive stars. One possible application of such an understanding would be to determine whether the brightest stars can be used as reliable distance indicators (e.g., Humphreys & Aaronson 1987).

In this paper, we revisit the modified Eddington limit scenario proposed by Lamers and collaborators. We utilize up-to-date stellar atmosphere calculations to evaluate the radiation pressure stability of low surface gravity stars and the most recent stellar evolution calculations to transform the stellar atmosphere parameters (T_{eff} and $\log g$) to the H-R diagram (T_{eff} and L). The model atmosphere calculations and the transformation to the H-R diagram are described in § 2. In § 3, the modified Eddington limit is compared with the observed upper H-R diagram of the LMC. Concluding remarks are given in § 4.

2. LOW-GRAVITY STELLAR PHOTOSPHERE MODELS AND THE H-R DIAGRAM

As in Lamers & Fitzpatrick (1988), our basic procedure is to compute line-blanketed, plane-parallel LTE stellar photosphere models for many T_{eff} 's corresponding to OB stars. At each temperature, we compute models for surface gravities, g , ranging from those appropriate for the main sequence ($\log g \simeq 4.0$) down to the lowest values for which a model in hydrostatic equilibrium can be computed. We then determine the luminosity, L , corresponding to each model atmosphere from stellar evolution calculations and thus can place the atmosphere models on the H-R diagram (L vs. T_{eff}) and compare with observations.

For calculating stellar atmospheres, we employ the ATLAS9 model atmosphere code of Kurucz (1995), kindly provided by R. L. Kurucz. The opacity distribution functions (ODFs) needed to compute the models were obtained from the CCP7 library (Jeffery 1990). We produced grids of models for four different metallicities, $Z/Z_{\odot} = 2.0, 1.0, 0.3$, and 0.1 , using the ODFs corresponding to a microturbulence velocity of $v_{\text{turb}} = 8 \text{ km s}^{-1}$. These values of the metal abundance can be taken as crude approximations for the massive-star populations in M31, the Milky Way, the LMC, and the SMC (in order of decreasing abundance), although these galaxies almost certainly exhibit a range of metallicities. Note that in the Kurucz ODFs all metals are taken to scale together. Thus, e.g., in the $Z/Z_{\odot} = 2.0$ models all the elemental abundances (excluding H and He) are taken at twice their solar values. Since Fe in its various ionic states is a major source of ultraviolet opacity for the early-type stars, the observational parameter most relevant for comparison with these models is probably Fe/H. For super-

giants that, through mass loss, have exposed their processed, helium-enriched cores, the opacities are clearly altered, and none of our scaled solar abundance models may be directly applicable. According to stellar evolution codes (e.g., Schaller et al. 1992), however, the cores are generally revealed only after the stars have hit the HD limit and retreated blueward (the exception is for massive stars with $M \geq 100 M_{\odot}$ and $Z/Z_{\odot} \geq 1$). Therefore, the helium-enriched stars are generally not the stars that define the HD limit.

In a study of the energy distributions of early-type stars using low-dispersion *IUE* data, Fitzpatrick & Massa (1998) find that large values of v_{turb} ($\sim 8 \text{ km s}^{-1}$) are required to reproduce the observed UV opacity in O stars and high-luminosity B stars. It is likely that the large equivalent widths of the strong stellar “photospheric” absorption features, which require the high values of v_{turb} to reproduce, are actually caused by a physical mechanism very different from microturbulence, namely, systematic velocity gradients due to increasingly deep penetration of the stellar wind into the photosphere (e.g., Massa, Shore, & Wynne 1992). Nevertheless, the important point for this investigation is that the $v_{\text{turb}} = 8 \text{ km s}^{-1}$ models represent the *observed opacities* remarkably well.

In all, we computed several thousand low-gravity models (which are available on request) at 35 different values of T_{eff} between 10,000 and 50,000 K. We characterize each atmosphere with the parameter Y_{max} —suggested in Davidson (1989) and Humphreys & Davidson (1994)—which is the maximum value of the ratio of the outward radiative force to the inward Newtonian gravitational force found within the optical depth range $10^{-2} < \tau < 10^2$, i.e., $Y_{\text{max}} = g_{\text{rad, max}}/g_{\text{grav}}$. A value of $Y_{\text{max}} = 1$, which formally defines the modified Eddington limit, corresponds to the case where the radiative and gravitational forces are equal.

A model in hydrostatic equilibrium cannot be computed for $Y_{\text{max}} = 1$, nor can models be computed arbitrarily close to this value. For most models, Y_{max} is obtained at optical depths of $\tau \simeq 1$ –15. Close to the modified Eddington limit, however, the region with highest radiative acceleration generally shifts to the surface, at $\tau < 10^{-3}$, and this constrains the lowest surface gravity for which a hydrostatic model can be computed. As noted by Lamers & Fitzpatrick (1988) and Lamers & Noordhoek (1993), this is not considered to represent the modified Eddington limit because the highest levels of the photospheres of normal OB stars merged with stellar winds are not in hydrostatic equilibrium, although such stars are considered to be stable. Both Lamers & Fitzpatrick (1988) and Lamers & Noordhoek (1993) extrapolated to estimate the value of g corresponding to the hypothetical case where $Y_{\text{max}} = 1$ (see Fig. 3 in Lamers & Fitzpatrick 1988) from models in which Y_{max} was determined at $\tau > 10^{-2}$. In this paper, we generally restrict our attention to values of Y_{max} less than 0.95; extrapolations are required only in a small number of cases and will be noted where appropriate.

Figure 1 demonstrates how the surface gravity, g , of models approaching the modified Eddington limit compare to those at the classical electron-scattering Eddington limit, defined by

$$g_{\text{Edd}} = \frac{4\pi\sigma T_{\text{eff}}^4 G}{L_{\text{Edd}}/M_*} \approx 6.55 \times 10^{-16} T_{\text{eff}}^4 \left(\frac{\mu_e}{1.15} \right)^{-1} \text{ cm s}^{-2}, \quad (1)$$

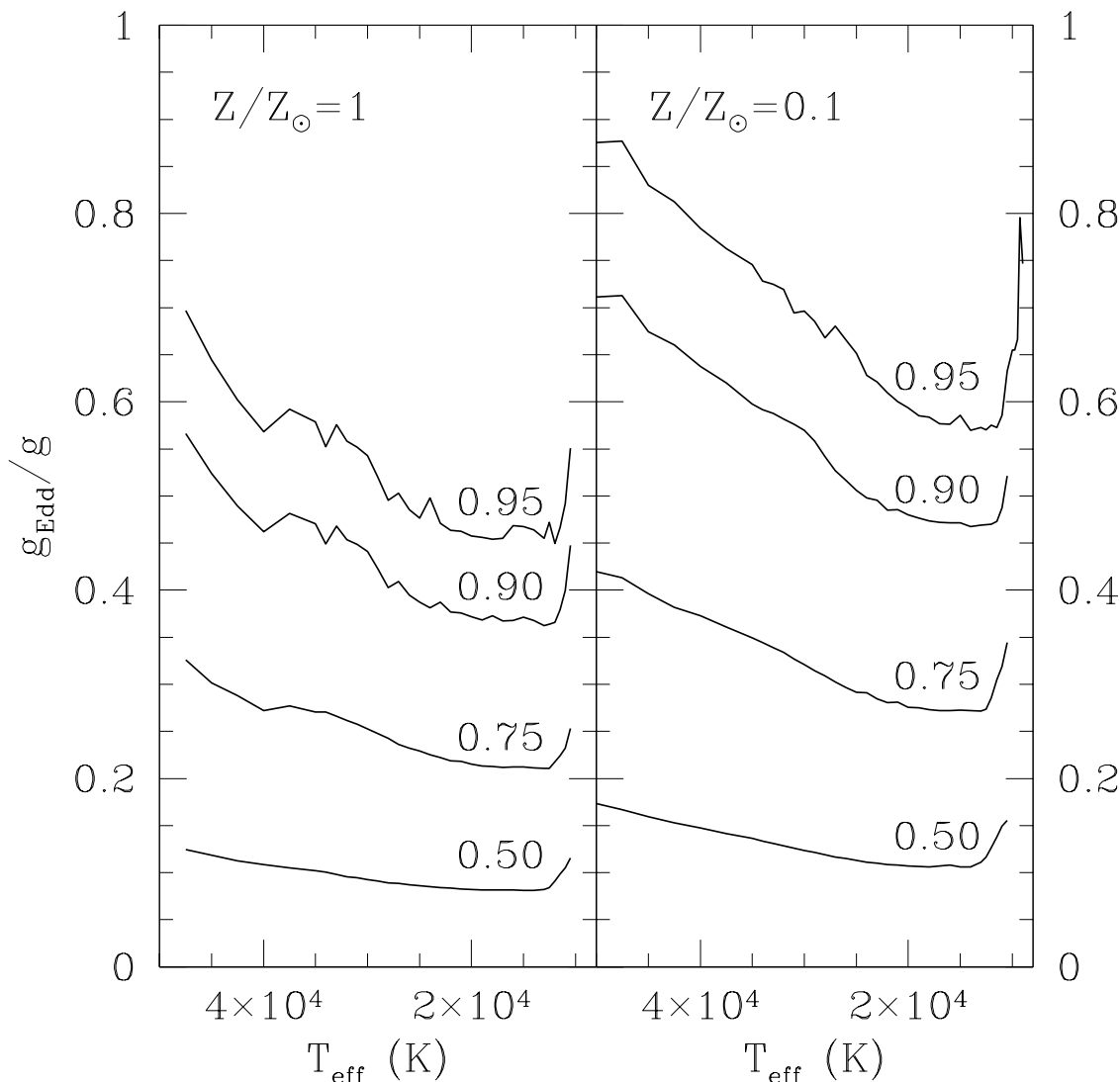


FIG. 1.—Ratio of Newtonian surface gravity g to classical Eddington gravity g_{Edd} (see eq. [1]) for metal line-blanketed atmosphere models with $Y_{\text{max}} = 0.95, 0.90, 0.75$, and 0.50 , where Y_{max} is the maximum value of the ratio of outward radiative force to inward Newtonian gravitational force within the optical depth range $2 \times 10^{-2} < \tau < 10^2$, i.e., $Y_{\text{max}} = g_{\text{rad,max}}/g_{\text{grav}}$. Results for a range of effective temperatures and two metallicities (solar and 0.1 times solar) are shown. The value of $Y_{\text{max}} = 0.90$ most closely follows the LMC. The limiting luminosities are significantly lower than those of the electron-scattering Eddington limit and correspond to $0.3\text{--}0.5L_{\text{Edd}}$ and $2\text{--}3g_{\text{Edd}}$, where $g_{\text{Edd}} = 4\pi\sigma T_{\text{eff}}^4 G/(L_{\text{Edd}}/M_*)$.

where μ_e is the mean atomic weight per electron. The two panels show the ratio of the surface gravities over a range of effective temperatures for two different metallicities and four representative values of Y_{max} (0.95, 0.90, 0.75, and 0.50). At all metallicities, the highest temperature models come closest to g_{Edd} since their atmospheric opacity is dominated by electron scattering. At lower temperatures, metal line blanketing in the UV becomes increasingly important and the classical and modified limits diverge. The rise in g_{Edd}/g below about 11,000 K is likely caused by the shifting of the emergent energy distributions out of the UV and into the relatively unblanketed optical region. Not surprisingly, the modified Eddington limit comes closest to the electron-scattering limit in the lowest metallicity models. Modest extrapolation is required outside the range $\sim 13,000\text{--}30,000$ K to reach $Y_{\text{max}} = 0.95$. The extrapolation is largest for models with $T_{\text{eff}} > 40,000$ K and solar metallicity.

To determine the luminosities corresponding to our atmosphere models, we use the stellar evolution grids published by Schaller et al. (1992) for $Z/Z_{\odot} = 1.0$ and 0.05 ;

Schaerer et al. (1993b) for $Z/Z_{\odot} = 0.4$; Schaerer et al. (1993a) for $Z/Z_{\odot} = 2.0$; and Charbonnel et al. (1993) for $Z/Z_{\odot} = 0.2$. These models were computed with the most recent updates of the relevant physical parameters (e.g., opacities), include the effect of mass loss by winds, and were tabulated explicitly for ease of interpolation within and between the grids. For simplicity, previous studies (Lamers & Fitzpatrick 1988; Lamers & Noordhoek 1993; Lamers 1997) used a mass-luminosity relation based on the end of the core hydrogen burning (CHB) evolutionary phase to map the atmosphere models onto the H-R diagram. Lamers & Noordhoek (1993) noted that many of the models considered actually correspond to stars still in the CHB phase, and that this procedure limits the ability to make quantitative comparisons with observations. We take a different approach here and interpolate within a grid of stellar evolution calculations (of the appropriate metallicity) to find the initial masses of all models that pass through a given set of T_{eff} and $\log g$ values, as well as the stellar luminosity at the desired T_{eff} and $\log g$. In this way we achieve an essentially

exact mapping of the T_{eff} and $\log g$ values onto the H-R diagram without simplifying assumptions.

Figure 2 shows the results of this mapping onto the M_{bol} vs. $\log T_{\text{eff}}$ diagram for the calculations done with solar metallicity for representative values of $Y_{\text{max}} = g_{\text{rad,max}}/g_{\text{grav}} = 0.95, 0.90, 0.75$, and 0.50 . Figure 2 yields two important results. First, the shape of the curves with Y_{max} values less than 1 are very similar to each other and to that derived by Lamers & Noordhoek (1993) for the extrapolated case of $Y_{\text{max}} = 1.0$. This characteristic shape, dubbed the “Eddington trough” by Lamers & Noordhoek (1993), is thus not unique to the hypothetical point of radiative instability, but rather represents the locus of constant Y_{max} values. Second, during the CHB phase the atmospheres of massive stars evolve in the direction of increasing Y_{max} , i.e., toward decreased stability against radiation pressure. These points will be discussed further in the following section, where the modified Eddington limit is compared with observations.

3. COMPARISON WITH OBSERVATIONS

Studies of the upper H-R diagram have often focused on the LMC for well-known observational reasons, including the uniform and well-determined distance of the stars, the low line-of-sight reddening, and the nearly complete census of the most luminous stars. In Figure 3, we reproduce the LMC H-R diagram published by Fitzpatrick & Garmany (1990). The observational errors are discussed in Fitzpatrick & Garmany (1990) and are small compared to the 2 mag difference between maximum luminosities observed for the

bluest and reddest stars. Two changes have been made for this paper. First, we adjusted the values of M_{bol} to reflect an LMC distance modulus of 18.6 mag (e.g., Whitelock, van Leeuwen, & Feast 1997). Second, we added data for about 80 O stars near the 30 Doradus region from a recent paper by Walborn & Blades (1998). The various features of the LMC H-R diagram, and the details of its construction, are discussed in Fitzpatrick & Garmany (1990). For our purposes here, the important aspect of the diagram is that there are many stars more luminous than $M_{\text{bol}} = -10$ for $T_{\text{eff}} \gtrsim 25,000$ K, while there are few, if any, for $T_{\text{eff}} \lesssim 25,000$ K, including the M supergiants, which have maximum luminosities of $M_{\text{bol}} \simeq -9.5$ (Humphreys 1979).

In Figure 3 we also show the results of the stellar atmosphere calculations for $Y_{\text{max}} = 1, 0.90, 0.75$, and 0.50 . These were computed for $Z/Z_{\odot} = 0.3$, appropriate for the LMC, and converted to the H-R diagram using a grid of stellar evolution models interpolated between the Schaerer et al. (1993b; $Z/Z_{\odot} = 0.4$) and Charbonnel et al. (1993; $Z/Z_{\odot} = 0.2$) grids. The luminosity at $Y_{\text{max}} = 1$ was estimated only for temperatures in the range 13,000–18,000 K, for which stable models could be computed out to $Y_{\text{max}} \simeq 0.98$. The extrapolation to the modified Eddington limit ($Y_{\text{max}} = 1$) is thus relatively secure in this region. The temperature dependence of the modified Eddington limit outside these temperatures may be inferred from the shapes of the other curves.

Figure 3 shows that the bottom of the trough of the modified Eddington limit ($M_{\text{bol}} \simeq -11$) is about 1 mag more luminous than the brightest LMC stars with $T_{\text{eff}} \lesssim 25,000$ K. This result is actually quite similar to those found by Lamers & Fitzpatrick (1988) and Lamers & Noordhoek

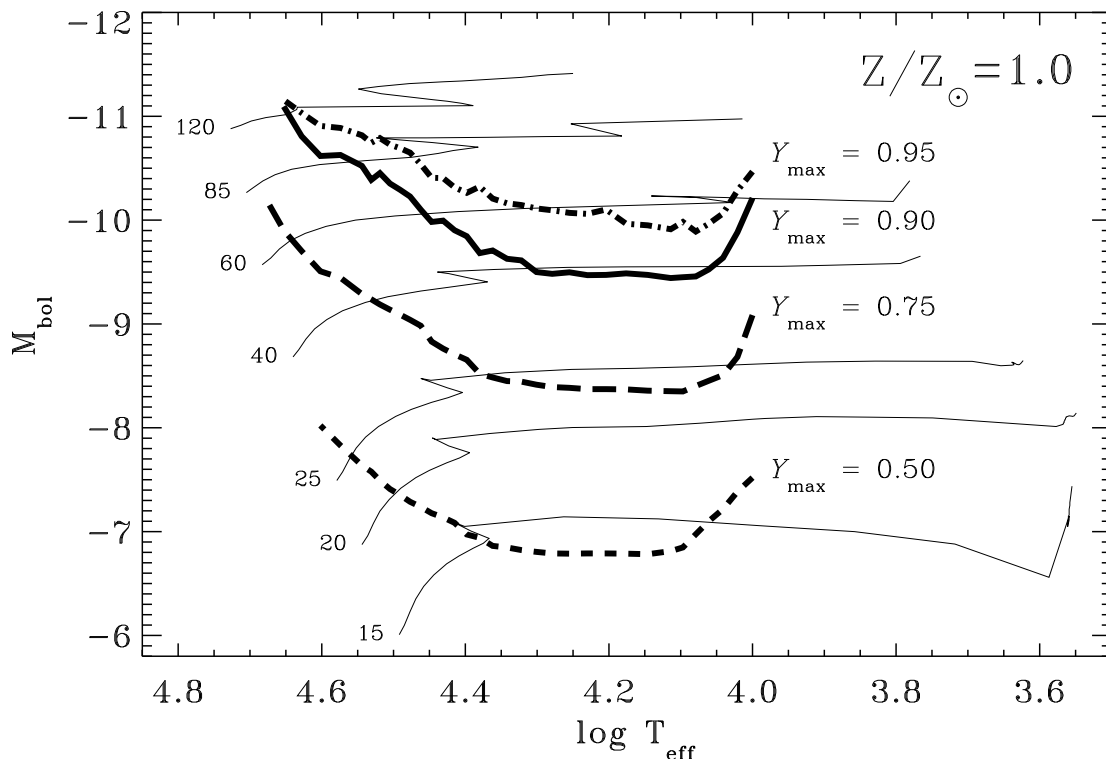


FIG. 2.—Locus of atmosphere models for representative values of $Y_{\text{max}} = g_{\text{rad}}/g_{\text{grav}} = 0.95, 0.90, 0.75$, and 0.50 are shown as thick solid and dashed lines. The thin lines show evolution tracks and are labeled with their respective initial masses (in units of $M_{\odot}$) (Schaller et al. 1992). For the models with $M_i < 25 M_{\odot}$ we show the tracks from the zero-age main sequence to the end of core helium burning. For the more massive stars we truncate the tracks at the coolest point in the evolution before the tracks double back to the blue.

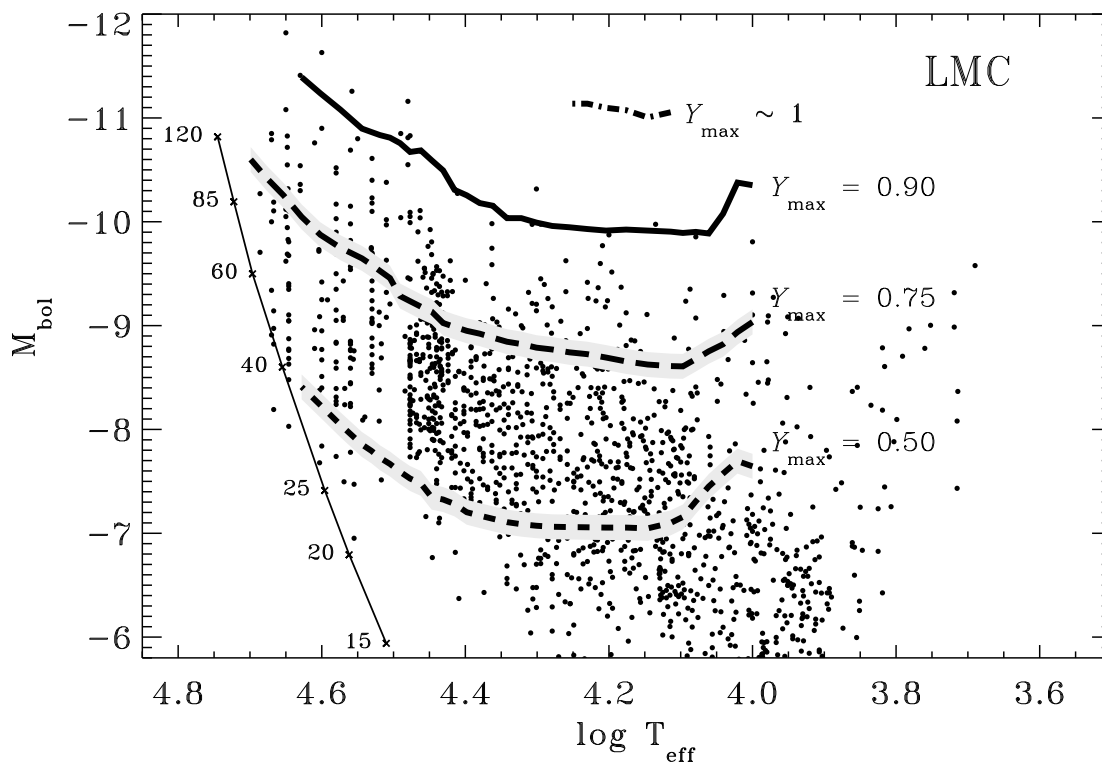


FIG. 3.—LMC upper H-R diagram from Fitzpatrick & Garmany (1990) with additional O stars from Walborn & Blades (1998). The thick solid and dashed lines show the locus of atmosphere models for $Y_{\max} = g_{\text{rad,max}}/g_{\text{grav}} = 1.0, 0.90, 0.75$, and 0.50 . The locus for $Y_{\max} = 0.90$ most closely resembles the upper luminosity limit. The points for $Y_{\max} = 1$ were estimated as described in the text and show that the modified Eddington limit, i.e., $Y_{\max} = 1$, is about 1 mag higher than the brightest stars with $T_{\text{eff}} \lesssim 25,000$ K. A few stars are above the 0.90 limit, in accord with expectations of a few misidentified effective temperatures, unresolved binaries, or observational error. Additionally, the physical depth of the LMC may induce a scatter of up to 0.5 mag if the LMC's depth is comparable to its 10 kpc width.

(1993); however, in those papers known deficiencies in the model atmosphere opacities (Lamers & Fitzpatrick 1988) and inadequate transformations to the H-R diagram (Lamers & Fitzpatrick 1988 and Lamers & Noordhoek 1993) obscured the significance of the discrepancy. Thus, in contrast to previous conclusions, we believe that Figure 3 shows quite clearly that *the modified Eddington limit does not coincide with the maximum luminosity observed for LMC stars*. Rather, we suggest that the observed temperature-dependent luminosity limit is much better defined by the locus of model atmospheres with $Y_{\max} \approx 0.90$. Only those stars whose luminosities never exceed the minimum value for these models ($M_{\text{bol}} \approx -9.9$) are able to complete their redward evolution and become red supergiants.

We can estimate the metallicity dependence of the upper luminosity limit by comparing, as in Lamers & Noordhoek (1993), the luminosities of the $Y_{\max} = 0.90$ models at various metallicities. Figure 4 shows such a comparison for $Z/Z_{\odot} = 2.0, 1.0, 0.3$, and 0.1 . The curves for the four metallicities have nearly identical shapes, the Eddington trough, and differ only by simple displacements. From this comparison we might expect the luminosity limit for cool stars in low-metallicity systems such as the SMC ($Z/Z_{\odot} \approx 0.1$) to be higher by 0.3 mag than the LMC, and those for higher metallicity systems such as the Milky Way ($Z/Z_{\odot} \approx 1.0$) and M31 ($Z/Z_{\odot} \approx 2.0$) to be lower by 0.4 and 0.75 mag, respectively.

The relatively small difference between the maximum luminosities expected for the LMC-like and SMC-like abundances is consistent with the lack of any obvious offset

between the LMC and SMC H-R diagrams (e.g., Garmany & Fitzpatrick 1989). Humphreys (1988) actually finds the SMC M supergiants to be somewhat less luminous than for the LMC, but this result may be affected by small-number statistics as noted by Humphreys. In addition, some studies find the metal abundances of luminous SMC supergiants to be as high as 0.13–0.25 times the solar value (e.g., Hill 1997), which would also tend to equalize the LMC and SMC luminosity limits.

Despite the observational challenge in resolving individual stars in M31 (cf. Massey et al. 1995), comparison of that galaxy with the LMC and SMC may offer the best hope for testing the predictive value of the $Y_{\max} \approx 0.9$ curves. Additionally, a strong metallicity gradient exists in M31 (a factor of ~ 5 from the center to 20 kpc; Blair, Kirshner, & Chevalier 1982a, 1982b), so it may be possible to observe the variation of the upper luminosity within that galaxy. More distant galaxies may also be useful to evaluate the metallicity dependence, but care must be taken to determine the metallicity appropriate to each star (in cases where strong galactic metallicity gradients are present) and to ensure that the stars are indeed single and not multiple systems (Humphreys & Aaronson 1987).

4. CONCLUDING COMMENTS

In summary, we have determined the location of the modified Eddington limit for stars in the LMC, using the most recent atmosphere models combined with a precise mapping to the H-R diagram through up-to-date stellar evolution calculations. We find that the modified Edding-

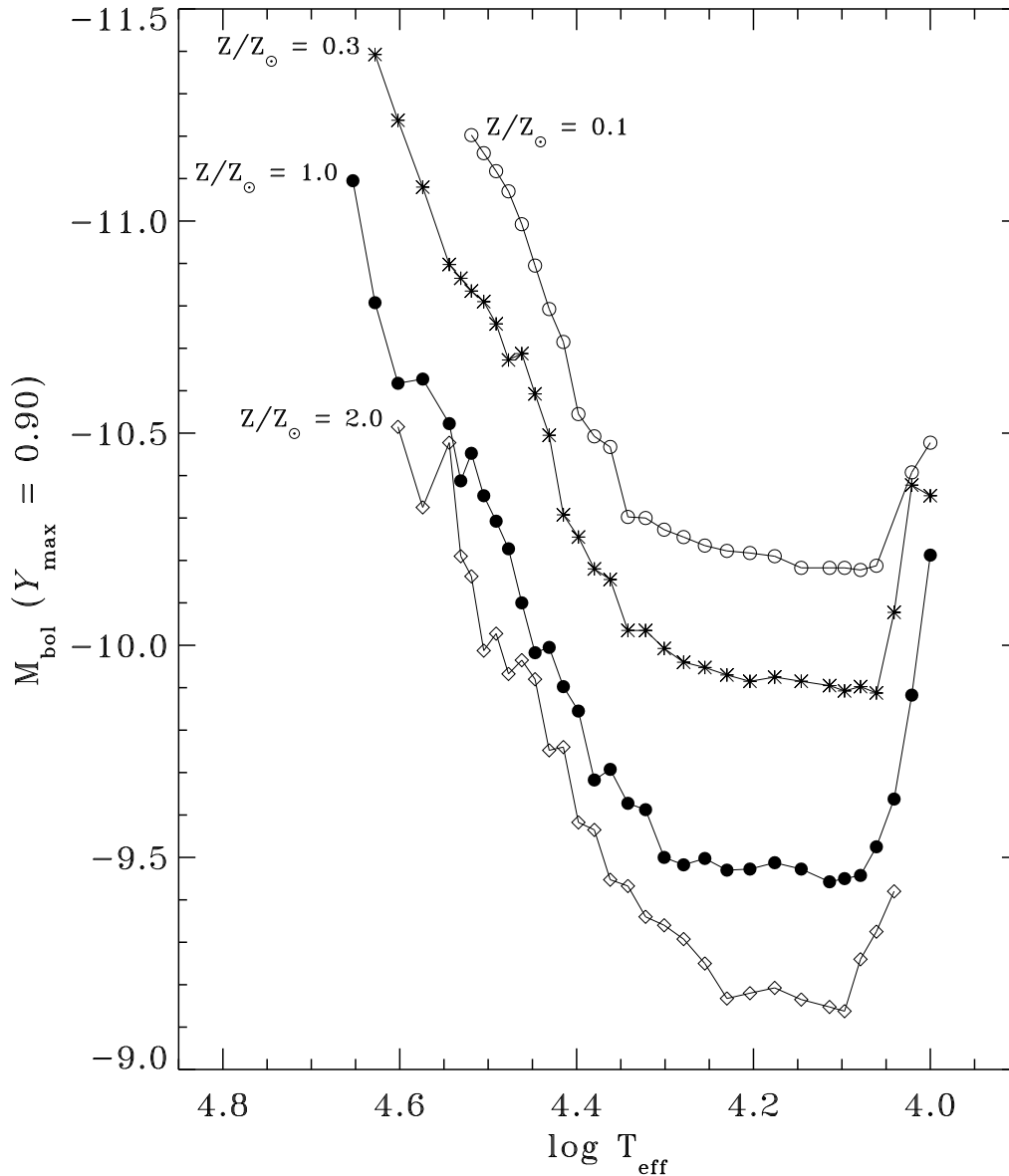


FIG. 4.—Metallicity dependence of the luminosity limit corresponding to $Y_{\max} = 0.90$ for $\log(Z/Z_{\odot}) = 1.3, 1, -0.5$, and -1 , which are approximately representative of M31, our Galaxy, the LMC, and the SMC, respectively. The shape of the curves, the Eddington trough, remains nearly constant as a function of metallicity.

ton limit is actually *a full magnitude higher* than the upper luminosity limit observed for LMC stars. The observed limit is consistent with atmosphere models in which the maximum value of the ratio of the radiation force outward to the gravitational force inward ($Y_{\max}$) is ~ 0.9 ; i.e., the photospheres of stars at the observed luminosity limit are bound. This result is consistent with the results of Lamers (1997) for LBVs despite some differences in methodology.

With some caution, we thus suggest that the simple picture in which a massive star evolves redward until its photosphere reaches the modified Eddington limit and becomes unbound is invalid. Although the stars do evolve from the zero-age main sequence in the direction of increasing $Y_{\max}$, i.e., *toward* the modified Eddington limit, very high mass-loss rates (capable of halting the redward evolution) must set in *before* the atmospheres reach the formal limit at $Y_{\max} = 1.0$. In Davidson (1989) and Humphreys & Davidson (1994), it is noted that if radiative instabilities them-

selves are the cause of the high mass-loss rates, then they may more naturally occur at a value of $Y_{\max}$ less than 1. Our results do not identify the nature of the instability driving the mass loss. They do, however, indicate that whatever mechanism is responsible is operating in a stellar photosphere which is only tenuously bound because of the effects of radiation pressure.

Our conclusions are necessarily tentative, since this analysis, like others before, relies on plane-parallel, hydrostatic atmosphere models, while the atmospheres of real stars near the observed luminosity limit are likely to exhibit neither of these properties; however, it seems very unlikely to be a coincidence that the temperature dependence of the luminosity limit should so closely match that of the $Y_{\max}$ curves seen in Figures 1–4 whose shapes are nearly invariant to metallicity and to the precise value of $Y_{\max}$ itself. The degree of stability against radiation pressure of the photospheres clearly plays an important role in shaping the upper

stellar luminosity limits, although the current characterization of that stability may leave something to be desired. The $Y_{\max}$ parameterization may well turn out to correlate with some more critical property, such as the depth of the "boundary" between a stellar wind and the underlying photosphere. A firm understanding of the upper luminosity limits and of the outbursts observed in LBVs will almost certainly require a melding of stellar wind, stellar photosphere, and stellar evolution calculations. Fortunately,

progress in this area is being made (e.g., Sellmaier et al. 1993; Schaerer et al. 1996).

We thank B. Paczyński for helpful discussions and R. Humphreys, K. Davidson, and the anonymous referee for comments on the manuscript. A. U. was supported by an NSF graduate fellowship and NSF grants AST 93-13620 and AST 95-30478.

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