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COLLISIONAL PROBABILITY OF PERIODIC COMETS WITH THE TERRESTRIAL PLANETS: AN INVALID CASE OF ANALYTIC FORMULATION

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ABSTRACT

Orbits of 228 known periodic comets ($P < 1000$ yr) were numerically integrated over $\pm 30{,}000$ yr, and thereby mean collision rates (CRs) of those comets with the planets from Venus through Neptune were calculated. For Venus through Mars, past calculations of CRs by analytic methods such as Öpik's are shown to overestimate by factors of 20–45, whereas for the outer planets agreement between numerical and analytic estimates is fairly good (within a factor of ~ 2). This is because analytic methods calculate CRs of comets as if they occupied their current orbits for an unlimited period of time, while in actuality observable periodic comets (the majority of which are of low inclination and interact with Jupiter) stay in the inner planetary regions for much shorter durations than the orbital evolution timescale of periodic comets. It is argued that, assuming a steady state population, the ratio between analytic and numerical estimates of collision rates for periodic comets with the terrestrial planets should be approximately equal to the ratio of observable to unobservable (but still Jupiter-interacting) comets. Implications of our CRs for impact frequencies on the terrestrial and outer planets are also briefly discussed. In particular, we show that, for Jupiter-interacting comets of greater than 1 km diameter, a Jupiter impact takes place every 500–1000 yr, and an Earth impact every 2–4 Myr.

Key words: celestial mechanics, stellar dynamics — Comets: general

1. INTRODUCTION

Near-Earth objects with potential danger of Earth impact are considered to be of either asteroidal or cometary origin. Although statistical analyses of the known population of asteroids and comets indicate that the Apollo-Amor asteroids are a few to several times more hazardous than comets in terms of impact frequency (Olsson-Steel 1987; Shoemaker, Weissman, & Shoemaker 1994), estimates of the total population of objects of cometary origin still have large ambiguity. It has been suggested that if we fairly take account of dormant and/or extinct comets, the contribution of cometary bodies to terrestrial impacts is likely to be equivalent to that of asteroids (Wetherill 1988; Shoemaker et al. 1994; Shoemaker & Shoemaker 1997). Therefore, there are good reasons to scrutinize the impact rates of periodic comets with Earth and the other planets.

"Impact frequency" is here defined as the mean interval of consecutive impacts on a planet. Calculations of impact frequencies for a group of comets classified by given ranges of orbital elements are obtained by multiplication of two factors: (1) the planetary collision rate (CR) (per year per comet) of the group's comets and (2) the total number of the group's comets with potential collision danger (including those currently unobservable because of their large perihelia).

In this paper we will focus on the first factor, for Earth and the other planets. Here we are interested in the periodic comets that predominantly contribute to impacts on the terrestrial planets and Jupiter, which can be collectively identified in several different ways. The traditional classification, short-period (SP) comets defined by $P < 200$ yr, is adopted by Olsson-Steel (1987) and Nakamura & Yoshikawa (1995, hereafter NY95). Carusi et al. (1987), however, suggested that classification by the values of the Tisserand parameter (J) is more suitable for long-term orbital evolution. Following Carusi et al. (1987), Levison & Duncan (1997, hereafter LD97) defined the Jupiter-family comets (JFCs) as those with $2 < J < 3$ (see also Levison 1996) and calculated collision rates of JFCs with the outer planets.

Although Levison's classification is dynamically clear, we are reluctant to follow his definition and terminology of JFCs for two reasons: (1) the name "Jupiter family" is somewhat confusing, because it has often been employed in the literature to describe comets with $P < 15$–20 yr; (2) as we found in this work, impact frequencies with Jupiter and the terrestrial planets for some low-inclination comets with $J < 2$ are not necessarily small compared with those for Levison's JFCs.

Therefore, we here treat as a group the low-inclination (defined in § 2) comets with periods roughly less than 1000 yr, and with perihelia less than 6–6.5 AU. In this paper, we term them "Jupiter-interacting comets" (JICs) for convenience, though this terminology may not be very appropriate for those comets that have relatively weak interactions with Jupiter. Note that this definition cannot always be rigorous, because there are several borderline comets (such as the one seen in Fig. 5*b* below) whose perihelia temporarily exceed the specified range; these have been classified by visual inspection. We also note that our JICs include almost all of the JFCs in LD97.

The impact probability of a small body on a planet has long been calculated using analytic models whose earliest formulation was proposed by Öpik (1951; see also Wetherill 1967; Kessler 1981). Although a fairly good agreement between analytic and numerical estimates in CRs has been shown for the case of SP comets and Jupiter (NY95), it is not a priori clear whether that is the case for other planets, because of the very chaotic nature of orbital motions of SP comets.

Hence, this paper aims at numerically estimating new planetary CRs from only known JICs. Section 2 briefly treats methods of numerically integrating orbits of JICs and of extrapolating the resultant close-encounter statistics to the planetary radii, from which CRs are calculated. In § 3, our obtained CRs are compared with an analytic estimate. We also discuss causes of the large discrepancy found between numerical and analytic CRs for the terrestrial planets, and how our numerical results affect the cometary contribution to the planetary impact frequencies, including those for the outer planets.

2. METHODS OF NUMERICAL INTEGRATION AND ESTIMATES OF COLLISION RATE

In 1991, we calculated 4400 yr orbital evolutions of 160 SP ($P < 200$ yr) comets and thereby estimated the total population of SP comets and their CR with Jupiter (Nakamura & Yoshikawa 1992, 1995). The present paper is an extension of our previous work.

2.1. *Numerical Integration of Orbits of Periodic Comets*

After propagating the orbits of 228 known periodic comets (180 with $P < 200$ yr and 48 with 200 yr $< P <$ 1000 yr) with different epochs (Marsden & Williams 1995) to the standard one of J2000.0, we made N-body numerical integrations of those orbits for $\pm 30{,}000$ yr. The adopted integrator is an extrapolation method of Bulirsch-Stoer type (Bulirsch & Stoer 1966) with variable step-size control (the basic coding is due to T. Fukushima, H. Nakai, & M. Yoshikawa). Venus through Neptune are taken into account as perturbing planets, and their initial conditions are from the JPL DE200 ephemerides. We considered the effects of Mercury only by adding its mass to the Sun. During a run for each comet, all close encounters with all the planets considered and the Sun were monitored and output.

Our choice of an integration time span of $\pm 30{,}000$ yr (which is about 15 times longer than that in Nakamura & Yoshikawa 1991) merits comment. According to the classical model of cometary nuclei (the so-called icy conglomerate model) of Whipple (1950), the physical lifetime of nuclei of 1 km diameter is estimated to be 5000–8000 yr for orbits with perihelion distance (q) of 1 AU (Weissman 1980). If SP comets have such a short physical lifetime, very long orbital integrations will be of little use. However, recent studies on nongravitational effects and the development of nonvolatile crusts on the cometary nuclei (see, e.g., Rickman et al. 1990) indicate that physical lifetimes of SP comets are likely to be 10–20 times longer than the classical estimate. If this is the case, we may integrate as long as, say, hundreds of thousands of years. On the other hand, such a long integration will cause inconvenience from a practical viewpoint. Integrations of hundreds of thousands of years allow a non-negligible fraction of comets to escape from the orbital region in question; this situation is inappropriate to keep the sample number large enough to do statistical analysis, because our basic assumption is that the total JIC population is in steady state, and any new supply of JICs was not modeled in our calculations. We therefore adopted the time span of $\pm 30{,}000$ yr as a reasonable compromise.

LD97 have performed an extensive numerical simulation of the orbital evolution from the Kuiper belt to JFCs (defined in § 1) and compared it with the observed JFC distributions. They found that physical lifetimes of the JFCs are 3000–30,000 yr at a 90% confidence level. Hence our choice of $\pm 30{,}000$ yr is not inconsistent with LD97, either.

2.2. *Steady State Assumption*

Whether or not the assumption of steady state in the JIC population is justified is not clear per se, because there are several processes leading to a non–steady state (such as the periodic crossing of the solar system through the Galactic plane, or stochastic injection of fragments caused by the disruption and splitting of Kuiper belt giant comets; see, e.g., Bailey et al. 1994). However, with the current lack of apparent observational evidence against steady state, the most reasonable measure is to assume that the JIC population is in steady state as a whole; we proceed along this line, as many researchers have done. If some evidence of a non–steady state is found in the future, the steady state population will still be useful as a standard for comparison.

In the course of our $\pm 30{,}000$ yr integrations, a small fraction of initial comets progressively continued to escape from the orbital region of JICs or to be ejected out of the solar system. In order to strictly keep the calculated JIC population in steady state, the number of lost comets must be supplied in compensation. However, we performed our integrations without compensatory additions, because we wanted to derive our statistics from the behavior of known comets only, and to avoid introducing hypothetical supply comets. This approach can be justified as long as the degree of loss is permissively small. Indeed, although the number fraction of depleted comets during $\pm 30{,}000$ yr was some 15%, the influence of depletion on the effective total time span in which the initial comets exist as JICs is much smaller. This effect will be taken into account later, in obtaining CRs. In addition, the 15% loss itself does not substantially affect the resultant CRs.

2.3. *Calculation of Collision Rate*

We calculate a CR for each planet by extrapolating to the planetary radius the cumulative distribution of closest encounters of JICs as a function of encounter distance. In an attempt to reduce noises that appeared in the close-encounter statistics, we used fairly large limits for encounter distances: 0.15–0.3 AU for Venus through Mars and the Sun, and 1–2 AU for Jupiter through Neptune. Relative speeds at closest encounters were also recorded.

Table 1 summarizes the close-encounter number statistics (within the limits mentioned above) for each planet with our low-inclination ($i \leq 65°$) and high-inclination ($i > 65°$) periodic comets. The value of 65° was adopted because inclinations of a few low-inclination comets attained 65° for

TABLE 1

CLOSE-ENCOUNTER NUMBERS OF PERIODIC COMETS WITH PLANETS

PLANET	ENCOUNTER LIMIT (AU)	NUMBER $i \leq 65°$	NUMBER $i > 65°$
Venus	0.15	3018	335
Earth	0.15	3724	274
Mars	0.30	15245	280
Jupiter........	1.00	64453	462
Saturn	1.50	9344	369
Uranus	2.00	1162	153
Neptune......	2.00	191	35

TABLE 2

IN-REGION TIME FOR LOW-INCLINATION ($i \leq 65°$) JICs

In Region (%)	Number of Comets
50–60	6
60–70	9
70–80	4
80–90	14
90–95	12
95–100......	3
100	139

a limited period of time during $\pm 30{,}000$ yr. However, since inclinations of most of those comets seldom exceed 40°–50°, the choice of 40°–50° as the boundary between low-inclination and high-inclination comets would not appreciably affect the following statistics.

We see from Table 1 that close encounters for low-inclination orbits are generally about 10–100 times more frequent than those for high-inclination orbits. As numbers of comets of low inclination and high inclination are respectively 187 (82%) and 41 (18%), a low-inclination comet is found to make close encounters with planets 3–25 times more frequently than a high-inclination comet. It is noted that five comets approached the Sun within 0.1 AU, including some cases of solar impacts.

In calculating CRs, the number of close encounters must be divided by the mean time interval during which comets stay in the Jupiter-interacting region. Table 2 shows the distribution of how much time the calculated low-inclination ($i \leq 65°$) comets spent in the JIC region. The "in region" time is expressed as a percentage of the full integration interval, 60,000 yr. We see in Table 2 that, for instance, nine comets stayed for 36,000–42,000 yr (60%–70%) in the JIC region. From this data, we obtained 95.3% (57,200 yr) as the mean in-region time; the value is afterward used in calculating CRs.

If close encounters of comets with a planet take place randomly in space without gravitational pull, like motions of free molecules contained in a hard-walled box (particle-in-a-box model, or PIAB), it can be shown that the cumulative encounter number within the distance of R is proportional to geometric cross section, or R^2. However, because of the planetary gravity, the incoming flux of comets, or collisional cross section, is increased by a factor of f^2. This f is called the gravitational focusing factor and expressed as

$$f^2 = 1 + 2(m/M)(V_p/v_0)^2(a_p/R) , \tag{1}$$

where m is the planetary mass, M the solar mass, V_p orbital speed of the planet, v_0 incident speed of a comet, a_p the planetary orbital radius, and R closest encounter distance. The speed at closest encounter v_c (NY95) is given by

$$v_c = f v_0 . \tag{2}$$

2.3.1. *Terrestrial Planets*

Figure 1 shows the cumulative number distributions of close encounters of JICs ($i \leq 65°$) with Venus, Earth, and Mars during $\pm 30{,}000$ yr. For the encounters less than 0.01 AU (0.02 AU for Mars), we find that mean encounter speeds are approximately 30, 25, and 18 km s^{-1} for Venus, Earth, and Mars, respectively; in this distance range, there was no appreciable correlation between encounter speed and encounter distance for the three planets. Then the corresponding f^2-values at the planetary surfaces are 1.08–1.20, comparable to the statistical noises of the cumulative distributions. Hence we calculated the collision number with each planet by fitting a straight line by least squares to the cumulative plot and extrapolating the line simply down to the planetary radius.

The resultant collision rate for Earth was thus found to be 5.0×10^{-11} yr^{-1} per comet. Similarly, the collision rates (p) for Venus and Mars were 4.0×10^{-11} and 5.7×10^{-12} yr^{-1} per comet, respectively.

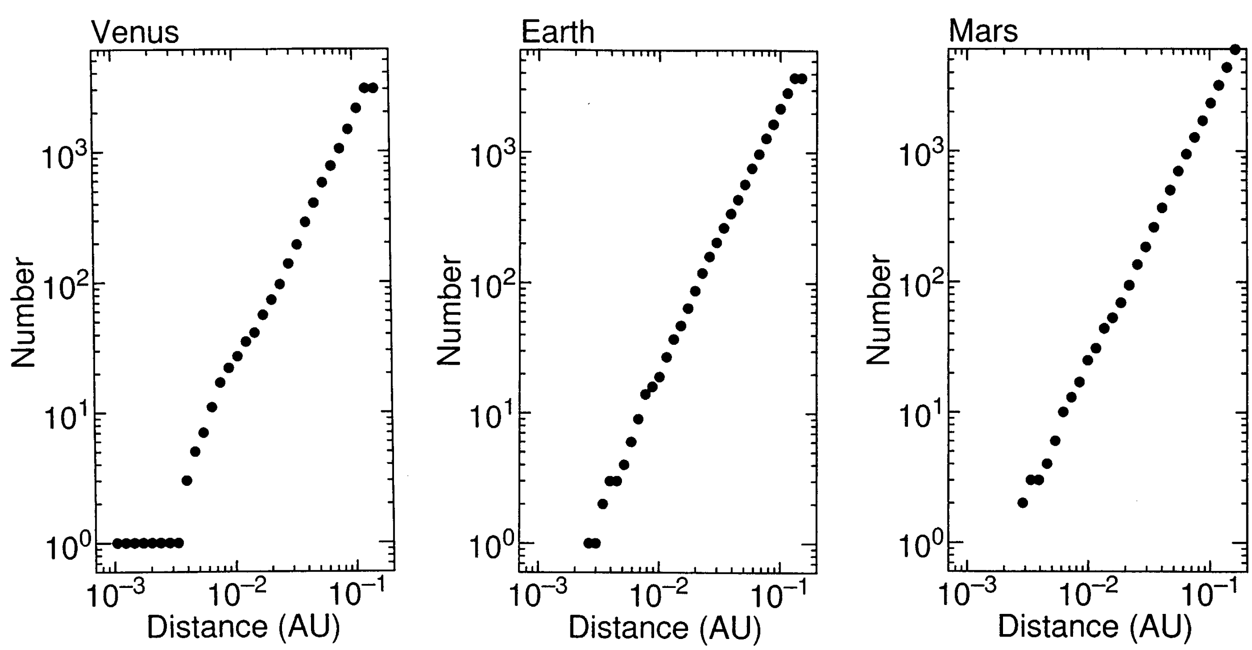

FIG. 1.—Cumulative number distributions of encounters for low-inclination JICs with the three terrestrial planets as a function of encounter distance. The plots are from $\pm 30{,}000$ yr integrations of 187 periodic comets with $i \leq 65°$.

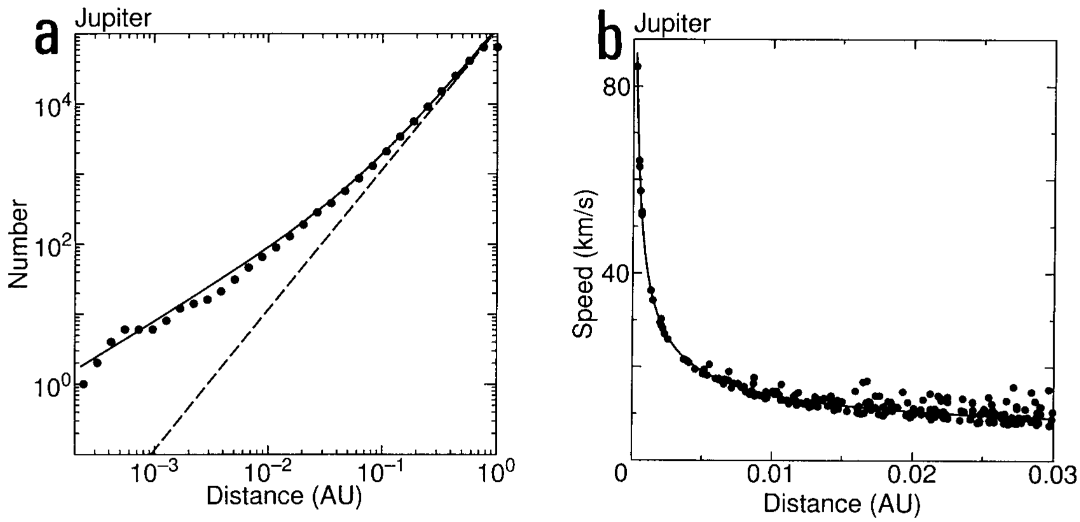

FIG. 2.—(*a*) Cumulative encounter number distribution and (*b*) plot of encounter speed for Jupiter, as a function of encounter distance. The solid curve in (*a*) is a gravitationally focused theoretical curve fitted for $v_0 = 5\ \mathrm{km\ s^{-1}}$, with which the curve in (*b*) is also drawn. The dashed line in (*a*) corresponds to the PIAB model. As for the calculated comets, refer to explanations for Fig. 1.

2.3.2. *Jupiter*

Figure 2*a* shows the cumulative encounter number distribution of low-inclination JICs with Jupiter as a function of encounter distance. Figure 2*b* represents the plot of encounter speed (v_c) versus encounter distance for the same comets. In contrast to the cases for the terrestrial planets, acceleration due to the gravity of Jupiter is manifest for small distances in Figure 2*b*. Although the gravity-increased speed is fairly insensitive to the variation of incident speed for distances less than 0.01–0.02 AU, a least-squares curve fitting seems to prefer $v_0 \simeq 5\ \mathrm{km\ s^{-1}}$; this is consistent with the v_0 obtained in NY95. From this v_0 and equation (1), the solid curve in Figure 2*a* was fitted at 0.434 AU. Extrapolating this curve down to the equatorial radius of Jupiter (0.00048 AU; $f^2 = 142.8$), the mean impact rate of JICs ($i \leq 65°$) with Jupiter is found to be 3.78/(57,200 yr)/(187 comets), or $p = 3.6 \times 10^{-7}\ \mathrm{yr^{-1}}$ per comet.

In order to check sensitivity of the obtained collision rate to the integration time span, we made diagrams similar to Figure 2 for $\pm 15{,}000$ yr. The resultant p was $4.5 \times 10^{-7}\ \mathrm{yr^{-1}}$ per comet, so our p seems to be reasonably robust against variation of integration time span.

2.3.3. *Other Outer Planets*

Figures 3*a* and 3*b* show the cumulative encounter distance and encounter speed distributions between JICs and Saturn. Similar to the case of Jupiter, the effects of gravitational focusing are seen in both plots. The solid curves in both panels are the best-fit gravity-focused curves corresponding to $v_0 = 4\ \mathrm{km\ s^{-1}}$, and the dashed line in Figure 3*a* is for the PIAB model. By extrapolating the solid curve in Figure 3*a* down to the Saturn's equatorial radius, where $f^2 = 79.9$, we have $p = 8.6 \times 10^{-9}\ \mathrm{yr^{-1}}$ comet.

As for Uranus and Neptune, in Figure 4 one can see that the close encounters so close as for gravitational focusing to become clear did not take place in our $\pm 30{,}000$ yr integrations. However, this does not imply that gravitational effects are negligibly small for these planets. For distances less than 0.2–0.5 AU, the encounter speeds of JICs with Uranus and Neptune were respectively $v_0 = 4$–5 and ~ 4

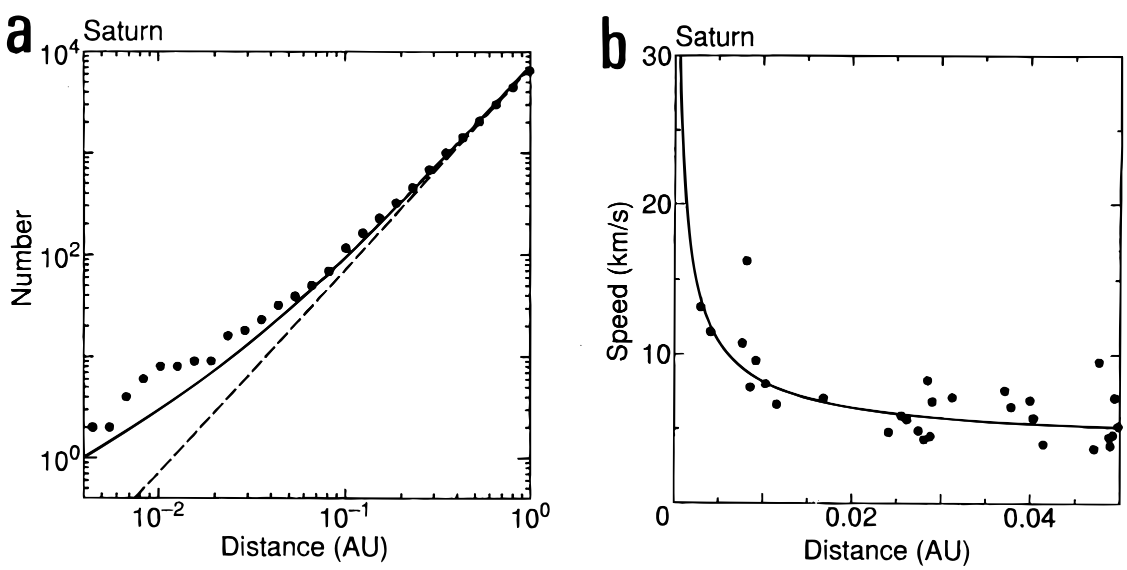

FIG. 3.—Same as Fig. 2, but for Saturn. The curve in (*b*) is for $v_0 = 4\ \mathrm{km\ s^{-1}}$. See Fig. 1 for other explanations.

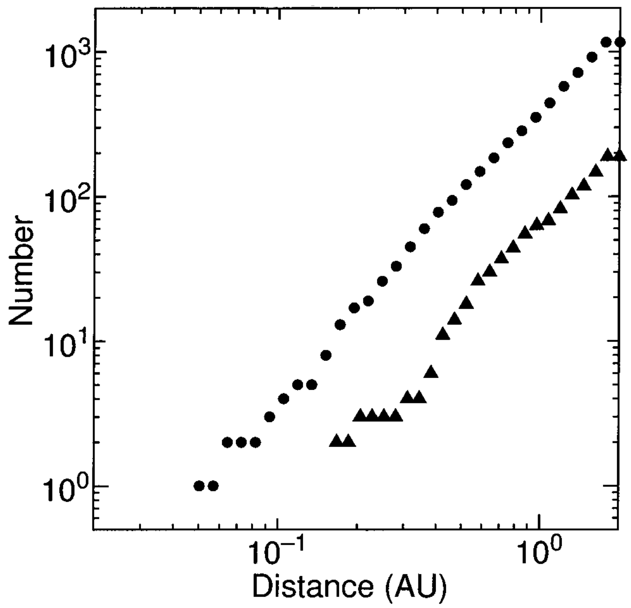

FIG. 4.—Cumulative encounter number distributions as a function of encounter distance for Uranus (*circles*) and Neptune (*triangles*). See Figs. 1 and 2 for other details.

km s^{-1}. With these values and the planetary parameters, the f^2 at the planetary surface is found to be 23.5 for Uranus and 35.6 for Neptune. By applying the f^2-values, we obtain $p = 2.4 \times 10^{-11}$ yr^{-1} per comet for Uranus and $p = 5.3 \times 10^{-12}$ for Neptune.

3. COMPARISON WITH ANALYTIC ESTIMATES

Here we compare the impact rates obtained in the previous section with those in NY95 and analytic estimates. However, comparison in a strict sense is difficult, and sometimes meaningless, because of differences in sample numbers, integration time spans, specified orbital regions of periodic comets as a group, and so forth. It should therefore be understood that being equal in comparison here can include errors of several times 10%.

3.1. *Comparison with NY95*

Bearing the above constraints in mind, comparison of the p for Jupiter will first be made between our work and NY95. Our result is $p = (3.6–4.5) \times 10^{-7}$ yr^{-1} per comet, whereas the p in NY95 was 8.7×10^{-7}, a difference of a factor of ~ 2. There are several reasons for this discrepancy.

The Jupiter p in NY95 was obtained by dividing the collision number extrapolated from their cumulative encounter distribution by the number of their adopted SP comets (165). On the other hand, our p is for JICs with $i \leq 65°$. This difference in classification of comets nevertheless causes errors of no more than a few times 10% in p at most. This is because the number of comets with $i > 65°$ (high inclination) in NY95 is 5%, the encounter number for high-inclination comets is about 1% of that for low-inclination comets (see Table 1), and the 200 yr $< P <$ 1000 yr comets in JICs are 8%.

We infer that the main cause of the discrepancy in p between this paper and NY95 is the difference in integration time spans. In the 4400 yr integration of NY95, the fraction of comets ejected outside the SP comet region was about 5%. By contrast, our 60,000 yr integration witnessed the temporary or permanent departure of some 15% of 187 comets from the Jupiter-interacting orbital region. We must, however, emphasize that the difference in p between NY95 and the present work is not due to the difference in loss fractions, since this effect has already been compensated for by the adoption of an effective mean time span of 57,200 yr, instead of 60,000 yr.

Indeed, considering that the 4400 yr integration in NY95 is somewhat too short for SP comets to experience substantial orbital evolution, it is not surprising that p was overestimated to some extent. Our p has a lower value than theirs, and is preferable.

3.2. *Comparison with Analytic Estimates*

3.2.1. *Outer Planets*

Next we compare our p-values with analytic estimates. To our knowledge, the most extensive analytic p-calculations of SP comets with all the planets have been made by Olsson-Steel (1987), using the Kessler (1981) model. His results are characterized by the inclusion of the effects of nonrandom distribution in argument of perihelion (ω) for SP comets.

Table 3 presents our p-values alongside those of Olsson-Steel (1987). The fourth column (OS87-2) gives his analytic p-values with nonrandom ω-distribution for SP comets, and the fifth column contains the ratios between our p-values and those of OS87-2.

Let us first examine the outer planets. One can see that analytic estimates are generally larger than numerical estimates, within a factor of 2. In the case of Jupiter (and probably the other outer planets), this trend will be interpreted similarly to the discrepancies between NY95 and this paper: since the analytic estimates for Jupiter and other planets are obtained from the current orbital distribution of SP comets,

TABLE 3

NUMERICAL AND ANALYTIC COLLISION RATES OF JICS WITH PLANETS

Planet	Present Work	OS87-1	OS87-2	Ratio
Venus	4.0×10^{-11}	1.5×10^{-9}	1.8×10^{-9}	45
Earth	5.0×10^{-11}	7.5×10^{-10}	1.1×10^{-9}	22
Mars	5.7×10^{-12}	8.1×10^{-11}	1.1×10^{-10}	19
Jupiter[a]	$(3.6–4.5) \times 10^{-7}$	7.7×10^{-7}	9.0×10^{-7}	~ 2
Saturn	8.6×10^{-9}	9.3×10^{-9}	9.3×10^{-9}	~ 1
Uranus	2.4×10^{-11}	1.4×10^{-11}	1.4×10^{-11}	~ 0.5
Neptune	5.3×10^{-12}	1.3×10^{-11}	1.3×10^{-11}	~ 2

NOTES.—OS87-1: for uniform distribution of argument of perihelion; OS87-2: for non-random argument of perihelion (Olsson-Steel 1987). The fifth column is the ratio between OS87-2 and the present work.

[a] The original OS87-1 value for Jupiter is modified in NY95, for direct comparison with the NY95 value.

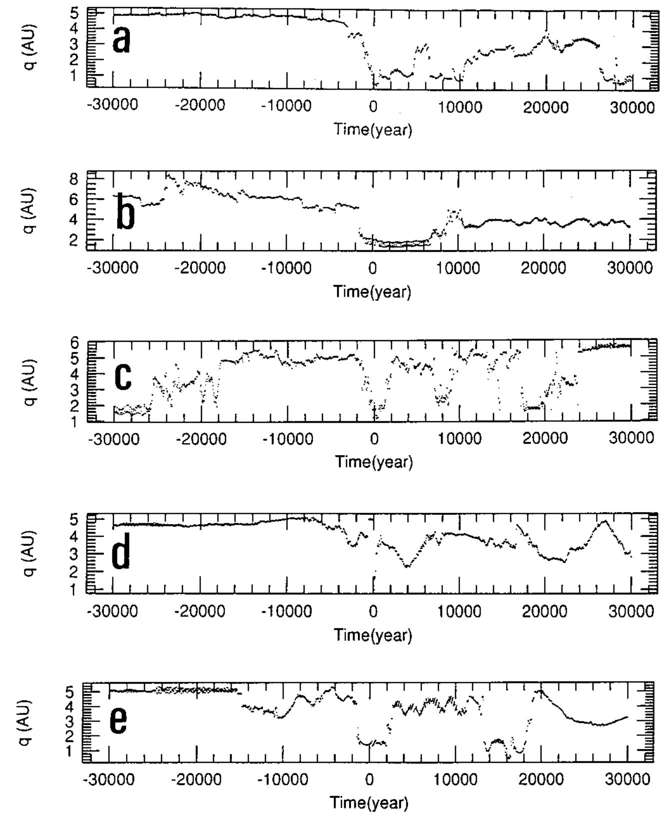

FIG. 5.—Time history of perihelion distance q for several periodic comets that showed typically chaotic and large variations: (*a*) 45P/Honda-Mrkos-Pajdusakova; (*b*) 71P/Clark; (*c*) 54P/de Vico–Swift; (*d*) 76P/West-Kohoutek-Ikemura; (*e*) 49P/Arend-Rigaux. The origin of time corresponds to C.E. 2000.0.

it is natural that the analytic p for Jupiter is nearly equal to the value in NY95, where substantial orbital evolution did not occur; this explanation also applies to the other outer planets. As for Uranus, Table 3 shows that, unlike that of the other outer planets, its analytic p is smaller than the numerical one. This is likely caused by statistical fluctuations due to the small sample number of the relevant data.

In summary, it may be stated that, considering possible nonrigorous conditions for comparison, analytic estimates of p are generally consistent with numerical ones for the outer planets.

3.2.2. *Terrestrial Planets*

We now compare analytic and numerical p-values for the terrestrial planets. Table 3 indicates that the analytic approach yields p-values exceeding those of our numerical estimates by factors of a few to several tens. In particular, an inner planet seems to give a larger discrepancy. Regarding Earth, the analytic p of Weissman (1982), 8.2×10^{-10} yr^{-1} per comet, is nearly the same as that of Olsson-Steel (see Table 3); this is natural, as both adopted similar methods and samples of SP comets. The large discrepancies between their and our findings can never be explained by only the difference in the orbital regions between SP comets and JICs.

Although the differences between analytic and numerical methods may appear embarrassingly large at first sight, they may be reasonably understood as follows: Analytic formulation calculates CRs by assuming that the perihelion and node of the orbit of a periodic comet have precessed long enough to circulate at nearly invariable rates, with other orbital elements kept almost constant. However, numerical integrations of orbits of SP comets (or JICs) show that, for the majority of such comets, their perihelion and nodal motions are very chaotic on timescales of a few thousand years. Figure 5 shows the time history of perihelion distance q for five arbitrarily selected comets that follow typical chaotic orbits. Approximately 80% of 187 JICs with $i \leq 65°$ belong to this category of chaotic orbits.

The typical q-behavior in Figure 5 is characterized by the perihelia spending most of their time near the orbit of Jupiter (except for the present epoch, in which their perihelia are near 1–2 AU). Thus, the majority of Jupiter-interacting periodic comets actually have no chance of colliding with the terrestrial planets during most of their dynamical lifetime. As such, this trend is stronger for smaller q. In other words, it could be said that the currently observed SP comets (or JICs) are a population heavily biased toward small q-values. Nevertheless, the analytic method calculates CR under the assumption that the current orbits of JICs are maintained indefinitely. This is why the analytic approach overestimated CRs by factors as large as 20–45.

4. DISCUSSION

In the previous section, we argued that analytic overestimates of p-values for the terrestrial planets are caused by those comets that spend most of their lifetime in orbits with perihelia near Jupiter. This understanding may be viewed in a different way, as follows: The discrepancy ratio (the fifth column in Table 3) in analytic and numerical p-values reflects the ratio between the period of time when JICs stay in the inner planetary region and the one when they are near Jupiter's orbit. The latter ratio is essentially none other than the ratio between the numbers of observable and unobservable JICs, so far as the total population is in steady state.

4.1. *Impact Frequencies*

The total population (the second factor mentioned in § 1) can be obtained from the above ratio multiplied by the number of observable JICs. Nakamura & Yoshikawa (1992) estimated this unobservable-to-observable number ratio for SP comets as a function of Tisserand parameter, calling it the "invisibility factor." NY95 calculated its mean value as about 10. For the purpose of inferring the total population of JICs, the mean value in NY95 can be somewhat underestimated, since the value was obtained by averaging over all the classes of comets classified by Tisserand parameter.

In this work we have refined the invisibility factors for low-inclination JICs using results of $\pm 30{,}000$ yr integration. They are found to be between some 30 and 50, depending on the Tisserand parameter values, though the details will be published elsewhere. This implies that for each observable JIC, there exist 30–50 unseen background comets of the same class. Then, by multiplying this invisibility factor and the number of observed JICs with the p for Jupiter obtained in § 2, one can find the impact frequency for the total population of Jupiter-interacting comets with Jupiter to be $(3.6\text{–}4.5) \times 10^{-7}(187)(30\text{–}50) = (2.0\text{–}4.2) \times 10^{-3}$ yr^{-1}, or one impact per 250–500 yr, over the entire size spectrum of JICs. This value is double the frequency given in NY95. If impacts are limited to comets larger than 1 km diameter (NY95), the above frequencies should be halved to once per 500–1000 yr.

Likewise, similar multiplications can provide impact frequencies for other planets. For example, the impact frequency of JICs with Earth is calculated as $5.0 \times 10^{-11}(187)(30\text{–}50) = (3\text{–}5) \times 10^{-7}$ yr, or once per 2–4 Myr over all the sizes, and should be half the above values for comets larger than 1 km.

It will be interesting to compare our impact frequencies for the outer planets with those of LD97. In doing so, however, we must be careful again of the difficulty in comparison with the same conditions, because LD97 considered the comets with $q \leq 2.5$ AU as "observable" and their CRs were based on the statistics of "only once per planet" closest encounter for each planet. With such limitations in mind, we compare the case of Jupiter first. The Jovian impact frequency of JFCs ($H < 9$ mag) by LD97 is calculated from their Table 1 to be once per 420 yr, whereas ours is once every 500–1000 yr for nuclei with $D > 1$ km. This reasonably good agreement, despite the different approaches in LD97 and this work, supports the conclusion that the long-term evolution of comets by LD97 and our steady state population of JICs are both basically correct and consistent.

For Saturn through Neptune, a comparison between LD97's and our findings allows us to know the approximate ratio of JIC to non-JIC impacts on these planets. For Saturn, our calculations predict that a Saturnian impact of JICs takes place every 12,000–20,000 yr, whereas LD97 give a frequency of once per 1300 yr (see their Table 1); the latter is 8–10 times more frequent. One might be able to check LD97's impact frequency in the crater statistics of the *Voyager* imaging data for the Saturnian satellites. As for Uranus and Neptune, it is found that the relative contribution of JICs to impacts is negligibly small.

4.2. *Shoulders Seen in Cumulative Encounter Plots*

By careful inspection of Figure 1, we note that each plot has a slight shoulder near 0.01 AU, whose degree becomes less clear toward outer planets. Figure 4 also seems to show a similar trend for Uranus and Neptune around 0.3–0.5 AU; interestingly, no such characteristics can be seen in Jupiter and Saturn.

These shoulders may be regarded in two ways. The first view is that each curve consists of two straight lines with different slopes; naturally, the outer slope should have that for the PIAB. The second view is that the curve is a straight line with the PIAB slope plus a midway hump. In any case, it is now unclear whether the shoulder is real or caused by some numerical artifacts, and so it will be a target for future investigation. However, the existence of the shoulders does not affect, so much beyond the statistical noise, the p-values of the terrestrial planets and the outer planets calculated in § 2.

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