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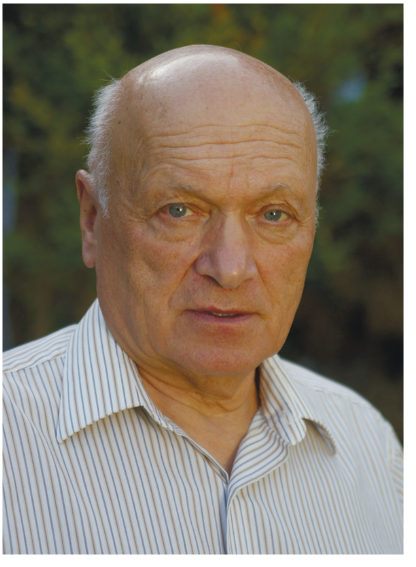
MATHEMATICAL LIFE

Boris Rufimovich Vainberg (on his 80th birthday)



Boris Rufimovich Vainberg was born on 17 March 1938 in Moscow. His father was a leading engineer at an aircraft design office and his mother was a housewife. From his early years Boris was interested in mathematics, took part in mathematical study groups (very popular at that time), and participated in the regularly held Moscow mathematical olympiads. His first mathematics library consisted of the books he received as prizes won in olympiads.

On graduating from secondary school, Boris enrolled in 1955 in the Faculty of Mechanics and Mathematics (Mech-Math) of Moscow State University, and in 1960, after the usual 5-year course, he began postgraduate studies in the Department of Differential Equations, where Professor S. A. Gal'pern was his scientific advisor. At that time the department was led by Ivan Georgievich Petrovsky, the rector of Moscow University, who was an embodiment of what we can now call 'the golden age of 'Mech-Math'. Such prominent researchers as V. I. Arnold, M. I. Vishik, E. M. Landis, and O. A. Oleinik were teaching then in the department. Other departments also featured such star researchers as A. N. Kolmogorov (probability theory), P. S. Alexandroff (topology), I. M. Gelfand and D. E. Men'shov (theory of functions and functional analysis), . . . ; the list goes on and on.

At these times of a political 'thaw' Mech-Math and particularly its Department of Differential Equations were very responsive to the very latest scientific ideas. First and foremost was the theory of distributions, founded by S. L. Sobolev as long ago as the 1930s, extended by L. Schwartz in the 1950s, and then further developed by Gelfand, G. E. Shilov, and others. Here we should also mention L. Hörmander's revolutionary works (on pseudodifferential operators, integral Fourier operators, hypoelliptic operators). One of Vainberg's important publications at the beginning of his career in research was devoted precisely to an analysis

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of Dirichlet–Neumann-type maps from the standpoint of pseudodifferential operators (see below).

In 1963 Vainberg defended his Ph.D. thesis, and with the essential support of Petrovsky was retained in the department as an assistant professor. He worked there for 28 years and for many years was the academic secretary of the department. He also made great contributions to the functioning of a correspondence school of mathematical education which was organized by Gelfand under the auspices of Mech-Math: namely, he attracted Mech-Math undergraduates to help with checking work sent by students of the correspondence school, and he supervised all communications between checkers and distant students. In August 1991 Vainberg moved to the USA. He spent his first year there as an invited professor at the University of Delaware, and since 1992 he has been a professor at the University of North Carolina in Charlotte.

Vainberg is the author of three monographs ([9], [11], and [22]), a chapter in the book [10], and about 170 research papers, mostly concerned with mathematical physics and partial differential equations.

In his Ph.D. thesis *Conditions at infinity which ensure that hypoelliptic equations are solvable in the whole space* he found [1] Sommerfeld-type radiation conditions for general elliptic operators (including the equations of elasticity theory). Much later he used these results in the joint paper [21] with W. Shaban (his postgraduate student) to analyze the discrete Laplace operator when the radiation conditions are significantly different from the usual conditions: there exist several scattered waves and the limit absorption principle fails for some frequencies.

In 1968 he and V. V. Grushin shared a prize of the Moscow Mathematical Society (their annual prize for young mathematicians) for the papers [2] and [3] on uniformly non-coercive problems for elliptic equations. One important consequence of these papers was the assertion that the Dirichlet–Neumann map is a pseudodifferential operator together with a calculation of its full symbol.

In 1970 Vainberg presented his D.Sc. thesis *Elliptic problems in external domains and the large-time asymptotic behaviour of solutions of hyperbolic equations*. The thesis was rejected because of the rapidly growing antisemitism in the late 1960s. Slightly earlier, when the defence of the D.Sc. thesis of the remarkable mathematician G. I. Eskin (now a professor at UCLA) went wrong, it was regarded as a casual aberration. Boris Vainberg was the next, and after that it was recognized that the atmosphere at Mech-Math had deteriorated sharply.

In 1973 Vainberg and Maz'ya described the waves produced by steady-state oscillations of a body put in a stratified fluid of variable depth or by the uniform motion of a body immersed in a fluid ([5], [6]). They found geometric conditions on the inhomogeneities (rises on the bottom of the fluid and the shape of the body) for which there are no eigenvalues embedded in the continuous spectrum, which results in the unique solvability of the problem. For operators with periodic coefficients Vainberg proved the non-existence of embedded eigenvalues in a subsequent joint paper with P. A. Kuchment [18]. In 1981 Vainberg and Maz'ya investigated the characteristic Cauchy problem for general hyperbolic equations [8]. Ten years later this result was reproduced by Hörmander, but only for equations of the second order.

In 1987 Vainberg presented and then successfully defended another D.Sc. thesis, *Decrease of local energy for exterior hyperbolic problems and quasi-classical approximation.* There he proposed a direct method for finding the large-time asymptotic behaviour of the local energy and solutions of non-stationary equations in the exterior of non-trapping obstacles ([4], [7], [9], [11]). This method is based on the high-frequency bounds and low-frequency asymptotic formulae which he found for solutions of the corresponding stationary problems. In particular, the results in his thesis covered all the consequences derived from the Lax–Phillips scattering theory. He later extended his approach to time-periodic media and obstacles ([12], [13]). Then he and his co-authors were able to use these results on the decrease of local energy in their investigations of the asymptotic stability of steady states of non-linear wave equations, and of solitons in the case of a particle interacting with its own wave field and a Klein–Gordon field ([14], [35]).

After moving to the USA, Vainberg established many deep results in conjunction with S. A. Molchanov. They found spectral asymptotic expansions for equations in domains with a fractal boundary ([15], [16]), equations with sparse potentials ([17], [19], [23]), and operators in other important classes ([27], [30], [36]), and they investigated the spectrum of Schrödinger operators with slowly decreasing and random potentials ([20], [24], [33], [34], [37], [38], [40]). Their series of papers [25], [26], [31], [32] was devoted to the propagation of waves in complicated systems of thin waveguides and had applications to fibreglass optics. They found the asymptotic behaviour of the solutions as the diameters of the waveguides tend to zero, obtained boundary matching conditions at the vertices of the limiting one-dimensional graph, and used this simplified problem on the graph to describe wave propagation in the original problem. In conjunction with other colleagues they investigated mathematical models for homopolymers ([28], [29]) and the spectral properties of non-local Schrödinger operators [44]. They also proved global limit theorems for random walks with 'heavy tails' and used these theorems to describe population fronts and to study intermittency phenomena in biological models [45].

Vainberg published a number of joint papers with E. L. Lakshtanov ([39], [41], [46]) on interior transmission eigenvalues (which arise in scattering on obstacles). In particular, they derived a new Weyl law, in which eigenvalues are counted with signs 'plus' or 'minus' depending on the direction of rotation of the corresponding eigenvalue of the scattering matrix. In another paper they and R. G. Novikov showed that a global Riemann–Hilbert problem can always be used in solving two-dimensional inverse scattering problems, even when there are exceptional points [43]. In this way they were able to solve some important non-linear equations in soliton theory in dimension $2+1$ (in particular, the focusing Davey–Stewartson II equation) without assuming that the initial data are small ([42], [43], [47], [48]). Vainberg has also made significant contributions to the solution of other important problems in mathematical physics.

Boris Vainberg has a wonderful family, he keeps on playing tennis, skiing, and successfully solving mathematical problems. We wish him good health and new achievements in mathematics.

Yu. V. Egorov, A. I. Komech, P. A. Kuchment, E. L. Lakshtanov, V. G. Maz'ya, S. A. Molchanov, R. G. Novikov, and M. I. Freidlin

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